Exercises on digital representations

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1 Tessellations

Why is it impossible to pave the space $\mathbb{R}^2$ with convex regular polygons having 10 edges?

2 Digital topology

2.1 Connectivity

What is the number of connected components and holes of the object in Figure 1, for the two types of connectivity on the square grid? Recall that a hole is a connected component of the background that does not touch the border of the image.

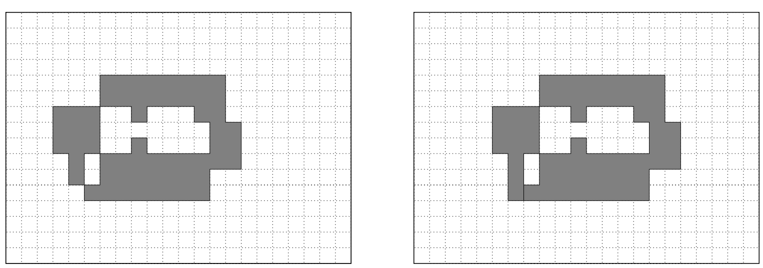

Figure 1: Connected components and holes.

2.2 Euler number

Compute the Euler number (in 4- and 8-connectivity for the object) in Figure 2 by using the global method and the method based on counting local configurations.

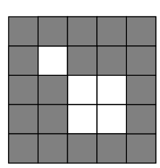

Figure 2: Euler number.

2.3 Adjacency

Let C be a 4-connected component of objects in a binary image. Let D be a 8-connected component of the background (complement of the objects) in this image. Prove that if C and D are 8-adjacent, they are also 4-adjacent.

3 Discrete distances

3.1 Distances between two points

Let us consider $P = (0, 0)$ and $Q = (4, 5)$ on a square grid (Figure 3). What is the discrete distance between P and Q when using:

- the elementary mask corresponding to 4-connectivity (with coefficients 1)?
- the elementary mask corresponding to 8-connectivity (with coefficients 1)?
- the 3×3 mask with coefficients 3 and 4?
- the 5×5 mask with coefficients 5, 7 and 11?

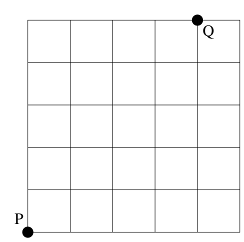

Figure 3: Definition of points P and Q.

3.2 Distance from a point to an object

Let us consider object X and point M as defined in Figure 4. What is the discrete distance from M to X and which is the closest point when using the four masks of the previous exercise?

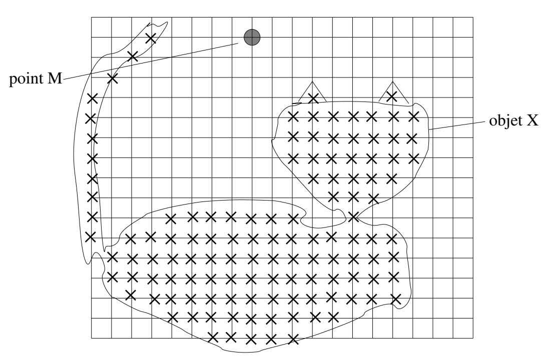

Figure 4: Definition of object X and point M.