Symbolic Artificial Intelligence
Lecture 3: Fuzzy Description Logics

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Dealing with uncertainty: Fuzzy (Description) Logics and Fuzzy Ontologies

Why Fuzzy Logic?

- Real life is not black or white
- Classical (crisp) logic: *true/ false*
- Fuzzy Logic: [0, 1]. **Ex**. *blond, tall, cheap*
- For automatic reasoning about uncertain, vague, incomplete or imprecise knowledge
- For near natural language expressions [2]

Fuzzy Logic and Fuzziness [10]

Fuzzy statements:

- involve context sensitive concepts with no exact definition, no binary decision/membership function:
  Ex. *small, close, far, cheap, expensive, is about, similar to, warm, cold*.
  Ex. *Find me a good hotel close to the conference venue*
  If a hotel is close to the leaning tower of Pisa, then it is a touristic hotel
- are true to some degree, taken from a truth space (usually [0, 1])

Types of Logic

Language	Ontological Commitment[1]	Epistemological Commitm.[2]
Propositional Logic	Facts	True/False/Unknown
First-order Logic	Facts, objects, relations	True/False/Unknown
Temporal Logic	Facts, objects, relations, times	True/False/Unknown
Probability Theory	Facts	Degree of belief (0..1)
Fuzzy Logic	Degree of truth	Degree of belief (0..1)

[1] What exists?-facts?, objects?, time? beliefs? What exists in the world

[2] What states of knowledge? What an agent believes about facts. [U. Straccia]

Fuzzy Description Logics (DL)

Fuzzy Description Logics

Fuzzy Knowledge Base (FKB) or fuzzy ontology: a finite set of axioms that comprises a fuzzy ABox A and a fuzzy TBox T [3].

Fuzzy ABox: a finite set of fuzzy (concept or role) assertions

Fuzzy TBox: a finite set of fuzzy General Concept Inclusions (GCIs), with a min. fuzzy degree of subsumption.

Logical operators of conjunction, disjunction and complement are special cases of the three fuzzy operators:

1. A possibilistic product is a t-norm: a $\bigotimes$ b, conjunction, $\wedge$
2. A possibilistic sum is a t-conorm: a $\bigoplus$ b; disjunction, $\vee$
3. Fuzzy complement: $\neg$ c

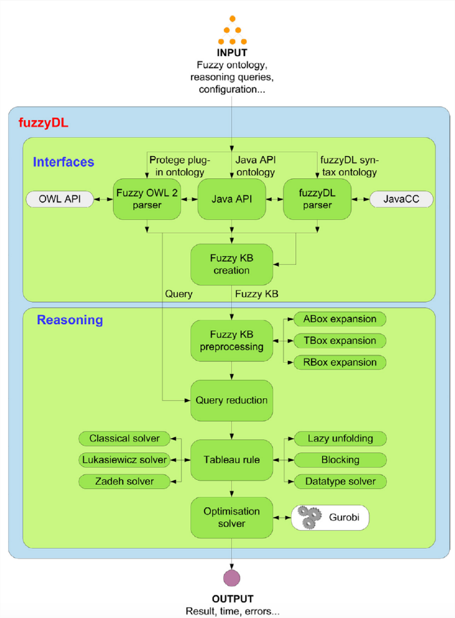
A fuzzy KB K is *consistent* if there is a model of K that satisfies each axiom in K.

Fuzzy operators supported by *fuzzyDL*

Operator	Łukasiewicz logic	Gödel logic	Zadeh logic
Conjunction $\alpha \wedge \beta$	$\max(\alpha + \beta - 1, 0)$	$\min(\alpha, \beta)$	$\min(\alpha, \beta)$
Disjunction $\alpha \vee \beta$	$\min(\alpha + \beta, 1)$	$\max(\alpha, \beta)$	$\max(\alpha, \beta)$
Negation $\neg\alpha$	$1 - \alpha$	$\begin{cases} 1 & \text{if } \alpha = 0 \\ 0 & \text{otherwise} \end{cases}$	$1 - \alpha$
Implication $\alpha \rightarrow \beta$	$\min(1 - \alpha + \beta, 1)$	$\begin{cases} 1 & \text{if } \alpha \leq \beta \\ \beta & \text{otherwise} \end{cases}$	$\max(1 - \alpha, \beta)$

Reasoner	Fuzzy DL	Event Subscript.	SPARQL	Cardinality Restr.	Fuzzy Sets	Concept Modifier	Fuzzy Data Type	Defuzzification	Fuzzy Rule	Satisfiab. Degree
FiRE [194, 193, 189]	$\mathcal{F}-\mathcal{SHIN}$			x						x
GURDL [84]	$\mathcal{F}-\mathcal{ALC}$									x
De-Lorean [29]	$\mathcal{F}-\mathcal{SROIQ}$			x	x	x	x			x
GERDS [85]	$\mathcal{F}-\mathcal{ALC}$									
fuzzyDL [30]	$\mathcal{F}-\mathcal{SHIF}(\mathbf{D})$				x	x	x	x	x	x
YADLR [119]	SLG algorithm									x
Fuzzy OWL Plugin[Fuz, 31]	$\mathcal{SROIQ}(\mathbf{D})$									
FRESG [87]	$\mathcal{F}-\mathcal{ALC}(\mathbf{G})$						x			x
SoftFacts	$\mathcal{F}$–DLR-lite									

FuzzyDL Architecture



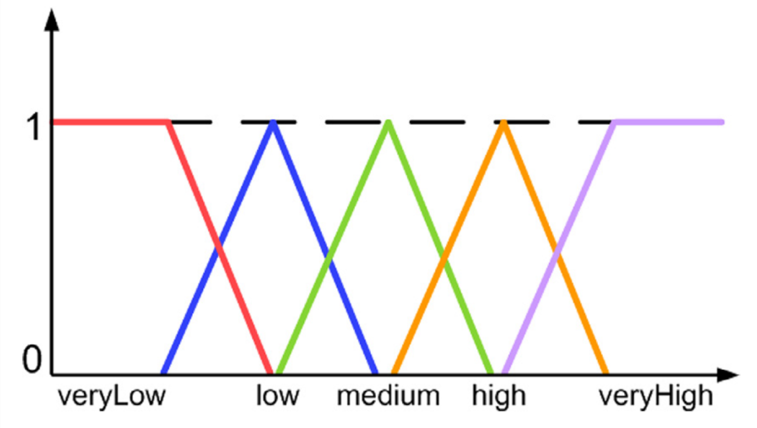
fuzzyDL answers queries by solving an MILP problem: minimising a linear function wrt a set of constraints (linear inequations in which rational and integer variables cannot occur); MILP problems will be bounded with rational variables ranging over a subset of [0,1] and integer variables ranging over {0,1}

FuzzyDL reasoner syntax Example[3]

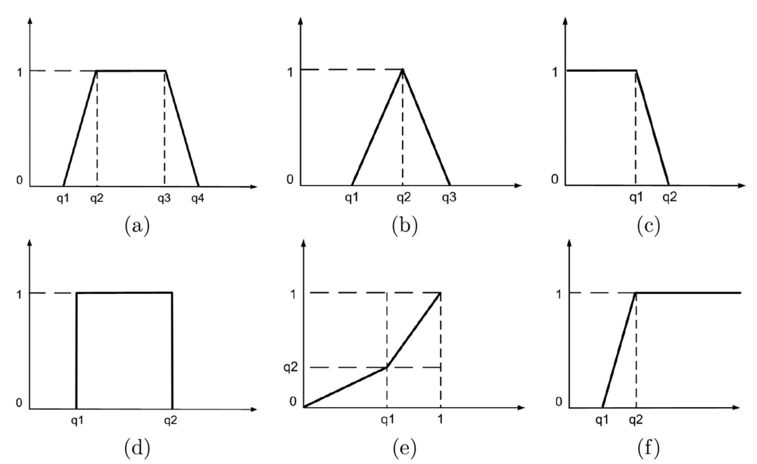
```
(define-primitive-concept Tall *top*)

(instance fernando *top*1.0)

(instance umberto Tall 0.9)

(related fernando umberto isFriendOf 0.8)
```

[3] ∗***top***∗ denotes the universal concept (similar to OWL2 class `Thing`. `Tall` is a fuzzy concept, `isFriendOf` a fuzzy relation. `umberto` and `fernando` are individuals) [4]

Partitioning a domain with fuzzy membership functions



Fuzzy Membership Functions (in *fuzzyDL* [4])



a) Trapezoidal function; b) Triangular; c) Left-shoulder; d) Crisp interval e) Linear f) Right-shoulder

FuzzyDL Reasoning Services

- *KB consistency*. A fuzzy KB $\mathcal{K}$ is *consistent* if there is a model of $\mathcal{K}$ that satisfies each axiom in $\mathcal{K}$.
- *Concept satisfiability*. A fuzzy concept `C` is `D`-satisfiable w.r.t. a fuzzy KB $\mathcal{K}$ if there exists a model of $\mathcal{K}$ where `C` can have some instance with degree greater or equal than `D`, where `D` is a degree of truth. In *fuzzyDL*, this task can also consider some particular individual `o` instead of an arbitrary one.
- *Best satisfiability degree* (BSD) of a fuzzy concept `C` w.r.t. a fuzzy KB $\mathcal{K}$ is the maximal degree `D` such that `C` is `D`-satisfiable w.r.t. $\mathcal{K}$.
- *Minimal satisfiability degree* (MSD) of a fuzzy concept `C` is similar to the BSD but considering the minimal degree rather than the maximal one.
- *Concept subsumption*. `C2` `D`-subsumes `C1` w.r.t. a fuzzy KB $\mathcal{K}$ if in every model of $\mathcal{K}$, `C1` is included in `C2` with degree greater or equal than `D`. The degree of inclusion is computed using a fuzzy implication.
- *Entailment*. A fuzzy KB $\mathcal{K}$ entails an axiom if every model of $\mathcal{K}$ satisfies it. *fuzzyDL* computes entailments of assertions and GCIs.
- *Best Entailment Degree* (BED) of a non-graded axiom with respect to a fuzzy KB $\mathcal{K}$ is the maximal degree `D` such that the axiom is satisfied in every model of $\mathcal{K}$ with degree greater or equal than `D`.
- *Maximal Entailment Degree* (MED) of a non-graded axiom is similar to the BED but considering some model rather than any model.
- *Instance retrieval*. Given a concept `C` and a fuzzy KB $\mathcal{K}$, the instance retrieval problem computes the individuals that belong to `C` with a non-zero degree together with the minimal degree of membership in every model of $\mathcal{K}$.
- *Variable maximisation*. Given a fuzzy KB $\mathcal{K}$ and a variable `x`, maximise `x` such that $\mathcal{K}$ is consistent.
- *Variable minimisation*. Given a fuzzy KB $\mathcal{K}$ and a variable `x`, minimise `x` such that $\mathcal{K}$ is consistent.
- *Defuzzification*. Given a fuzzy KB $\mathcal{K}$, a concrete role `t`, a concept `C`, and an individual `o`, compute the BSD of `C` for the individual `o` and then defuzzify the value of `t` for the individual `o` using some defuzzification method: largest of maxima (LOM), smallest of maxima (SOM), or the middle of maxima (MOM).
- *Best Non-Fuzzy Performance (BNP)*. Given a triangular fuzzy number `F = (triangular q1 q2 q3)`, $BNP(\texttt{F}) = (\texttt{q1} + \texttt{q2} + \texttt{q3})/3$. This task is particularly useful when fuzzy numbers are arithmetically combined.

Query languages: SPARQL Query Example:

```
SELECT  ?calendar1 ?phone2
WHERE{ ?user0 a ha:User.
      ?user0 ha:hasName "Natalia"^^xsd:string.
      ?user0 ha:hasCalendar ?calendar1.
      ?user0 ha:hasPhone ?phone2.
      ?user0 ha:isInLocation ?location3.
      ?phone2 ha:isInLocation ?location3.
      ?location3 ha:isNear ?office4.
      ?user5 a ha:User.
      ?user5 ha:hasName "Johan"^^xsd:string.
      ?user5 ha:hasOffice ?office4.}
```

Fuzzy DL Query Syntax [4]

(Q1)	`(sat?)`	Consistency
(Q2)	`(min-sat? C [o])`	Minimal Satisfiability Degree of a concept
(Q3)	`(max-sat? C [o])`	Best Satisfiability Degree of a concept
(Q4)	`(min-instance? o C)`	Best Entailment Degree of a concept assertion
(Q5)	`(max-instance? o C)`	Maximal Entailment Degree of a concept assertion
(Q6)	`(min-related? o1 o2 R)`	Best Entailment Degree of a role assertion
(Q7)	`(max-related? o1 o2 R)`	Maximal Entailment Degree of a role assertion
(Q8)	`(min-subs? C D)`	Best Entailment Degree of a GCI
(Q9)	`(max-subs? C D)`	Maximal Entailment Degree of a GCI
(Q10)	`(min-g-subs? C D)`	BED of a GCI using Gödel implication
(Q11)	`(max-g-subs? C D)`	MED of a GCI using Gödel implication
(Q12)	`(min-l-subs? C D)`	BED of a GCI using Łukasiewicz implication
(Q13)	`(max-l-subs? C D)`	MED of a GCI using Łukasiewicz implication
(Q14)	`(min-kd-subs? C D)`	BED of a GCI using Kleene-Dienes implication
(Q15)	`(max-kd-subs? C D)`	MED of a GCI using Kleene-Dienes implication
(Q16)	`(all-instances? C)`	Instance retrieval
(Q17)	`(max-var? var)`	Variable maximisation
(Q18)	`(min-var? var)`	Variable minimisation
(Q19)	`(defuzzify-lom? C o t)`	LOM defuzzification
(Q20)	`(defuzzify-som? C o t)`	SOM defuzzification
(Q21)	`(defuzzify-mom? C o t)`	MOM defuzzification
(Q22)	`(bnp? F)`	Best Non-Fuzzy Performance

Fuzzy Wine Ontology v 1.00

Choose context:
Candle
Choose food:
Game

Submit

This Fuzzy Wine Ontology is based on 601 wines

You picked: Candle and Game

The most suitable wines for this combination are:

0.883 Villages_Cuvee_3_Fleurs

0.881 Abadal Cabernet Sauvignon Reserva

0.823 Domaine Depeyre

0.717 Belleruche

0.713 Baron_de_Ley_Reserva

0.709 Terres de Berne

0.704 Beringer_Clear_Lake_Zinfandel

0.703 Beringer_Founders_Estate_Merlot

0.699 Amarone_della_Valpolicella_Classico_I_Castei_2

0.699 Amarone della Valpolicella Classico I Castei

Fuzzy DL Example: Wine ontology [4]

C:\Documents and Settings\usuario\Escritorio\FuzzyWine.fdl

```

# Fuzzy logic
(define-fuzzy-logic zadeh)

# Datatypes
(define-fuzzy-concept MediumAlcoholForWine triangular(0.0, 20.0, 12.0, 13.0, 14.0) )
(define-fuzzy-concept HighPriceForWine right-shoulder(0.0, 10000.0, 15.0, 30.0) )

# TBox axioms
(implies (and SparklingWine (some hasSugar DrySugarContentForSparklingWine) ) DrySparklingWine 1.0)
(define-primitive-concept PinotNoir (some hasColor RedWineColor ))
(define-primitive-concept Chianti (some locatedIn ChiantiRegion ))
(define-concept RedWine (and Wine (some hasColor RedWineColor ) ) )
(define-concept Beaujolais (and Wine (some locatedIn BeaujolaisRegion ) ) )
(define-concept HighPriceWine (some hasPrice HighPriceForWine) )

# RBox axioms
(implies-role madeFromGrape madeFromFruit 1.0)
(transitive locatedIn)
(symmetric adjacentRegion)
(functional hasQualitativeSugar)
(inverse hasMaker producesWine)
(domain madeFromGrape Wine )
(range madeFromGrape WineGrape )

# ABox axioms
(related RemyPannier2009 DAnjouWinery hasMaker 1.0)
(instance RemyPannier2009 (= hasAlcohol 12.0) 1.0 )
(instance RemyPannier2009 (= hasPrice 8.0) 1.0 )

# Query
(min-instance? RemyPannier2009 HighPriceWine )

```

Query languages: Triple patterns in SPARQL → fuzzyDL query:

Subscription pattern	fuzzyDL query
(?, ?, ?)	*∀ Concept C: (all-instances? C)*
(s, ?, ?)	*If s is a Concept: (min-sat? s)* *If Individual s ∈ Concept C: (min-instance? s C)*
(?, p, ?)	*If D is p's Domain and R is p's Range; ∀ Individual d ∈ D and ∀ Individual r ∈ R: (min-related? d r p)*
(?, ?, o)	*If o is a Concept: (min-sat? o)* *If Individual o ∈ Concept C: (min-instance? o C)*
(s, p, ?)	*If R ∈ p.Range: ∀ Individual i ∈ R: (min-related? s i p)*
(?, p, o)	*If D ∈ p.Domain: ∀ Individual i ∈ D: (min-related? i o p)*
(s, ?, o)	*∀ Role r, (min-related? s o r)*
(s, p, o)	*(min-related? s o p)*

Practical tools for fuzzy logic and fuzzy ontologies:

- *fuzzyDL* reasoner[4]: A DL reasoner supporting Fuzzy Logic and Fuzzy Rough Set[5] reasoning.
- Scikit-fuzzy[6][11]

[4] https://tinyurl.com/ya8l9y9h
[5] Useful for rule induction from incomplete datasets, a generalization of fuzzy membership
[6] https://github.com/scikit-fuzzy/scikit-fuzzy

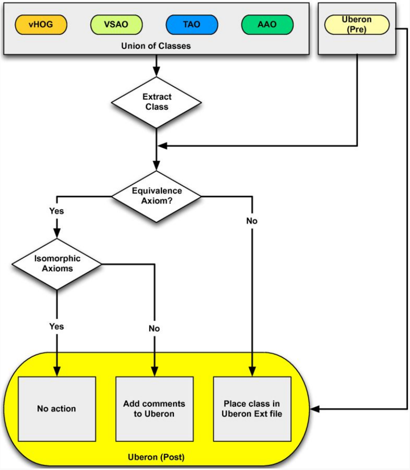
Research problems in Description Logics

Research challenges in (approximated) reasoning

- Scalability (subsumption algorithms [1]: classifying large graphs)
- Reasoning under inconsistency-tolerant semantics: inherently intractable (even for very simple DLs [9] or for tractable DLs).
- Automatic ontology learning
- Can we provide near real time reasoning answers via
  - KR learned with deep learning?
  - Genetic algorithm approximations?

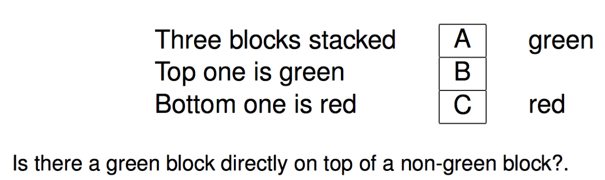
Research challenges in (approximated) reasoning

- Ontology evolution, merging, matching, unification of different specializations
  Ex.: cross-taxon resource unification ontology for policy consensus decision making [8].



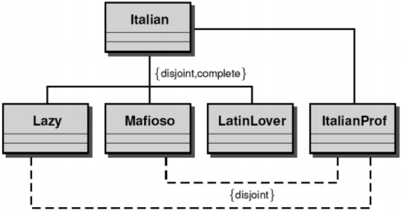
Research challenges in (approximated) reasoning

Neural-symbolic learning and reasoning (NeSy community)



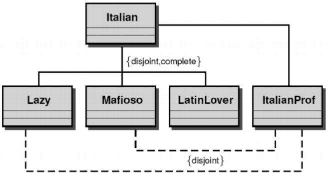
Description Logics in practice!

Description Logics icebreaker problem [Straccia]



Encode it into Description logics and prove that $KB \models ItalianProf \sqsubseteq LatinLover$

Description Logics icebreaker solution [Straccia]

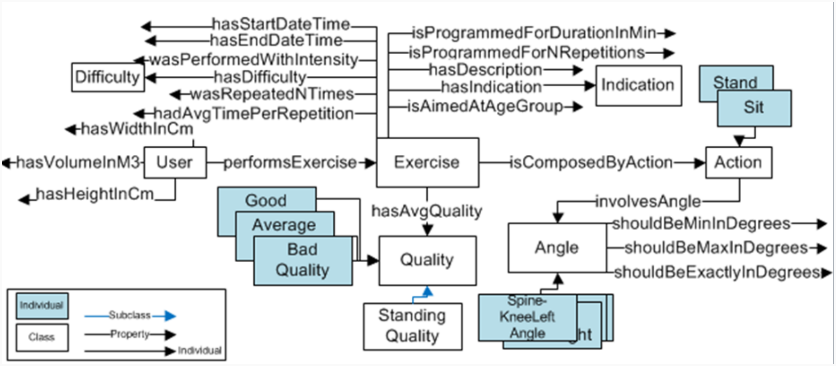


Encode it into Description logics and prove that $KB \models ItalianProf \sqsubseteq LatinLover$

Solution:

$$
\begin{aligned}
Lazy &\sqsubseteq Italian \\
Mafioso &\sqsubseteq Italian \\
LatinLover &\sqsubseteq Italian \\
Italian &\sqsubseteq (Lazy \sqcup Mafioso \sqcup LatinLover) \\
ItalianProf &\sqsubseteq Italian \\
Lazy &\sqsubseteq \neg Mafioso \\
Lazy &\sqsubseteq \neg LatinLover \\
Mafioso &\sqsubseteq \neg LatinLover \\
Mafioso &\sqsubseteq \neg ItalianProf \\
Lazy &\sqsubseteq \neg ItalianProf
\end{aligned}
$$

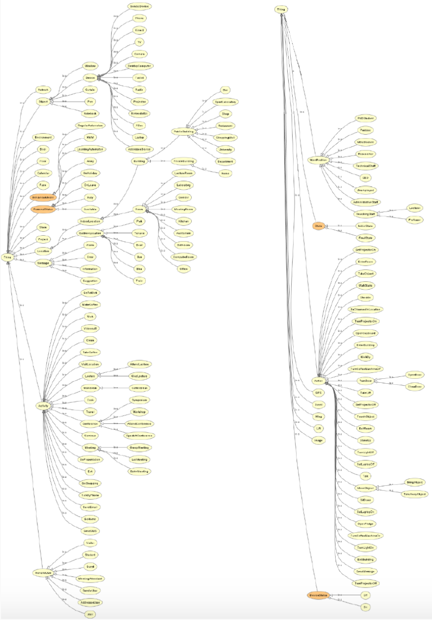
Ontology examples: Kinect movement and interaction ontology [7]



Human Activity Recognition: Data and Object properties and classes [6]

Fuzzy Human Activity Recognition [6]

Rule 1	*(define-concept antecedent1 (w-sum (0.17 reachMilkOrBowlOrBox)(0.41 moveMilkOrBowlOrBox)(0.24 placeMilkOrBowlOrBox)(0.01 openMilkOrBox)(0.16 pourMilkOrBox))) (define-concept consequent1 (g-and User (some performsActivity cereal)))*
Rule 2	*(define-concept antecedent2 (w-sum (0.29 reachCupOrMedicineBox)(0.3 moveCupOrMedicineBox)(0.1 placeCupOrMedicineBox)(0.1 openMedicineBox)(0.1 eatMedicineBox)(0.1 drinkCup))) (define-concept consequent2 (g-and User (some performsActivity medicine)))*
Rule 3	*(define-concept antecedent3 (w-sum (0.26 reachStackable)(0.27 moveStackable)(0.27 placeStackable)(0.20 nullSA))) (define-concept consequent3 (g-and User (some performsActivity stacking)))*
Rule 4	*(define-concept antecedent4 (w-sum (0.26 reachStackable)(0.27 moveStackable)(0.27 placeStackable)(0.20 nullSA))) (define-concept consequent4 (g-and User (some performsActivity unstacking)))*
Rule 5	*(define-concept antecedent5 (w-sum (0.32 reachMicroOrDrinkingKitchenware)(0.11 moveDrinkingKitchenware)(0.11 placeDrinkingKitchenware)(0.12 openMicro)(0.11 closeMicro)(0.23 nullSA))) (define-concept consequent5 (g-and User (some performsActivity microwaving)))*
Rule 6	*(define-concept antecedent6 (w-sum (0.26 reachPickable)(0.27 movePickable)(0.47 nullSA))) (define-concept consequent6 (g-and User (some performsActivity bending)))*
Rule 7	*(define-concept antecedent7 (w-sum (0.27 reachMicroOrCloth)(0.23 moveCloth)(0.1 placeCloth)(0.1 openMicro)(0.1 closeMicro)(0.1 cleanMicroOrCloth)(0.1 nullSA))) (define-concept consequent7 (g-and User (some performsActivity cleaningObjects)))*



Let's get started!

Learning to model fuzzy ontologies with *fuzzyDL* reasoner:

- *FuzzyDL* syntax:
  `http://www.umbertostraccia.it/cs/software/fuzzyDL/fuzzyDL.html`
- *FuzzyDL* syntax and semantics cheeatsheet:
  `https://tinyurl.com/y8slmcck`
- How to write ontologies in *fuzzyDL*:
  `http://www.umbertostraccia.it/cs/software/FuzzyOWL/index.html`
  → Study matchmaking ontology and query examples in *fuzzyDL* web[7]

[7] https://tinyurl.com/ya8l9y9h

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