PATCH-BASED NONLOCAL DENOISING FOR MRI AND ULTRASOUND IMAGES

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ABSTRACT

Image model plays a critical role in recovering diagnosis-relevant information from noisy observation data. Unlike conventional denoising techniques based on local models, a patch-based *nonlocal* image model is presented and its applications into restoring medical images are demonstrated. We introduce geometric resampling techniques for obtaining redundancy representations which facilitate the exploitation of nonlinear manifold constraint in the patch space. We extend existing locally linear embedding (LLE) into locally linear transform (LLT) to impose sparsity constraint on the uncorrupted images. A nonlocal denoising algorithm based LLT thresholding and adaptive fusion is proposed for removing Rician noise from MRI data and speckle noise from ultrasound images. Encouraging experimental results are achieved, which confirms the value of nonlocal processing as a supplementary tool.

1. INTRODUCTION

Mathematical modeling of image signals is a fundamental task in medical image processing. Although the content of medical images are constrained to human organs, there is still a great deal of uncertainty due to the inherent variability (e.g., soft tissue vs. hard bones). The modeling task is further complicated by the high dimensionality of image data especially when high-resolution imaging is desirable. Moreover, noise contamination is often inevitable due to the physical constraints of medical imaging. Sometimes distinguishing noise from signal is tricky - e.g., speckle noise in ultrasound images has been long thought of tissue microstructure carrying useful information to diagnosis [1].

To overcome the curse of dimensionality, locality assumption has been widely used during the modeling of image signals. In statistical approaches [2], [3] [4], local statistics such as mean, median or variance are essential tools for adapting the filtering tasks. In PDE-based approaches [5], diffusion process is often driven by local gradients calculated from the image intensity values. In wavelet-based approaches [6], [7], linear basis with good space-frequency localization property is often used and local statistical models are developed for wavelet coefficients. Localized models have found effective on removing a wide range of noise for various imaging modalities.

Despite the popularity of localized models, nonlocal dependency is also important to our understanding of images especially under the context of medical imaging. Nonlocal dependency arises from geometric symmetry of natural objects (e.g., self-similarity of repeating patterns, rotation invariance of circular shapes). Nonlocal dependency is particularly useful for medical images because human subjects often approximately observe specific geometric constraints (e.g., bilateral symmetry of human body). Existing local models can not exploit such nonlocal dependency which could be useful to suppress noise components especially in the presence of heavy noise. The main objective of this work is to demonstrate the potential of nonlocal models in restoring medical images.

In this work, we present a patch-based nonlocal denoising scheme based on geometric resampling techniques. The redundancy introduced by geometric resampling improves the sampling density of local neighborhood of a patch, which facilitates the discovery of local geometry on a manifold in the high-dimensional space. Conceptually similar to locally linear embedding (LLE) [8], we propose to exploit the nonlinear manifold constraint by locally linear transforms (LLT). Unlike existing 2D transforms, LLT is a 3D transform whose third dimension codes the geometric resampling information. Note that the locality concept in LLT is NOT the same as that used in conventional local models and LLT can be viewed as a "bilateral" transform defined on both geometric domain and photometric range [9]. It can be shown that such bilateral property of LLT is particularly useful for exploiting geometric symmetry related constraints which are beyond the reach of local models. We consider two applications of nonlocal model here: removal of Rician noise from MRI images and removal of speckle noise from ultrasound images.

2. NONLOCAL IMAGE REPRESENTATION IN THE PATCH SPACE

Let $\mathcal{I}$ denote the collection of all images with a specified size $H \times W$. It has been widely recognized that $\mathcal{I}$ does not span over the full space $R^{H\times W}$ but forms a low-dimensional manifold. To overcome the curse of dimensionality, one might consider a patch-based representation which decomposes an image into overlapping patches (refer to Fig. 1a) [10], [11]. For simplicity, we only consider patches of square shape (say $P \times P, P = 2T + 1$) in this paper. Due to the overlap among patches, the total number of patches is in the same order as that of pixels (boundary extension is required). Such patch-based representation can be viewed as the projection of $\mathcal{I}$ onto a lower-dimensional subspace $R^{P\times P}$ in which nonlinear manifold constraint is preserved.

In the seminal work of local linear embedding (LLE) [8], the local geometry of a manifold is characterized by linear coefficients that reconstruct each data point from its neighbors in the patch space, i.e.,

$$\mathbf{x} = \sum_{i=1}^{N} w_i \mathbf{x}_i, \tag{1}$$

where $\mathbf{x}$ is the data point (patch of interest) and $\{\mathbf{x}_i\}_{i=1}^{N}$ denotes its local neighborhood. Note that the *locality* in the patch space is NOT equivalent to that in the spatial domain where images are acquired. Two neighboring points in $R^{P \times P}$ would have small Euclidean distance in photometric range (i.e., visually similar) but large Euclidean distance in the geometric domain (i.e., spatially distant). The idea of LLE is closely related to the celebrated bilateral filtering [9] where the weighting coefficients are jointly determined by the distances of domain and range.

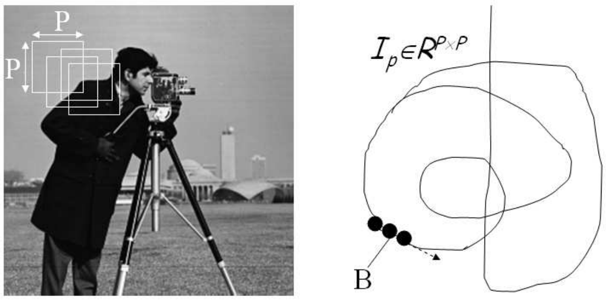

Fig. 1. a) Patch-based representation of image signals; b) nonlinear manifold constraint still holds in the patch space $\mathcal{I}_p$.

The motivations behind our approach are summarized into the following two aspects. On one hand, we propose to generalize the idea of linear prediction in Eq. (1) into linear transform. Such generalization is spiritually similar to the migration from predictive models to transform models. However, since we are considering patches instead of pixels here, the generalized transforms in the patch space are 3D where the third dimension reflects the nonlocal nature - i.e., we are packing patches far away from each other into a 3D array. To maximize the sparsity, it is natural to order the patches monotonically based on the photometric distance. We call such 3D transform applied to packed patches "locally linear transform" (LLT). Similar to LLE, our LLT is capable of exploiting nonlocal dependency in image signals.

On the other hand, we propose to improve the sampling density of local neighborhood in the patch space by geometric resampling. Specifically, geometrically resampled version of a given image X is defined by $Y = \mathcal{T}[X]$ where $\mathcal{T}$ is the concatenation of various geometric operators (e.g., translation, reflection, transpose, rotation and scaling). For the convenience of implementation, we only consider lossless operators translation, reflection and transpose in this work. It is easy to see that geometric resampling generates a redundant representation of image signal (refer to Fig. 2). As the redundancy ratio increases, the manifold in the patch space is better-sampled.

Putting things together, we argue that LLT powered by geometric resampling is capable of better exploiting the nonlinear manifold constraint or nonlocal dependency than the existing 2D linear transforms. Since geometric information of $\mathbf{x}_i$ (i.e., how it is related to patch $\mathbf{x}$) is implicitly coded into the third dimension of LLT, improved sparsity can be achieved. In other words, despite the linearity of 3D LLT, its derived image representation is nonlinear due to the embedded geometric information. Moreover, the redundancy brought by geometric resampling is beneficial to the denoising task because it allows us to pool together statistical inference results of the same pixel from multiple resampled signals (a generalization of translation invariant denoising [12]).

Fig. 2. Illustration of geometric resampling (redundancy ratio of 4 is due to reflections along the x and y directions).

3. PATCH-BASED NONLOCAL DENOISING

Image denoising refers to the task of removing unwanted noise from a given observation image. The priors of both image and noise signals are important to the effectiveness of denoising algorithm. In biomedical imaging, noise modeling is a challenging task itself. However, if nonlocal image representation developed in the previous section offers a powerful prior model for medical images such as MRI and ultrasound, we conjecture that image denoising can still be solved even with little assumption about the noise. In other words, we attempt to show that nonlocal image models lead to a fairly general class of $blind$ denoising algorithms. Our patch-based nonlocal denoising algorithm consists of two components: 1) how to denoise a single patch? 2) how to pool together the denoised result of a single pixel contained in multiple patches?

For the first question, we observe that sparseness of transform-domain representations has been long recognized useful for denoising applications. In the classical wavelet shrinkage [13], simple nonlinear thresholding operation is shown effective on separating image structures from additive white Gaussian noise (AWGN). The power of nonlinear thresholding lies in that it works as long as noise signals do not produce significant coefficients in the transform space. Therefore, although the noise model in practical imaging is often more complex (e.g., non-Gaussian or signal-dependent), wavelet filtering has still been successfully used in denoising MRI images [6] and despeckling ultrasound images [7]. Similar to wavelet thresholding, we can envision a LLT-based thresholding counterpart, which can be viewed as an extension of existing patch-based denoising algorithm [10]. Due to the blindness assumption made about the noise, we opt to empirically adjust the threshold.

An ad-hoc solution to the second question is simple (uniform) averaging. However, uniform weights are suboptimal because both the location of a pixel within a patch (at the center or at the border) and the sparsity of 3D array affect the confidence of statistical inference result from the given patch. Loosely speaking, the assigned weights in fusing multiple patches containing the same pixel should be inversely proportional to the distance of that pixel to the patch center and the sparsity of 3D array. In our current implementation, a bell-shape window (e.g., Gaussian or Kaiser) similar to bilateral filtering [9] is used to reflect the distance consideration and the total number of nonzero LLT coefficients after thresholding similar to [11] is adopted as the sparsity index. Another tricky issue related to the implementation is that geometric resampling information of each patch $\mathbf{B}_k$ (i.e., how it is related to $\mathbf{B}$ by translation, transpose and reflection) has to be recorded carefully because every denoised patch after LLT thresholding should be placed back at the right position and orientation before weighted fusion.

The proposed patch-based nonlocal denoising algorithm is summarized as follows.

Patch-based Nonlocal Image Denoising

Input: noisy image $Y(i,j)$, $P=2T+1$

Output: denoised image $\hat{X}(i,j)$

• Apply geometric resampling to Y to obtain redundant representation $\{\mathcal{T}_1[Y],...,\mathcal{T}_R[Y]\}$ and collect all patches into $\mathcal{B}$;

• At the chosen locations (e.g., $(i,j)=(4k+1,4l+1)$):

- formulate the patch $\mathbf{B}=Y(i-T,i+T,j-T,j+T)$ and search its N nearest neighbors $\mathbf{B}_i$ in $\mathcal{B}$;

- order $Q=N+1$ patches in the order of monotonically increasing distance and pack them into a noisy 3D array $\mathbf{Y}$;

- denoise $\mathbf{Y}$ by LLT-based thresholding to obtain denoised version $\hat{\mathbf{Y}}$;

• Unpack the 3D array $\hat{\mathbf{Y}}$ to obtain denoised redundant representation $\{\mathcal{T}_1[\hat{Y}],...,\mathcal{T}_R[\hat{Y}]\}$ and fuse them to obtain the denoised image $\hat{X}$.

4. EXPERIMENTAL RESULTS

In our current implementation, geometric transform $\mathcal{T}$ contains the concatenation of translation, transpose and reflections (both left-right and up-down); LLT is chosen to be 3D fast Fourier transform (similar to [11] which is developed for removing AWGN). The thresholding parameter of LLT coefficients is empirically tuned for each individual image and noise characteristics. The proposed nonlocal denoising algorithm is tested for two types of noise: Rician noise in MRI and speckle noise in ultrasound. Our objective is to show that the proposed nonlocal technique offers a unified solution to handle different types of noise.

A. Rician Noise Removal from MRI images

In our first experiment, we compare the denoising performance between local (PDE-based) and nonlocal algorithms on MRI image. The Rician noise insertion is implemented by taking the FFT of original, adding complex Gaussian white noise and then computing the magnitude of inverse FFT [6]. To simulate low SNR MRI, a relatively large variance of 30 is chosen. Figs. 3 and 4 include the comparison among original, noisy and denoised images by different schemes for simulated and real MRI data. It can be observed that nonlocal algorithm achieves lower MSE and comparable subjective quality to variational denoising [14]. Although PDE-denoised image is arguably sharper (due to contrast enhancing capability), nonlocal denoised image contains less artifact especially around the contour and within the smooth region.

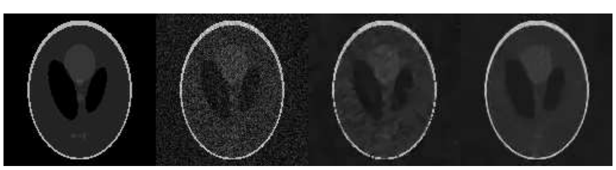

Fig. 3. From left to right: synthetic 128×128 $phantom$ image; noisy image (MSE=739); PDE scheme (MSE=305); nonlocal scheme (MSE=299).

B. Speckle Noise Suppression from Ultrasound Images

Speckles generated by interference of subresolution scatters in ultrasound imaging are more difficult to handle due to their encoded spatial information (note that from such perspective multiplicative speckle noise model is no different from additive one up to homomorphic transformation). In the second experiment, we use the Field-II simulated phantom data provided by the authors of [4] to test our nonlocal algorithm. To objectively evaluate the performance of various despeckling schemes, a metric $\hat{Q}$ called ultrasound despeckling assessment index (USDSAI) was proposed in [4]. A larger $\hat{Q}$ value indicates a more desirable restoration or enhancement result. We have found that our nonlocal denoising algorithm produces similar $\hat{Q}$ performance to SRAD [5] and SBF [4]. However, when we concatenate nonlocal and local schemes for the purpose of combining the strength of global smoothing and local enhancement, superior $\hat{Q}$ performance can be achieved. Fig. 5 includes the comparion among noisy image and despeckled images by different schemes. We have also tested the performance of our nonlocal denoising algorithm on real ultrasound images. The benchmarks used here are two local schemes: geometric filtering [15] and Perona-Malik (PM) filtering [16]. Fig. 6 contains the comparison of ultrasound images before and after different despeckling techniques. Note that thresholding parameter in PT is manually adjusted to achieve a good tradeoff between noise suppression and structure preservation. Subjective evalution suggests that nonlocal despeckling gives better result than geometric filtering and comparable performance to PM filtering.

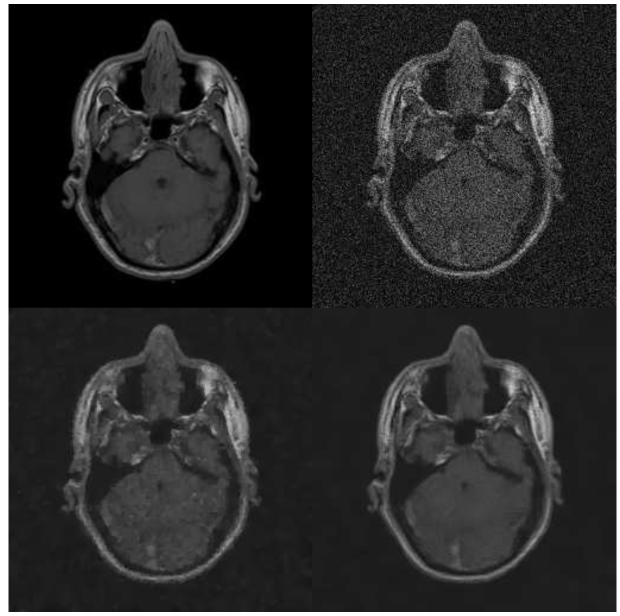

Fig. 4. Top-left: real 256×256 $head\_mri$ image; top-right: noisy image (MSE=724); bottom-left: PDE scheme (MSE=271); bottom-right: nonlocal scheme (MSE=255).

5. CONCLUSION

In this paper, we attempt to demonstrate the potential of patch-based nonlocal image models in restoring medical images. In contrast to existing local models such as PDE and wavelets, nonlocal models are motivated by geometric symmetry of natural objects and we introduce a new class of geometric resampling techniques

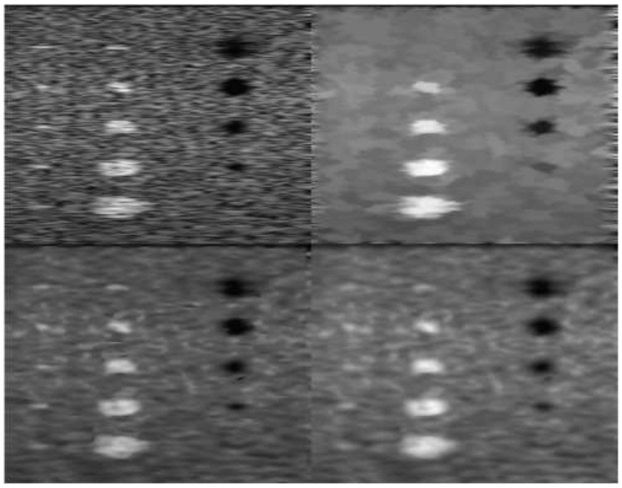

Fig. 5. top-left: Field II simulation; top-right: denoised image by SBF ($\hat{Q} = 2.11$); bottom-left: denoised image by our nonlocal scheme ($\hat{Q} = 2.02$); bottom-right: denoised image by our combined scheme ($\hat{Q} = 2.40$).

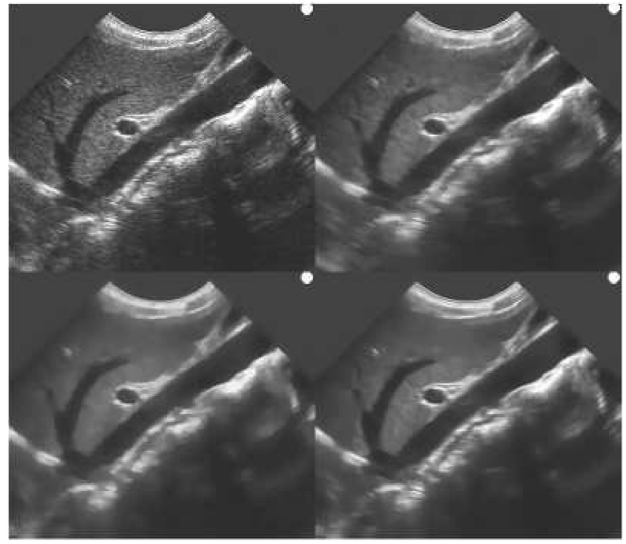

Fig. 6. top-left: original noisy ultrasound images; top-right: despeckled image by geometric filtering [15]; bottom-left: despeckled image by PM filter; bottom-right: despeckled image by our nonlocal technique.

to obtain redundant representations. The introduced redundancy is shown beneficial to exploit the nonlinear manifold constraint in the high-dimensional patch space (because the manifold is better sampled) and improve the statistical reference results by adaptive fusion (because of the diversity from resampling). We extend LLE into LLT whose third dimension encodes the geometric resampling information, which contributes to its capability of exploiting nonlocal dependency. Patch-based nonlocal denoising has achieved highly encouraging results for removing Rician noise from MRI images and reducing speckle noise from ultrasound images. We have achieved better objective and comparable subjective performance to known denoising schemes in the literature. More powerful denoising techniques are expected by expanding the set of lossless resampling operators into lossy and pursuing more sophisticated fusion schemes.

6. REFERENCES

[1] T. L. Szabo, *Diagnostic Ultrasound Imaging*. Elsevier, 2004.

[2] J.-S. Lee, "Digital image enhancement and noise filtering by use of local statistics," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 2, pp. 165–168, Mar. 1980.

[3] T. Loupas, W. N. McDicken, and P. L. Allan, "An adaptive weighted median filter for speckle suppression in medical ultrasonic images.," *IEEE Trans. Cir. Sys.*, vol. 36, no. 1, pp. 129–135, 1989.

[4] P. Tay, S. Acton, and J. Hossack, "A stochastic approach to ultrasound despeckling.," in *ISBI*, pp. 221–224, 2006.

[5] Y. Yu and S. T. Acton, "Speckle reducing anisotropic diffusion.," *IEEE Transactions on Image Processing*, vol. 11, no. 11, pp. 1260–1270, 2002.

[6] R. D. Nowak, "Wavelet-based rician noise removal for magnetic resonance imaging.," *IEEE Transactions on Image Processing*, vol. 8, no. 10, pp. 1408–1419, 1999.

[7] A. Achim, A. Bezerianos, and P. Tsakalides, "Novel bayesian multiscale method for speckle removal in medical ultrasound images.," *IEEE Trans. Med. Imaging*, vol. 20, no. 8, pp. 772–783, 2001.

[8] S. T. Roweis and L. K. Saul, "Nonlinear Dimensionality Reduction by Locally Linear Embedding," *Science*, vol. 290, no. 5500, pp. 2323–2326, 2000.

[9] C. Tomasi and R. Manduchi, "Bilateral filtering for gray and color images," in *ICCV*, pp. 839–846, 1998.

[10] A. Buades, B. Coll, and J.-M. Morel, "A non-local algorithm for image denoising," *CVPR*, vol. 2, pp. 60–65, 2005.

[11] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian, "Image denoising with block-matching and 3D filtering," in *Proc. SPIE Electronic Imaging: Algorithms and Systems V*, vol. 6064, (San Jose, CA, USA), January 2006.

[12] R. Coifman and D. Donoho, "Translation-invariant denoising," in *Wavelet and Statistics, Springer Lecture Notes in Statistics*, vol. 103, (New York), pp. 125–150, Springer-Verlag, 1994.

[13] D. Donoho and I. Johnstone, "Ideal spatial adaptation by wavelet shrinkage," *Biometrika*, vol. 81, pp. 425–455, 1994.

[14] G. Aubert and L. Vese, "A variational method in image recovery," *SIAM Journal on Numerical Analysis*, vol. 34, no. 5, pp. 1948–1979, 1997.

[15] T. R. Crimmins, "Geometric filter for speckle reduction," *Applied Optics*, vol. 24, pp. 1438–1443, May 1985.

[16] P. Perona and J. Malik, "Scale space and edge detection using anisotropic diffusion," *IEEE Trans. Patt. Anal. Mach. Intell.*, vol. 12, no. 7, pp. 629–639, 1990.