# Symbolic and structural models for image understanding

Part II: Ontologies and description logics

Jamal Atif - Isabelle Bloch - Céline Hudelot



IJCAI 2016

## Outline

- What is an ontology?
- Ontologies for image understanding: overview
- Oescription Logics
- Description Logics for image understanding
- Conclusion

# What is an ontology?

Example from F. Gandon, WIMMICS Team, INRIA

What is the last document that you have read?

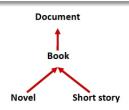


#### **Documents**



Your answer is based on a shared ontology

You can reason
I can understand



# Ontologies: Definition

### Ontology

ethymology: ontos (being, that which is) + logos (science, study, theory)

- Philosophy
  - Study of the nature of being, becoming and reality.
  - Study of the basic categories of being and their relations.
- Computer Science
  - Formal representation of a domain of discourse.
  - Explicit specification of a conceptualization [Gruber 95].





Ref: [Guarino 09]

# Ontologies: Definition

### ontology

Formal, explicit (and shared) specification of a conceptualization [Gruber 95, Studer 98]

- Formal, explicit specification:
  - a formal language is used to refer to the elements of the conceptualization, e.g. description logics
- Conceptualization:
  - Objects, concepts and other entities and their relationships

### Concept

### Denoted by:

- a name
- a meaning (intensional definition)
- a set of denoted objects (extensional definition)

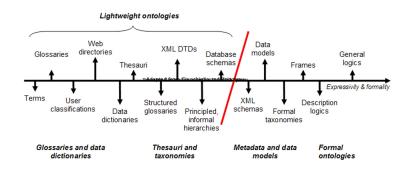
#### Relation

#### Denoted by:

- a name
- an intension
- an extension

# The different types of ontologies

### According to their expressivity



Source: [Uschold 04]

# The different types of ontologies

### According to their abstraction level

- Top (or Upper)-level ontology: very general concepts that are the same across all knowledge domains [Wikipedia] (e.g. DOLCE).
- Core ontology: minimal set of concepts and relations used to structure and describe a given domain (e.g. Dublin Core).
- **Domain ontology**: concepts and relations of a specific domain (e.g. FMA).

### In this tutorial, focus on:

- Domain ontologies
- Formal ontologies

## Outline

- What is an ontology ?
- 2 Ontologies for image understanding: overview
- Obscription Logics
- 4 Description Logics for image understanding
- 6 Conclusion

# Ontologies for image interpretation

A growing interest since 2001

#### Various objectives:

- Providing an unified vocabulary for the description and annotation of image content.
  - e.g. MPEG-7 ontologies.
- Structuring the vocabulary and the database for large-scale image problems.
  - e.g. visual ontologies (LabelMe, ImageNet, Visipedia).
- Representing the application domain knowledge for reasoning and for guiding the interpretation process.
  - e.g. formal ontologies based on description logics.

# Ontologies for an unified and standardized description of image content

- MPEG-7 ontologies: Boemie, AceMedia, Rhizomik... (see [Dasiopoulou 10b] for a recent review).
  - Main motivation: interoperability between applications.
- LSCOM (Large Scale concept ontology for multimedia) [Naphade 06], MediaMill [Habibian 13].
  - Main motivation: common vocabulary for video shot description.



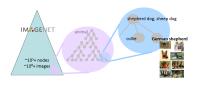


Mainly focused on the descriptive part of ontologies.

#### Ontologies for structuring the vocabulary and the learning database (1/3)

#### Main motivation: image classification, annotation and retrieval at large scale [Liu 07, Deng 10].

- Ontologies based on lexical resources (e.g. Wordnet) populated with images:
  - ImageNet [Russakovsky 15], LabelMe [Russell 08], Visipedia [Belongie 16]...



& Kulendt

ImageNet

Which concepts are closer?

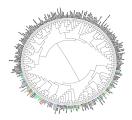
- Adequacy of the lexical resources for image interpretation problems?
- Mainly lightweight ontologies (non-formal, without reasoning capabilities).

### Ontologies for structuring the vocabulary and the learning database (2/3)

#### Main motivation: hierarchical image classification.

 Visual concept hierarchies inferred from image datasets: [Fei-Fei 05, Marszalek 08, Griffin 08, Sivic 08, Bart 08, Gao 11].





[Griffin 08]

- Mainly hierarchies (no other semantic relations than is-a).
- Concepts without semantics (except the leaves).
- Mainly lightweight ontologies (non-formal, without reasoning capabilities).

#### Ontologies for structuring the vocabulary and the learning database (3/3)

#### Main motivation: image classification and annotation.

Ontologies combining text and visual knowledge: [Li 10, Wu 12, Bannour 14].



Image hierarchy [Li 10]



VCNet [Wu 12]

- Dedicated knowledge models.
- Mainly lightweight ontologies (non-formal, without reasoning capabilities).

# Image interpretation as an ontological driven inference approach

Main motivation: explicit and formal representation of domain and contextual knowledge used to reason and infer the interpretation.

- Annotation and interpretation refinement using basic DLs inference services: [Simou 08, Dasiopoulou 09, Dasiopoulou 10a, Bannour 14].
- Ontologies to narrow the semantic gap: [Town 06, Bagdanov 07, Hudelot 08]
- Image interpretation as a non-monotonic reasoning process:
  - Image interpretation as a default reasoning service [Möller 99a, Neumann 08].
  - Abductive reasoning for image interpretation [Peraldi 07, Möller 08, Atif 14, Donadello 14].

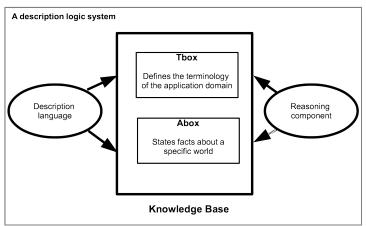
Often based on Description Logics (DLs).

## Outline

- What is an ontology ?
- 2 Ontologies for image understanding: overview
- Oescription Logics
- 4 Description Logics for image understanding
- 6 Conclusion

# Descriptions logics

- Family of logics for representing structured knowledge.
- Well understood semantics.
- Defined by a set of concepts and role forming operators.
- Compact and expressive and basis of OWL language to represent ontologies.



# Description logics

## A family of knowledge representation languages

 ${\mathcal L}$  a logic,  ${\mathbb C}{\mathcal L}$  a set of concept descriptions. Syntax

- A set of concept names and a set of role names Σ := (N<sub>C</sub>, N<sub>R</sub>) define a signature.
- Concept descriptions:

$$C ::= \underbrace{A \mid (C \sqcap C) \mid (\exists r.C) \mid \top}_{\mathcal{E}L} \mid (C \sqcup C) \mid (\forall r.C) \mid \neg C \mid \bot \mid \cdots$$

#### Semantics

- An interpretation  $\mathcal{I} := (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}), C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}, r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$  (binary relation).
- $\bullet \quad \top^{\mathcal{I}} = \Delta^{\mathcal{I}}, (C \sqcap D)^{\mathcal{I}} = (C^{\mathcal{I}} \cap D^{\mathcal{I}}), (\exists r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \exists e, (d, e) \in r^{\mathcal{I}} \text{ and } e \in C^{\mathcal{I}}\}, \cdots.$

#### Subsumption, equivalence

- C is subsumed by D ( $C \subseteq D$ ) if  $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$  for all interpretations  $\mathcal{I}$  (ex: Musician  $\subseteq$  Artist).
- C and D are equivalent (C  $\equiv$  D) if  $C^{\mathcal{I}} = D^{\mathcal{I}}$  for all interpretations  $\mathcal{I}$ .

#### Knowledge base

 $\mathcal{K}=(\mathcal{T},\mathcal{A}),\mathcal{T}$  a set of axioms (e.g.  $C\subseteq D,C\equiv D$ ) and  $\mathcal{A}$  a set of assertions (e.g. a:C,(a,b):R)

# Description logics for knowledge representation

#### Example in the medical domain

### Knowledge in brain imaging

- caudate nucleus: a deep gray nucleus of the telencephalon involved with control of voluntary movement
- the left caudate nucleus is inside the left hemisphere
- it is close to the lateral ventricle
- it is outside (left of) the left lateral ventricle

### Excerpt of a corresponding TBox

- LV 

  AnatomicalStructure
- ON □ GN
- LV ≡ RLV ⊔ LLV
- O CN = RCN ⊢ LCN
- LCN 

  GN □∃ closeTo.(LLV) □∃ leftOf.(LLV)
- etc.

# Description logics: concrete domains

- A way to integrate concrete and quantitative qualities (integers, strings,...) of real world objects with conceptual knowledge [Baader,91].
- A pair  $(\Delta_D, \Phi_D)$  where  $\Delta_D$  is a set and  $\Phi_D$  a set of predicates names on  $\Delta_D$ . Each predicate name P is associated with an arity n and an n-ary predicate  $P^D \subseteq \Delta_D^n$ .

### Examples

- Concrete domain N:
  - Domain: non negative integers.
  - Predicates:  $\leq$  (binary predicate)  $\leq$  *n* unary predicate.
  - Person⊓∃age. ≤ 20 denotes a person whose age is less than 20.
- Concrete domain AL, Allen's interval calculus:
  - Domain: intervals.
  - Predicates: built from Allen's basic interval relations.

# Description logics: reasoning services

- ⇒ Infer implicit knowledge from explicitly one.
- Terminological reasoning.
- Assertional reasoning.

# Description logics: reasoning services

### Terminological reasoning

### Satisfiability

*C* is satisfiable w.r.t. a TBox  $\mathcal{T}$  iff  $C^{\mathcal{I}} \neq \emptyset$  for some model  $\mathcal{I}$  of  $\mathcal{T}$ .

### Subsumption

C is subsumed by D w.r.t. a TBox  $\mathcal{T}$  ( $C \sqsubseteq_{\mathcal{T}} D$ ) iff  $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$  for all models  $\mathcal{I}$  of  $\mathcal{T}$ .

### Equivalence

*C* is equivalent to *D* w.r.t. a TBox  $\mathcal{T}$  ( $C \equiv_{\mathcal{T}} D$ ) iff  $C^{\mathcal{I}} = D^{\mathcal{I}}$  for all models  $\mathcal{I}$  of  $\mathcal{T}$ .

### Disjointness

Two concepts C and D are disjoint with respect to T if  $C^{\mathcal{I}} \cap D^{\mathcal{I}} = \emptyset$  for every model  $\mathcal{I}$  of T.

# Description logics: reasoning services

### Assertional reasoning

Let  $\mathcal{K} = (\mathcal{T}, \mathcal{A})$  be an ontology.

#### Consistency

 ${\cal A}$  is consistent with respect to a TBox  ${\cal T}$ , if there is an interpretation that is a model of both  ${\cal A}$  and  ${\cal T}$ .

#### Instance checking

a is an instance of C w.r.t.  $\mathcal{T}$  iff  $a^{\mathcal{I}} \in C^{\mathcal{I}}$  for all models  $\mathcal{I}$  of  $\mathcal{T}$ . We also write  $\mathcal{A} \models C(a)$ . The same holds for roles.

#### Retrieval problem

Given an ABox A and a concept C, find all individuals a such that  $A \models C(a)$ .

#### Realization problem (dual to the retrieval problem)

Given an individual a and a set of concepts, find the most specific concepts (msc) C from the set such that  $A \models C(a)$ . The mscs are the concepts that are minimal with respect to the subsumption ordering  $\sqsubseteq$ .

### Outline

- What is an ontology?
- 2 Ontologies for image understanding: overview
- Obscription Logics
- Description Logics for image understanding
  - Ontologies for interpretation refinement
  - Narrowing the semantic gap
  - Non-monotonic reasoning for image interpretation
    - Default reasoning
    - Abductive reasoning
- Conclusion

# Interpretation refinement using basic DLs inference services

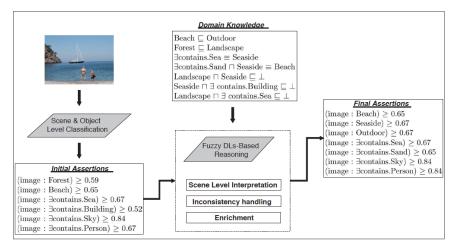
### Main principles

- Application domain knowledge is encoded into a TBox.
- A first interpretation of the targeted image is built using computer vision algorithms and translated into ABox assertions.
- Basic reasoning services of DLs such as consistency handling are used to revise the interpretation.
- Fuzzy DLs are used to take into account the imprecision of computer vision algorithms results.

Investigating fuzzy DLs-based reasoning in semantic image analysis [Dasiopoulou 10a]. Building and using fuzzy multimedia ontologies for semantic image annotation [Bannour 14].

# Interpretation refinement using basic DLs inference services

Dasiopoulou et al. [Dasiopoulou 10a]



### Outline

- What is an ontology?
- 2 Ontologies for image understanding: overview
- Oescription Logics
- Description Logics for image understanding
  - Ontologies for interpretation refinement
  - Narrowing the semantic gap
  - Non-monotonic reasoning for image interpretation
    - Default reasoning
    - Abductive reasoning
- Conclusion

# Narrowing the semantic gap

### Main approaches

- Building a dedicated visual concept ontology as an intermediate level between image features and application domain concepts: [Town 06, Bagdanov 07, Maillot 08, Porello 13, Mezaris 04].
- Using concrete domains to link high level concepts to their specific representations into the image domain:

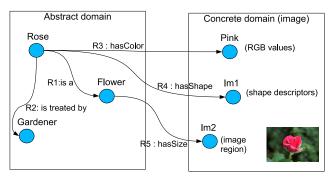
[Hudelot 08, Hudelot 14].

 $\Rightarrow$  operational ontologies for image interpretation.

A spatial relation ontology for semantic image interpretation Hudelot et al. [Hudelot 08, Hudelot 14]

# Ontologies, concrete domains and semantic gap

Hudelot et al. [Hudelot 08]

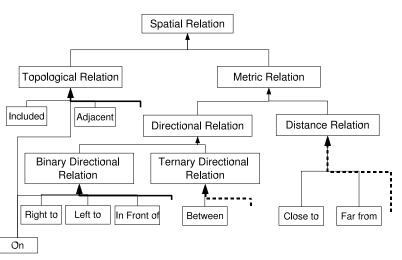


#### Idea

Each application domain concept is linked to its representation in the image domain: use of concrete domains.

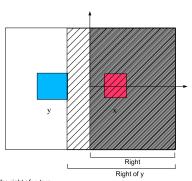
# A spatial relation ontology

Hudelot et al. [Hudelot 08]



## Formal representation of spatial relations

Hudelot et al. [Hudelot 08]



#### Abox:

- y:SpatialObject; x:SpatialObject
- $\begin{array}{l} \bullet \quad \text{Right\_Of\_y} \equiv \text{Right\_Of} \; \sqcap \\ \ni \text{hasReferentObject.\{y\}} \end{array}$
- x:SpatialObject □ ∃
   hasSpatialRelation.Right\_Of\_y and
   x:SpatiallyRelatedObject
- $C_0 \equiv \text{SpatialRelation} \sqcap$   $\ni \text{hasReferentObject.\{y\}} \sqcap$  $\ni \text{hasTargetObject.\{x\}}$

x is to the right of y: true

# A dedicated logic for spatial reasoning: $\mathcal{ALC}(F)$

Instantiation of the description logic  $\mathcal{ALCRP}(D)$  with the concrete domain  $F = (\Delta_F, \Phi_F)$ .

$$\Delta_F = (\mathcal{F}, \leq_{\mathcal{F}}, \wedge, \vee, \emptyset_{\mathcal{F}}, 1_{\mathcal{F}}, t, I)$$

A residuated lattice of fuzzy sets defined over the image space S, S being typically  $\mathbb{Z}^2$  or  $\mathbb{Z}^3$  for 2D or 3D images, with t a t-norm (fuzzy intersection) and I its residuated implication.

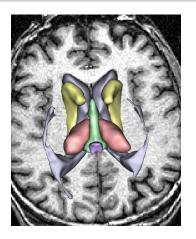
#### Main predicates of $\Phi_F$ :

- $\mu_X$ : degree of belonging to the spatial representation of the object X in the spatial domain.
- $\nu_R$ : fuzzy structuring element representing the fuzzy relation R in the spatial domain.
- $\delta_{\nu_R}^{\mu_X}$ : fuzzy dilation.
- $\varepsilon_{\nu_R}^{\mu_X}$ : fuzzy erosion.

# Application to brain imaging

### Objective:

Progressive recognition of anatomical structures using spatial information.

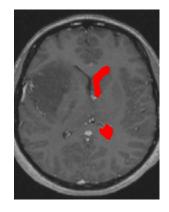


# Description of anatomical knowledge

#### Tbox:

- AnatomicalStructure 
   □ SpatialObject
- RLV ≡ AnatomicalStructure □ ∃ hasFR.μ<sub>RLV</sub>
- LLV  $\equiv$  AnatomicalStructure  $\sqcap \exists$  hasFR. $\mu_{LLV}$
- LV  $\equiv$  RLV  $\sqcup$  LLV
- $LV \equiv RLV \sqcup LLV$
- Right\_of  $\equiv$  DirectionalRelation  $\sqcap \exists$  hasFR. $\nu_{IN\_DIRECTION\_0}$
- Close\_to  $\equiv$  DistanceRelation  $\sqcap \exists$  hasFR. $\nu_{CLOSE\ TO}$
- Right\_of\_RLV  $\equiv$  DirectionalRelation  $\sqcap$   $\exists$  hasReferentObject.RLV  $\sqcap$   $\exists$  hasFR.  $\delta^{\mu_{RLV}}_{\nu_{IN\_DIRECTION\_0}}$
- Close\_To\_RLV  $\equiv$  DistanceRelation  $\sqcap$   $\exists$  hasReferentObject.RLV  $\sqcap$   $\exists$  hasFR. $\delta_{\nu_{CLOSE\_TO}}^{\mu_{RLV}}$
- $RCN \equiv GN \sqcap \exists hasSR.(Right\_of\_RLV \sqcap Close\_To\_RLV)$
- $CN \equiv GN \sqcap \exists hasSR.(Close\_To\_LV)$
- $CN \equiv RCN \sqcup LCN$

# Example



#### Abox:

- $c_1$ : RLV ,  $(c_1, \mu_{S_1})$ : hasFR
- $r_1$ : Right\_of,  $(r_1, \nu_{IN\_DIRECTION\_0})$ : hasFR
- $r_2$ : Close\_to,  $(r_2, \nu_{CLOSE\ TO})$ : hasFR

# Example

### Objective:

- Find some spatial constraints in the image domain on an instance *c*2 of the Left Caudate Nucleus.
- $\Rightarrow$  Find constraints on concrete domains to ensure the satisfiability of the assertions  $c_2$ : RCN,  $(c_2, \mu_{S_2})$ : hasFR

### Results using inference and properties

$$(\mu_{S_2})^F \leq_{\mathcal{F}} (\delta_{\nu_{\text{IN DIRFCTION 0}}}^{\mu_{S_1}})^F \wedge (\delta_{\nu_{\text{CLOSE TO}}}^{\mu_{S_1}})^F$$







### Inference details:

$$\mathcal{A} \cup \{c_2: \mathsf{GN} \,\sqcap\, \exists \mathsf{hasSR}. (\mathsf{Right\_of\_RLV} \,\sqcap\, \mathsf{Close\_to\_RLV}), (c_2, \mu_{S_2}) : \mathsf{hasFR} \} \\ \downarrow^{\sqcap - \mathit{rule}} \\ c_2: \mathsf{GN}, c_2: \exists \mathsf{hasSR}. (\mathsf{Right\_of\_RLV} \,\sqcap\, \mathsf{Close\_to\_RLV}) \\ \downarrow^{\exists - \mathit{rule}} \\ c_3: \mathsf{Right\_of\_RLV} \,\sqcap\, \mathsf{Close\_to\_RLV}, (c_2, c_3) : \mathsf{hasSR}, (c_3, \mu_{S_3}) : \mathsf{hasFR} \\ \downarrow^{\mathsf{Spatial}} \, \mathsf{Object} \, \mathsf{Conjunction} \, \mathsf{Rule} \, \mathcal{R}_{\sqcap} \\ ((\mu_{\mathsf{Right\_of\_RLV}}) \,\sqcap\, d \, (\mu_{\mathsf{Close\_to\_RLV}}))^F \\ \downarrow^{\mathsf{Spatial}} \, \mathsf{Object} \, \mathsf{Conjunction} \, \mathsf{Rule} \, \mathcal{R}_{\sqcap} \\ c_3: \, \mathsf{Right\_of\_RLV}, c_3: \, \mathsf{Close\_to\_RLV}) \\ \downarrow^{\mathsf{Spatial}} \, \mathsf{Conjunction} \, \mathsf{Rule} \, \mathcal{R}_{\sqcap} \\ c_3: \, \mathsf{Right\_of\_RLV}, c_3: \, \mathsf{Close\_to\_RLV} \\ \downarrow^{\mathsf{Spatial}} \, \mathsf{Relation} \, \mathsf{Rule} \, \mathcal{R}_{2R_X} \\ \mu_{S_3} = \delta^{\mu_{S_1}}_{\nu_{IN\_DIRECTION\_0}} \,\sqcap\, \delta^{\mu_{S_1}}_{\nu_{CLOSE\_TO}} \\ \downarrow^{\mathsf{Spatial}} \, \mathsf{constraints} \\ \mathit{fit}(\mu_{S_2}^F, \mu_{S_3}^F) = \mathit{fit}(\mu_{S_2}^F, (\delta^{\mu_{S_1}}_{\nu_{IN\_DIRECTION\_0}}) \,\sqcap\, \delta^{\mu_{S_1}}_{\nu_{CLOSE\_TO}})^F) = 1 \\ \downarrow^{\mathsf{Spatial}} \, \mathsf{constraints} \\ (\mu_{S_2})^F \leq_{\mathcal{F}} \, (\delta^{\mu_{S_1}}_{\nu_{IN\_DIRECTION\_0}})^F \wedge (\delta^{\mu_{S_1}}_{\nu_{CLOSE\_TO}})^F$$

## Outline

- What is an ontology?
- 2 Ontologies for image understanding: overview
- Oescription Logics
- Description Logics for image understanding
  - Ontologies for interpretation refinement
  - Narrowing the semantic gap
  - Non-monotonic reasoning for image interpretation
    - Default reasoning
    - Abductive reasoning
- Conclusion

# Non-monotonic reasoning for image interpretation

### Main principles:

Image interpretation is modeled as a non-monotonic reasoning process.

- Default reasoning: Non-monotonic logic to formalize reasoning with default assumptions [Reiter 80].
- Abductive reasoning: Backward reasoning: from observations to explanations, Charles Sanders Peirce in the late 19th century.

# Image interpretation as a default reasoning service

### Default rule

$$\frac{\alpha:\beta_1,\cdots,\beta_n}{\gamma}$$

- $\alpha$ : precondition of the rule.
- $\beta_i$ : justifications.
- $\gamma$ : consequent.

## Intuitive explanation

Starting with a world description  $\alpha$  of what is known to be true, i.e. deducible and it is consistent to assume  $\beta_i$  then conclude  $\gamma$ .

### Example

 $\forall x, plays\_instruments(x) : improvises(x)/jazz\_musician(x)^a$ 

 $^{a}$ For all x, if x plays an instrument and if the fact that x can improvise is consistent with all other knowledge then we can conclude that x is a jazz musician.

# Default reasoning in DL

### Reiter's default theory [Reiter 80]

A pair  $(W, \mathcal{D})$  where W is a set of closed first-order formulae (the world description) and W a set of default rules.

## Terminological default theory [Baader 92]

A pair (A, D) where:

- A: an ABox.
- D: a finite set of default rules whose preconditions, justifications and consequents are concept terms.

Maintaining decidability

- Default rules have to be closed over the ABox (instanciation with explicitly mentioned ABox individuals).
- Closed default rules:  $\alpha$ ,  $\beta_i$ ,  $\gamma$  are ABox concept axioms (no use of free variables, i.e. TBox concept axioms).

Moller et al. approach [Möller 99b, Neumann 08]

## Use case: topological reasoning for aerial image interpretation

### Main idea

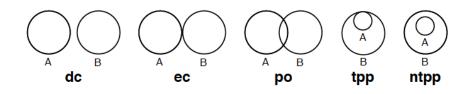
- Defaults are used for hypothesis generation regarding the classification of areas in an image.
- Default reasoning generates ABox extensions (hypothesized classifications) consistent with the rest of the knowledge base.

### **Preliminaries**

The description logic  $\mathcal{ALCRP}(S_2)$  for spatial information modeling and reasoning:  $\mathcal{ALC}$  with:

- predicate existence restriction:  $\exists u_1, ..., u_n.P$  with P a predicate name from  $S_2$  with arty n and  $u_1, ..., u_n$  feature chains.
- a concrete domain  $S_2$  defined w.r.t. the topological space  $(\mathbb{R}^2, 2^{\mathbb{R}^2})$ .

Moller et al. approach [Möller 99b, Neumann 08]

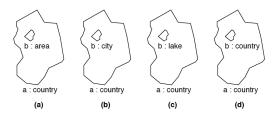


The concrete domain  $\mathcal{S}_2$  over the topological space  $\langle \mathbb{R}^2, 2^{\mathbb{R}^2} \rangle$ 

- $\Delta_{\mathcal{S}_2}$ : set of non-empty, regular closed subsets of  $\mathbb{R}^2$ : regions
- Set of predicate names:
  - Predicate is\_region with is\_region  $S_2 = \Delta_{S_2}$  and its negation is\_no\_region with is\_no\_region  $S_2 = 0_{S_2}$
  - 8 basic predicates dc, ec, po, tpp, ntpp, tppi, eq (RCC-8 relations)
  - Predicates to name disjunctions of base relations:  $p1 ... p_n$
  - The predicate dc-ec-po-tpp-ntpp-tppi-ntppi-eq is called spatially\_related
  - A binary predicate inconsistent\_relation with inconsistent\_relation  $S_2 = \emptyset$  (negation of spatially\_related).

Moller et al. approach [Möller 99b, Neumann 08]

## Example



Interpretation problem: generate hypotheses for object b.

### $\mathcal{S}_2$ predicates formalization

$$inside \equiv \exists (has\_area)(has\_area).tpp - ntpp$$
 $contains \equiv \exists (has\_area)(has\_area).tppi - ntppi$ 
 $overlaps \equiv \exists (has\_area)(has\_area).po$ 
 $touches \equiv \exists (has\_area)(has\_area).ec$ 
 $disjoint \equiv \exists (has\_area)(has\_area).dc$ 

Moller et al. approach [Möller 99b, Neumann 08]

## Example

### **TBox**

area	=	∃(has_area).is_region
natural_region	=	¬administrative_region
country_region	⊑	administrative_region □
		large_scale   area
city_region	⊑	administrative_region □
		¬large — scale □ area
lake_region	⊑	natural_region □ area
river region	⊏	natural region □ area

```
country = country_region □
∀ contains. ¬ country_region □
∀ overlaps. ¬ country_region □
∀ inside. ¬ country_region □
city = city_region □
∃inside. country_region
lake □ lake_region
river □ river_region □
∀ contains. ⊥ □
∀ inside. ¬ lake_region □
∀ contains. ⊥ □
∀ inside. ¬ lake_region □
∀ inside. ¬ lake_region □
∀ inside. ¬ lake_region □
```

Moller et al. approach [Möller 99b, Neumann 08]

## Example

### Abox

 $\{a: country, b: area, (a, b): contains, (b, a): inside\}$ 

### Spatioterminological default rules

$$d_1 = \frac{area: city}{city}$$
  $d_2 = \frac{area: lake}{lake}$   $d_3 = \frac{area: city}{city}$ 

## Closed spatioterminological default rules, $d_i(ind)$

e.g.

$$d_1(a) = \frac{\{a : area\} : \{a : city\}}{\{a : city\}}$$

6 different closed defaults can be obtained  $(d_1(a), d_1(b), d_2(a), d_2(b), d_3(a), d_3(b))$ 

Moller et al. approach [Möller 99b, Neumann 08]

## Example

### Default rules reasoning

$$d_1 = \frac{area: city}{city}$$

- d<sub>1</sub>(a): cannot be applied.
   Contradiction between a : city and a : country in the Abox. country\_region and city\_region are disjoint in the TBox (due to large\_scale and ¬large\_scale).
- d<sub>1</sub>(b): can be applied.
   Abox extension:

 $\{a: country, b: area, b: city, (a, b): contains, (b, a): inside\}$ 

Moller et al. approach [Möller 99b, Neumann 08]

## Example

### Default rules reasoning

$$d_2 = \frac{area: lake}{lake}$$

- d<sub>2</sub>(a): cannot be applied.
   Contradiction between a: lake and a: country in the Abox. administrative\_region and natural\_region are disjoint.
- d<sub>2</sub>(b): can be applied.
   Abox extension:

$$\{a : country, b : area, b : lake, (a, b) : contains, (b, a) : inside\}$$

But if Abox contains  $d_1(a)$ ,  $d_2(b)$  cannot be applied  $\implies$  two possible extensions.

Moller et al. approach [Möller 99b, ?]

## Example

Default rules reasoning, cont'd

$$d_3 = \frac{area: country}{country}$$

- $d_3(a)$  cannot be applied. Its conclusion is already entailed by the ABox.
- d<sub>3</sub>(b) cannot be applied. The consequent b : country makes the Abox inconsistent because a
  is already known as a country.

$$\mathcal{A} \models (a : \forall contains. \neg country\_region)$$
  
 $(a,b) : contains, b : country \implies b : country\_region$ 

# Abductive reasoning

- Abduction using safe rules (Peraldi et al. [Peraldí 09]).
- Concept abduction (Atif et al. [Atif 14]).

# Abductive reasoning

Sort of backward reasoning from a set of observations to a cause.

#### Definition

Given a knowledge base K and a formula  $\mathcal{O}$  representing an observation with  $K \not\models \mathcal{O}$ , we look for an explanation formula  $\mathcal{H}$  such that  $\mathcal{H}$  is satisfiable w.r.t. K and

$$\mathcal{K} \cup \mathcal{H} \models \mathcal{O}$$

holds.

### Case of image interpretation

- Scene = observation.
- Interpretation = look for the best explanation considering a terminological knowledge part about the scene context.

# Abductive reasoning and description logics

## Distinct abductive problems [Elsenbroich 06]

Let  $\mathcal{L}$  be a DL,  $\mathcal{K}$  a knowledge base in  $\mathcal{L}$ 

- Concept abduction
- ABox abduction
- TBox abduction
- Knowledge base abduction

# Abduction using safe rules

Multimedia interpretation as abduction Peraldi et al. [Peraldi 09]

Ontology-based reasoning techniques for multimedia interpretation and retrieval

Möller et al. [Möller 08]

# Multimedia interpretation as an abduction problem.

Peraldi et al. [Peraldi 09]

#### Main idea: Abduction as a non-standard retrieval inference service

Observations are used to constitute queries that have to be answered by acquiring what should be added to the knowledge base in order to positively answer to a query

### Use of conjunctive queries

Structure of the form {head | body}:

$$\{(X_1, \dots, X_n) \mid atom_1, \dots, atom_m\}, \text{ with}$$
  
 $atom = C(X), R(X, Y), (X = Y)$ 

- head: list of variables for which we like to compute bindings
- body: query atoms

Example:  $\{x \mid \exists y \exists z (ChildOf(x, y) \land ChildOf(x, z) \land Married(y, z))\}$ 

Query answer: set of bindings for variables in the head

## **Formalisation**

Peraldi et al. [Peraldí 09]

### Abduction inference

Given a set of ABox assertions  $\Gamma$  in form of a query and a KB,  $\Sigma = (\mathcal{T}, \mathcal{A})$ , derive all sets of ABox assertions  $\Delta$  (explanations) such that  $\Delta$  is consistent w.r.t the ontology  $\Sigma$  ( $\Sigma \cup \Delta$  is satisfiable) and:

- $\Sigma \cup \Delta \models \Gamma$ .
- $\Delta$  is a minimal explanation for  $\Gamma$ , i.e. there exists no other explanation  $\Delta'$  in the solution set that is not equivalent to  $\Delta$  and it holds that  $\Sigma \cup \Delta' \models \Delta$ .

## **Formalisation**

Peraldi et al.[Peraldí 09]

### Multimedia abduction:

- $\Sigma = (\mathcal{T}, \mathcal{A})$ , a knowledge base on the application domain with  $\mathcal{A}$  assumed empty.
- Γ = Γ<sub>1</sub> ∪ Γ<sub>2</sub>, set of Abox assertions, encoding low level extracted information from images (objects and their spatial relationships):
  - $\Gamma_1$ : bona fide assertions, assumed to be true by default.
  - $\Gamma_2$ : assertions requiring fiats (aimed to be explained).
- Abduction process : compute  $\Delta$ , a set of ABox explanations, such that

$$\Sigma \cup \Gamma_1 \cup \Delta \models \Gamma_2$$

The process is implemented as (boolean) query answering.

# Illustration on an example





#### ABox $\Gamma$ : low-level image analysis results

pole<sub>1</sub> : Pole human<sub>1</sub> : Human bar<sub>1</sub> : Bar

 $\{bar_1, human_1\}$  : near

### $\Sigma$ , a Tbox and DL-safe rules on the athletics domain

 $\begin{array}{c|cccc} Jumper & \sqsubseteq & Hum. \\ Pole & \sqsubseteq & Spor \\ Bar & \sqsubseteq & Spor \\ \hline Pole \sqcap Bar & \sqsubseteq \\ \hline Pole \sqcap Jumper & \sqsubseteq \\ Jumper \sqcap Bar & \sqsubseteq \\ Jumping\_Event & \sqsubseteq & \exists_{\leq 1}^{i} \\ \hline Pole\_Vault & \sqsubseteq & Jump. \\ High\_Jump & \sqsubseteq & Jump. \\ hasP & & hasP \\ \hline near(Y,Z) & \leftarrow & High. \\ \end{array}$ 

Human Sports\_Equipment Sports\_Equipment

 $\exists_{\leq 1}$ hasParticipant.Jumper Jumping\_Event  $\sqcap \exists$ hasPart.Pole  $\sqcap \exists$ hasPart.Bar Jumping\_Event  $\sqcap \exists$ hasPart.Bar Pole\_Vault(X), hasPart(X, Y), Bar(Y),

hasPart(X, W), Pole(W), hasParticipant(X, Z), Jumper(Z) $High\_Jump(X)$ , hasPart(X, Y), Bar(Y),

hasParticipant(X, Z), Jumper(Z)

# Illustration on an example





ABox  $\Gamma$  : low-level image analysis results

 $pole_1$  : Pole  $human_1$  : Human  $bar_1$  : Bar  $\{bar_1, human_1\}$  : near

- $\Gamma_1 = \{pole_1 : Pole, human_1 : Human, bar_1 : Bar\}$
- $\Gamma_2 = \{(bar_1, human_1) : near\}$
- Boolean query  $Q_1 := \{() \mid near(bar_1, human_1)\}$

Peraldi et al. [Peraldí 09]

### Possible explanations:

- $\Delta_1 = \{new\_ind_1 : Pole\_Vault, (new\_ind_1, bar_1) : hasPart, (new\_ind_1, new\_ind_2) : hasPart, new\_ind_2 : Pole, (new\_ind_1, human_1) : hasParticipant, human_1 : Jumper\}$
- Δ<sub>2</sub> = {new\_ind<sub>1</sub> : Pole\_Vault, (new\_ind<sub>1</sub>, bar<sub>1</sub>) : hasPart, (new\_ind<sub>1</sub>, pole<sub>1</sub>) : hasPart, (new\_ind<sub>1</sub>, human<sub>1</sub>) : hasParticipant, human<sub>1</sub> : Jumper}
- Δ<sub>3</sub> = {new\_ind<sub>1</sub> : High\_Jump, (new\_ind<sub>1</sub>, bar<sub>1</sub>) : hasPart, (new\_ind<sub>1</sub>, human<sub>1</sub>) : hasParticipant, human<sub>1</sub> : Jumper}

### Preference score :

$$S_p(\Delta) := S_i(\Delta) - S_h(\Delta)$$
, with  $S_i(\Delta) := |\{i \mid i \in inds(\Delta) \text{ and } i \in inds(\Sigma \cup \Gamma_1)\}|$   
 $S_h(\Delta) := |\{i \mid i \in inds(\Delta) \text{ and } i \in new\_inds\}|$ 

Peraldi et al. [Peraldí 09]

- $\Delta_1$  incorporates  $human_1$  and  $bar_1$  from  $\Gamma_1$ , then  $S_i(\Delta_1) = 2$ .
- $\Delta_1$  hypothesizes two new individuals:  $new\_ind_1$ ,  $new\_ind_2$ , then  $S_h(\Delta_1) = 2$ .

$$\Longrightarrow S_p(\Delta_1) = 0$$

- $S_p(\Delta_2) = 3 1 = 2$ .
- $S_p(\Delta_3) = 2 1 = 1$ .

 $\implies \Delta_2$  represents the 'preferred' explanation:

```
\Delta_2 = \{new\_ind_1 : Pole\_Vault, (new\_ind_1, bar_1) : hasPart, (new\_ind_1, pole_1) : hasPart, (new\_ind_1, human_1) : hasParticipant, human_1 : Jumper\}
```

The image should better be interpreted as showing a pole vault and not a high jump.

# Multimedia interpretation as concept abduction

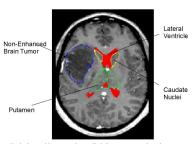
Explanatory reasoning for image understanding using formal concept analysis and description logics.

Atif et al. [Atif 14]

# Brain image understanding

Atif et al. [Atif 14]

### lmage interpretation



Pathological brain with small deforming peripheral tumor

### Interpretation as an abduction process

$$\mathcal{K} \models (\gamma \to \varphi)$$

Computing of the best explanation from observations  $\varphi$  given some a priori expert knowledge  $\mathcal K$  encoded in description logics.

# Knowledge representation

CerebralHemisphere Brain Anatomical Structure Cerebral Hemisphere AreaPeripheralCerebralHemisphere LargeDefTumor BrainTumor □ SubCorticalCerebralHemisphere CerebralHemisphereArea ∃hasI ocation CerebralHem GreuNuclei **BrainAnatomicalStructure** □∃hasComvonent.Edema I ateralVentricle BrainAnatomicalStructure BrainTumor  $\sqcap \exists \textit{hasComponent.Necrosis}$ Disease □ ∃hasEnhancement Enhanced □ ∃hasLocation Brain SmallDeformingTumor BrainTumor □ ∃hasBehavior . Infiltrating □ ∃hasEnhancement NonEnhanced SubCorticalSmallDeformingTumor SmallDeformingTumor □ ∃hasLocation .SubCorticalCerebralHemisphere □ ∃closeTo.GreyNuclei PeripheralSmallDeformingTumor BrainTumor □ ∃hasLocation . PeripheralCerebralHemisphere  $\sqcap \exists farFrom.LateralVentricle$ 

### Initial ABox $A_1$

 $\{t_1: BrainTumor; e_1: NonEnhanced; l_1: LateralVentricle; p_1: PeripheralCerebralHemisphere; (t_1, e_1): hasEnhancement; (t_1, l_1): farFrom; (t_1, p_1): hasLocation; ... \}.$ 

# Interpretation as a concept abduction process

 $\mathcal{K} \models \gamma \sqsubseteq O$ , with O, main specific concept of  $t_1$ , defined as

```
BrainTumor \sqcap \exists hasEnhancement.NonEnhanced \sqcap \\ \exists farFrom.LateralVentricle \sqcap \\ \exists hasLocation.PeripheralCerebralHemisphere
```

A set of possible explanations is:

{DiseasedBrain, SmallDeformingTumoralBrain, PeripheralSmallDeformingTumoralBrain}

The preferred solution according to minimality constraints is:

 $\gamma \equiv PeripheralSmallDeformingTumoralBrain$ 

# Abduction and logics

### Description logics

Where are we?

- Only a few works
- Rewriting approach (Modal logics Description Logics)

Propositional logics (morpho-logics, Bloch et al. [Bloch 02])

$$\llbracket \varepsilon(\varphi) \rrbracket := \varepsilon(\llbracket \varphi \rrbracket), \llbracket \delta(\varphi) \rrbracket := \delta(\llbracket \varphi \rrbracket)$$

Successive erosions of the set of models

- Erosion of the conjunction of the theory with the formula to be explained
- Erosion of the theory while maintaining the coherence with the formula to be explained

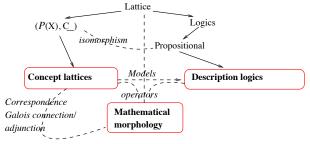




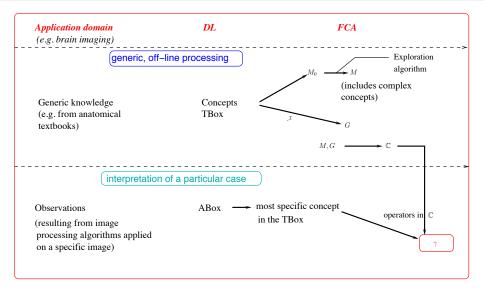
# Proposed approach

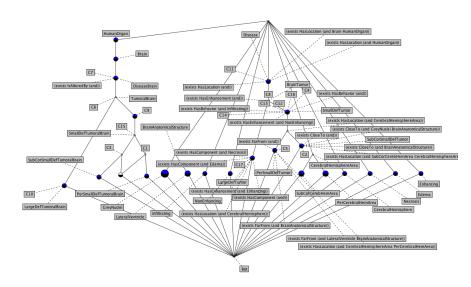
## Enrichment of description logics with abductive reasoning services

⇒ Association between three theories :

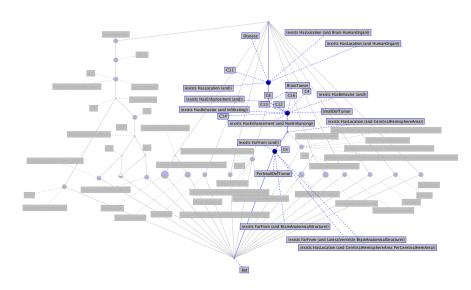


## Global scheme





### Concept lattice induced from $\mathbb{K}_{brain}$ .



Erosion path leading to compute a preferred explanation

## Outline

- What is an ontology ?
- 2 Ontologies for image understanding: overview
- 3 Description Logics
- 4 Description Logics for image understanding
- Conclusion

## Conclusion

## Ontologies and logic-based approaches for image interpretation

- A growing interest in the litterature.
- Main advantages: explicit knowledge encoding for reuse and reasoning processes.
- Need for more convergence between computer vision, machine learning and logics community.

Coming next: graphs, grammars and constraint satisfaction problems.

# Thanks for your attention





Lourdes Agapito, Michael M. Bronstein & Carsten Rother, editeurs. Computer vision - ECCV 2014 workshops - zurich, switzerland, september 6-7 and 12, 2014, proceedings, part II, volume 8926 of *Lecture Notes in Computer Science*. Springer, 2015.



J. Atif, C. Hudelot & I. Bloch.

Explanatory reasoning for image understanding using formal concept analysis and description logics.

IEEE Transactions on Systems, Man and Cybernetics: Systems, vol. 44, no. 5, pages 552–570, May 2014.



F. Baader & B. Hollunder.

Embedding Defaults into Terminological Knowledge Representation Formalisms.

In B. Nebel, C. Rich & W. Swartout, editeurs, Principles of Knowledge Representation and Reasoning: Proc. of the Third International Conference (KR'92), pages 306–317. Kaufmann, San Mateo, CA, 1992.



Andrew D. Bagdanov, Marco Bertini, Alberto Del Bimbo, Giuseppe Serra & Carlo Torniai.

Semantic annotation and retrieval of video events using multimedia ontologies.

In International Conference on Semantic Computing (ICSC'07), pages 713 –720, 2007.

Hichem Bannour & Céline Hudelot.

Building and using fuzzy multimedia ontologies for semantic image annotation.

Multimedia Tools Appl., vol. 72, no. 3, pages 2107–2141, 2014.

Evgeniy Bart, Ian Porteous, Pietro Perona & Max Welling. *Unsupervised learning of visual taxonomies*.

In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR'08), 2008.

Serge J. Belongie & Pietro Perona. Visipedia circa 2015. Pattern Recognition Letters, vol. 72, pages 15–24, 2016.

Isabelle Bloch & Jérôme Lang. Towards mathematical morpho-logics, pages 367–380. Physica-Verlag HD, Heidelberg, 2002.

Stamatia Dasiopoulou, Ioannis Kompatsiaris & Michael Strintzis. *Applying Fuzzy DLs in the Extraction of Image Semantics*.

In Stefano Spaccapietra & Lois Delcambre, editeurs, Journal on Data Semantics XIV, volume 5880 of *Lecture Notes in Computer Science*, pages 105–132. Springer Berlin / Heidelberg, 2009.



Trends and Issues in Description Logics Frameworks for Image Interpretation. Artificial Intelligence: Theories, Models and Applications, vol. 6040, pages 61–70, 2010.

Stamatia Dasiopoulou, Vassilis Tzouvaras, Ioannis Kompatsiaris & Michael G. Strintzis.

Enquiring MPEG-7 based multimedia ontologies.

Multimedia Tools and Applications (MTAP'10), vol. 46, pages 331–370, 2010.

Jia Deng, Alexander C. Berg, Kai Li & Li Fei-Fei.

What does classifying more than 10,000 image categories tell us?

In Proceedings of the European Conference on Computer Vision (ECCV'10), 2010.

Ivan Donadello & Luciano Serafini.

Mixing Low-Level and Semantic Features for Image Interpretation - A
Framework and a Simple Case Study.

In Agapito et al. [Agapito 15], pages 283–298.



Corinna Elsenbroich, Oliver Kutz & Ulrike Sattler.

A Case for Abductive Reasoning over Ontologies.

In Bernardo Cuenca Grau, Pascal Hitzler, Conor Shankey & Evan Wallace, editeurs, Proceedings of the OWLED\*06 Workshop on OWL: Experiences and Directions, Athens, Georgia, USA, November 10-11, 2006, volume 216 of CEUR Workshop Proceedings. CEUR-WS.org, 2006.



L. Fei-Fei & P. Perona.

A Bayesian hierarchical model for learning natural scene categories. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR'2005), volume 2, pages 524 – 531, 2005.



Tianshi Gao & Daphne Koller.

Discriminative learning of relaxed hierarchy for large-scale visual recognition. In Proceedings of the International Conference on Computer Vision (ICCV'11), 2011.



Gregory Griffin & Pietro Perona.

Learning and Using Taxonomies for Fast Visual Categorization.

In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR'08), 2008.



Toward principles for the design of ontologies used for knowledge sharing. International Journal of Human-Computer Studies, vol. 43, no. 5-6, pages 907–928, 1995.

Nicola Guarino, Daniel Oberle & Steffen Staab. What Is an Ontology?

In Steffen Staab & Rudi Studer, editeurs, Handbook on Ontologies, International Handbooks on Information Systems, pages 1–17. Springer, 2009.

Amirhossein Habibian, Koen E.A. van de Sande & Cees G.M. Snoek. Recommendations for Video Event Recognition Using Concept Vocabularies. In Proceedings of the 3rd ACM Conference on International Conference on Multimedia Retrieval, ICMR '13, pages 89–96, New York, NY, USA, 2013. ACM.

C. Hudelot, J. Atif & I. Bloch.

*Fuzzy Spatial Relation Ontology for Image Interpretation.* Fuzzy Sets and Systems, vol. 159, no. 15, pages 1929–1951, 2008.

- Céline Hudelot, Jamal Atif & Isabelle Bloch.
  - $ALC(\mathbb{F})$ : A New Description Logic for Spatial Reasoning in Images. In Agapito et al. [Agapito 15], pages 370–384.
- Li-Jia Li, Chong Wang, Yongwhan Lim, David M. Blei & Fei-Fei Li. Building and using a semantivisual image hierarchy.

  In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR'10), pages 3336 –3343, 2010.
- Ying Liu, Dengsheng Zhang, Guojun Lu & Wei-Ying Ma. A survey of content-based image retrieval with high-level semantics. Pattern Recognition, vol. 40, no. 1, pages 262 282, 2007.
- Nicolas Maillot & Monique Thonnat.

  Ontology based complex object recognition.

  Image Vision Comput., vol. 26, no. 1, pages 102–113, 2008.
  - Marcin Marszalek & Cordelia Schmid.

    Constructing Category Hierarchies for Visual Recognition.

    In Proceedings of the European Conference on Computer Vision (ECCV), pages 479–491, 2008.



Vasileios Mezaris, Ioannis Kompatsiaris & Michael G. Strintzis. Region-Based Image Retrieval Using an Object Ontology and Relevance Feedback.

EURASIP J. Adv. Sig. Proc., vol. 2004, no. 6, pages 886-901, 2004.



Ralf Möller & Bernd Neumann.

Ontology-based reasoning techniques for multimedia interpretation and retrieval, pages 55–98.

Springer London, London, 2008.



Ralf Möller, Bernd Neumann & Michael Wessel.

Towards Computer Vision with Description Logics: Some Recent Progress. Integration of Speech and Image Understanding, ICCV Workshop on, vol. 0, page 101, 1999.



Ralf Möller, Bernd Neumann & Michael Wessel.

*Towards Computer Vision with Description Logics: Some Recent Progress.* Integration of Speech and Image Understanding, ICCV Workshop on, vol. 0, page 101, 1999.



Ralf Möller & Bernd Neumann.

Ontology-Based Reasoning Techniques for Multimedia Interpretation and Retrieval.

In Yiannis Kompatsiaris & Paola Hobson, editeurs, Semantic Multimedia and Ontologies, pages 55–98. Springer London, 2008.

Milind Naphade, John R. Smith, Jelena Tesic, Shih-Fu Chang, Winston Hsu, Lyndon Kennedy, Alexander Hauptmann & Jon Curtis. Large-Scale Concept Ontology for Multimedia. IEEE MultiMedia, vol. 13, pages 86–91, July 2006.

Bernd Neumann & Ralf Möller.

On scene interpretation with description logics.

Image Vision Computing, vol. 26, no. 1, pages 82–101, 2008.

S. Espinosa Peraldi, A. Kaya, S. Melzer, R. Möller & M. Wessel. *Multimedia Interpretation as Abduction*. In International Workshop on Description Logics (DL'07), 2007.

Irma Sofía Espinosa Peraldí, Atila Kaya & Ralf Möller. Formalizing Multimedia Interpretation based on Abduction over Description Logic Aboxes. In Bernardo Cuenca Grau, Ian Horrocks, Boris Motik & Ulrike Sattler, editeurs, Proceedings of the 22nd International Workshop on Description Logics (DL 2009), Oxford, UK, July 27-30, 2009, volume 477 of CEUR Workshop Proceedings. CEUR-WS.org, 2009.



Proceedings of the Workshop on Neural-Cognitive Integration in German Conference on Artificial Intelligence, pages 1–15, 2013.

Raymond Reiter.

A Logic for Default Reasoning.

Artif. Intell., vol. 13, no. 1-2, pages 81-132, 1980.

Olga Russakovsky, Jia Deng, Hao Su, Jonathan Krause, Sanjeev Satheesh, Sean Ma, Zhiheng Huang, Andrej Karpathy, Aditya Khosla, Michael S. Bernstein, Alexander C. Berg & Fei-Fei Li.

ImageNet Large Scale Visual Recognition Challenge.

International Journal of Computer Vision, vol. 115, no. 3, pages 211–252, 2015.



Bryan C. Russell, Antonio Torralba, Kevin P. Murphy & William T. Freeman.

*LabelMe: A Database and Web-Based Tool for Image Annotation.* International Journal of Computer Vision, vol. 77, no. 1-3, pages 157–173, 2008.



N. Simou, Th Athanasiadis, G. Stoilos & S. Kollias. Image indexing and retrieval using expressive fuzzy description logics. Signal, Image and Video Processing, vol. 2, no. 4, pages 321–335, 2008.



J. Sivic, B. C. Russell, A. Zisserman, W. T. Freeman & A. A. Efros. *Unsupervised Discovery of Visual Object Class Hierarchies.* In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR'08), 2008.



Rudi Studer, V. Richard Benjamins & Dieter Fensel. *Knowledge Engineering: Principles and Methods.* Data Knowl. Eng., vol. 25, no. 1-2, pages 161–197, 1998.



Christopher Town.

Ontological inference for image and video analysis.

Machine Vision and Applications, vol. 17, no. 2, pages 94–115, 2006.



Michael Uschold & Michael Gruninger.

Ontologies and Semantics for Seamless Connectivity.

SIGMOD Rec., vol. 33, no. 4, pages 58–64, December 2004.



Lei Wu, Xian-Sheng Hua, Nenghai Yu, Wei-Ying Ma & Shipeng Li. *Flickr Distance: A Relationship Measure for Visual Concepts*. Pattern Analysis and Machine Intelligence, IEEE Transactions on, vol. 34, no. 5, pages 863 –875, 2012.