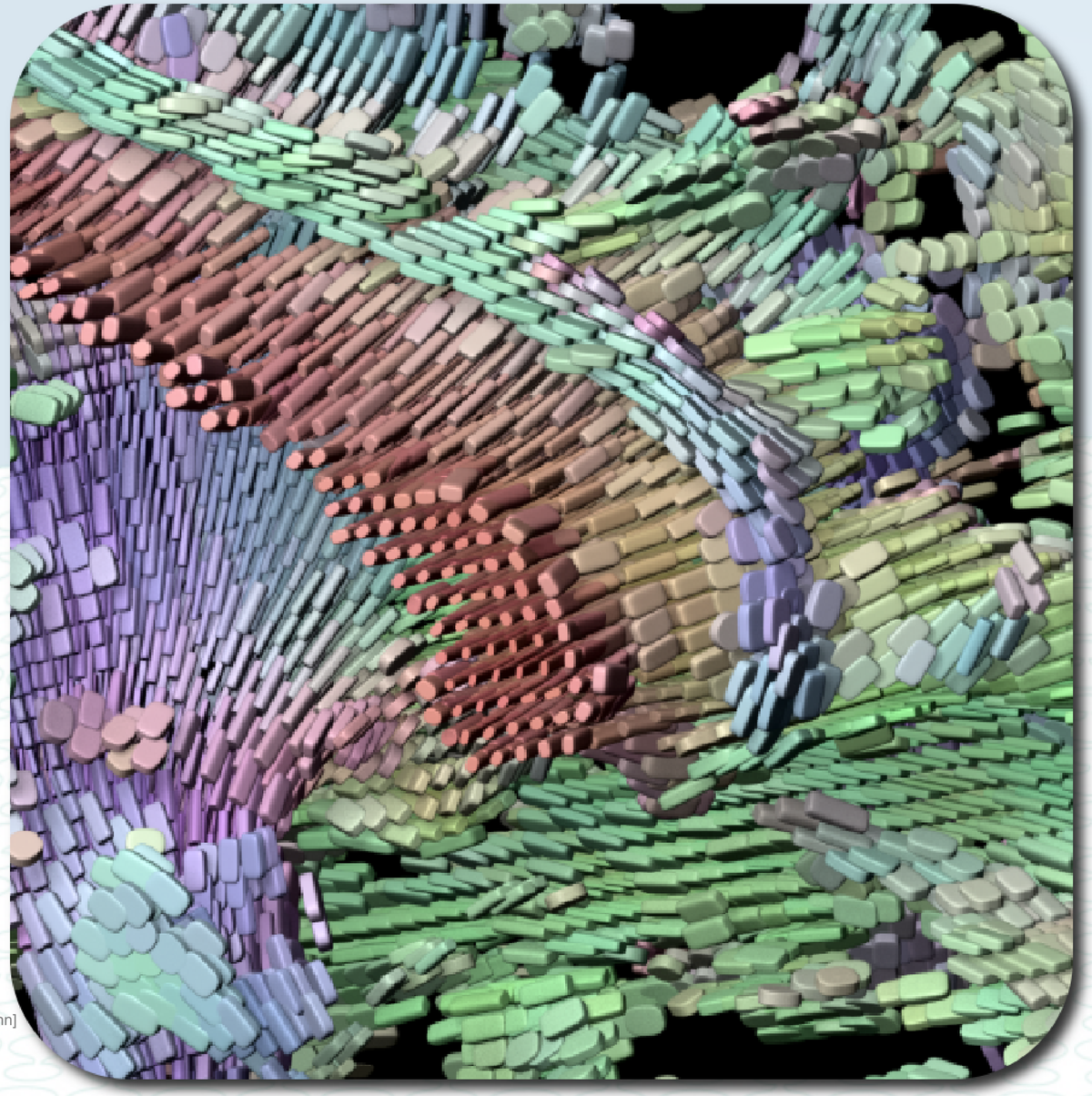


[Kindlmann]

Introduction to Scientific Visualization

Télécom ParisTech

tierny@telecom-paristech.fr

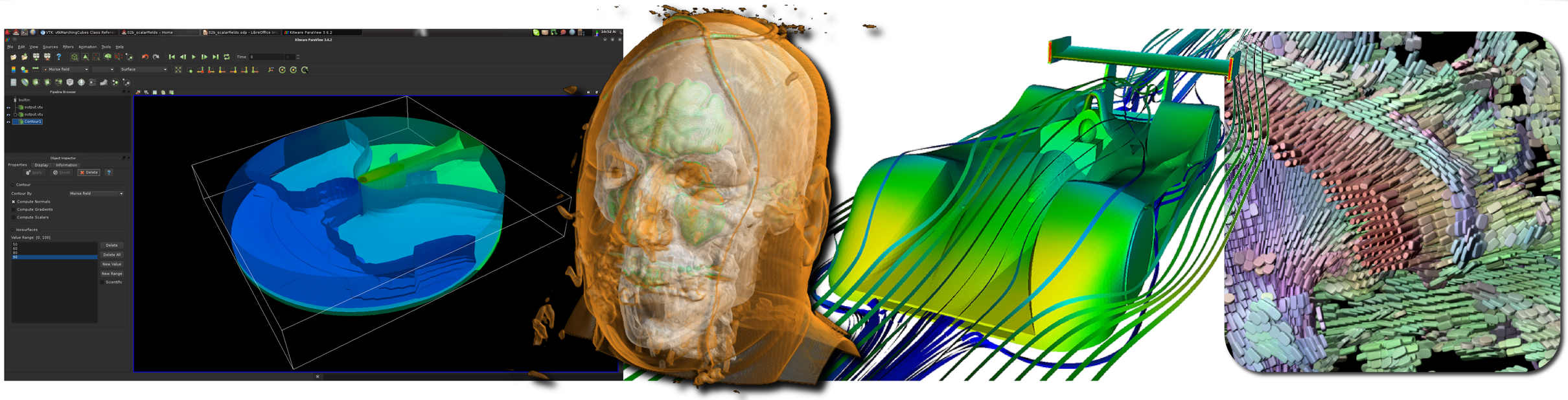
What?

- Generation of intelligible and interactive graphical representations

What?

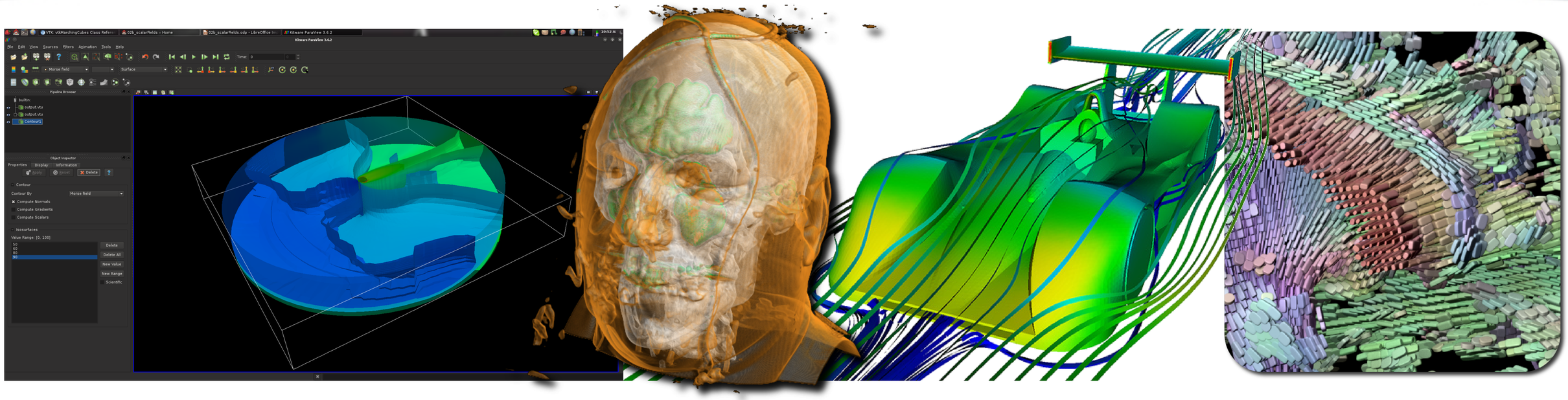
- Generation of intelligible and interactive graphical representations
  - Of scientific data

What?



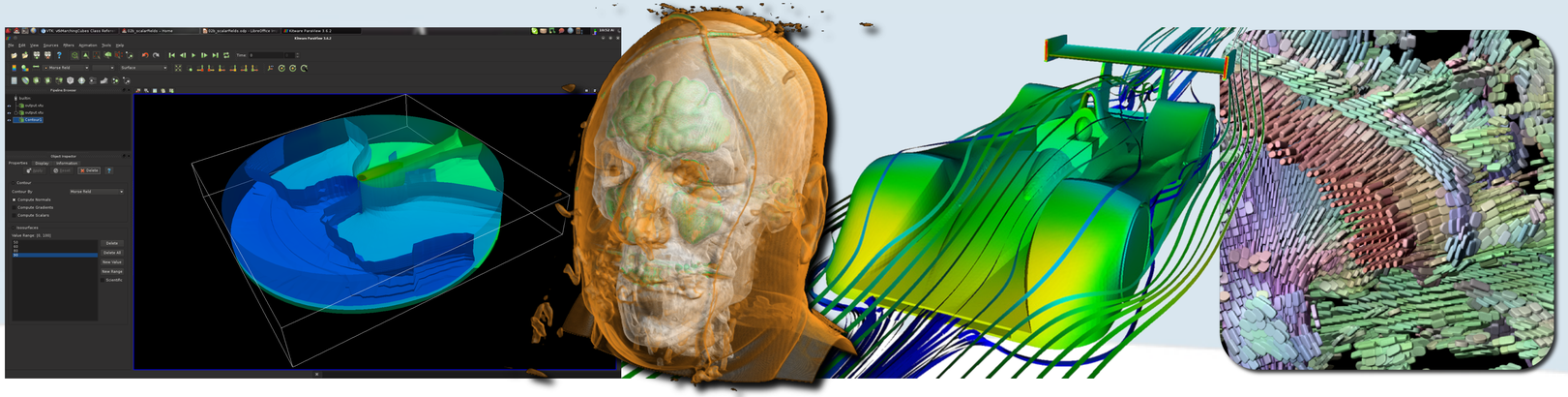
- Generation of intelligible and interactive graphical representations
  - Of scientific data

What?



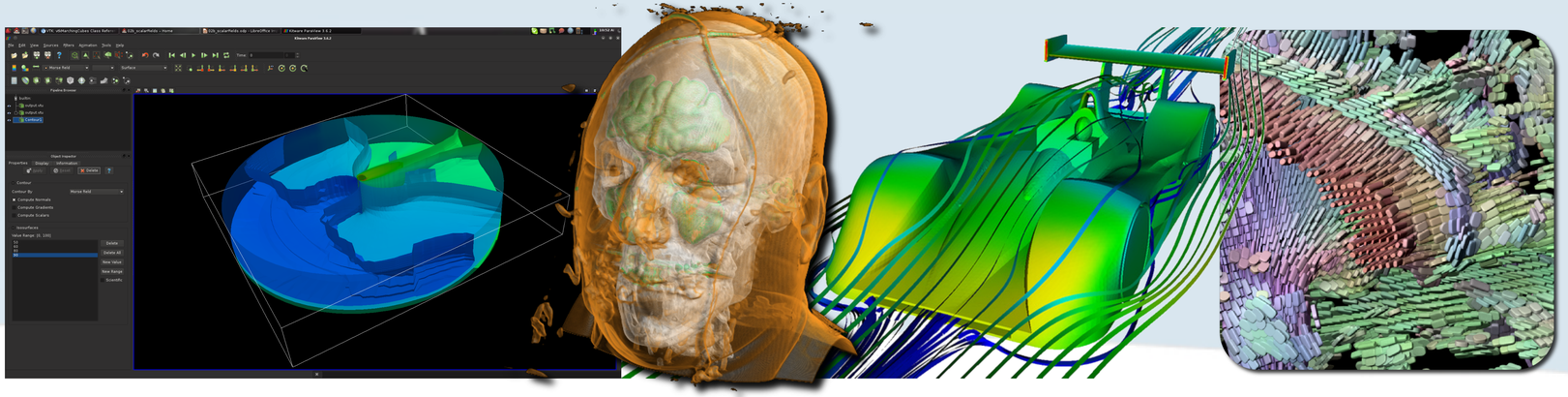
- Generation of intelligible and interactive graphical representations
  - Of scientific data
- Computer graphics for grown-ups

What?



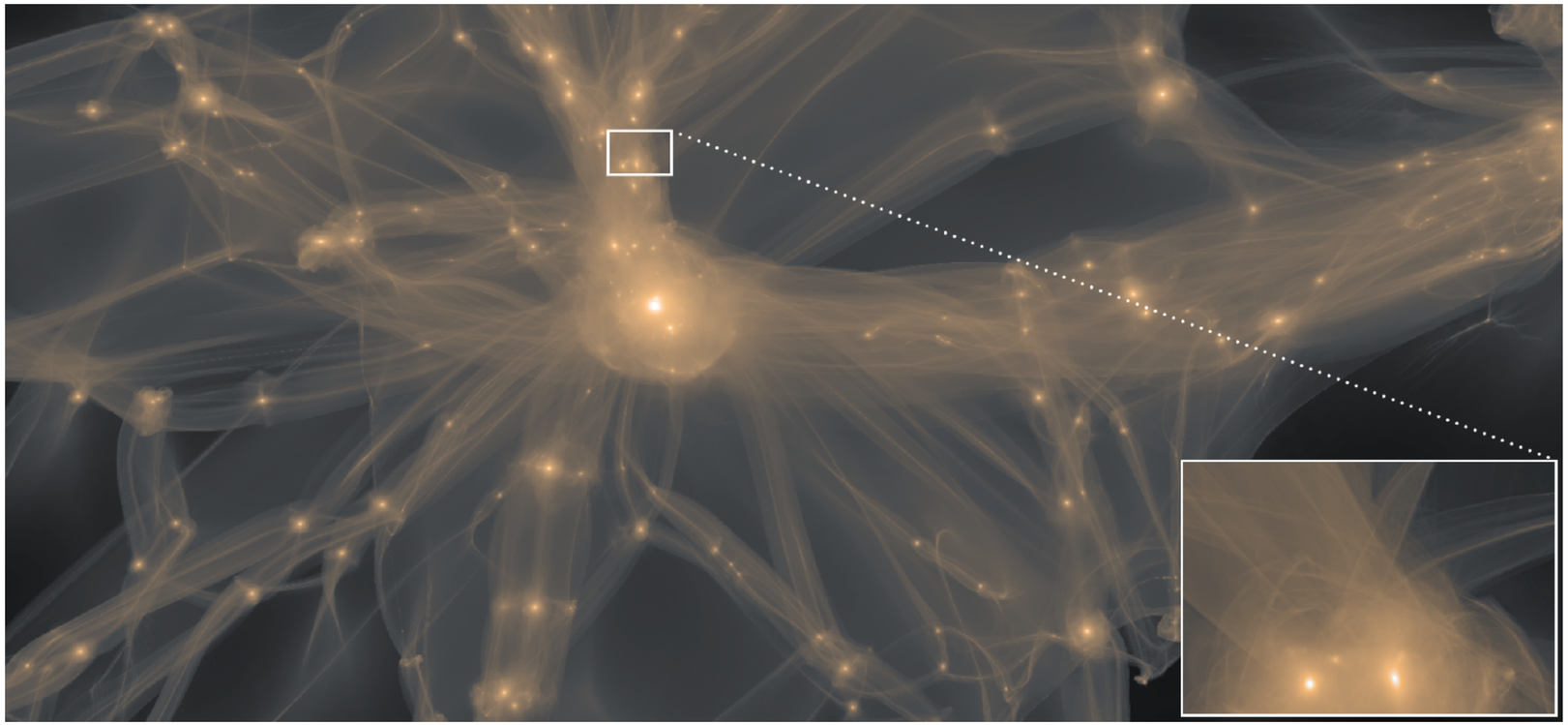
- Scientific data-sets
  - Results of simulations

What?



- Scientific data-sets
  - Results of simulations
    - Chemistry, Physics

[Kaehler]



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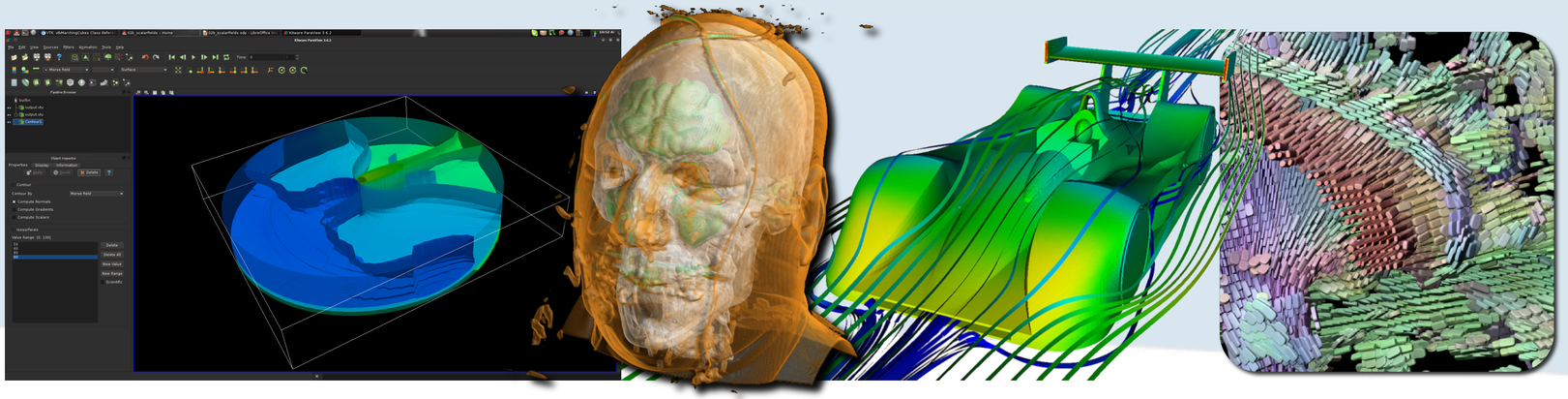
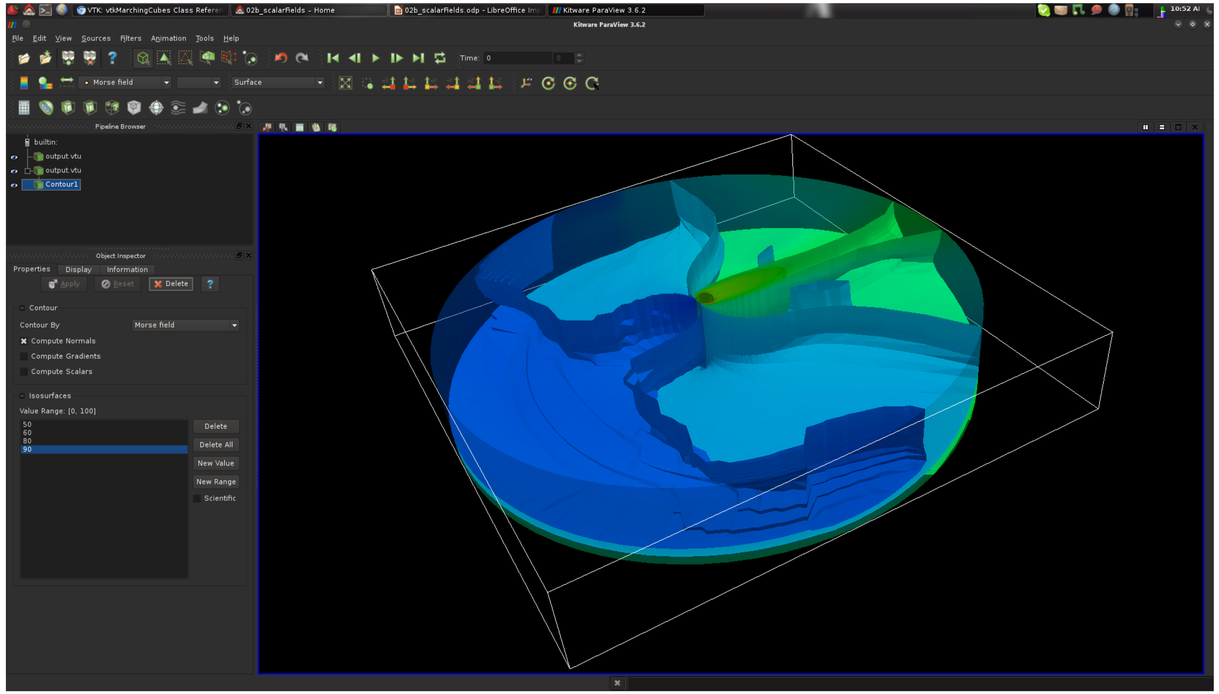


- Scientific data-sets
  - Results of simulations
    - Chemistry, Physics



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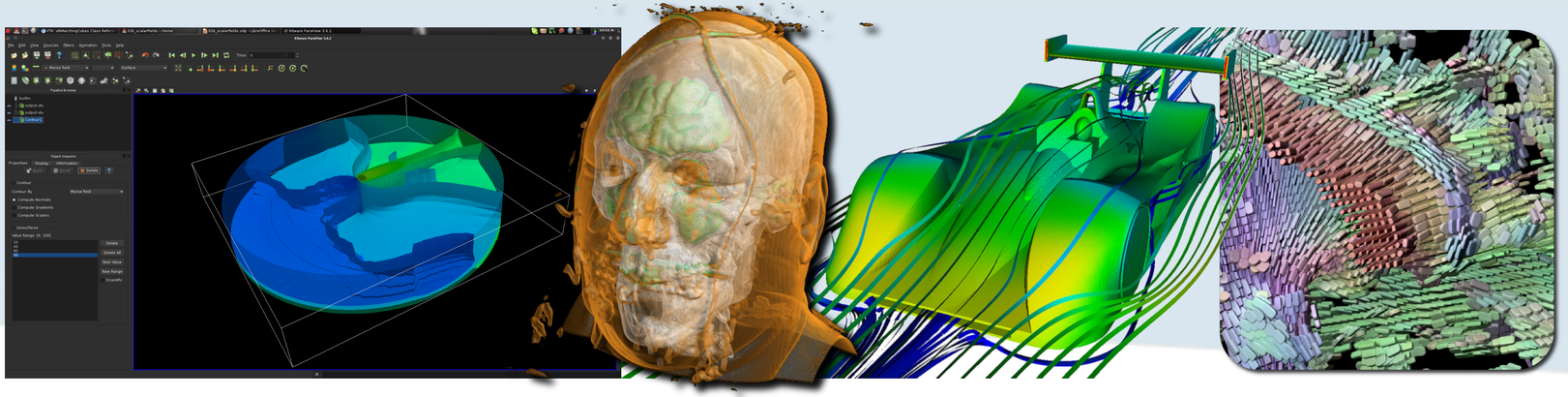
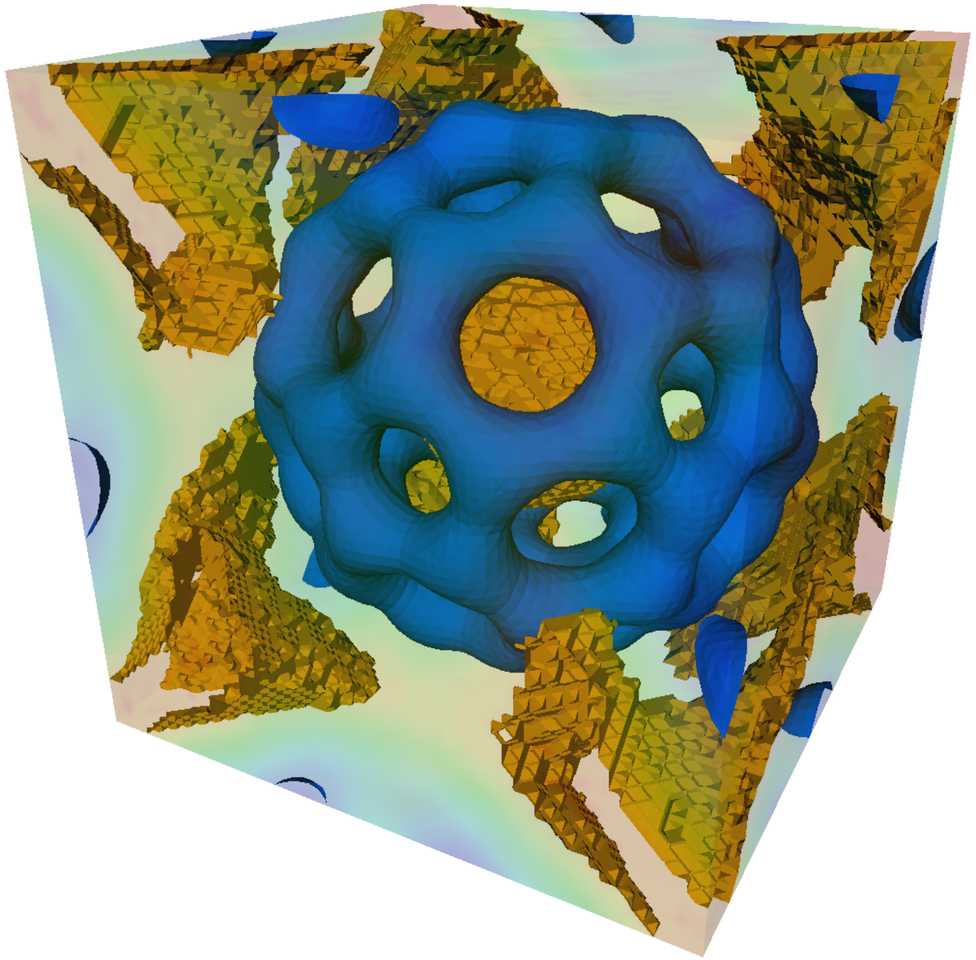


- Scientific data-sets
  - Results of simulations
    - Chemistry, Physics



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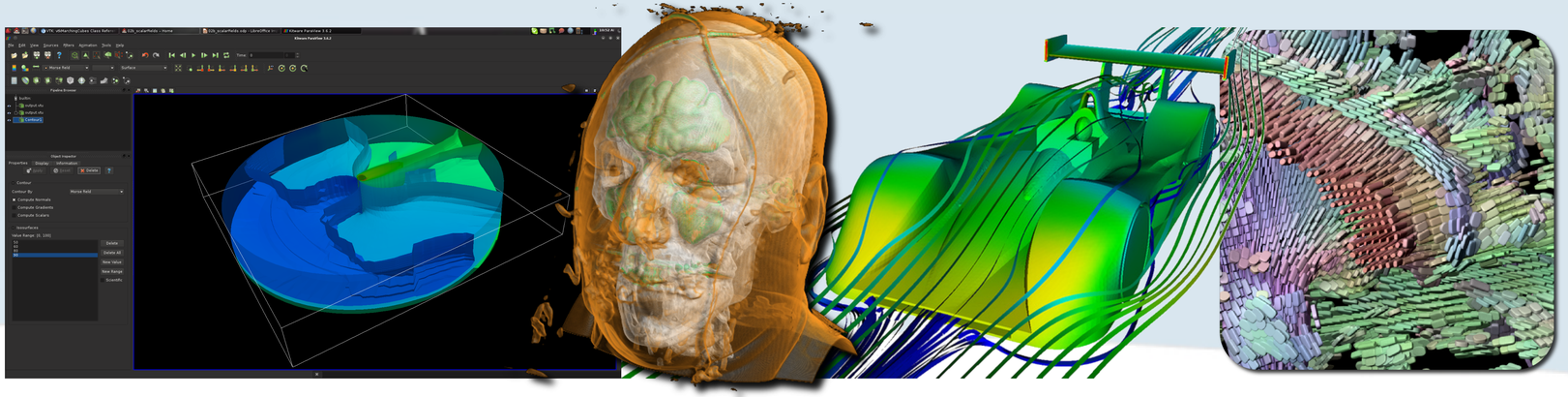
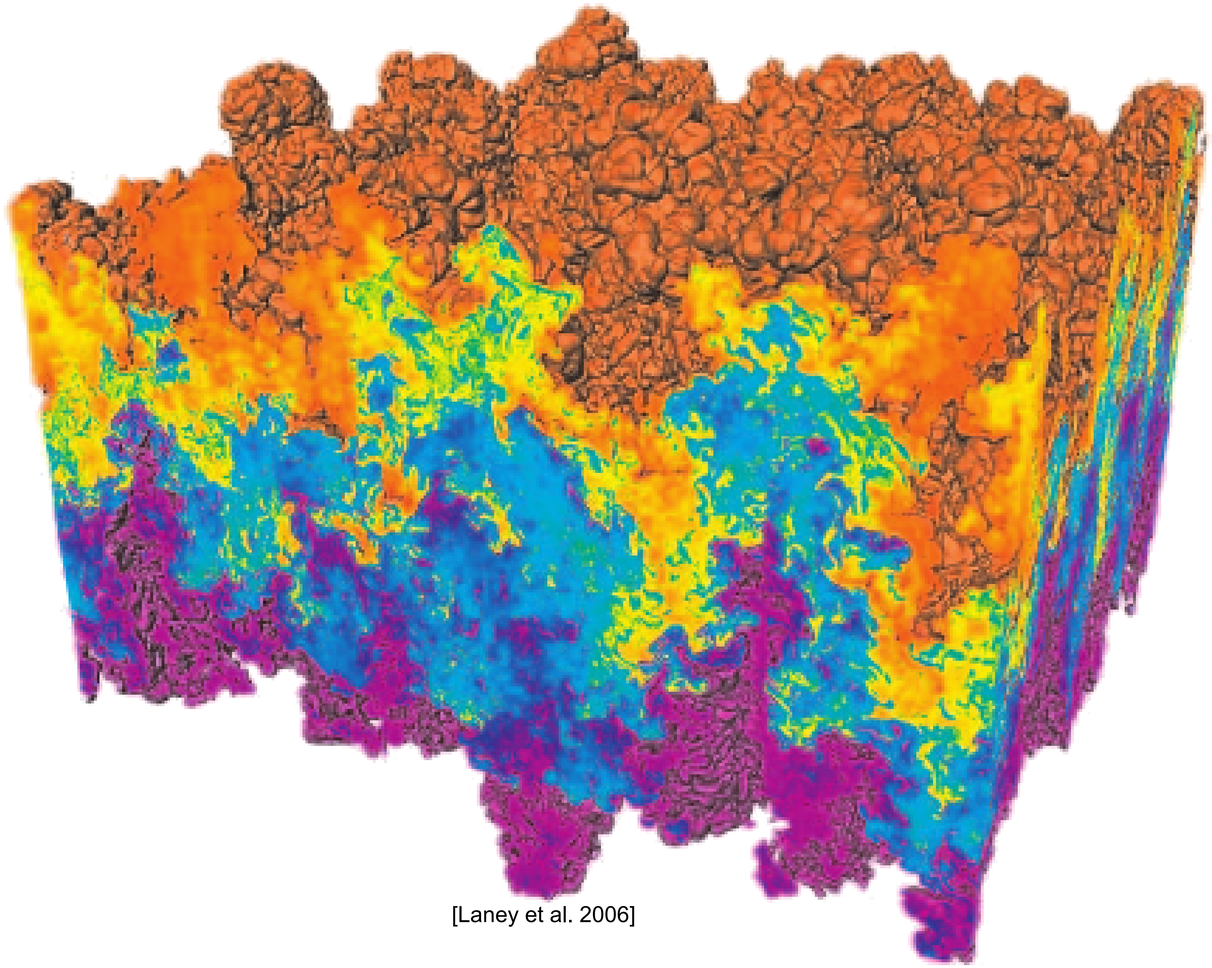


- Scientific data-sets
  - Results of simulations
    - Chemistry, Physics



[Laney et al. 2006]

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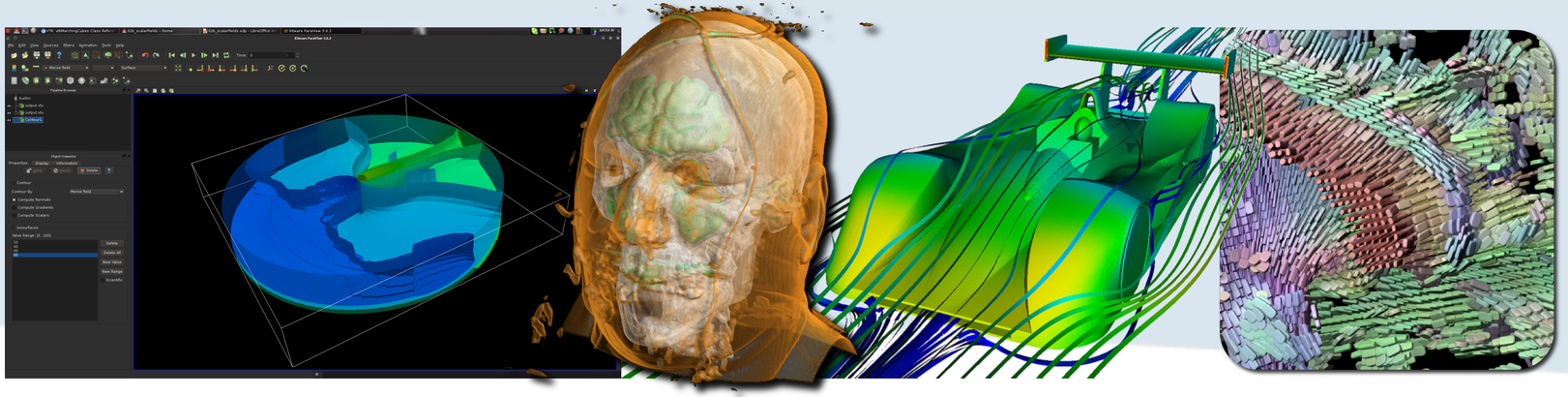
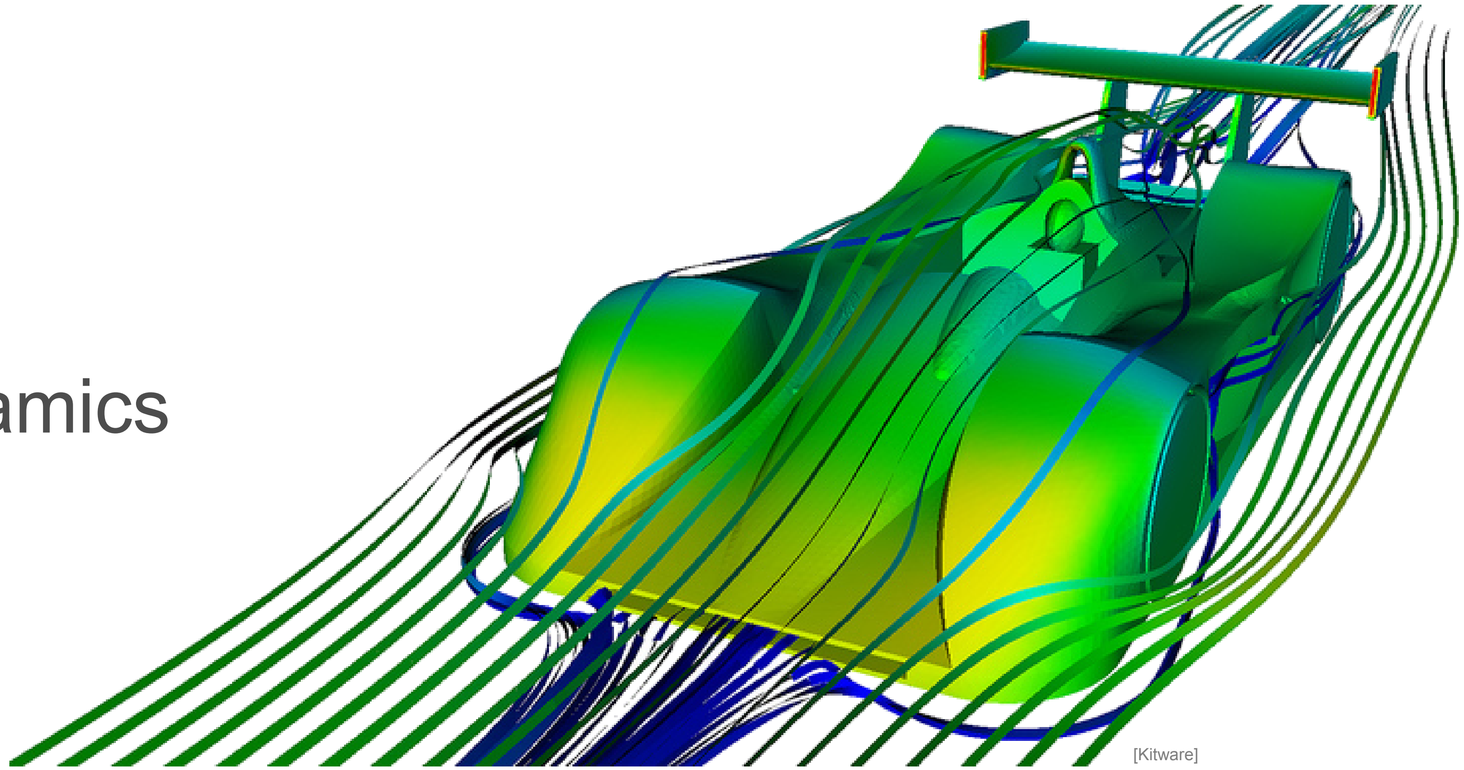
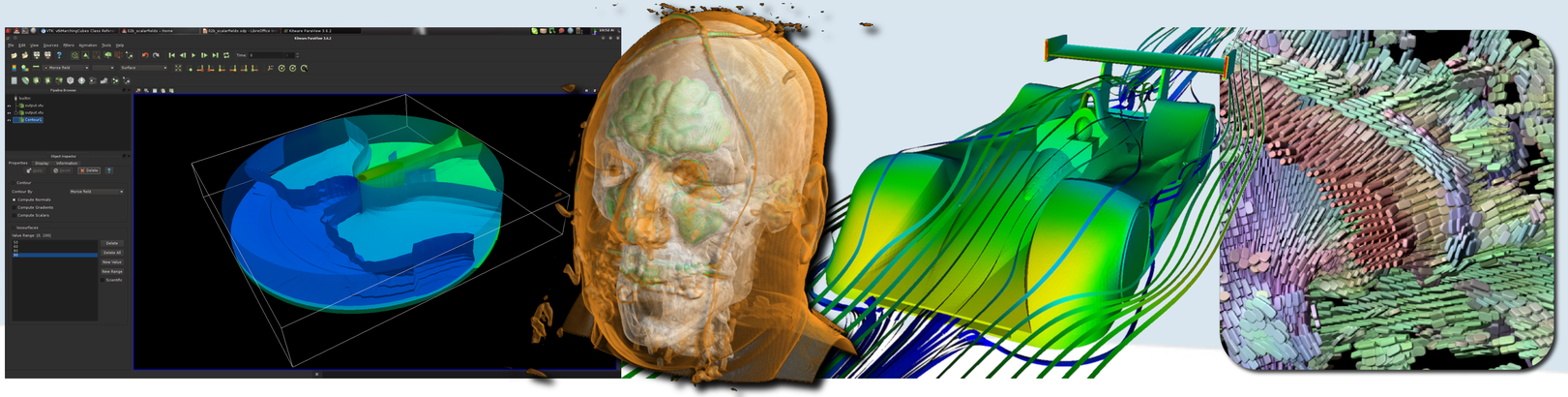


- Scientific data-sets
  - Results of simulations
    - Chemistry, Physics
    - Computational fluid dynamics



[Kitware]

What?



- Scientific data-sets
  - Results of simulations
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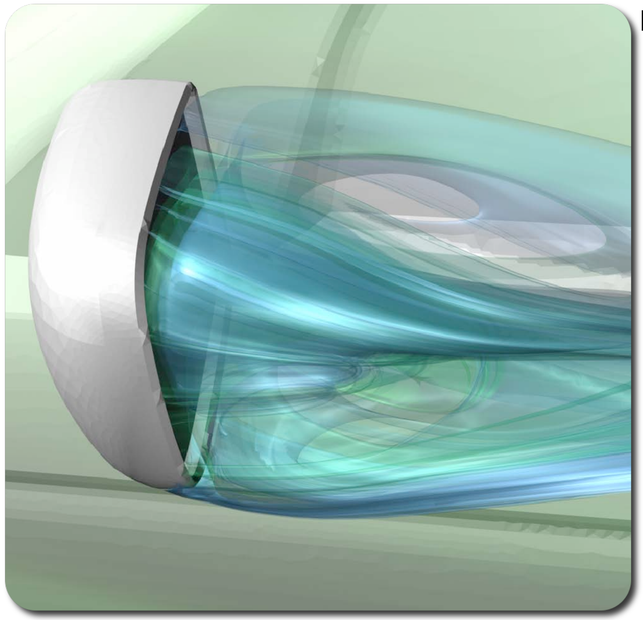

[GarthVIS08]

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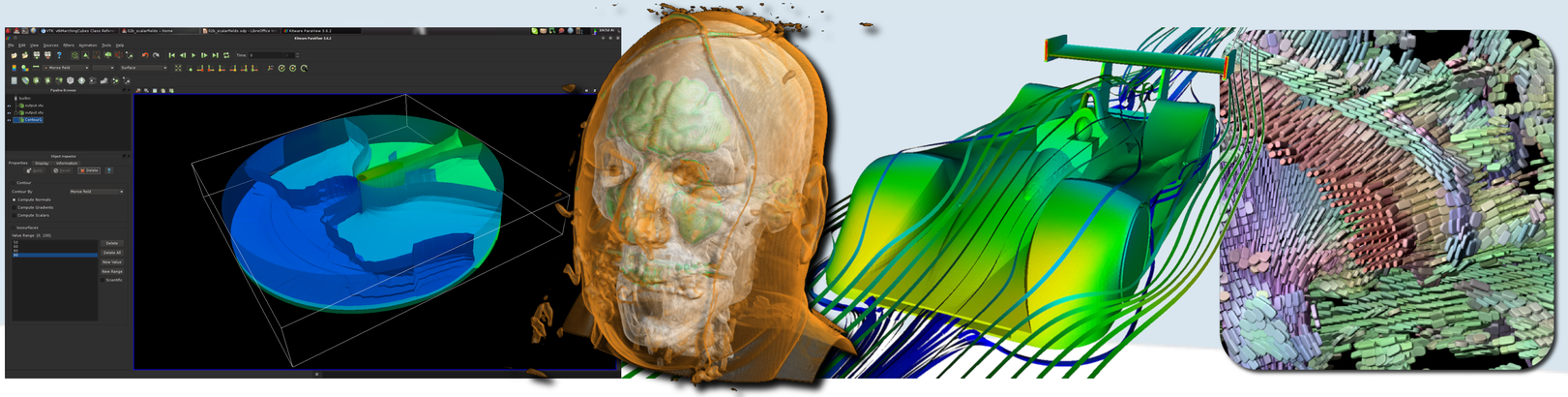

- Scientific data-sets
  - Results of simulations
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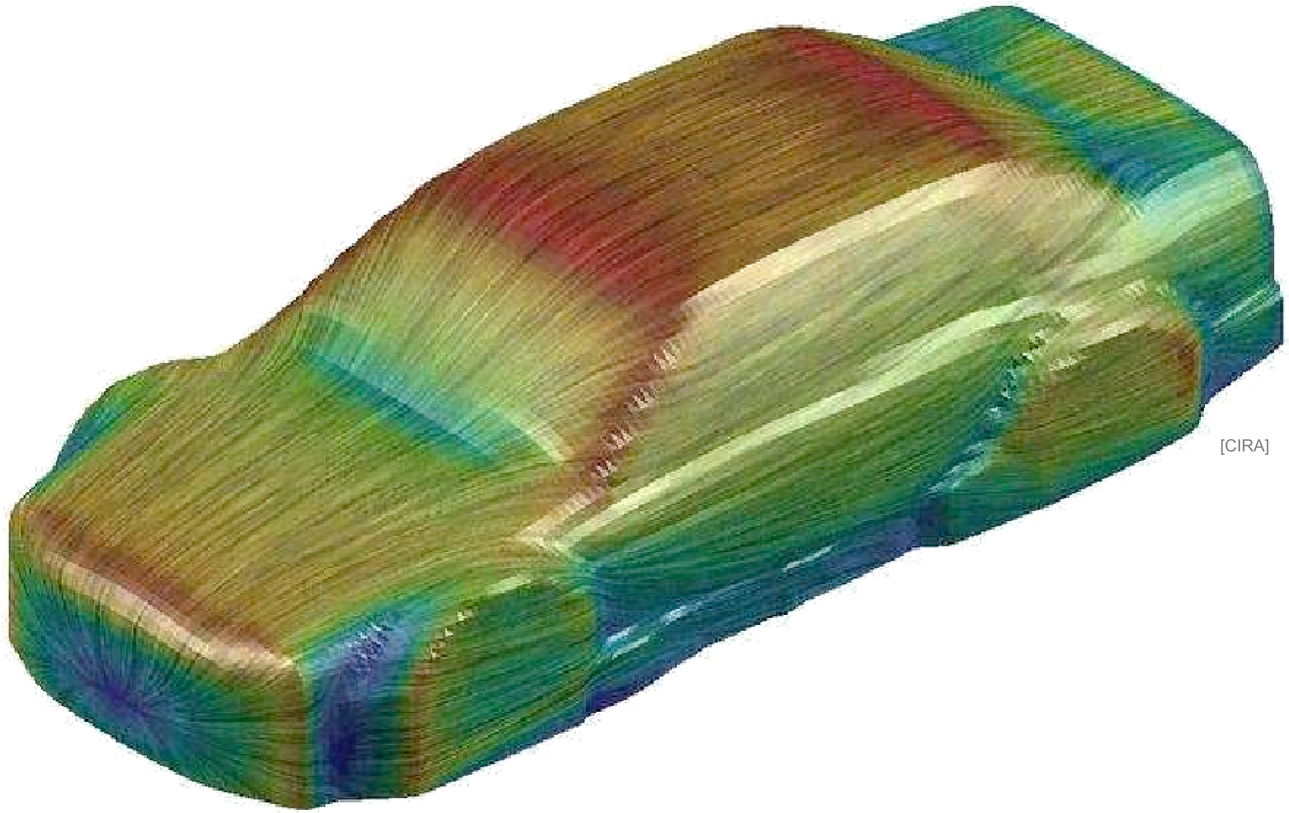

[CIRA]

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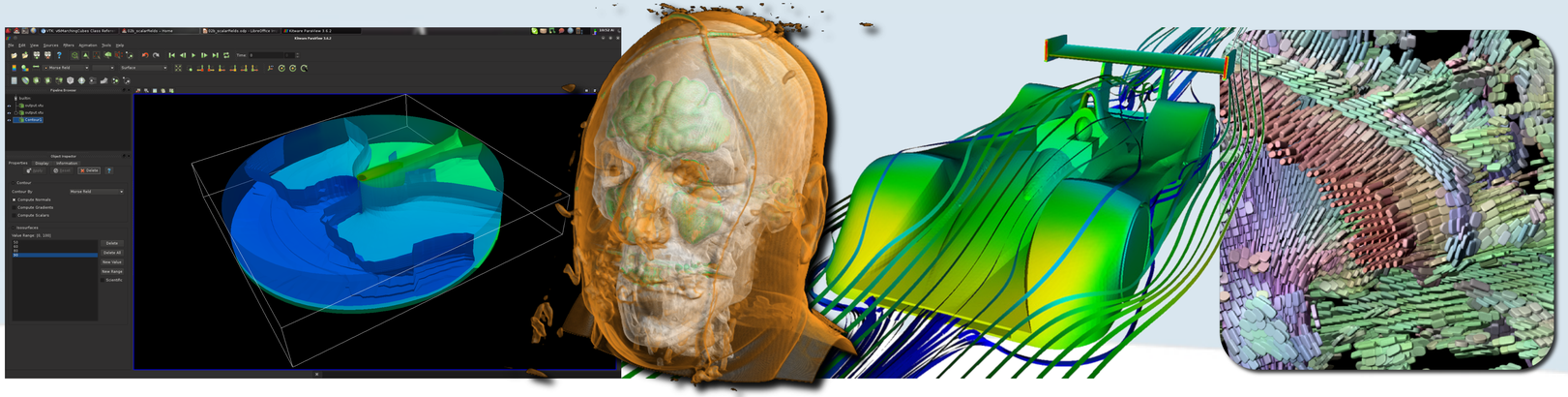

- Scientific data-sets
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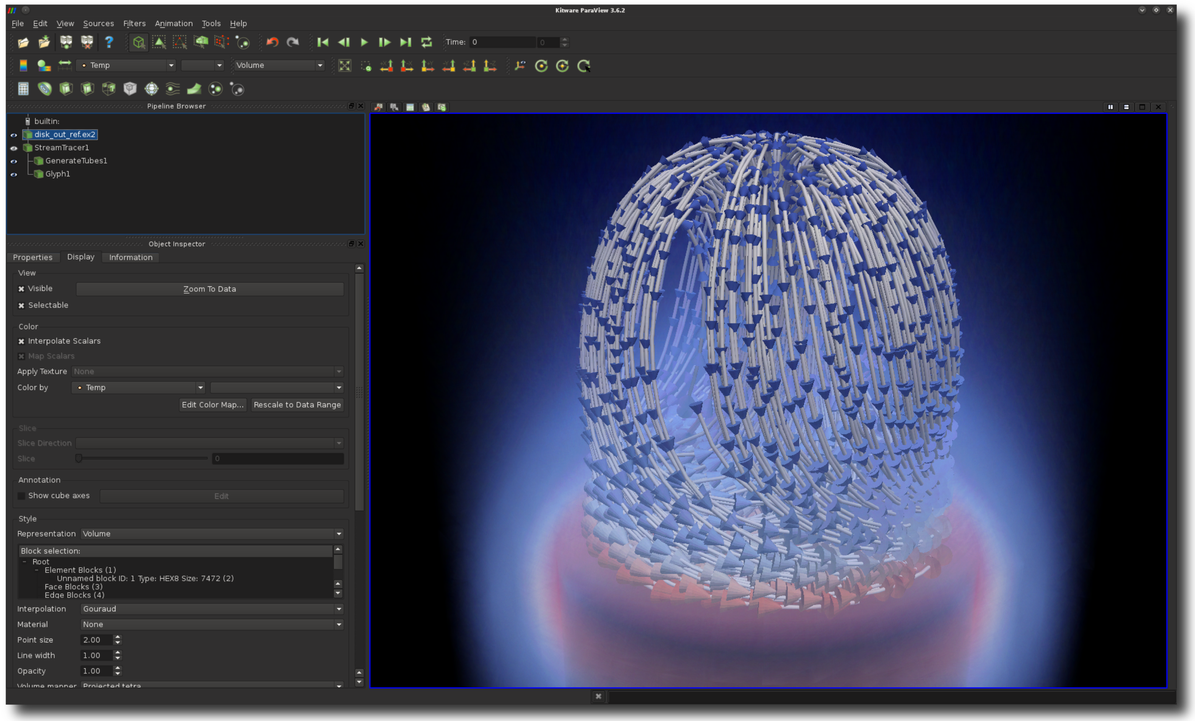

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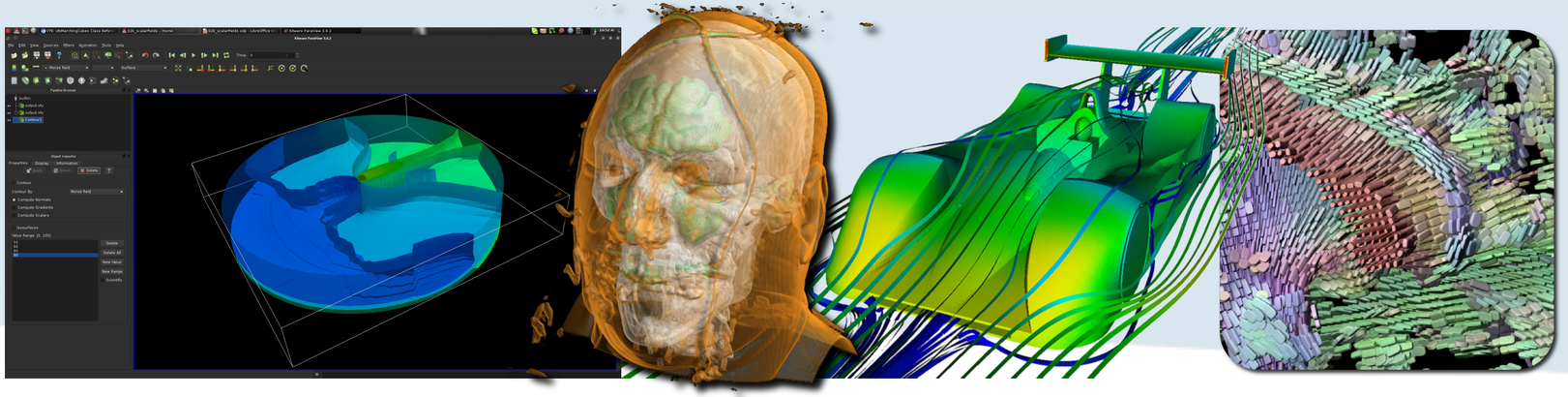

- Scientific data-sets
  - Results of simulations
    - Chemistry, Physics
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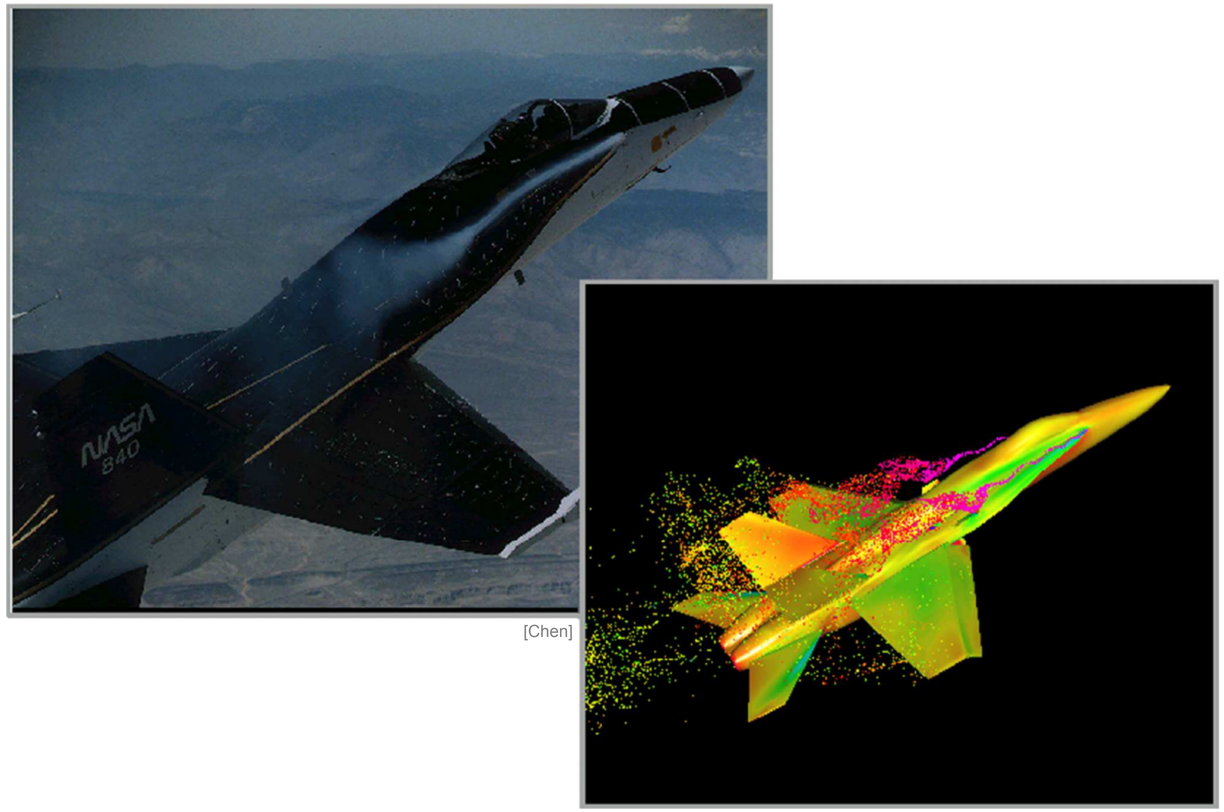
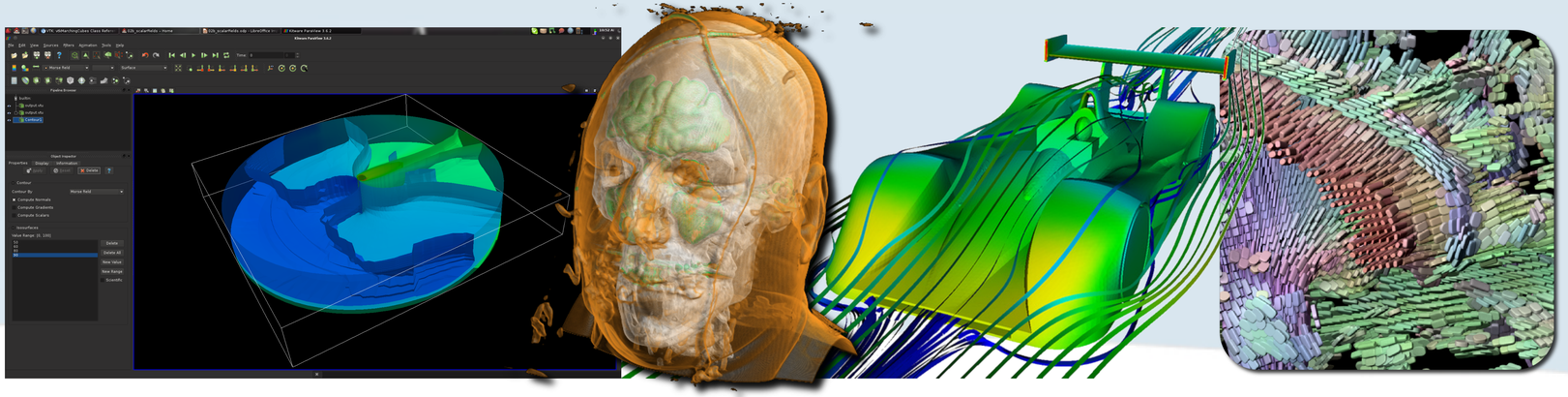
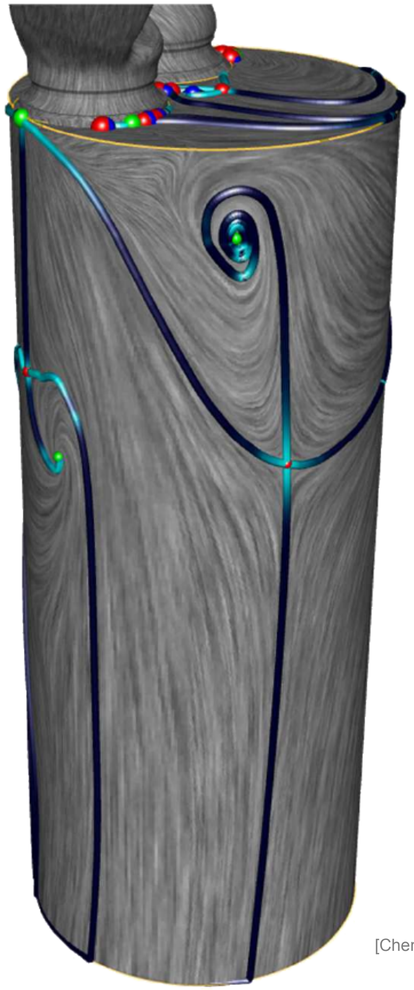


[Chen]

What?



- Scientific data-sets
  - Results of simulations
    - Chemistry, Physics
    - Computational fluid dynamics
    - Computer assisted design



[Chen]

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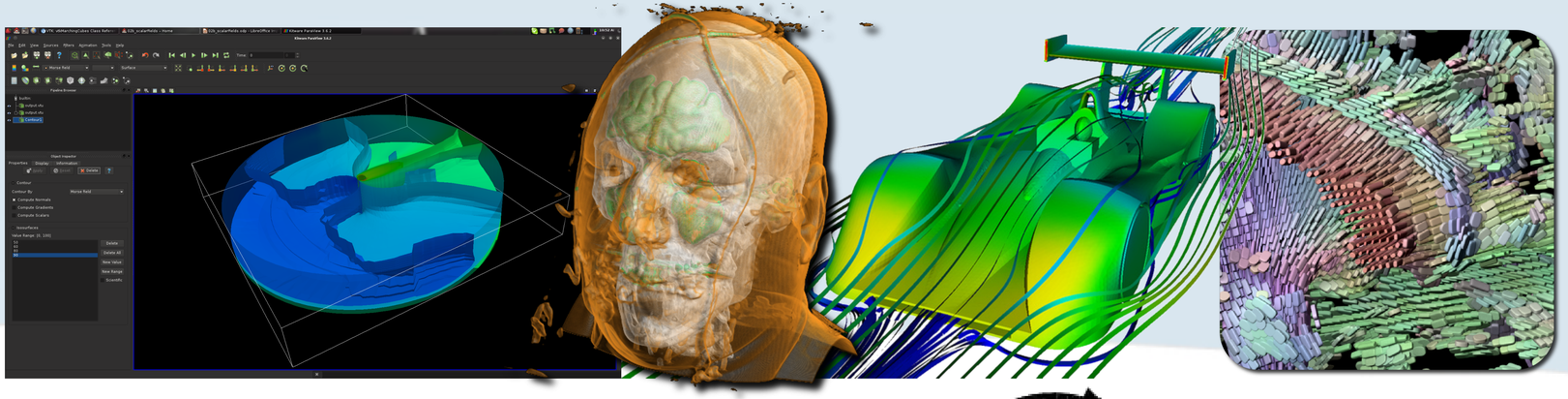
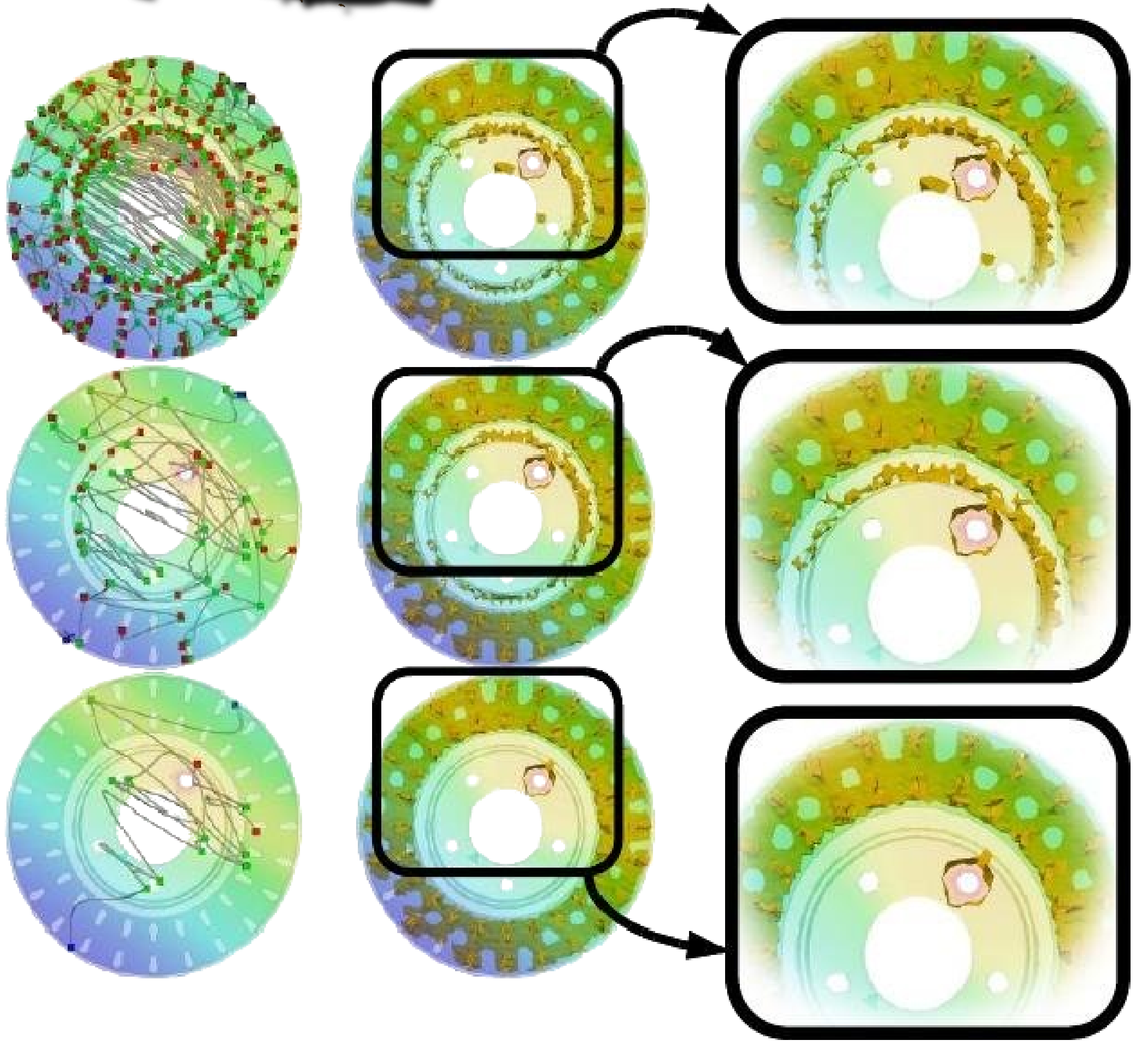


- Scientific data-sets
  - Results of simulations
    - Chemistry, Physics
    - Computational fluid dynamics
    - Computer assisted design



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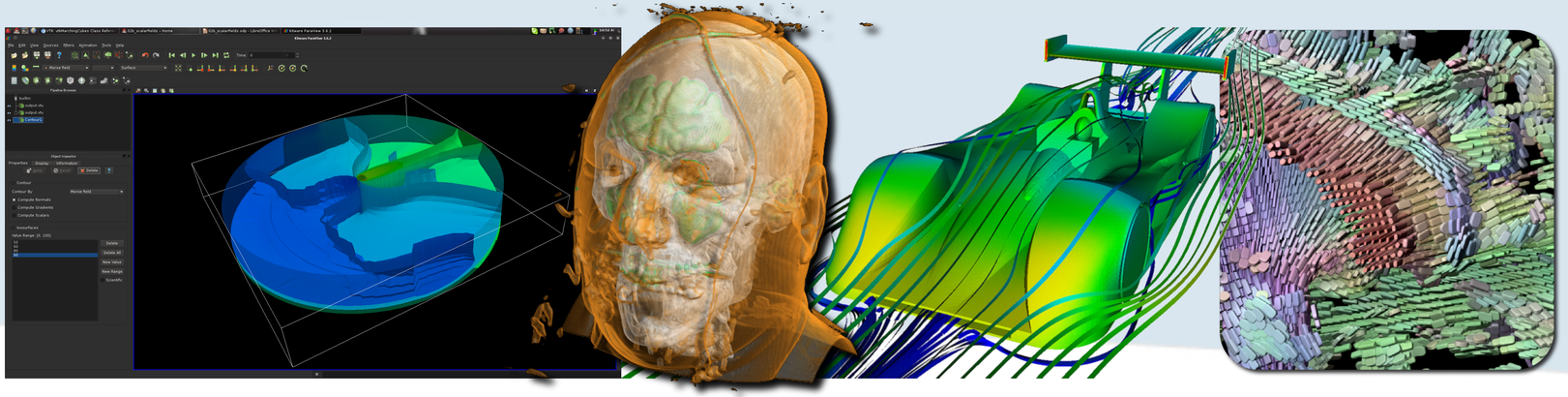

- Scientific data-sets
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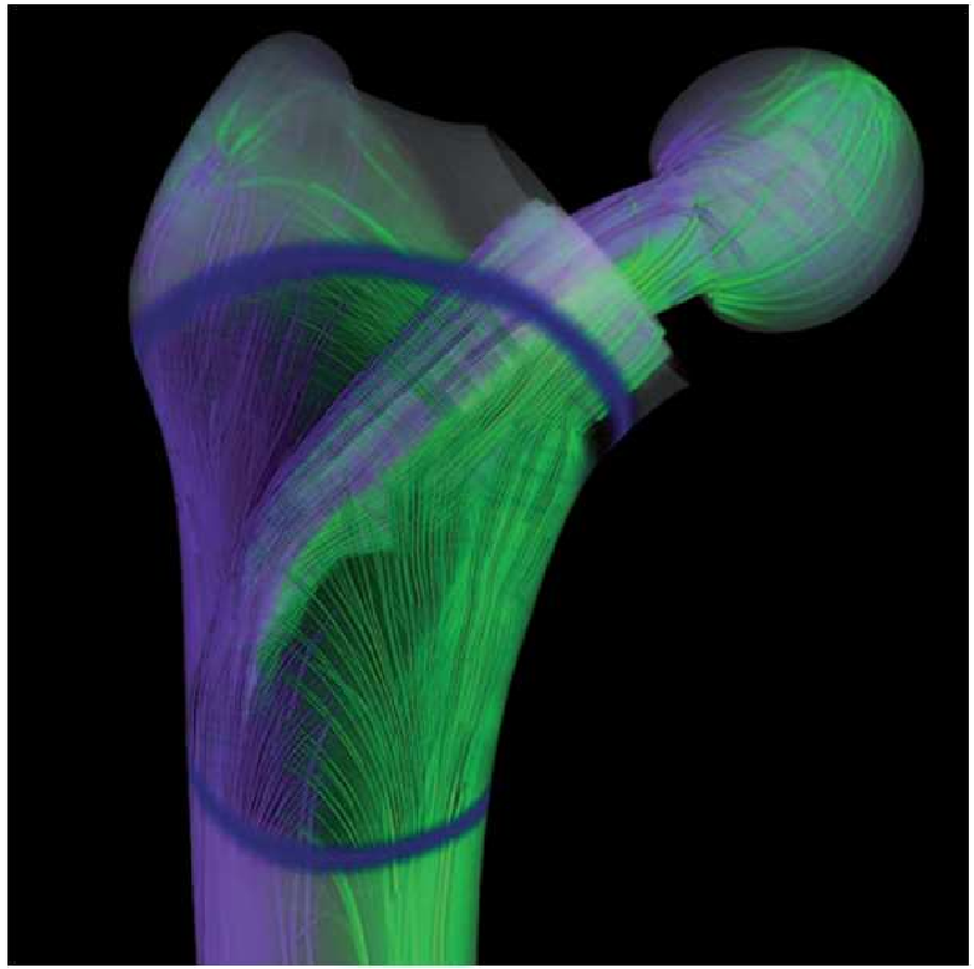
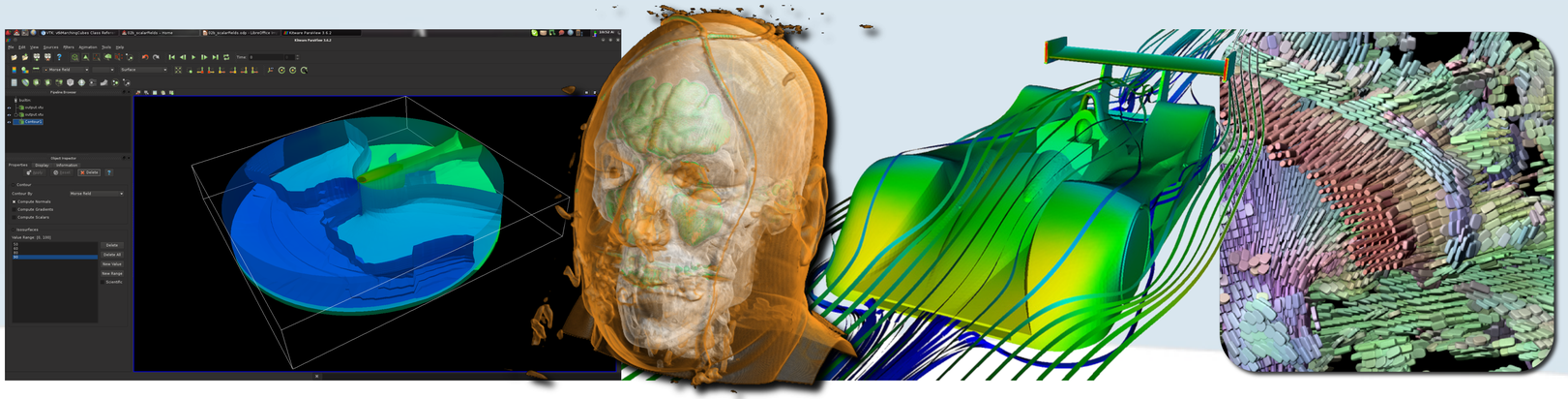
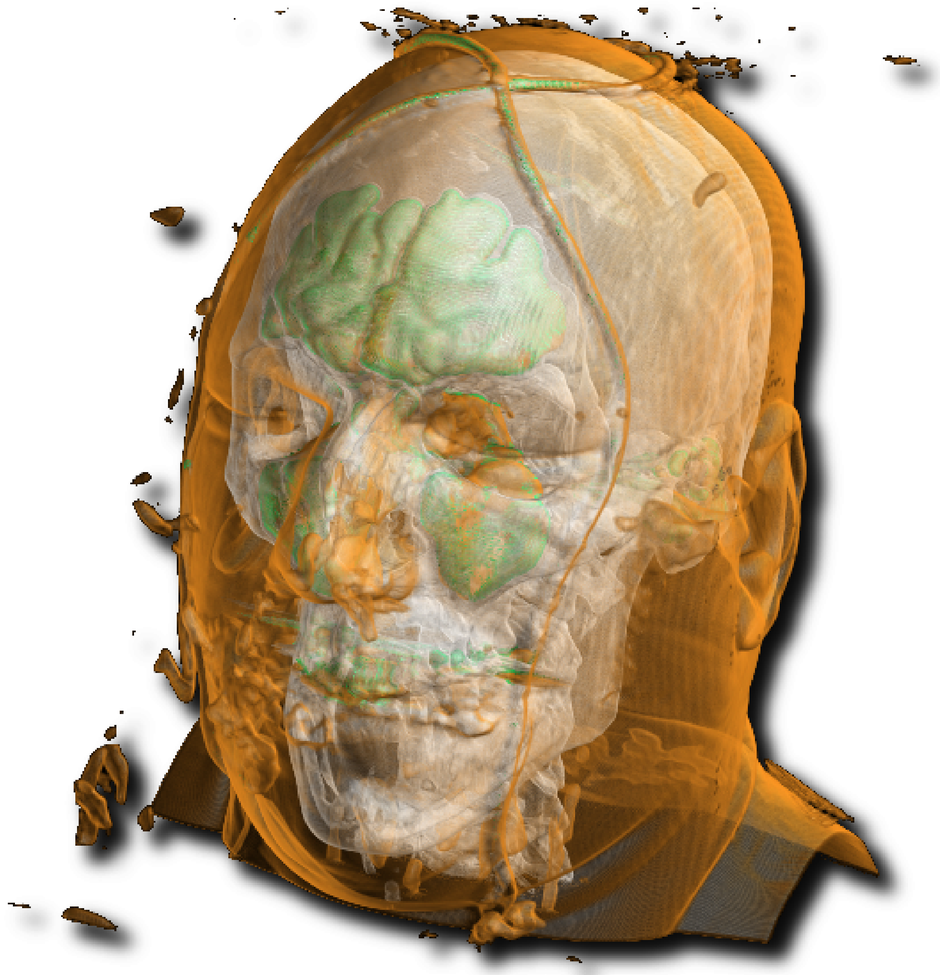


[Dick et al 09]

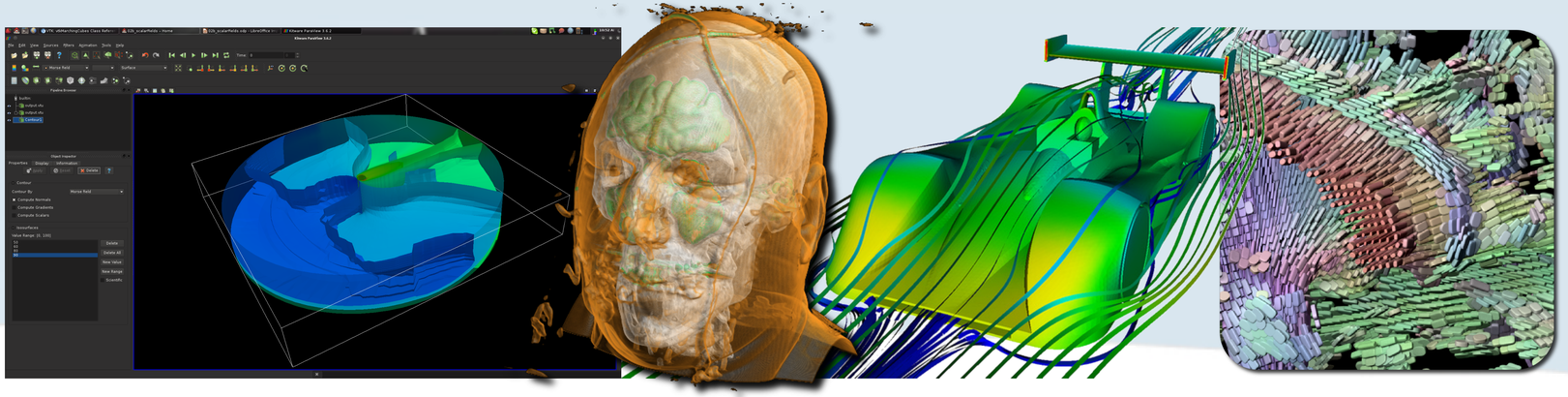
What?



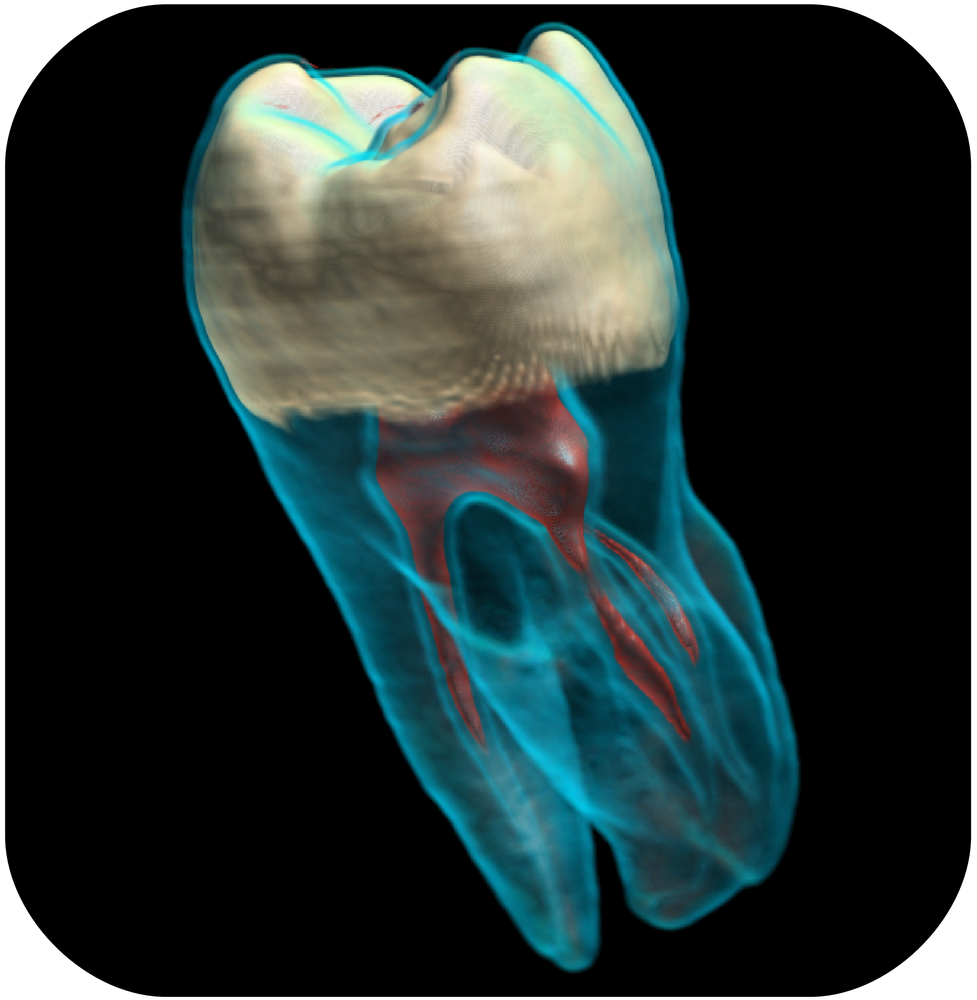
- Scientific data-sets
  - Results of simulations
    - Chemistry, Physics
    - Computational fluid dynamics
    - Computer assisted design
  - Results of acquisitions
    - Medical imaging



What?



- Scientific data-sets
  - Results of simulations
    - Chemistry, Physics
    - Computational fluid dynamics
    - Computer assisted design
  - Results of acquisitions
    - Medical imaging



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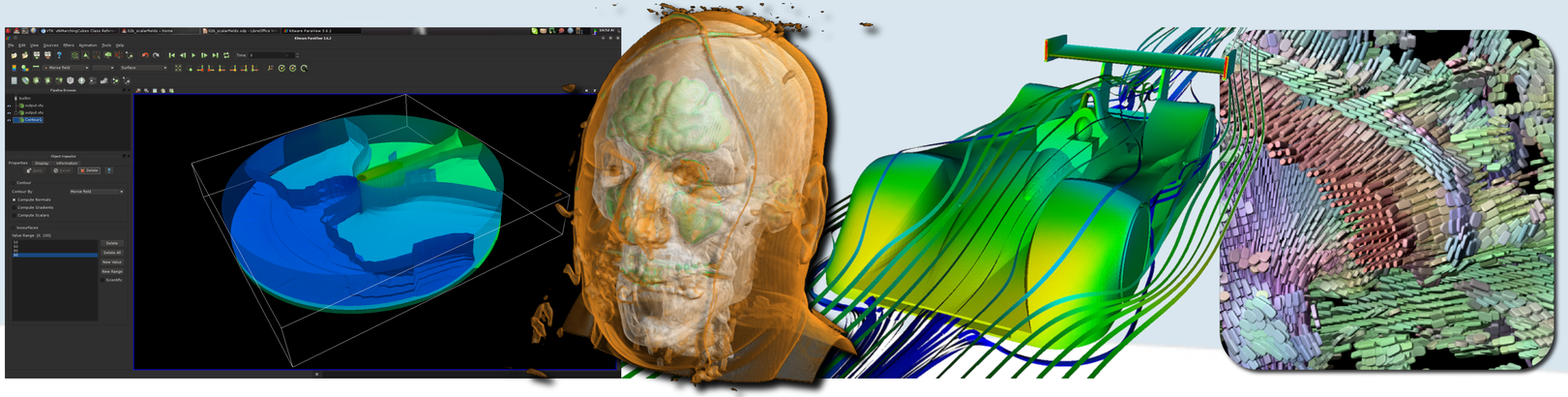

- Scientific data-sets
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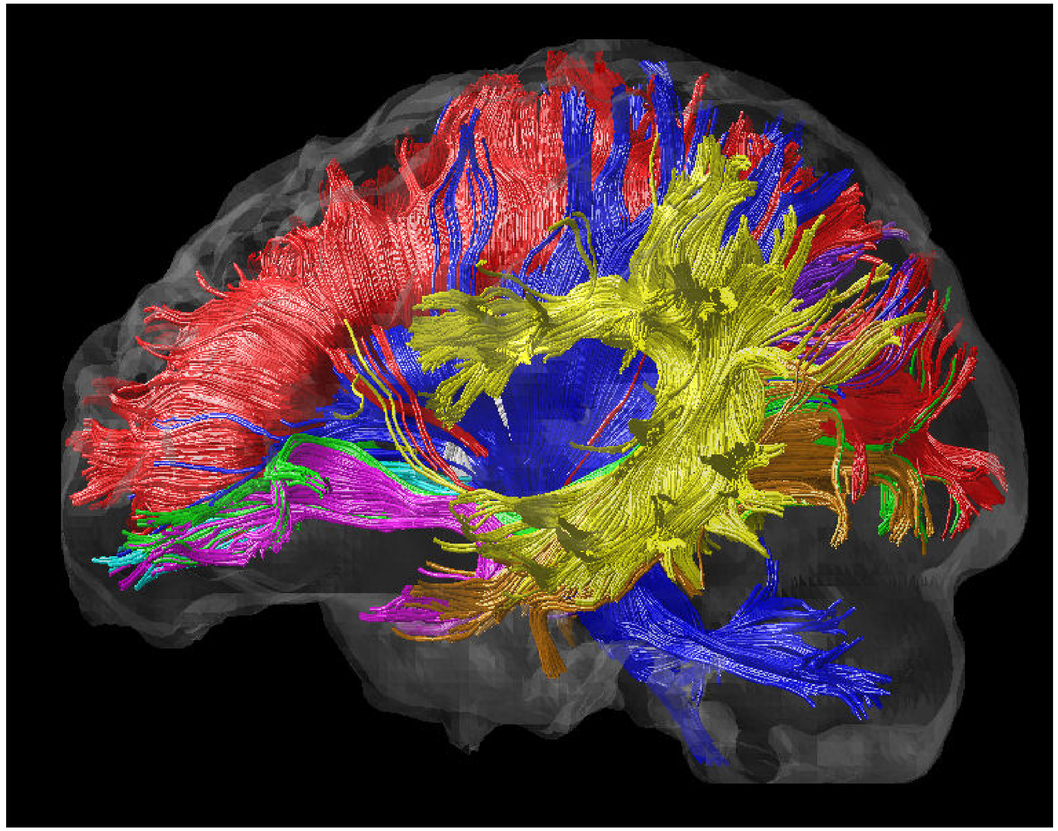
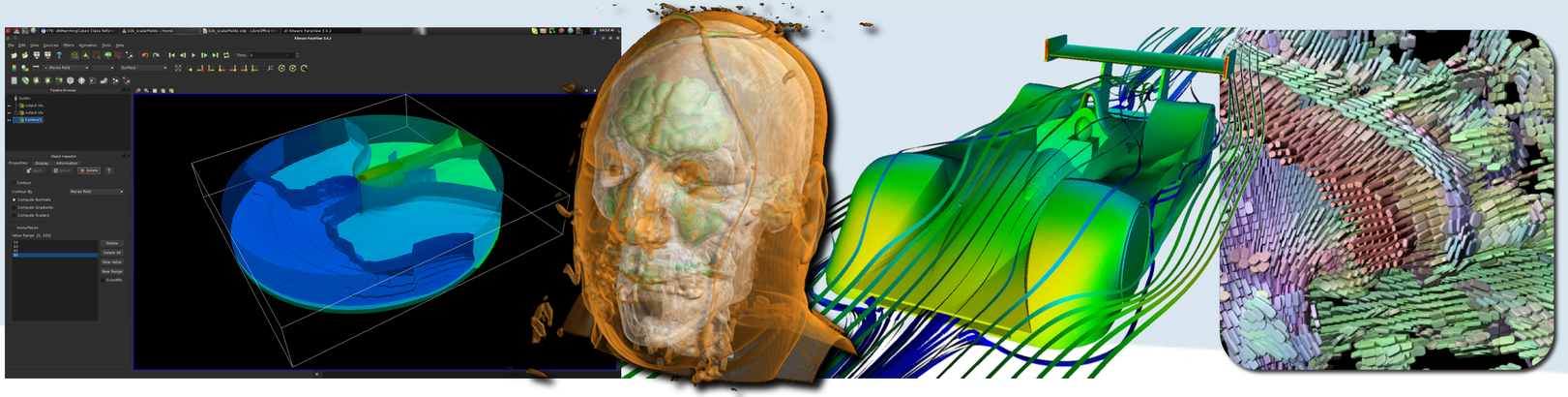
    - Chemistry, Physics
    - Computational fluid dynamics
    - Computer assisted design
  - Results of acquisitions
    - Medical imaging

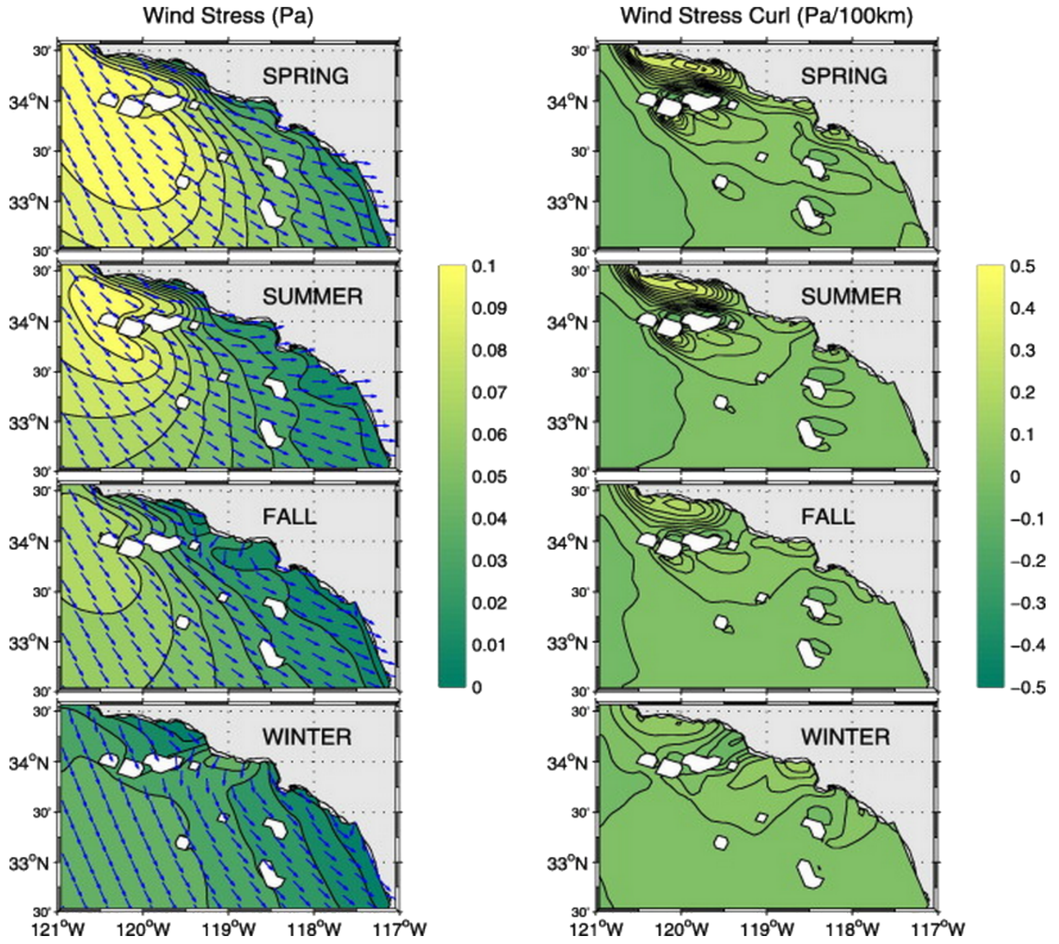


[http://www.neuroimaging.tau.ac.il]

What?



- Scientific data-sets
  - Results of simulations
    - Chemistry, Physics
    - Computational fluid dynamics
    - Computer assisted design
  - Results of acquisitions
    - Medical imaging
    - Seismology, oceanography, etc.



[Dong09]

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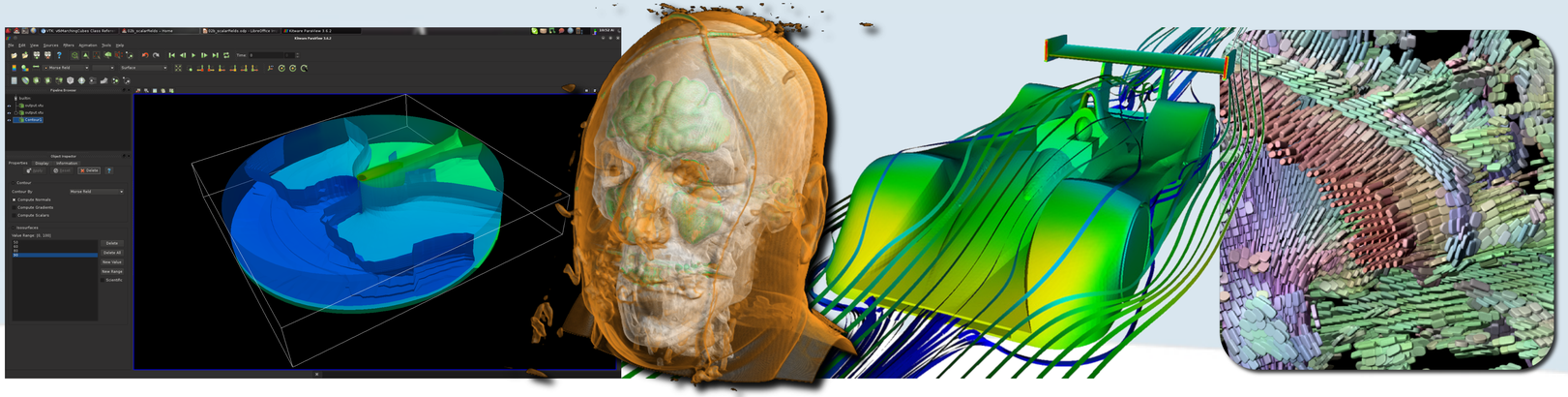
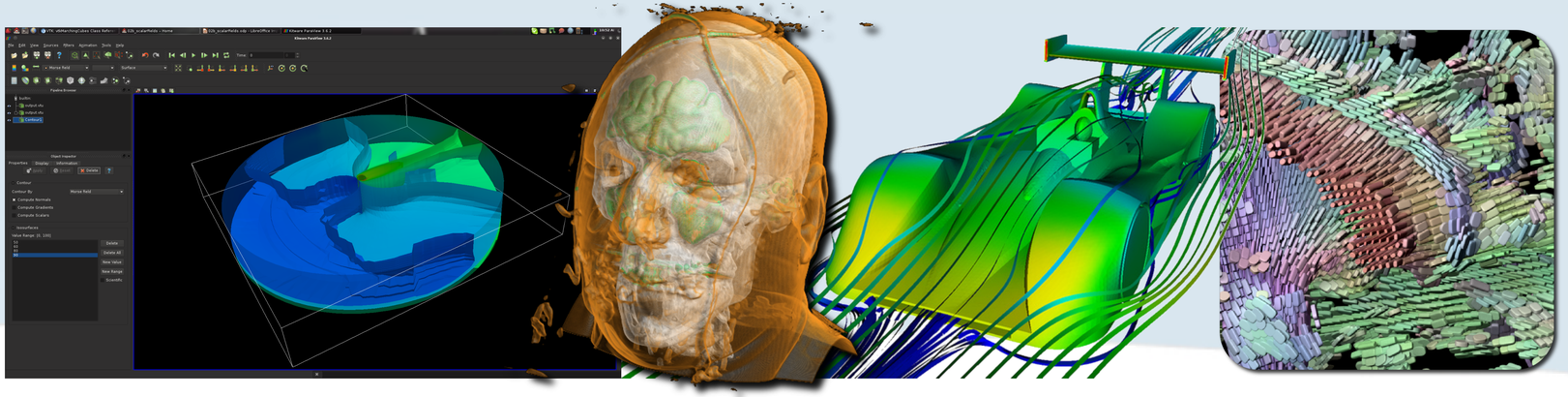


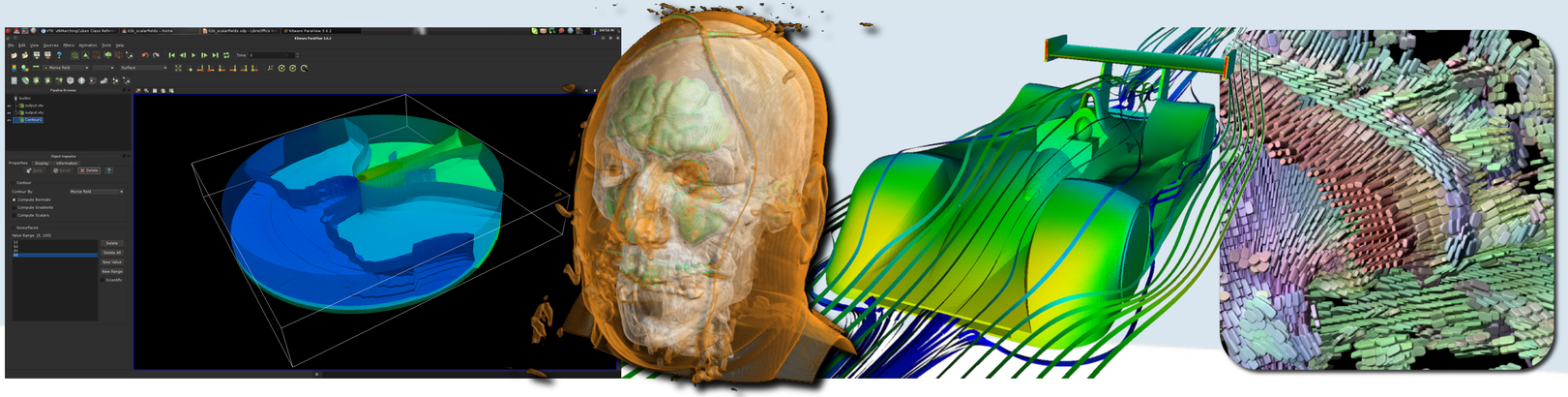
- Visual exploration of scientific data

What for?



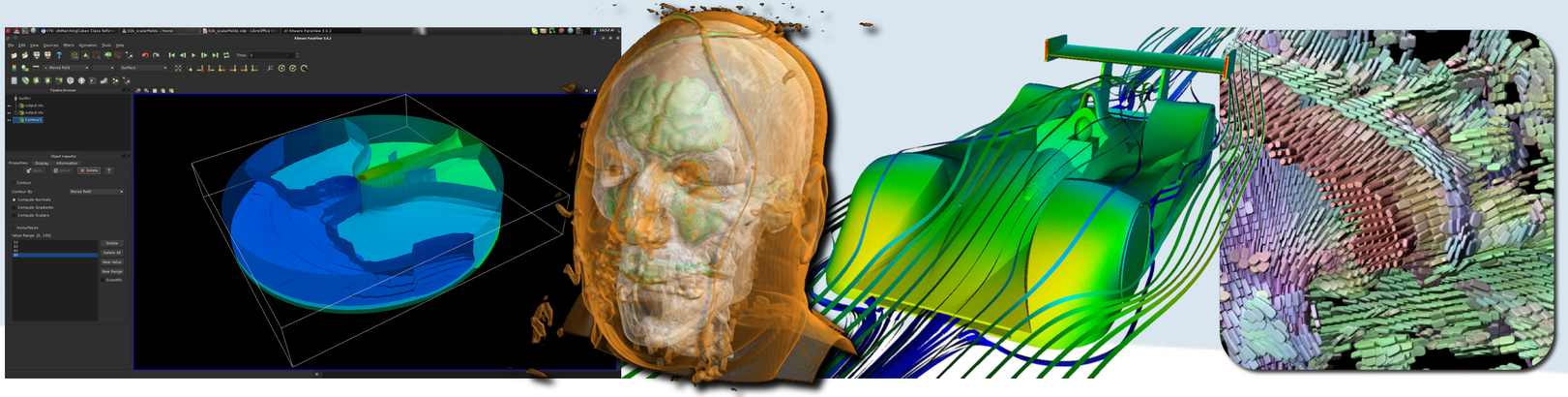
- Visual exploration of scientific data
  - Hypothesis formulation

What for?



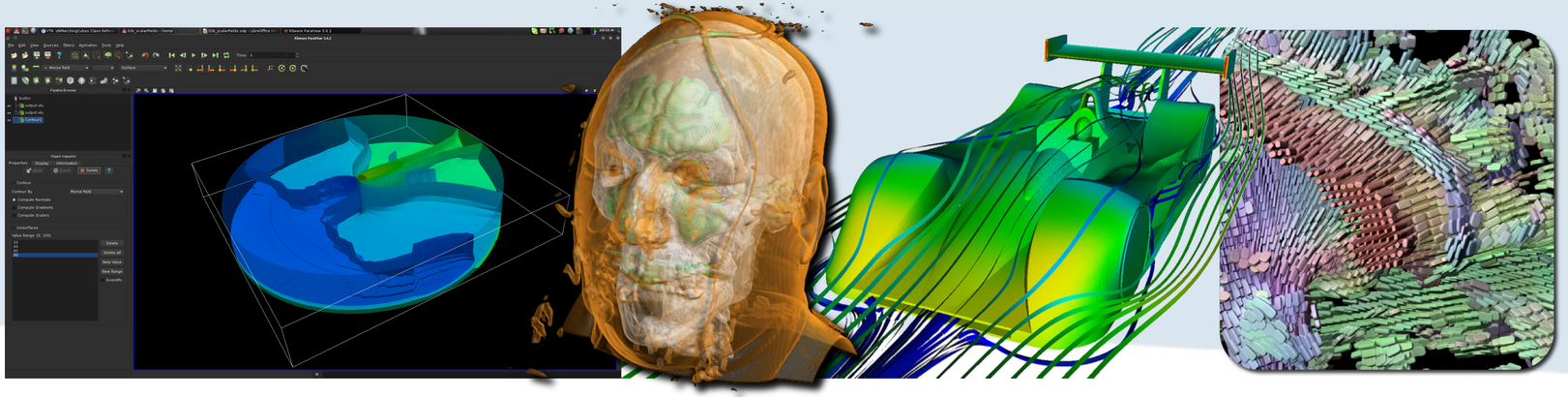
- Visual exploration of scientific data
  - Hypothesis formulation
  - Model verification

What for?



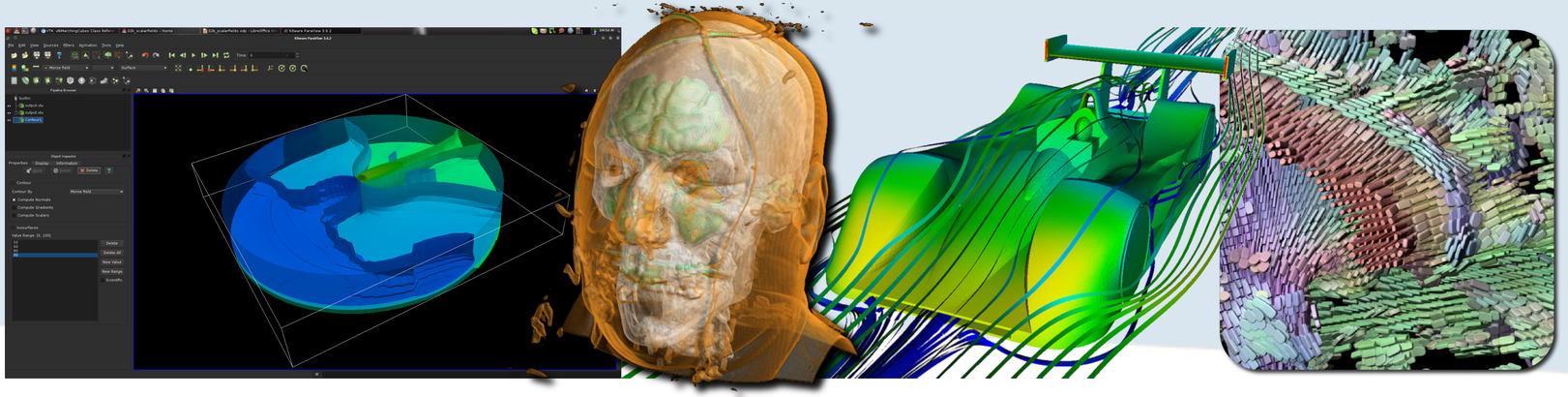
- Visual exploration of scientific data
  - Hypothesis formulation
  - Model verification
  - Intuition validation

What for?



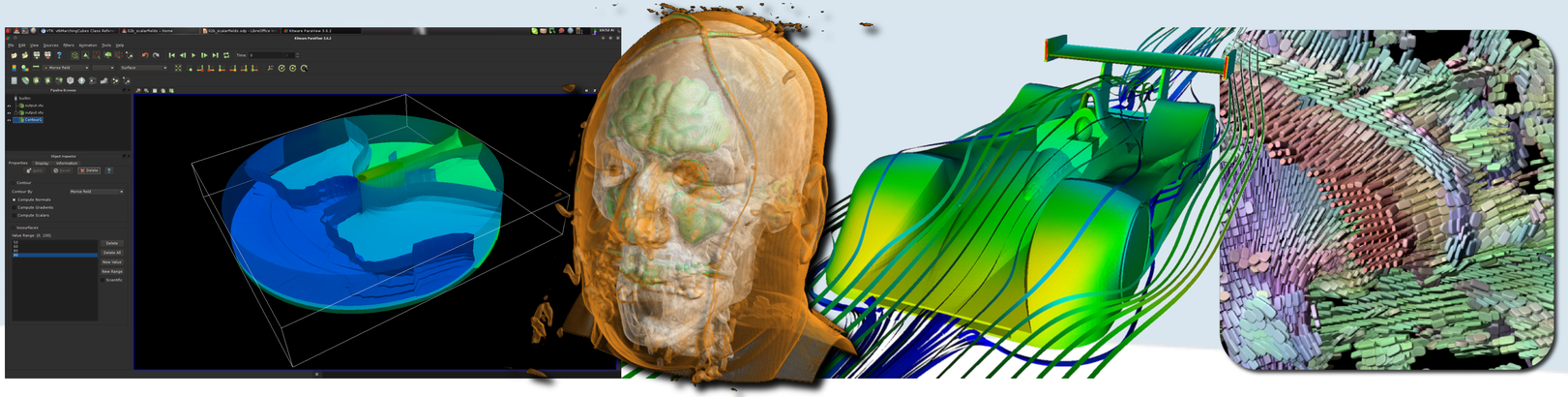
- Visual exploration of scientific data
  - Hypothesis formulation
  - Model verification
  - Intuition validation
- Geometrical analysis
  - Result interpretation

What for?

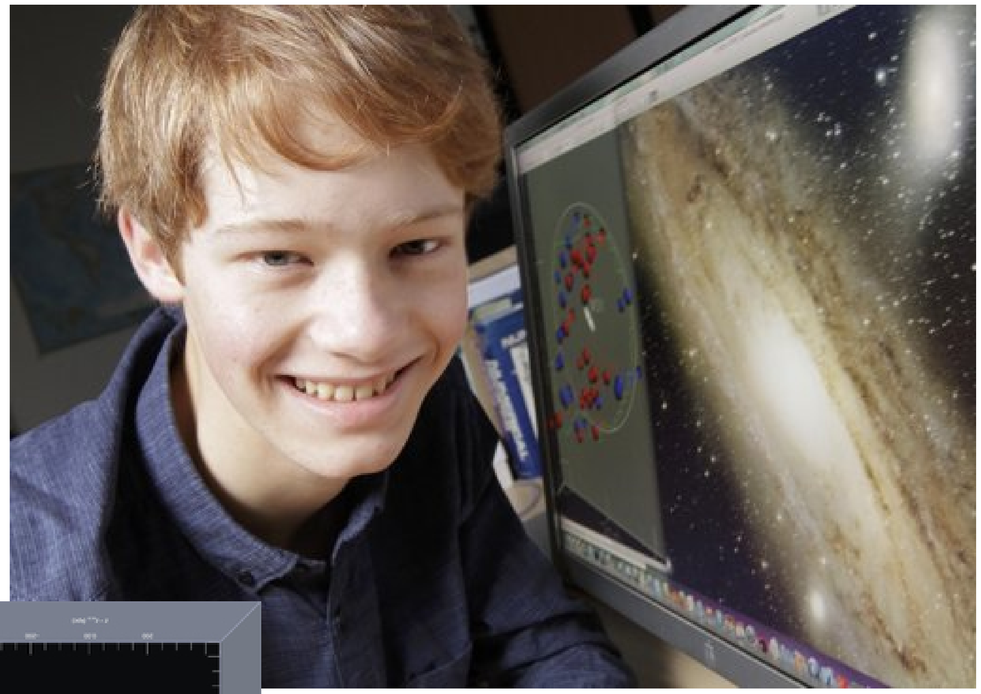


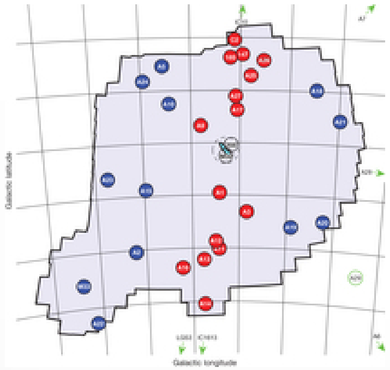
- Visual exploration of scientific data
  - Hypothesis formulation
  - Model verification
  - Intuition validation
- Geometrical analysis
  - Result interpretation
- Communication of scientific results
  - Interactive and graphical material

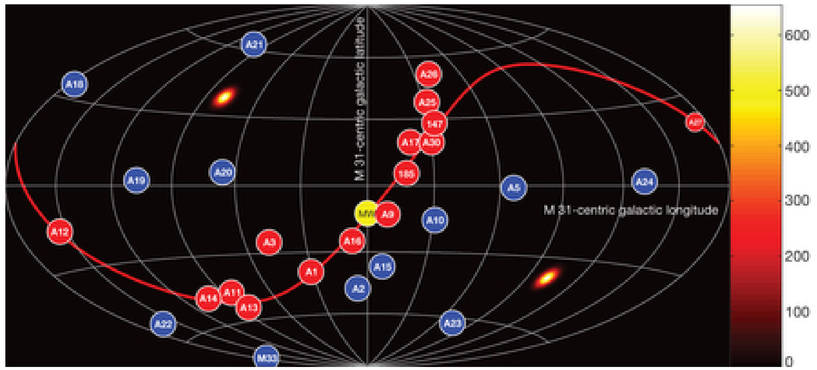
What for?

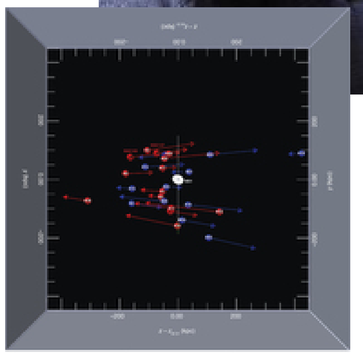


- Dwarf galaxies orbiting the Andromeda Galaxy
- *Nature*, January 3rd, 2013

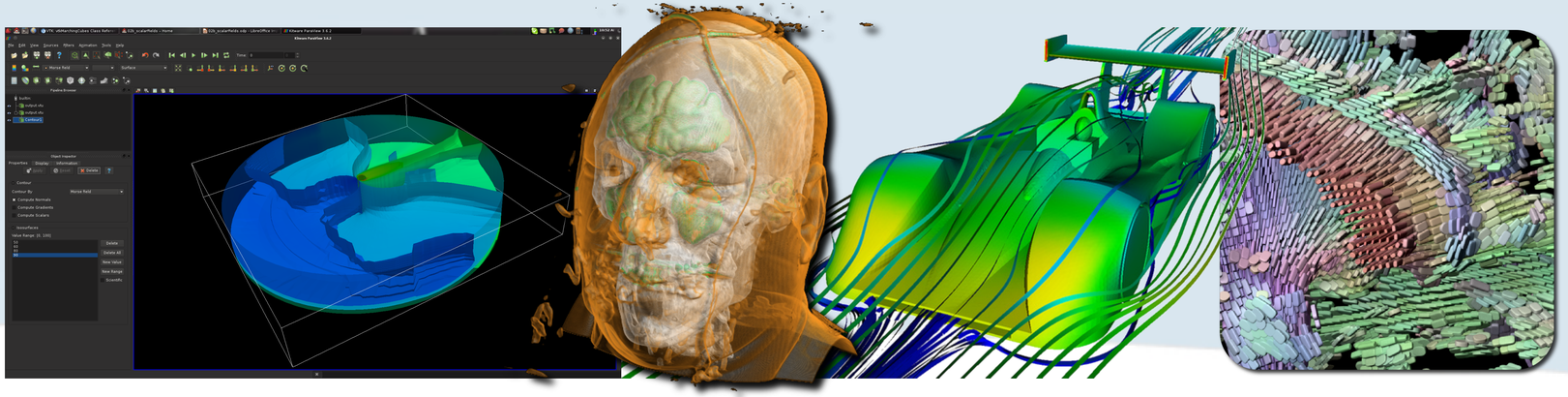




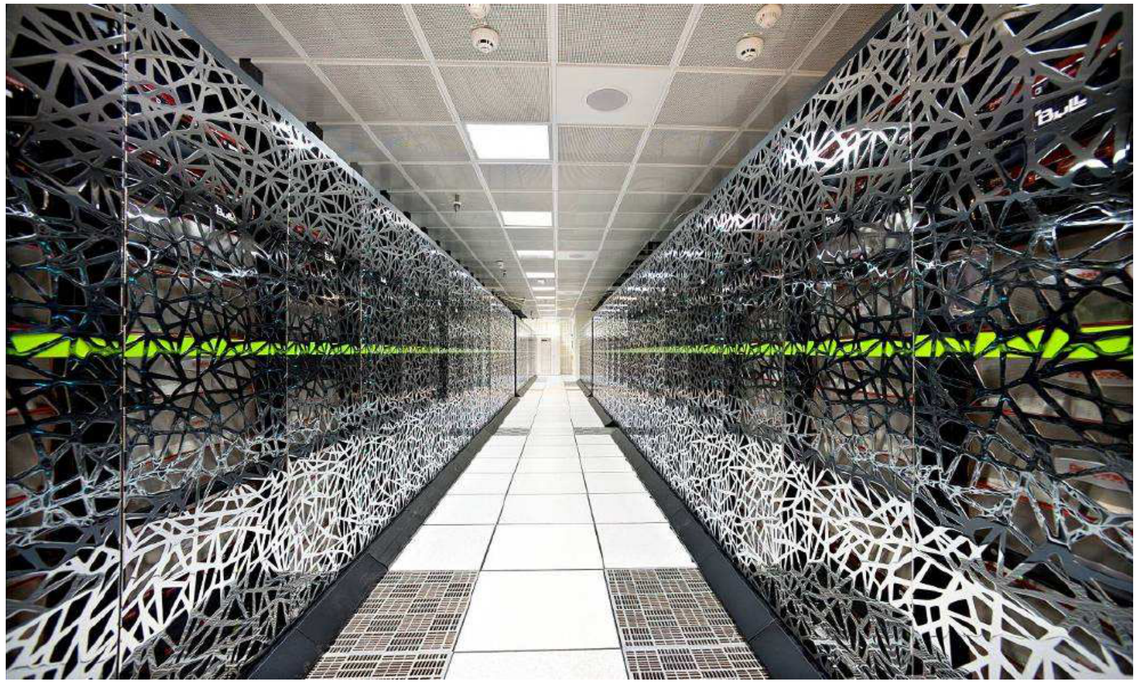




What for?



- Research and development activities



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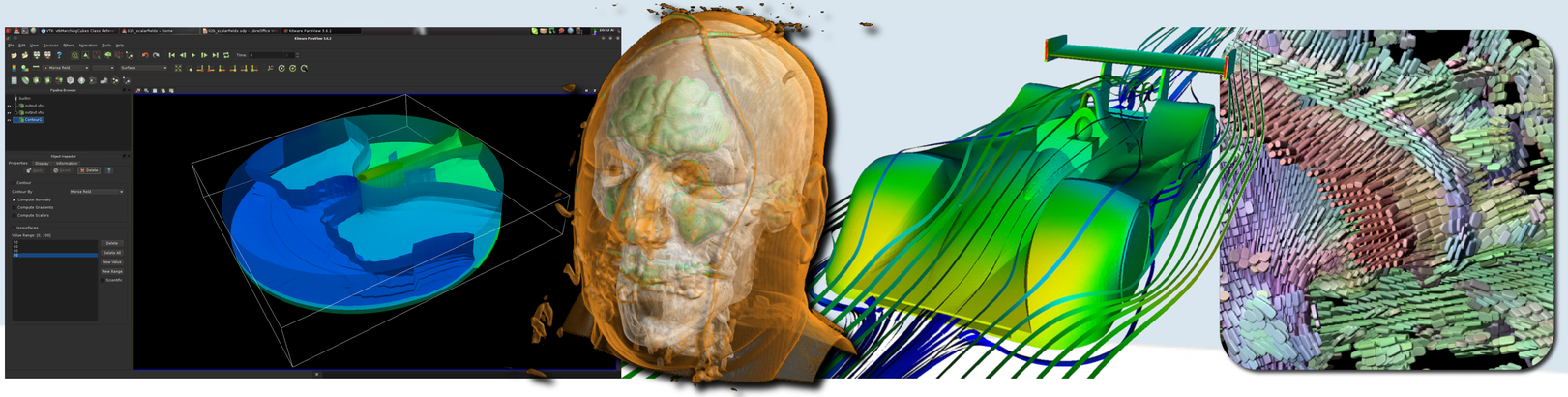
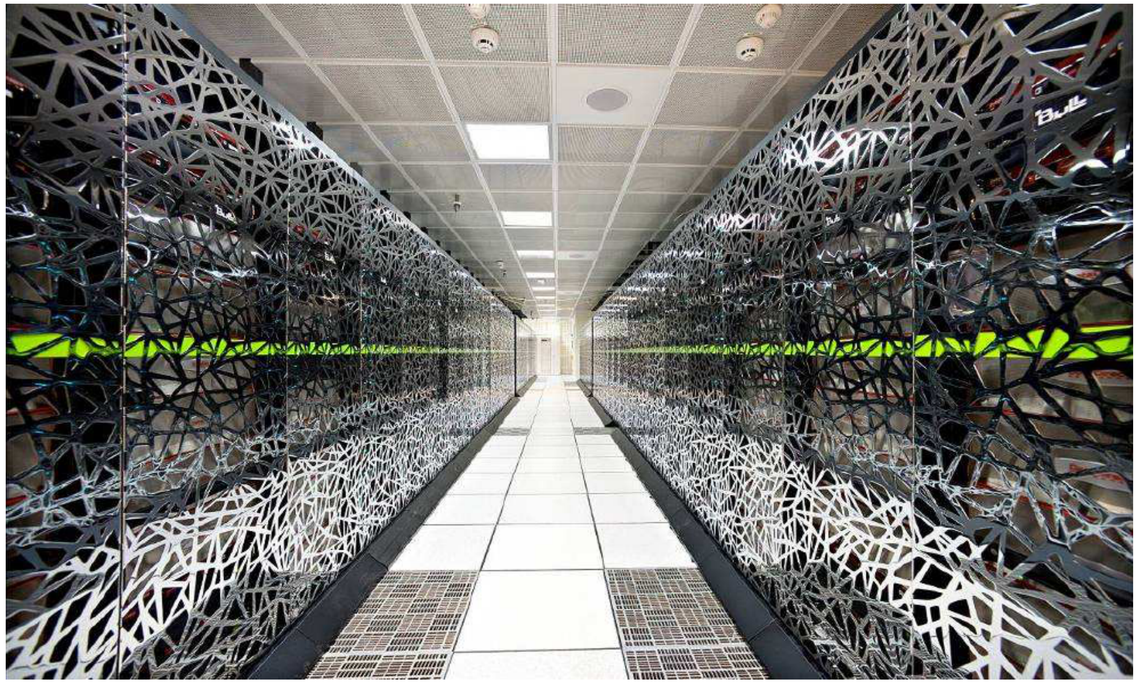


- Research and development activities
  - Academic
  - Industrial



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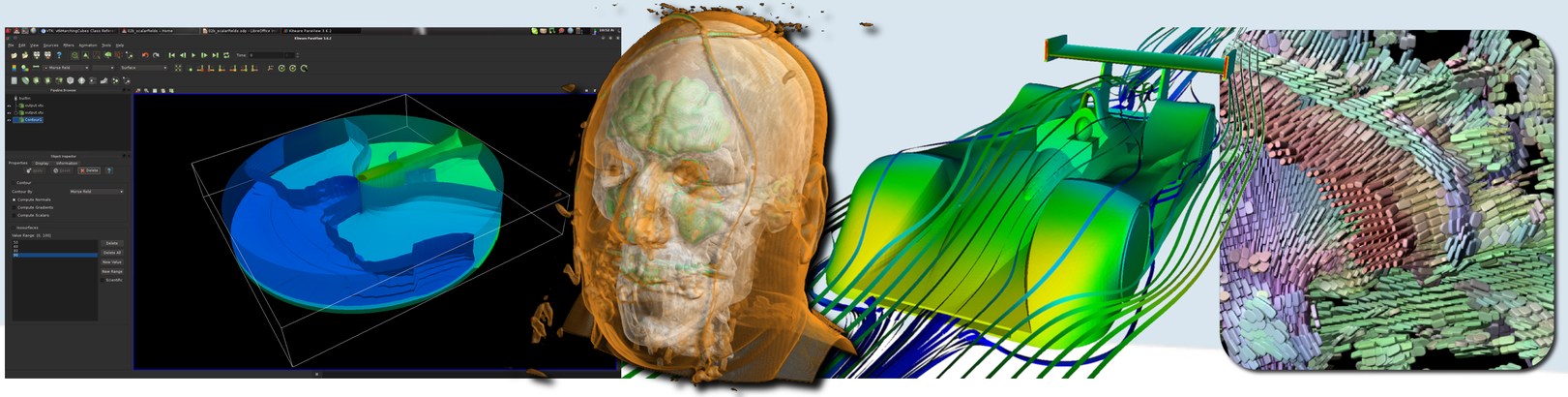
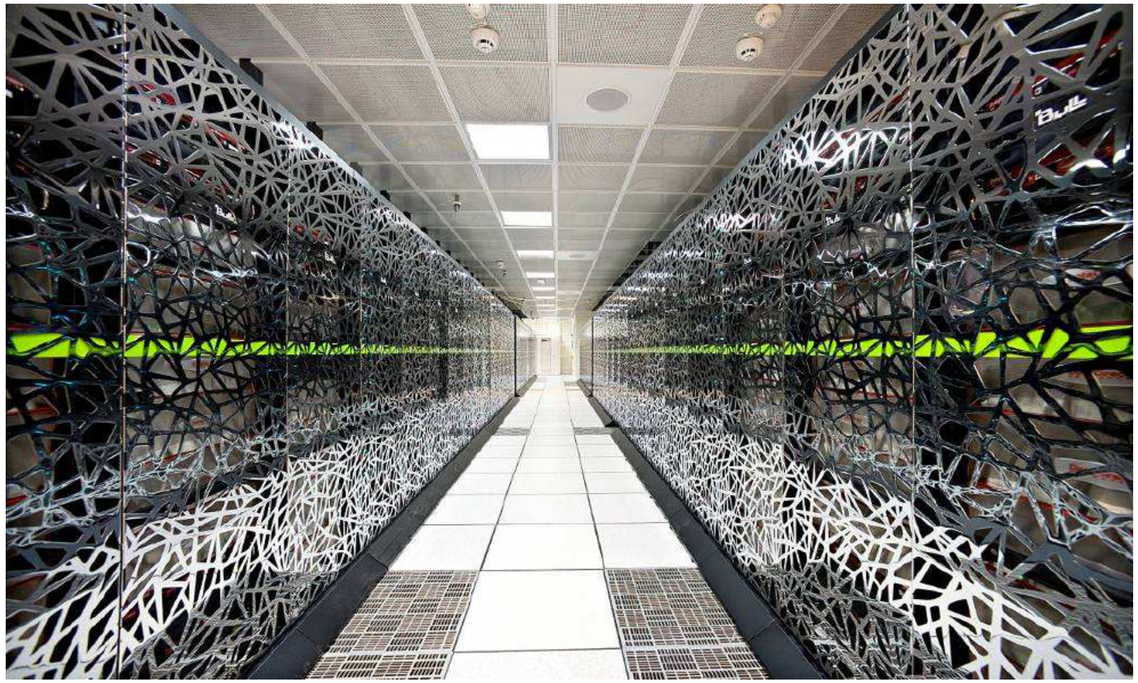


- Research and development activities
  - Academic
  - Industrial
    - CEA,
    - EDF,
    - Total,
    - Dassault Systèmes



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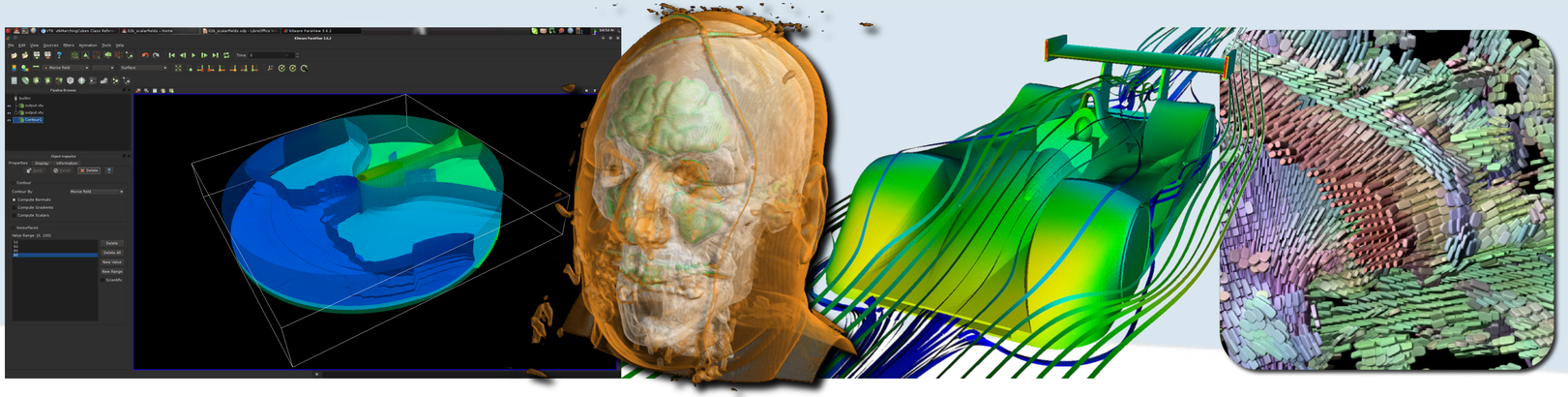

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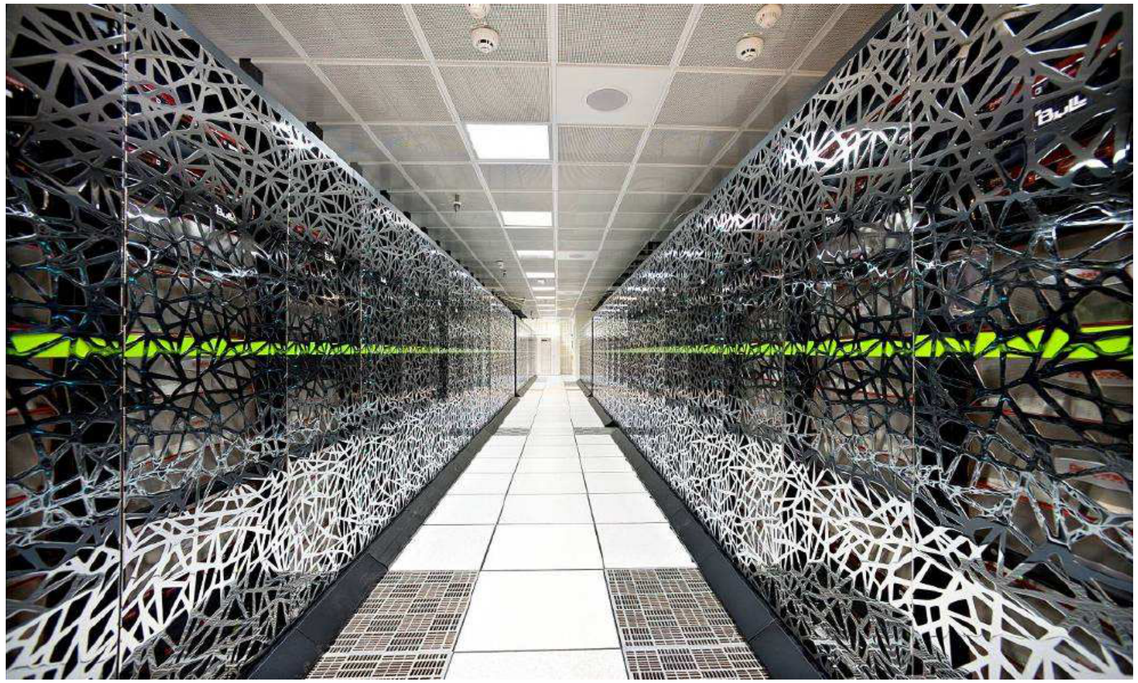
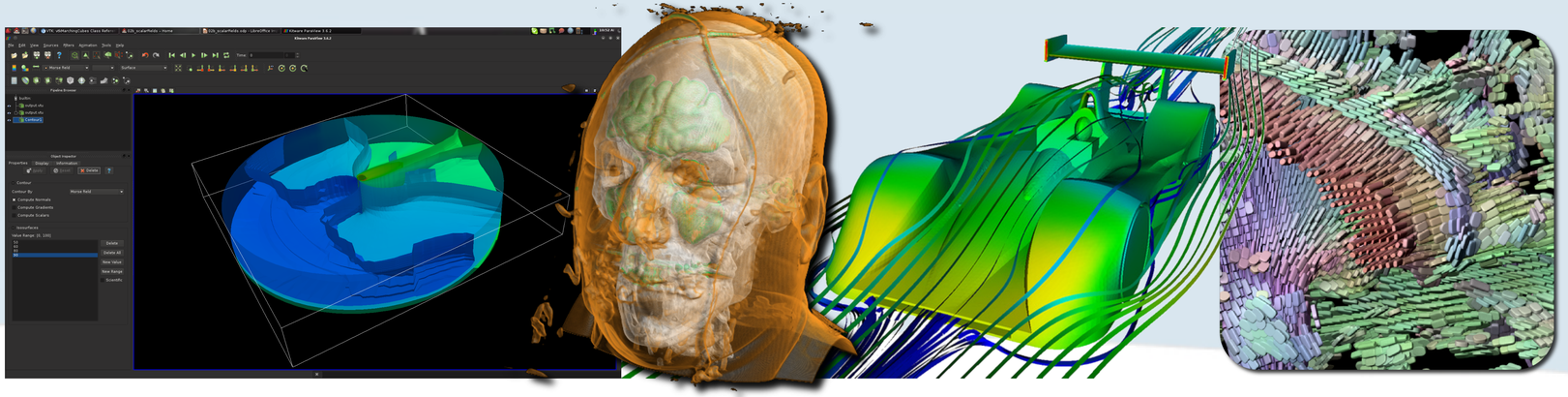


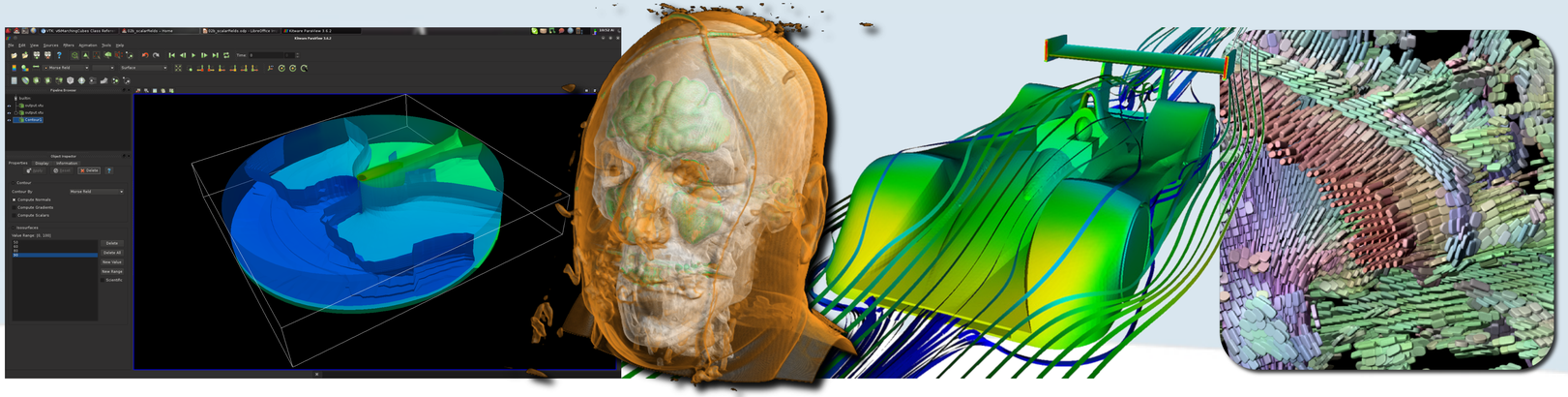
- 3D simulation, rendering and analysis software industry

What about?



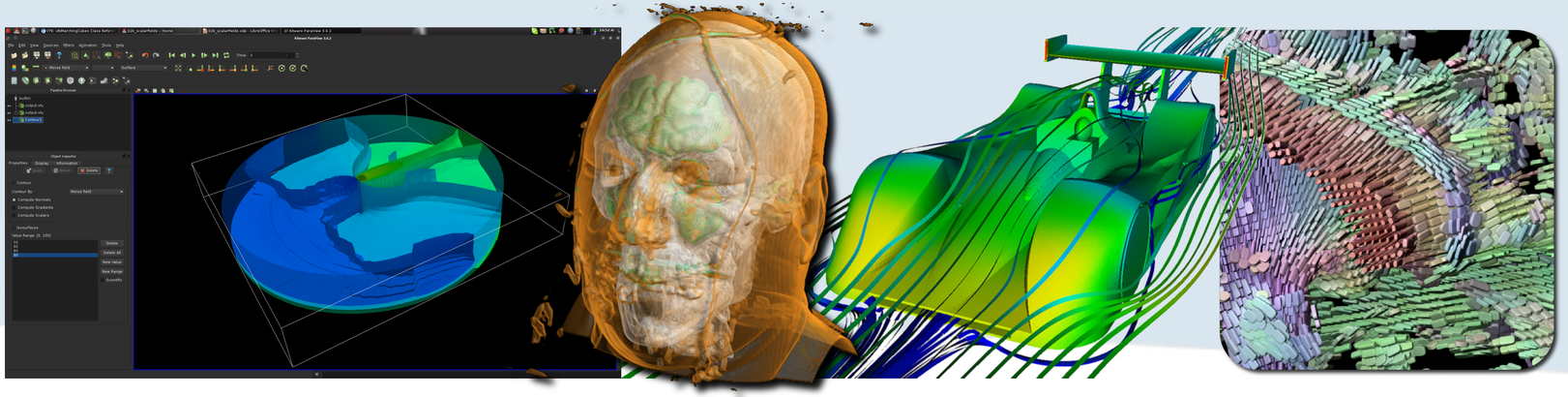
- Covered topics

What about?



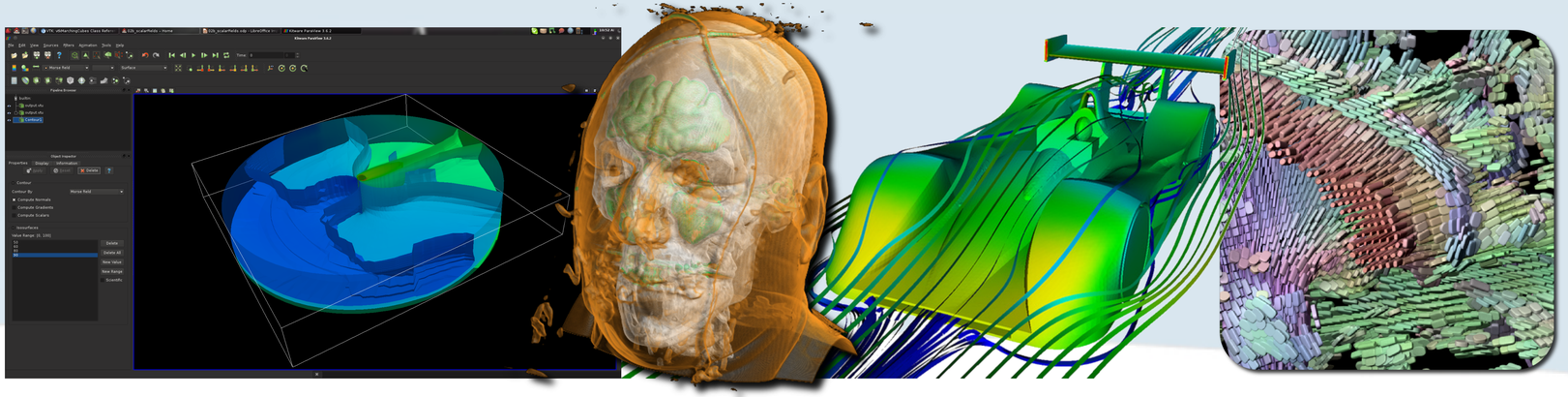
- Covered topics
  - 3D Rendering

What about?



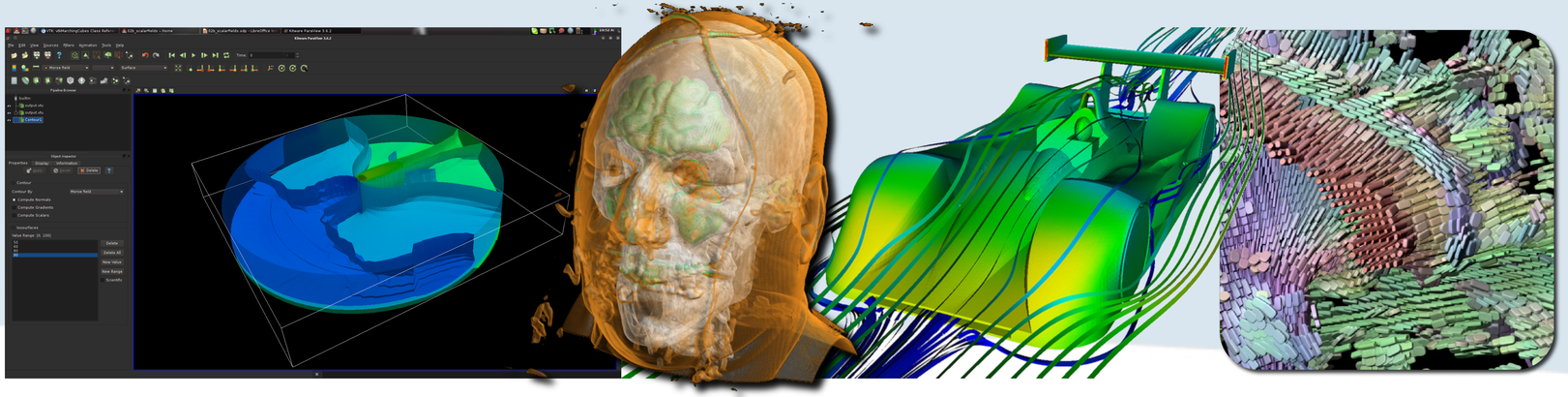
- Covered topics
  - 3D Rendering
  - Interactive systems

What about?



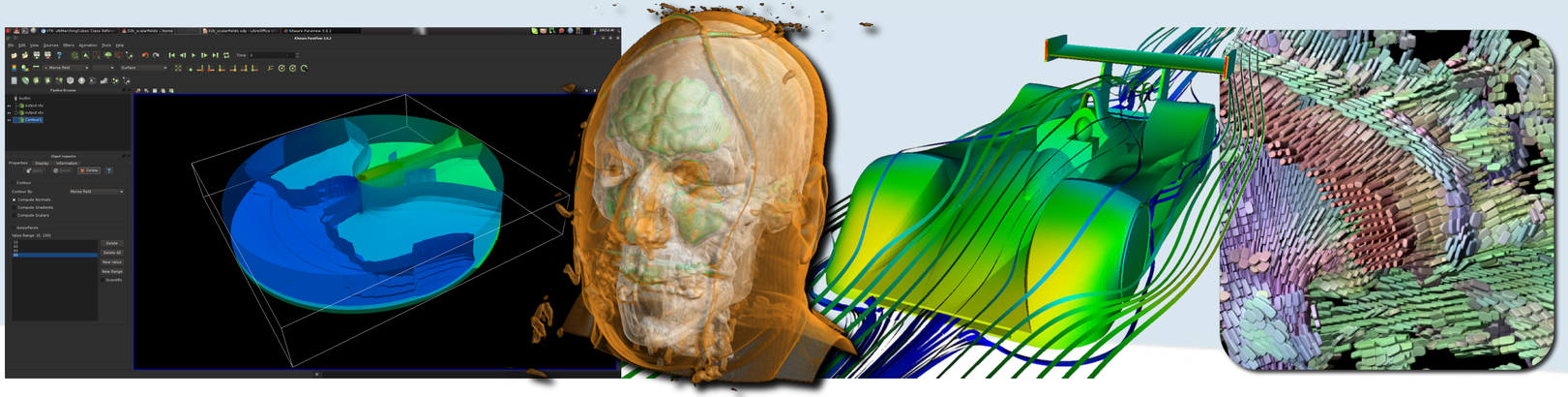
- Covered topics
  - 3D Rendering
  - Interactive systems
  - **Geometrical and topological analysis**

Who?



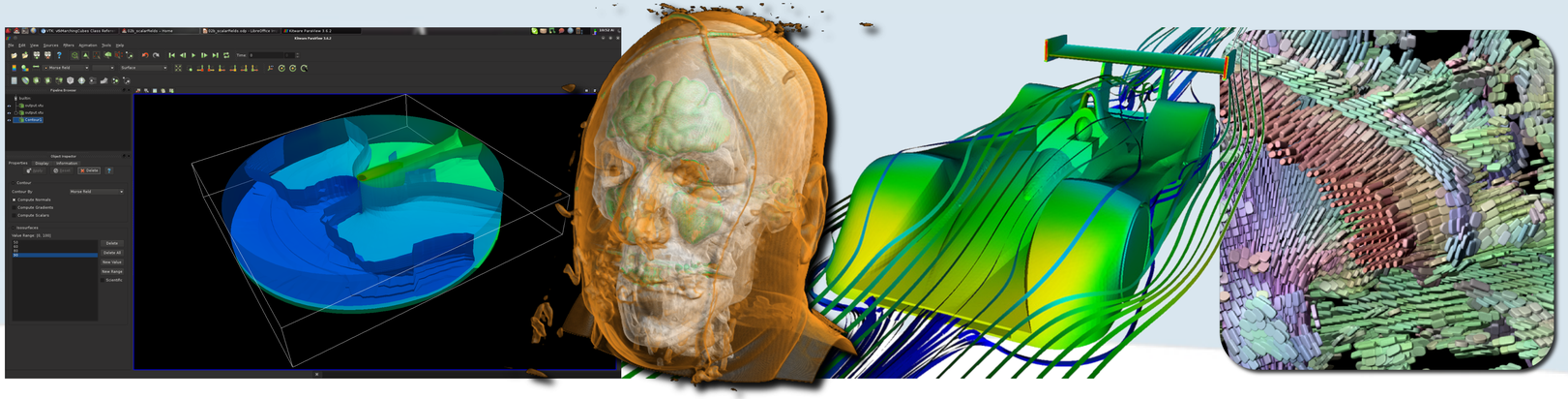
- Students targeting

Who?



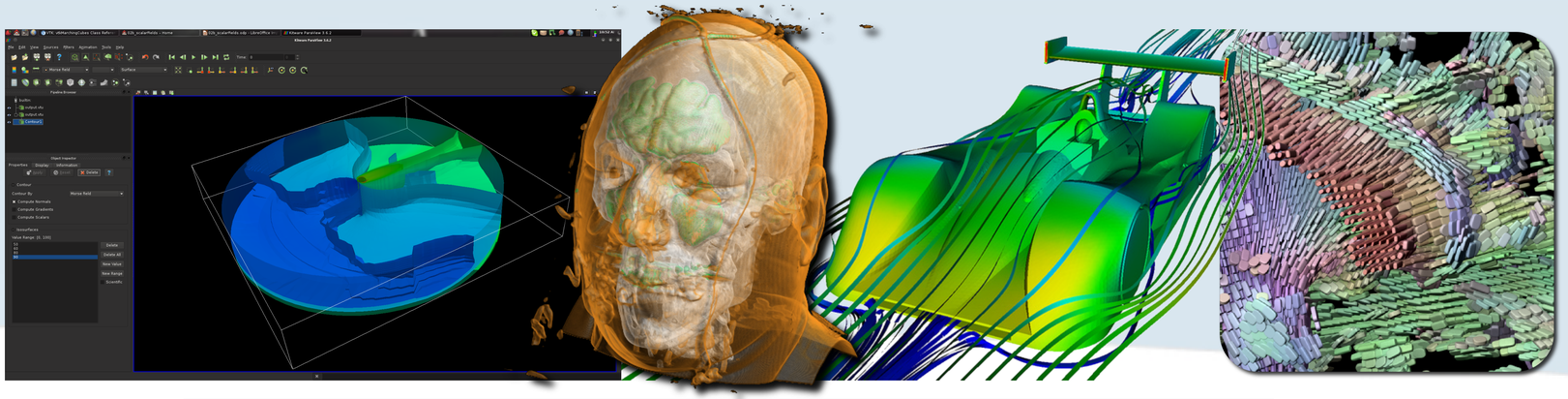
- Students targeting
  - R&D activities

Who?



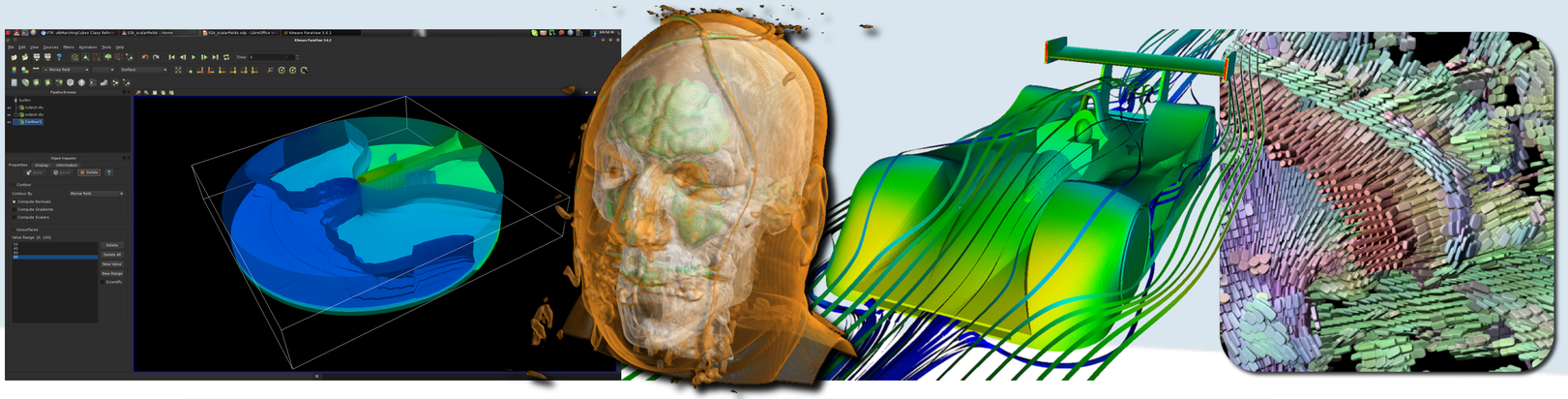
- Students targeting
  - R&D activities
- Students liking

Who?



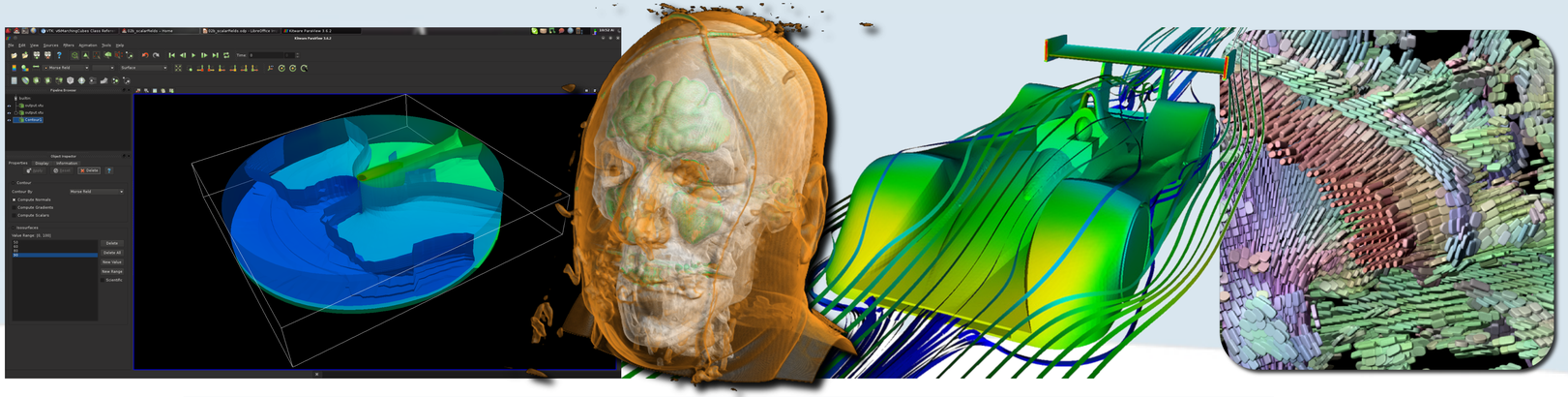
- Students targeting
  - R&D activities
- Students liking
  - Science

Who?



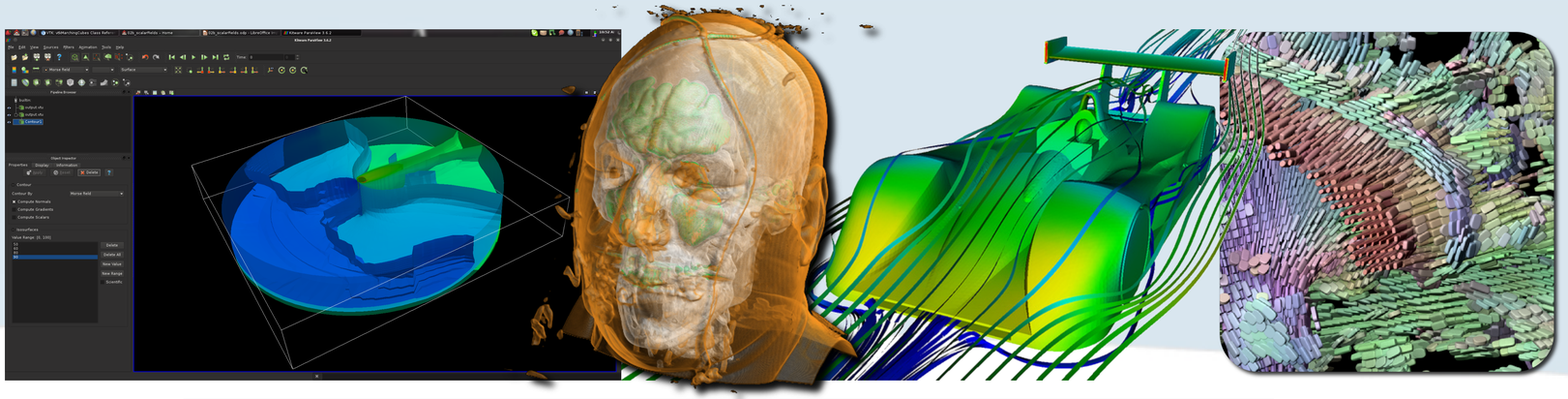
- Students targeting
  - R&D activities
- Students liking
  - Science
  - Geometry

Who?



- Students targeting
  - R&D activities
- Students liking
  - Science
  - Geometry
  - Coding

Who?



- Students targeting
  - R&D activities
- Students liking
  - Science
  - Geometry
  - Coding
  - Cool pictures

How?

How?

- First, follow this class

How?

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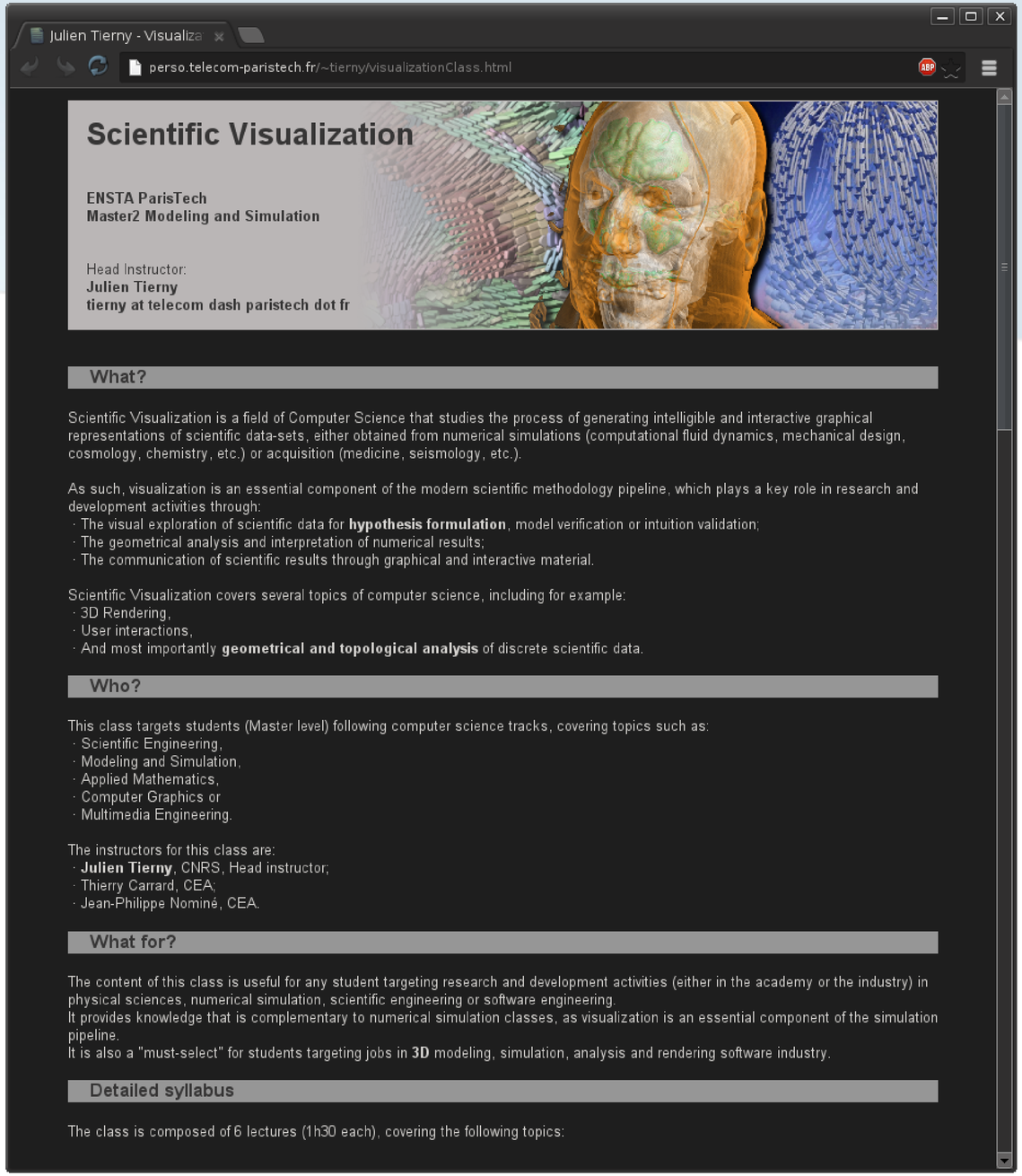

- Advanced class
  - http://www.telecom-paristech.fr/~tierny/visualizationClass.html

How?

- First, follow this class
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  - ENSTA ParisTech
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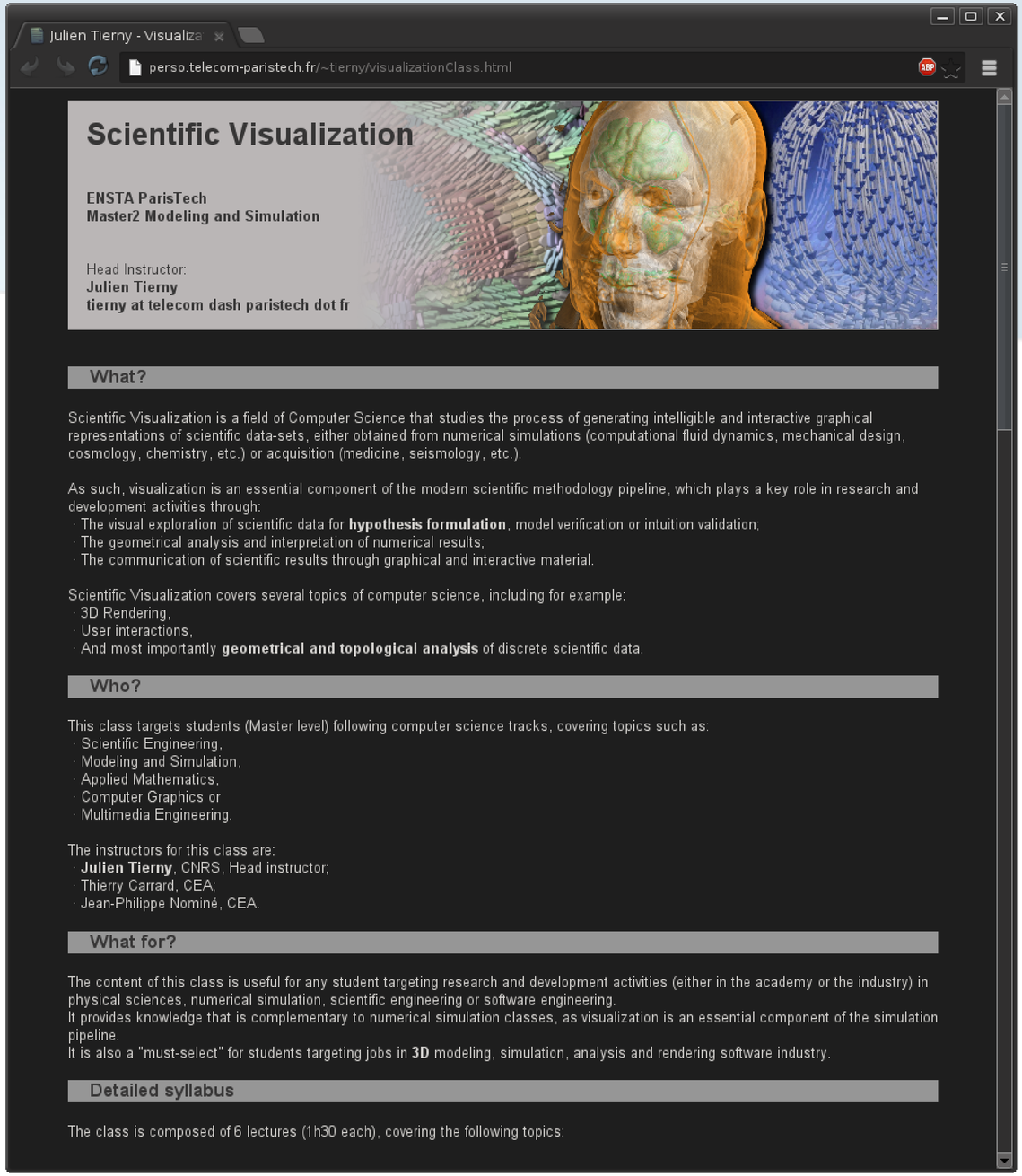

How?

- First, follow this class
- Modeling & Simulation Master
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- Projects, internships, Ph.D. thesis
- Advanced class
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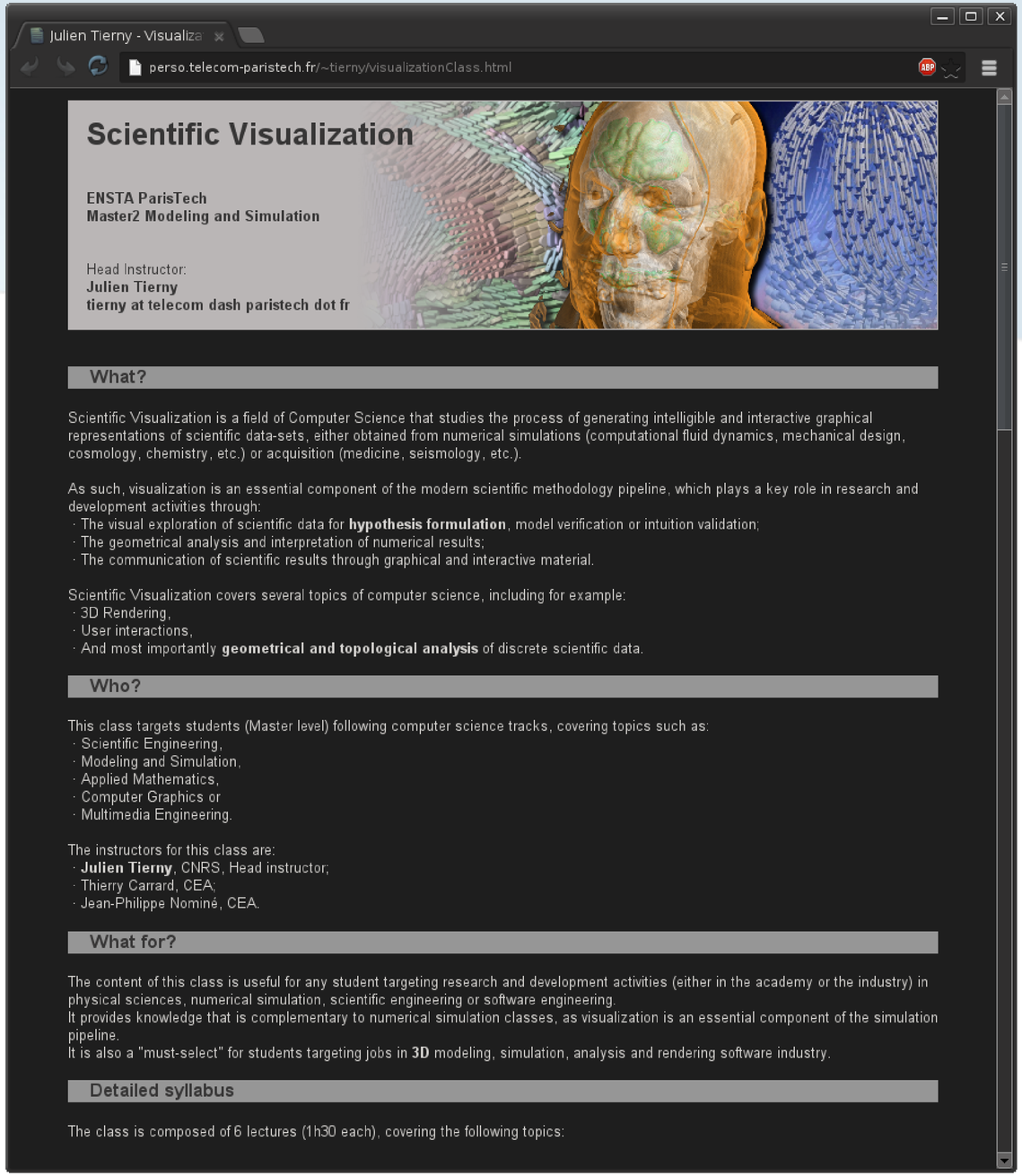

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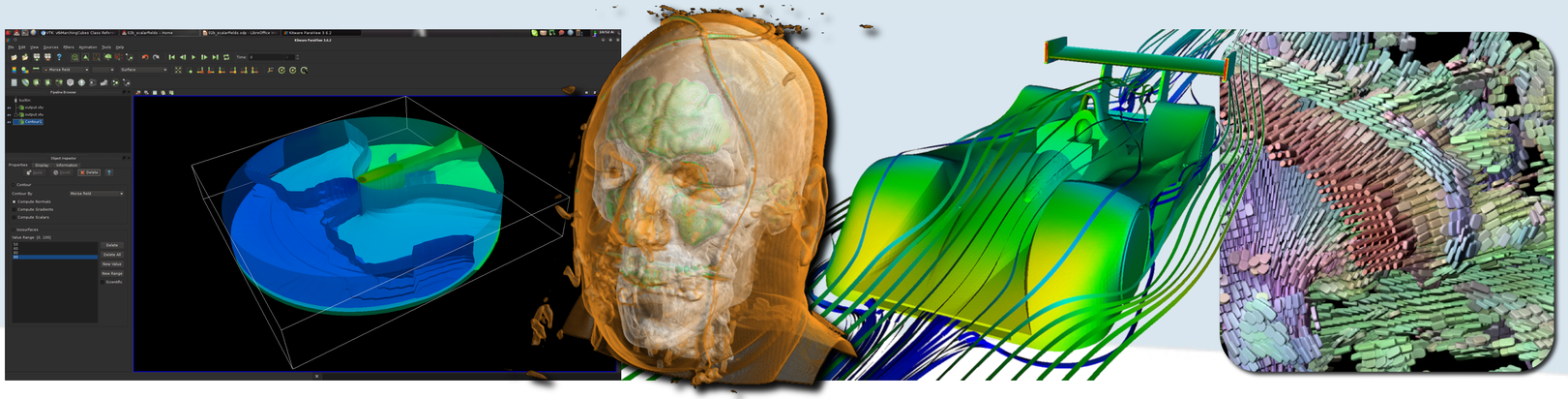
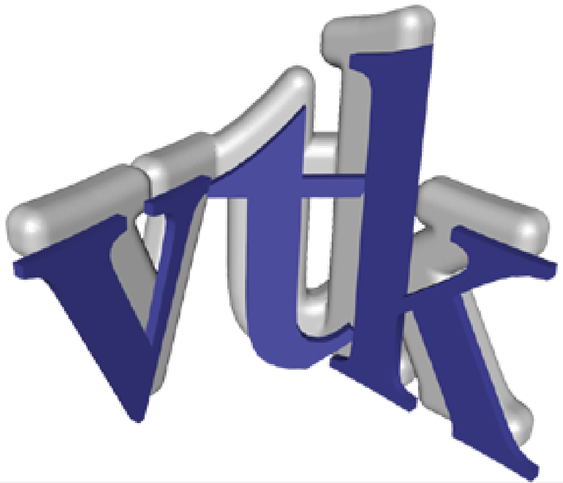
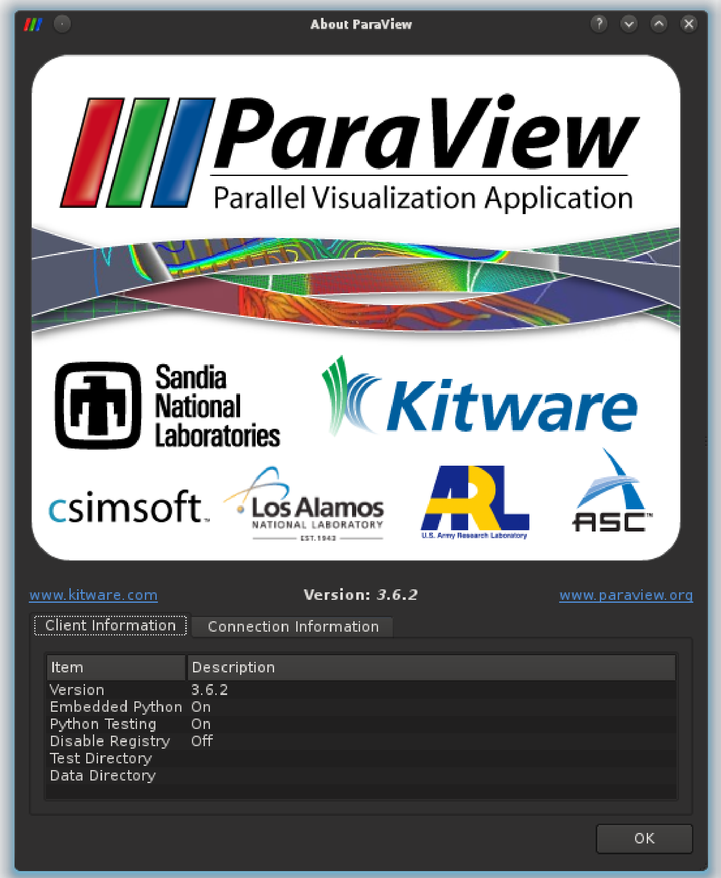


- Everything (well most of it) is already coded for you!





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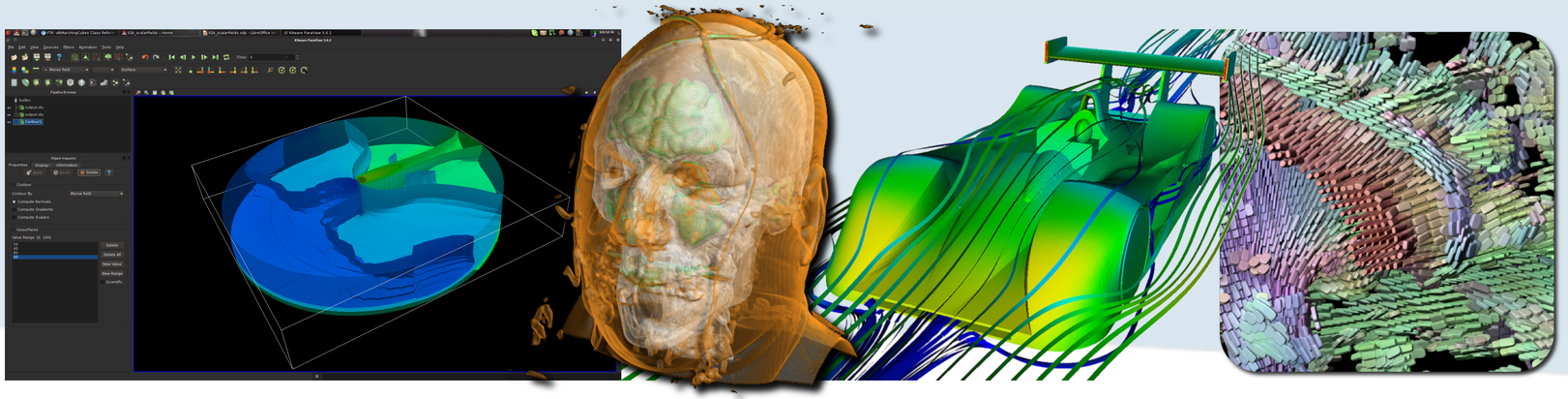
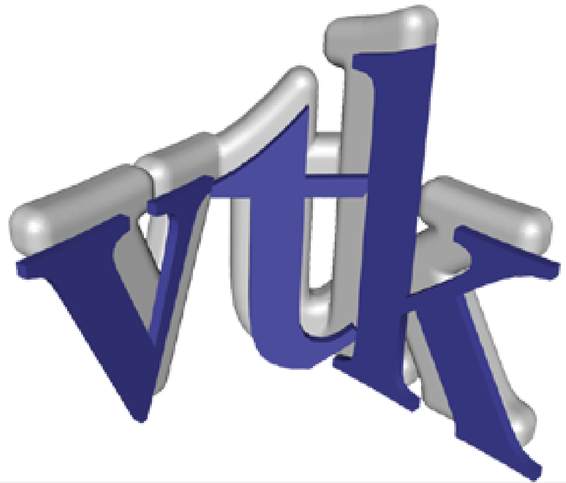
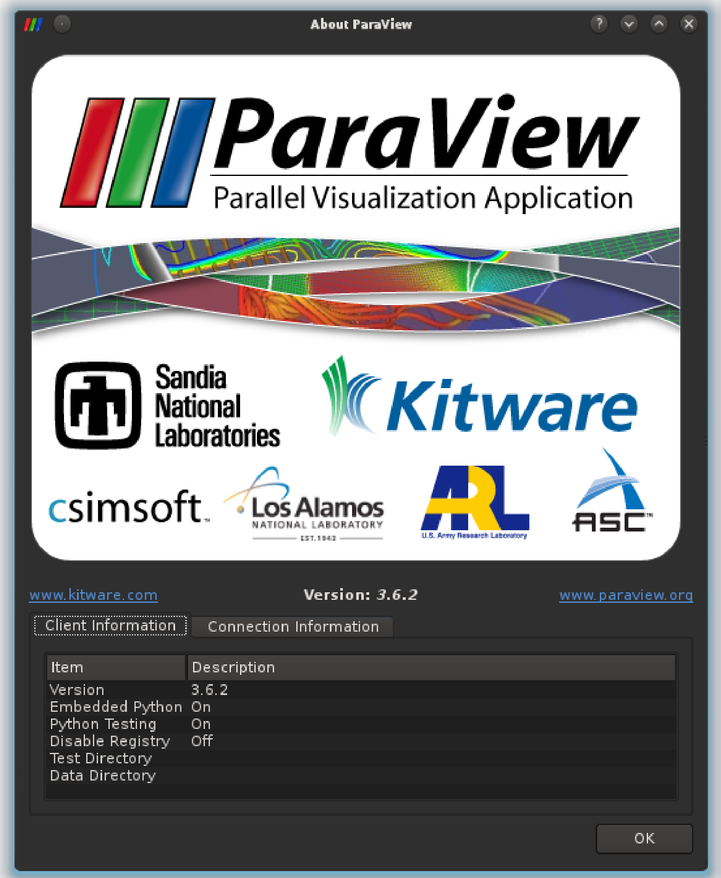


- Everything (well most of it) is already coded for you!
  - Visualization Tool Kit
    - Open Source C++ library
    - Started in 1993
    - Over a million lines of code
    - Tens of thousands of users





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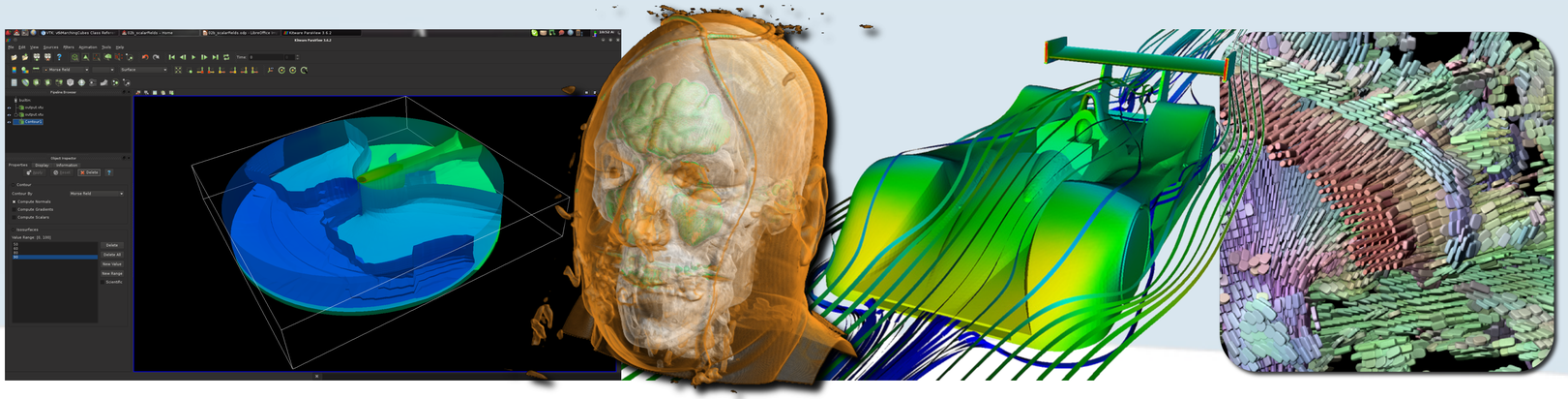

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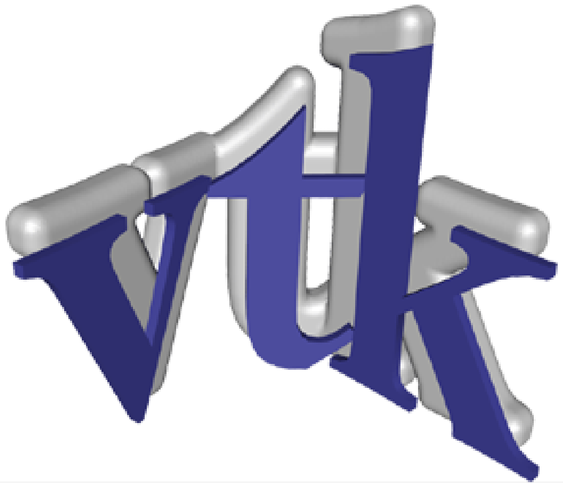
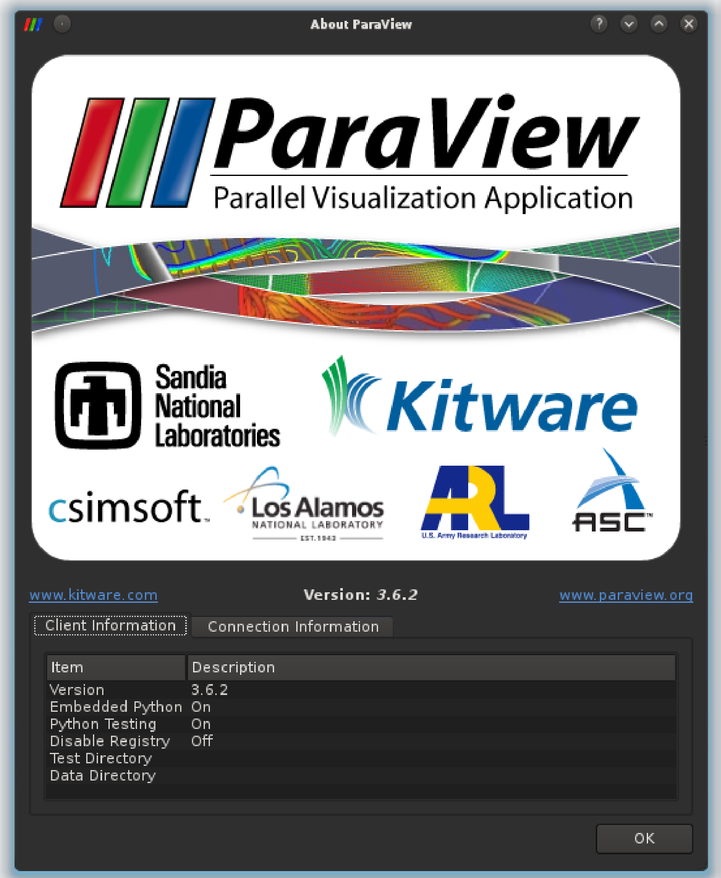
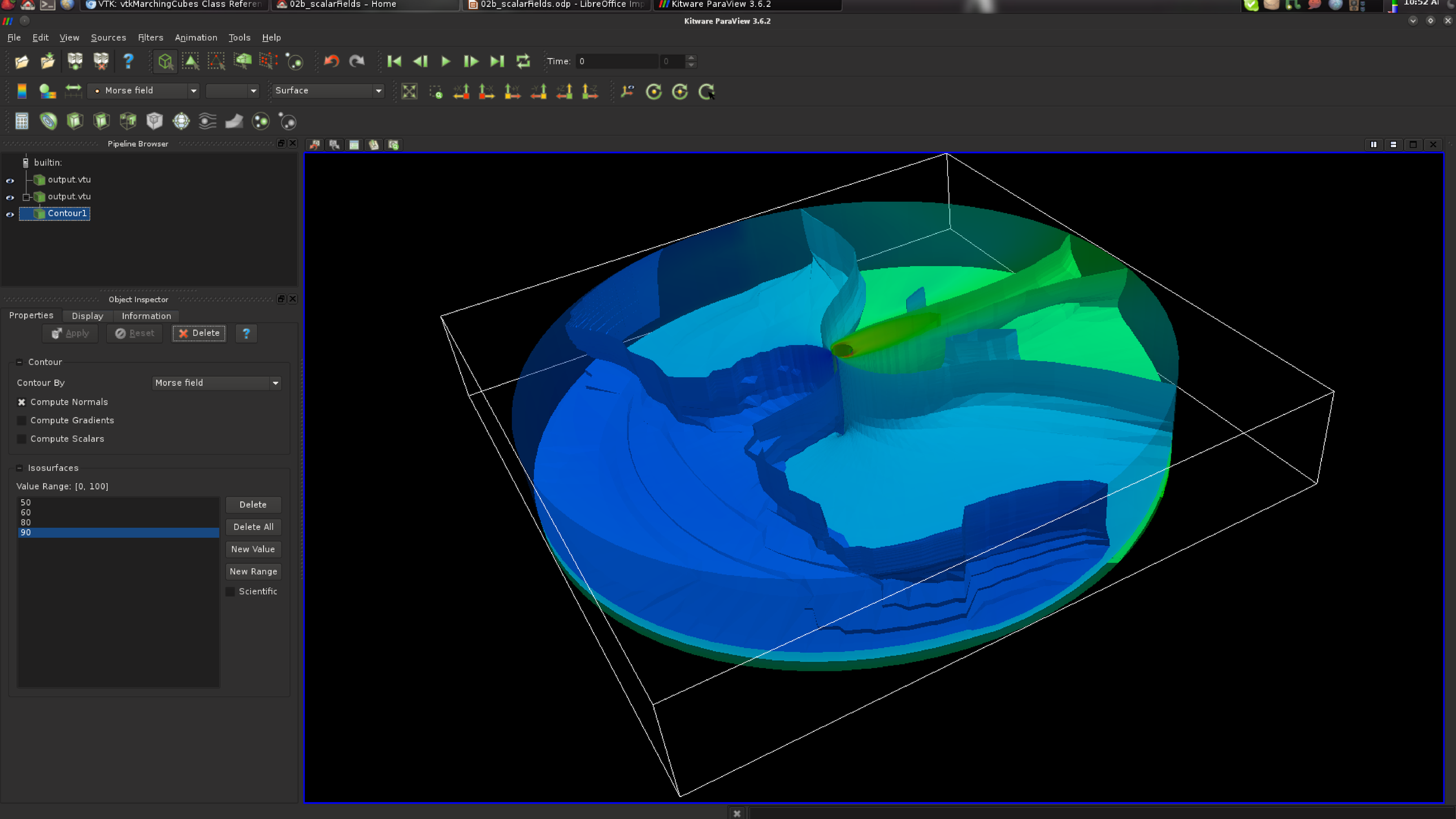
    - User interface front end



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Morse field
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Compute Gradients
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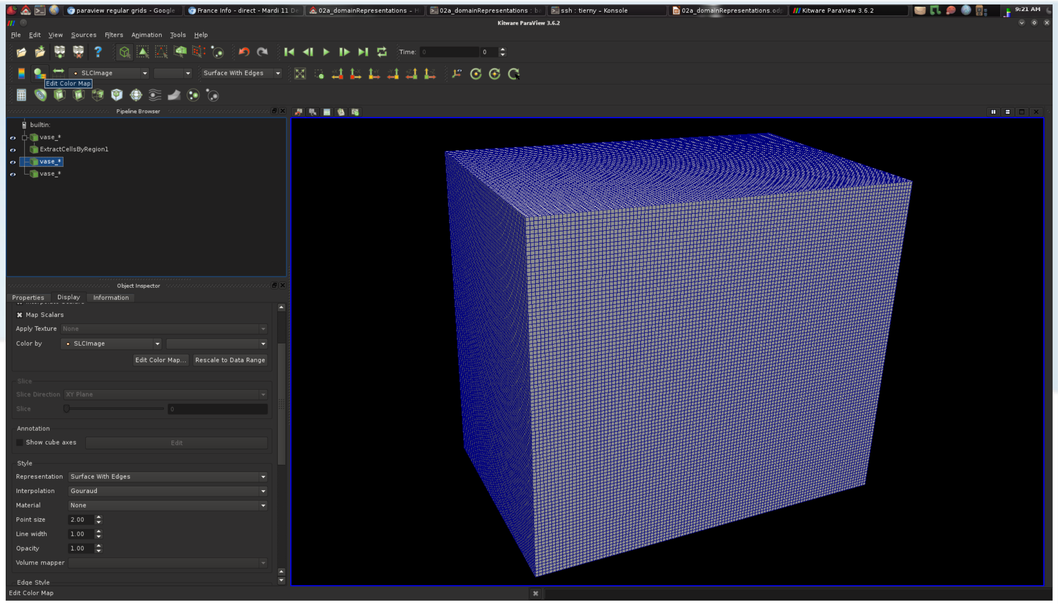
New Value
New Range
Scientific

How does it work?

- It all starts with a numerical domain

How does it work?



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  - Sub-set of euclidean spaces (1D, 2D, 3D, nD)
    - Regular Grids (pixels, voxels)

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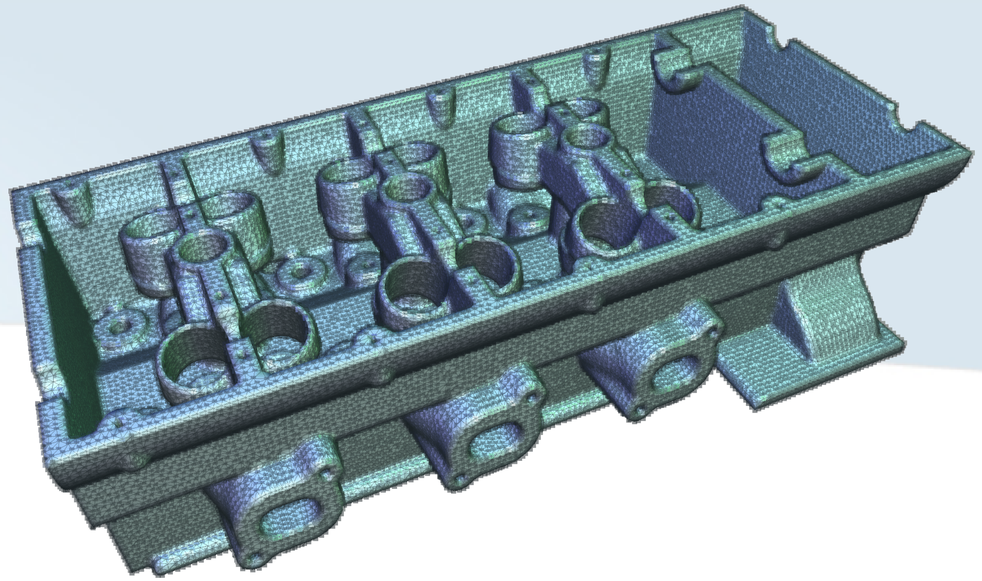
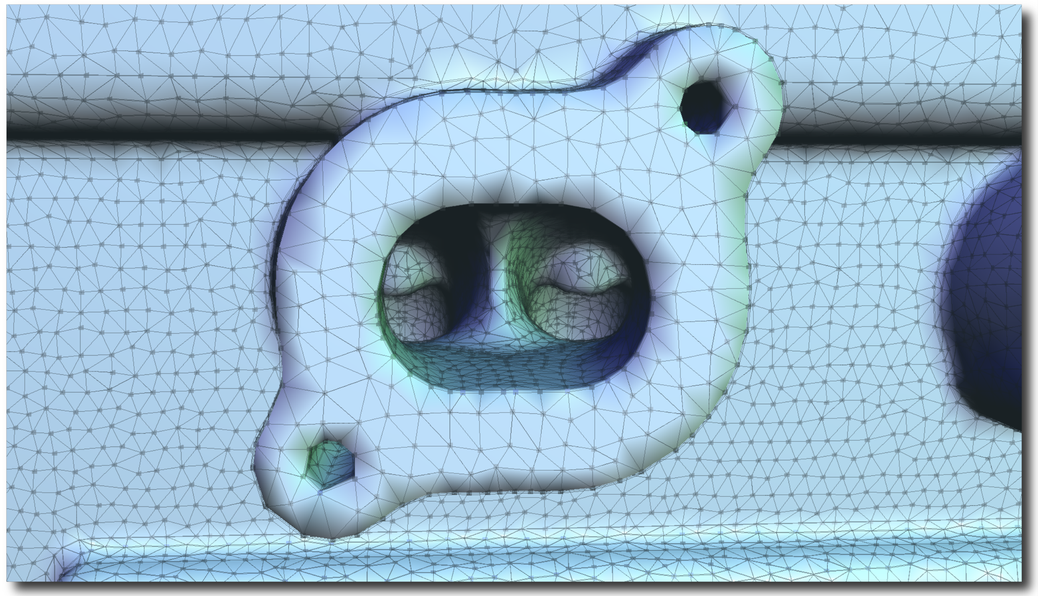


- It all starts with a numerical domain
  - Sub-set of euclidean spaces (1D, 2D, 3D, nD)
    - Regular Grids (pixels, voxels)
  - Non-euclidean spaces (1D, 2D, 3D, nD)
    - Notion of piecewise linear manifold
    - Triangle surface, Tetrahedral mesh



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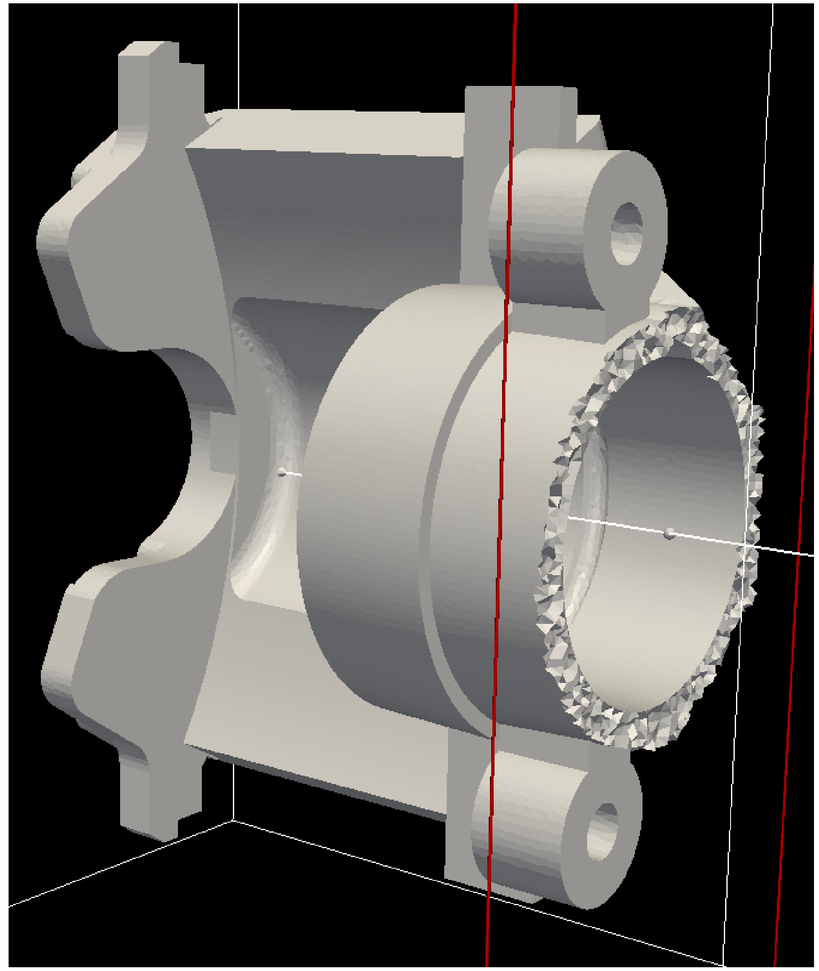
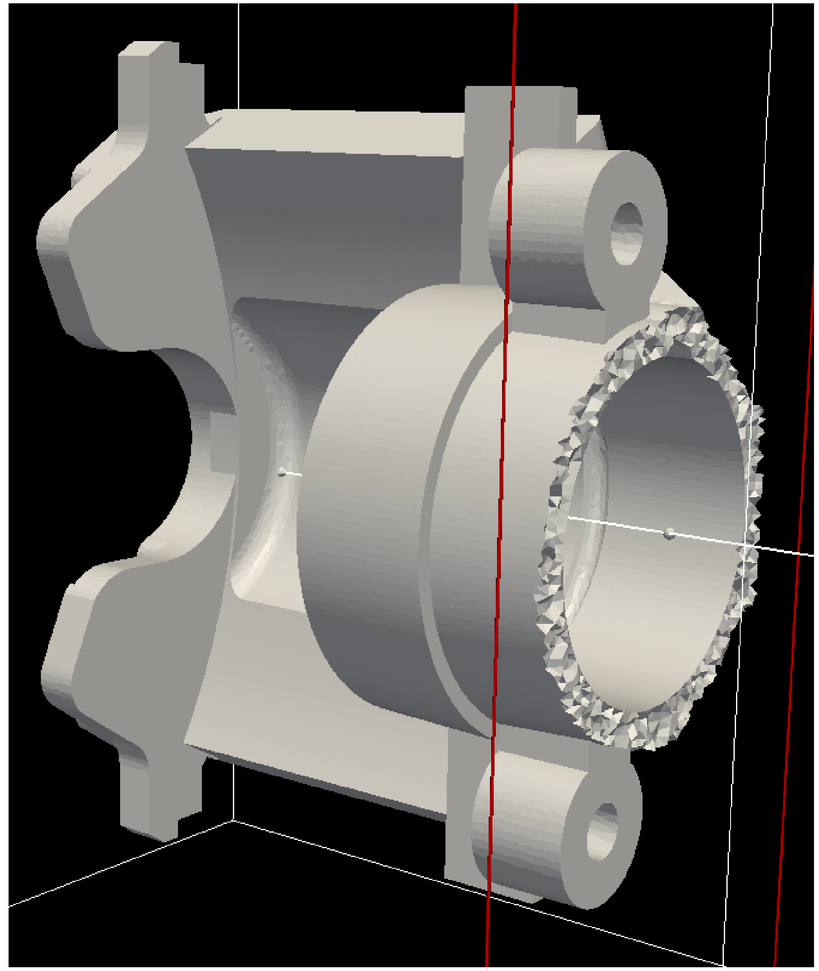


How does it work?

$$f : \mathcal{D} \rightarrow \mathcal{A}$$

- It all starts with a numerical domain
  - Sub-set of euclidean spaces (1D, 2D, 3D, nD)
    - Regular Grids (pixels, voxels)
  - Non-euclidean spaces (1D, 2D, 3D, nD)
    - Notion of piecewise linear manifold
    - Triangle surface, Tetrahedral mesh
- Simulation or acquisition
  - Compute **something** on the domain



How does it work?

$$f : \mathcal{D} \rightarrow \mathcal{A}$$

How does it work?

$f : \mathcal{D} \rightarrow \mathcal{A}$

- $f : \mathcal{D} \rightarrow \mathbb{R}$
  - Scalar field visualization

How does it work?

$f : \mathcal{D} \rightarrow \mathcal{A}$

- $f : \mathcal{D} \rightarrow \mathbb{R}$
  - Scalar field visualization
- $f : \mathcal{D} \rightarrow \mathcal{T}(\mathcal{D})$
  - Vector field visualization

How does it work?

$f : \mathcal{D} \rightarrow \mathcal{A}$

- $f : \mathcal{D} \rightarrow \mathbb{R}$
  - Scalar field visualization
- $f : \mathcal{D} \rightarrow \mathcal{T}(\mathcal{D})$
  - Vector field visualization
- $f : \mathcal{D} \rightarrow \mathbb{M}_{d \times d}$
  - Tensor field visualization

How does it work?

$$f : \mathcal{D} \to \mathcal{A}$$

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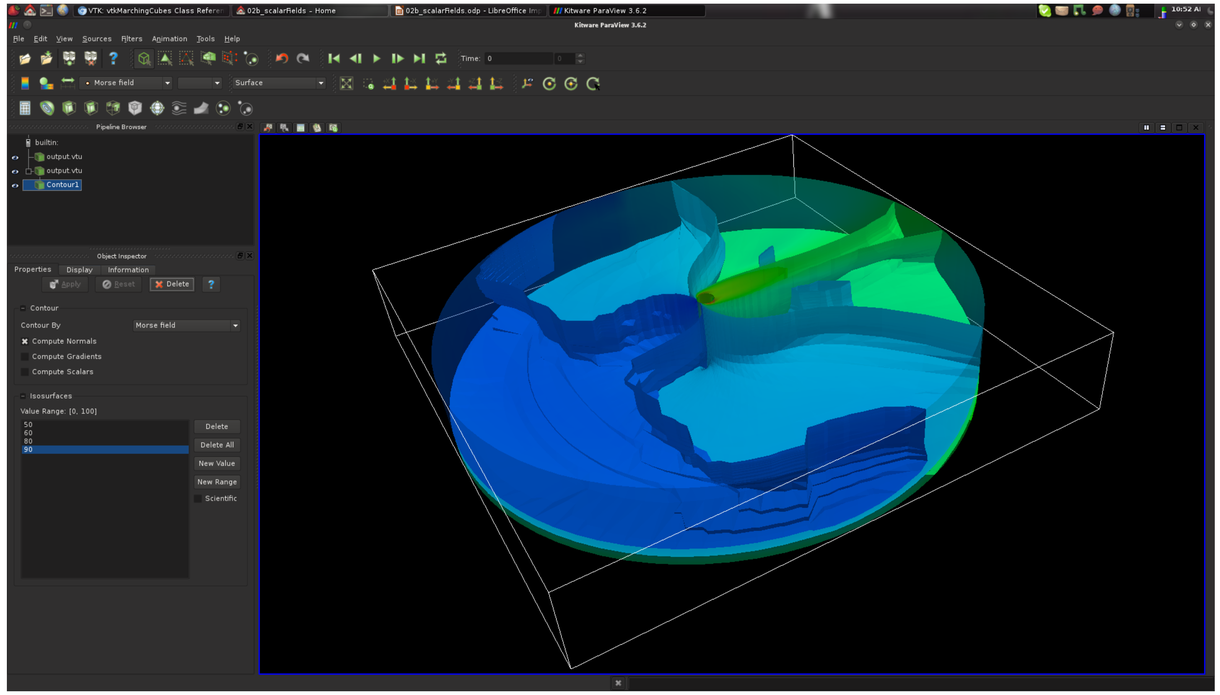

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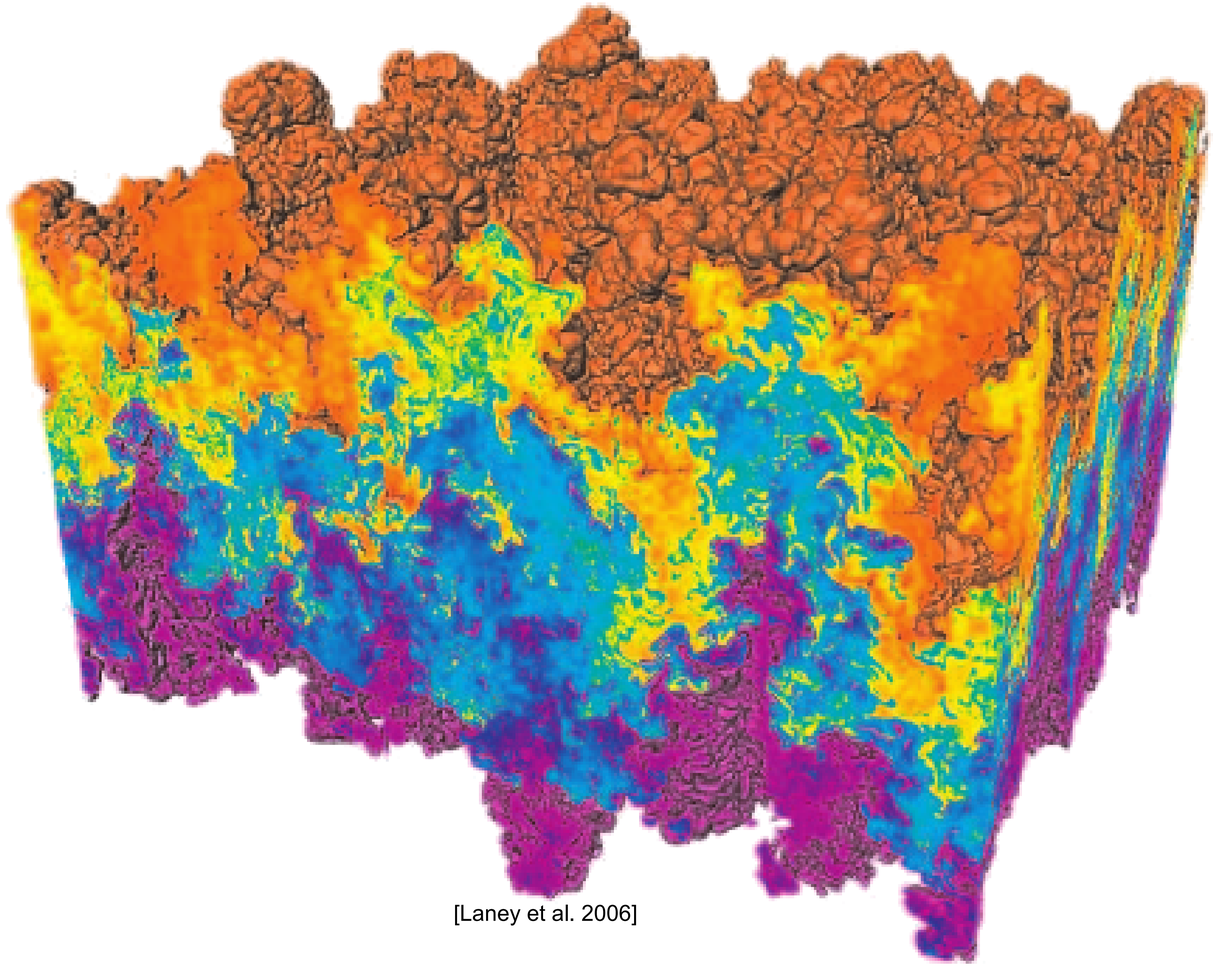

[Laney et al. 2006]

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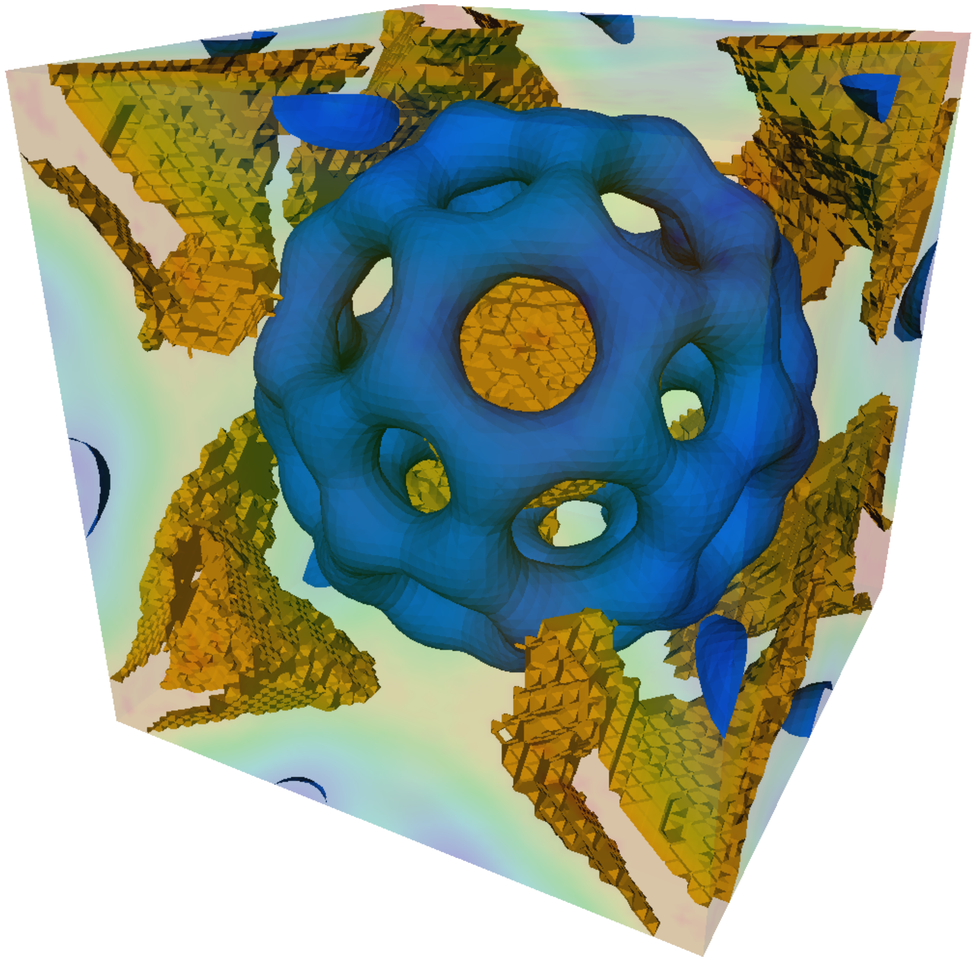

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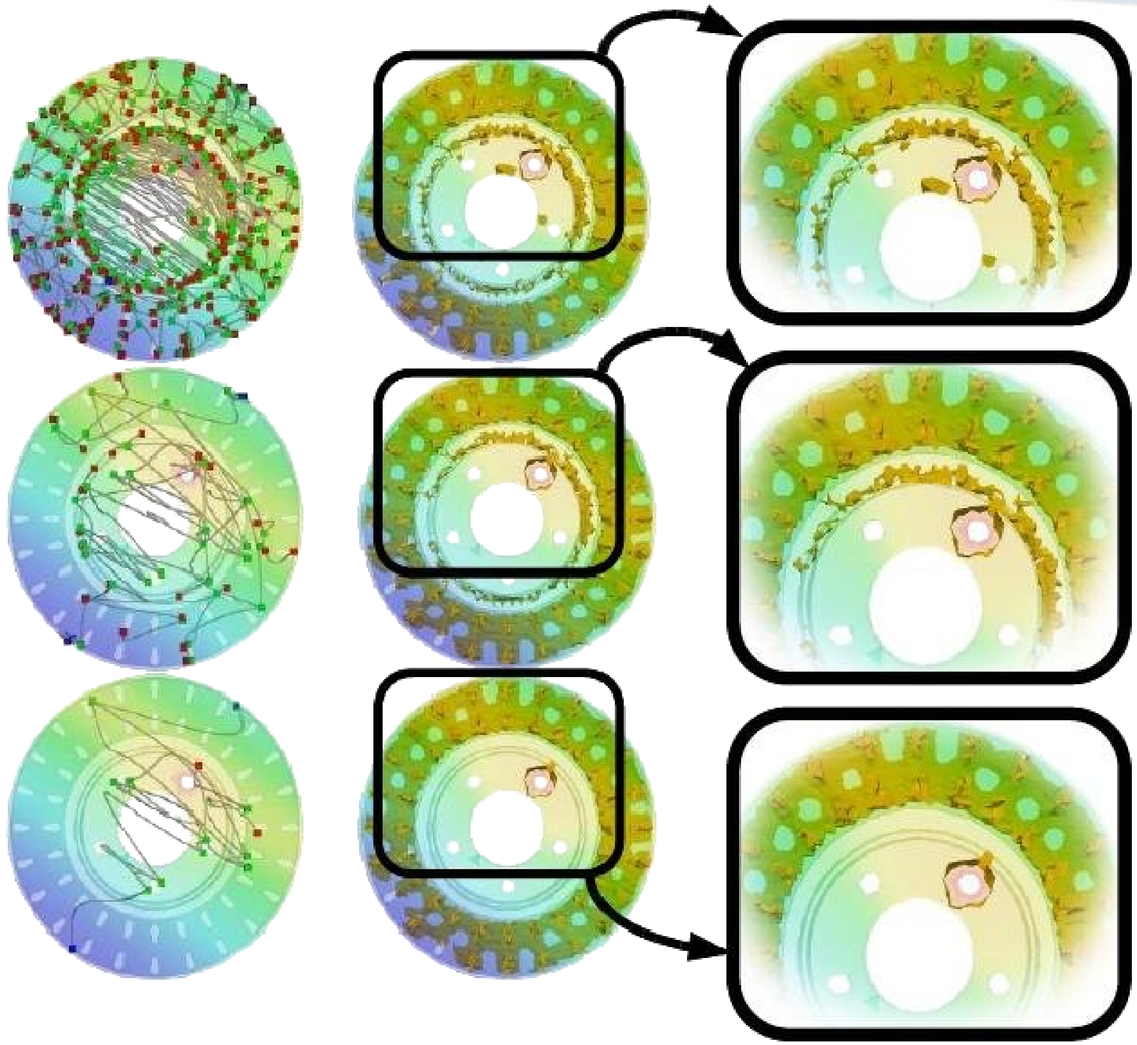

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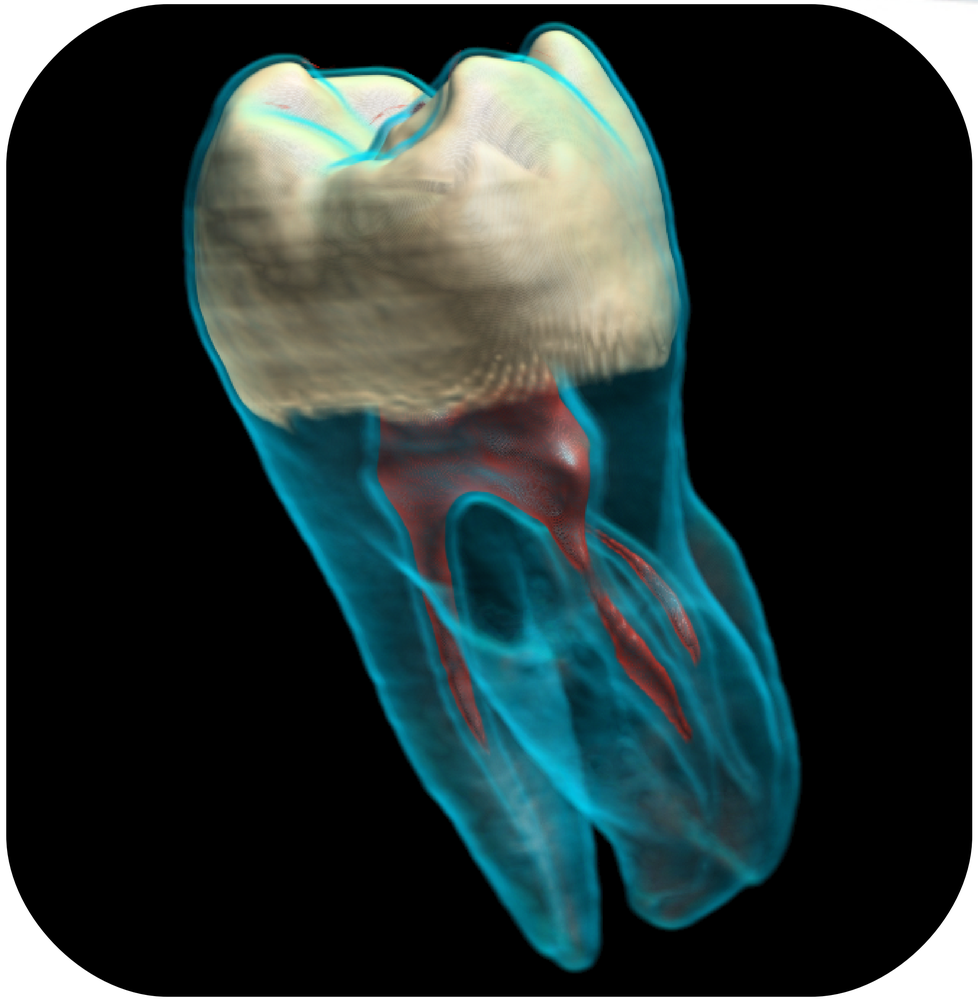

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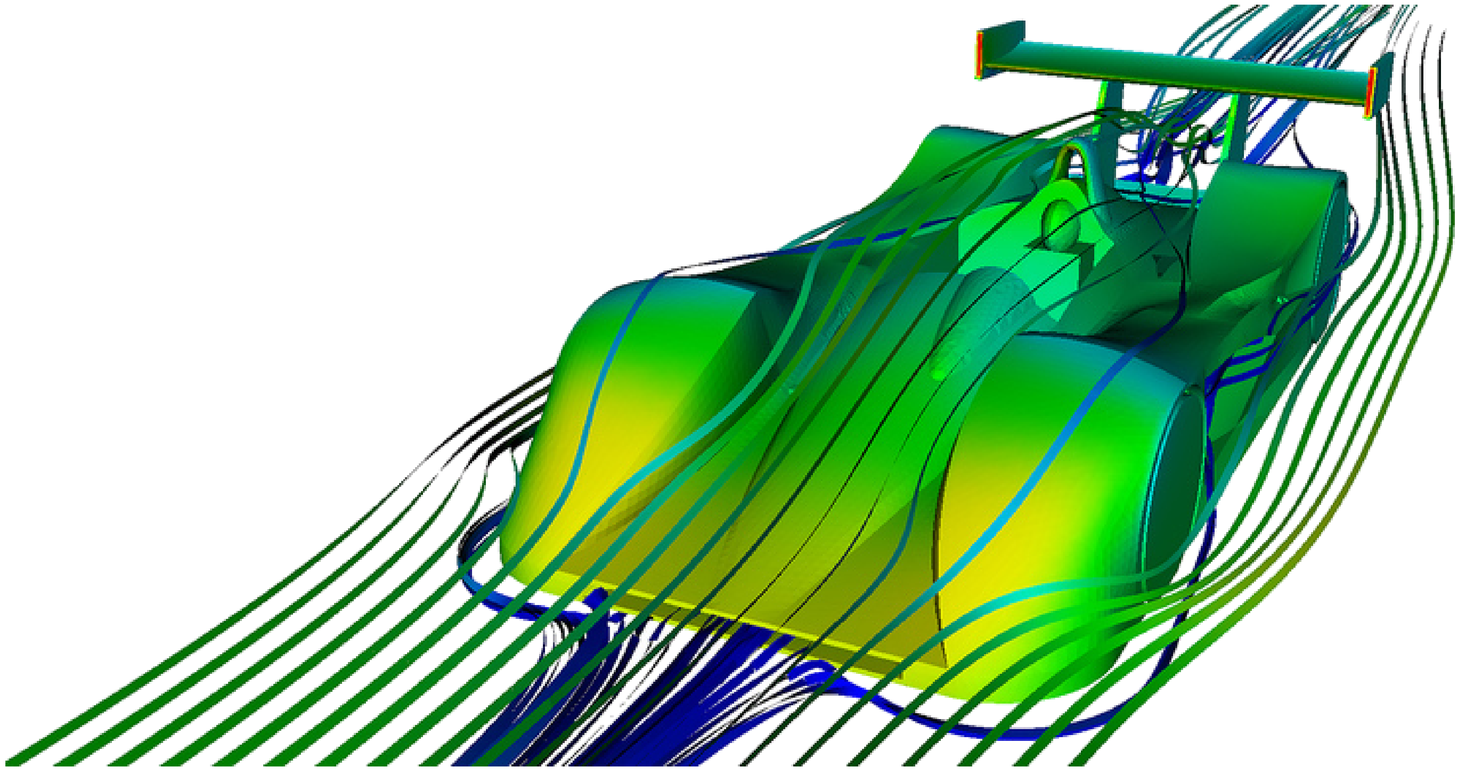

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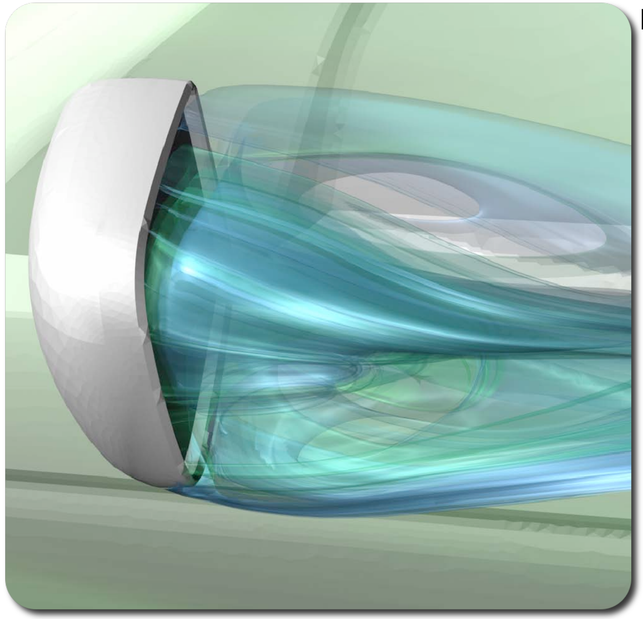

[GarthVIS08]

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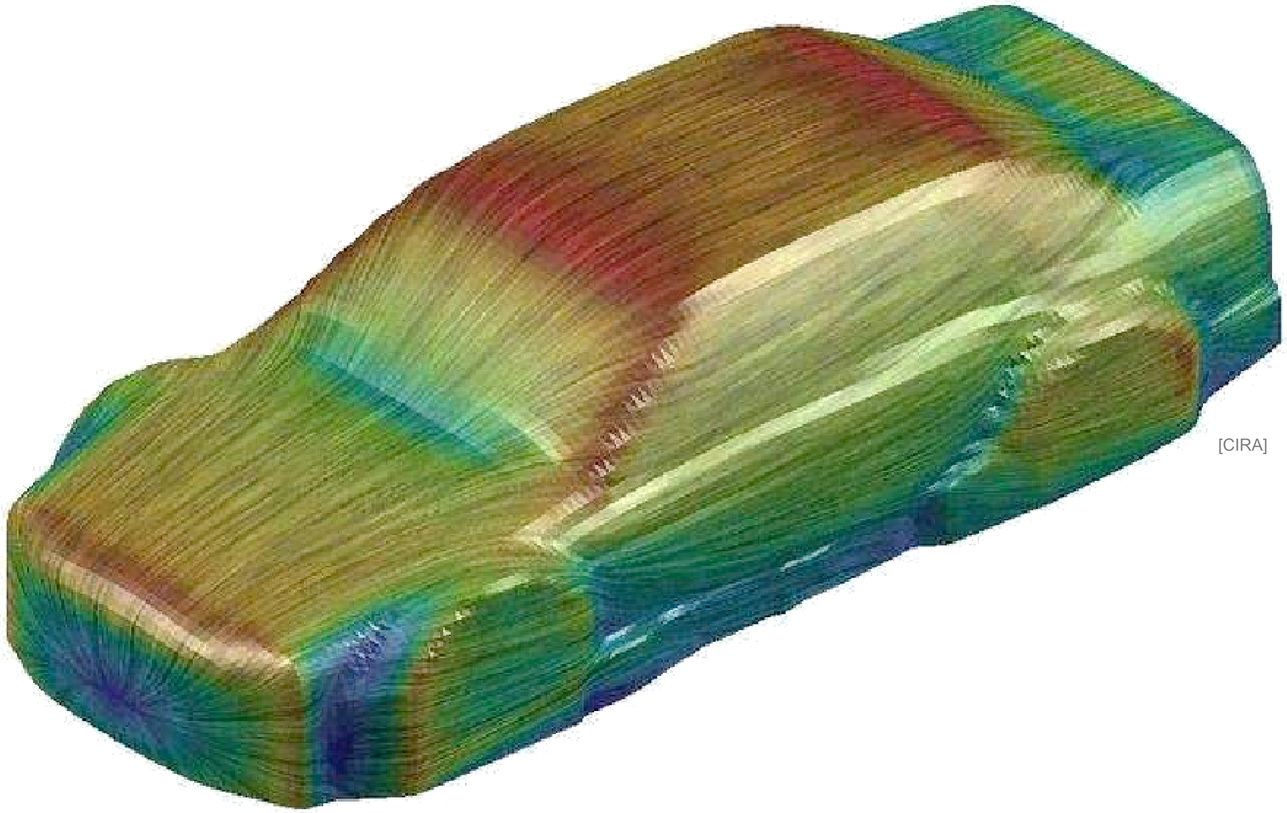

[CIRA]

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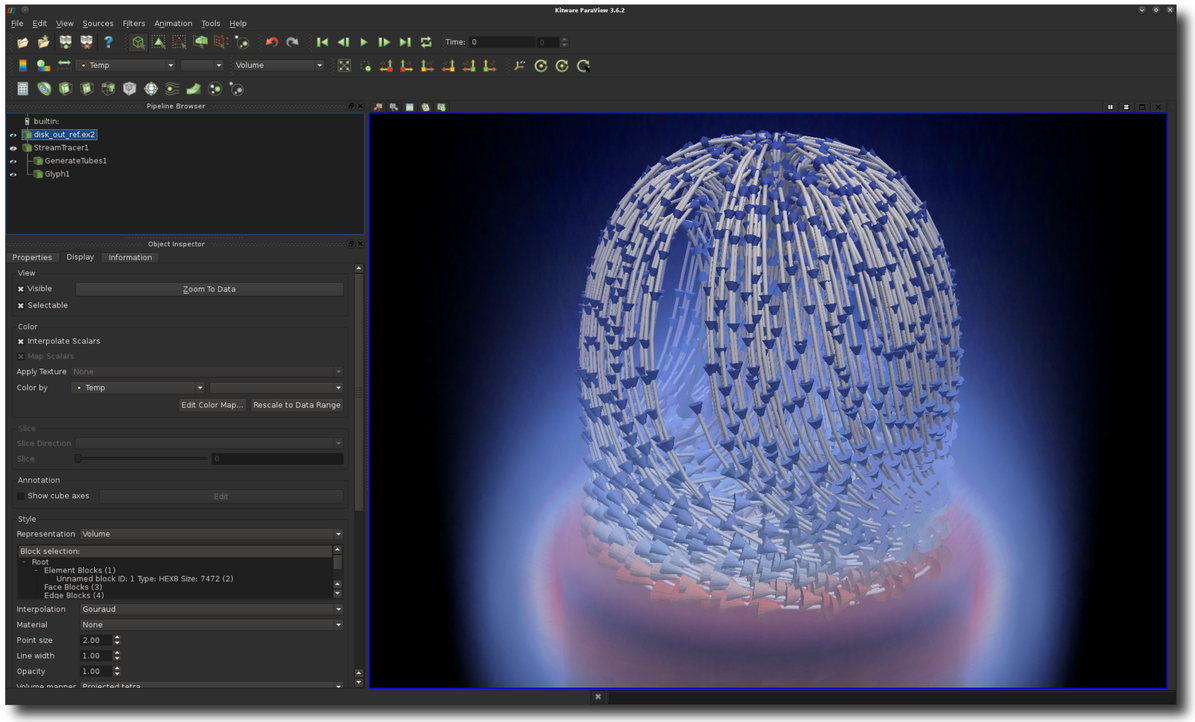

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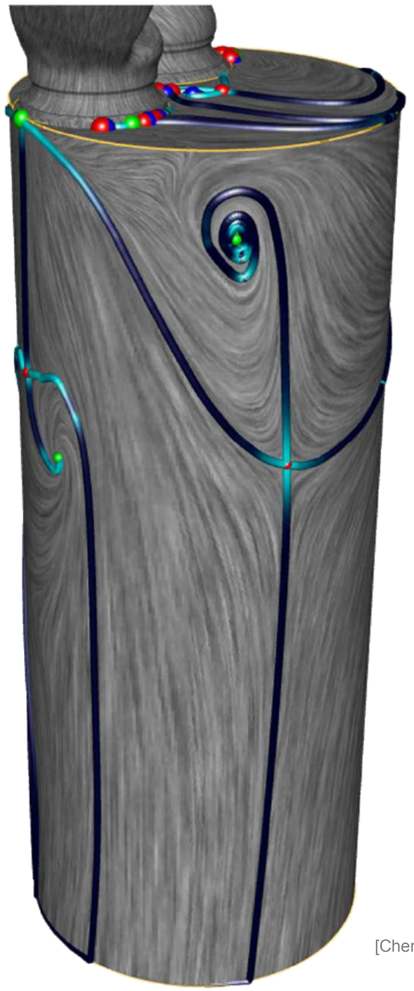

[Chen]

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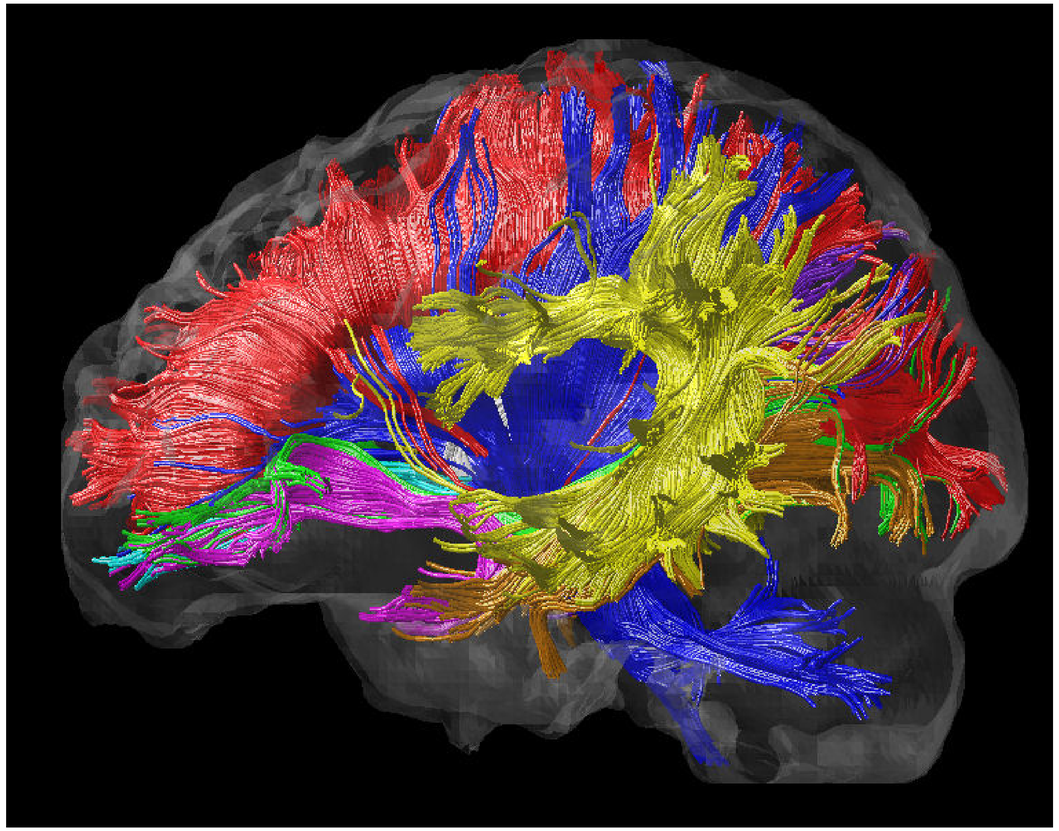

[http://www.neuroimaging.tau.ac.il]

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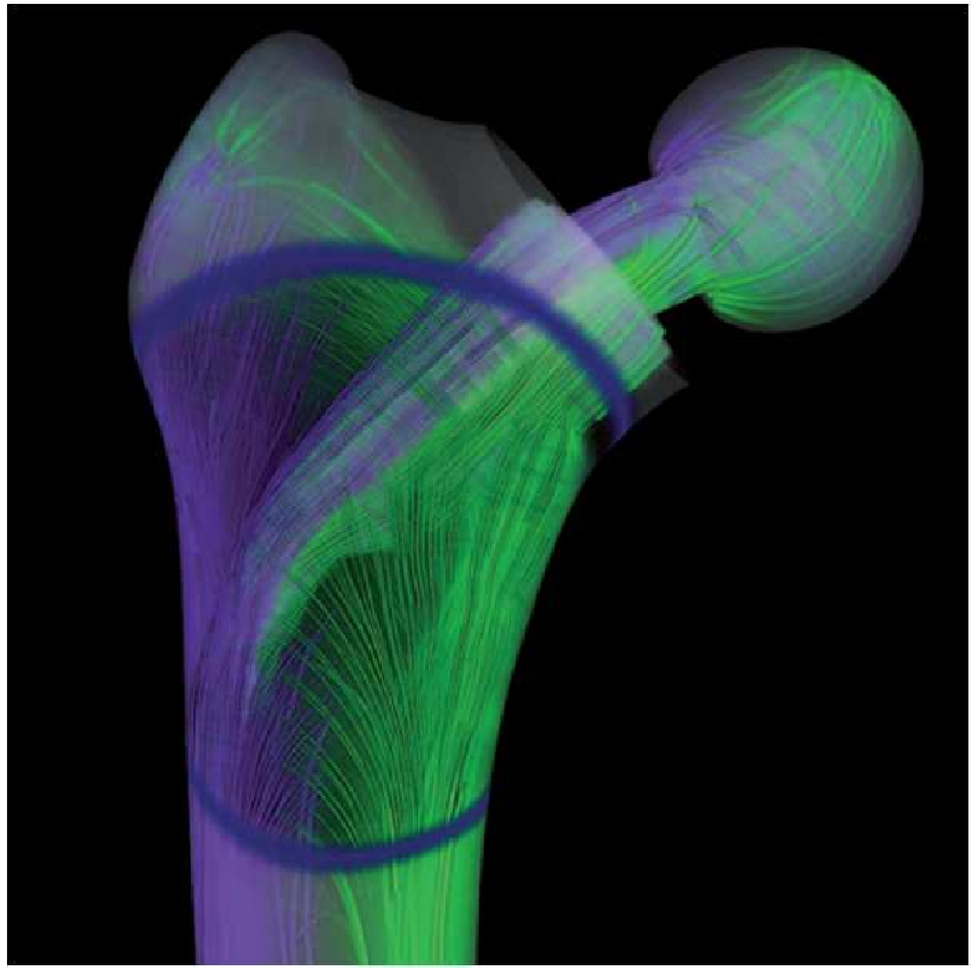

[Dick et al 09]

Challenges

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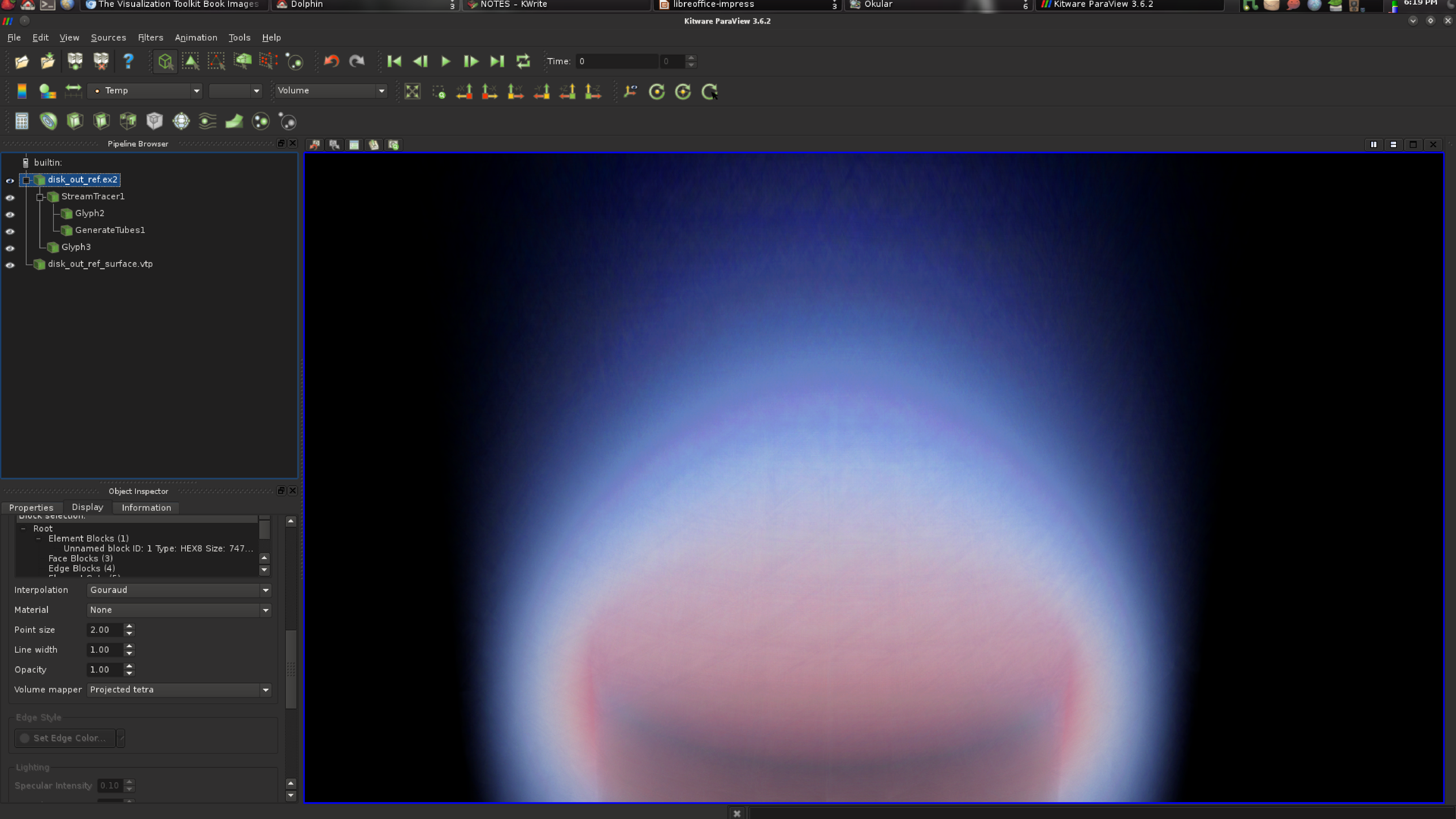
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Dolphin
NOTES - KWrite
libreoffice-impress
Okular
Kitware ParaView 3.6.2
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File Edit View Sources Filters Animation Tools Help
Time: 0
Temp
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Pipeline Browser
builtin:
disk_out_ref.ex2
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Object Inspector
Properties Display Information
Root
Element Blocks (1)
Unnamed block ID: 1 Type: HEX8 Size: 747...
Face Blocks (3)
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Interpolation Gouraud
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Line width 1.00
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Edge Style
Set Edge Color...
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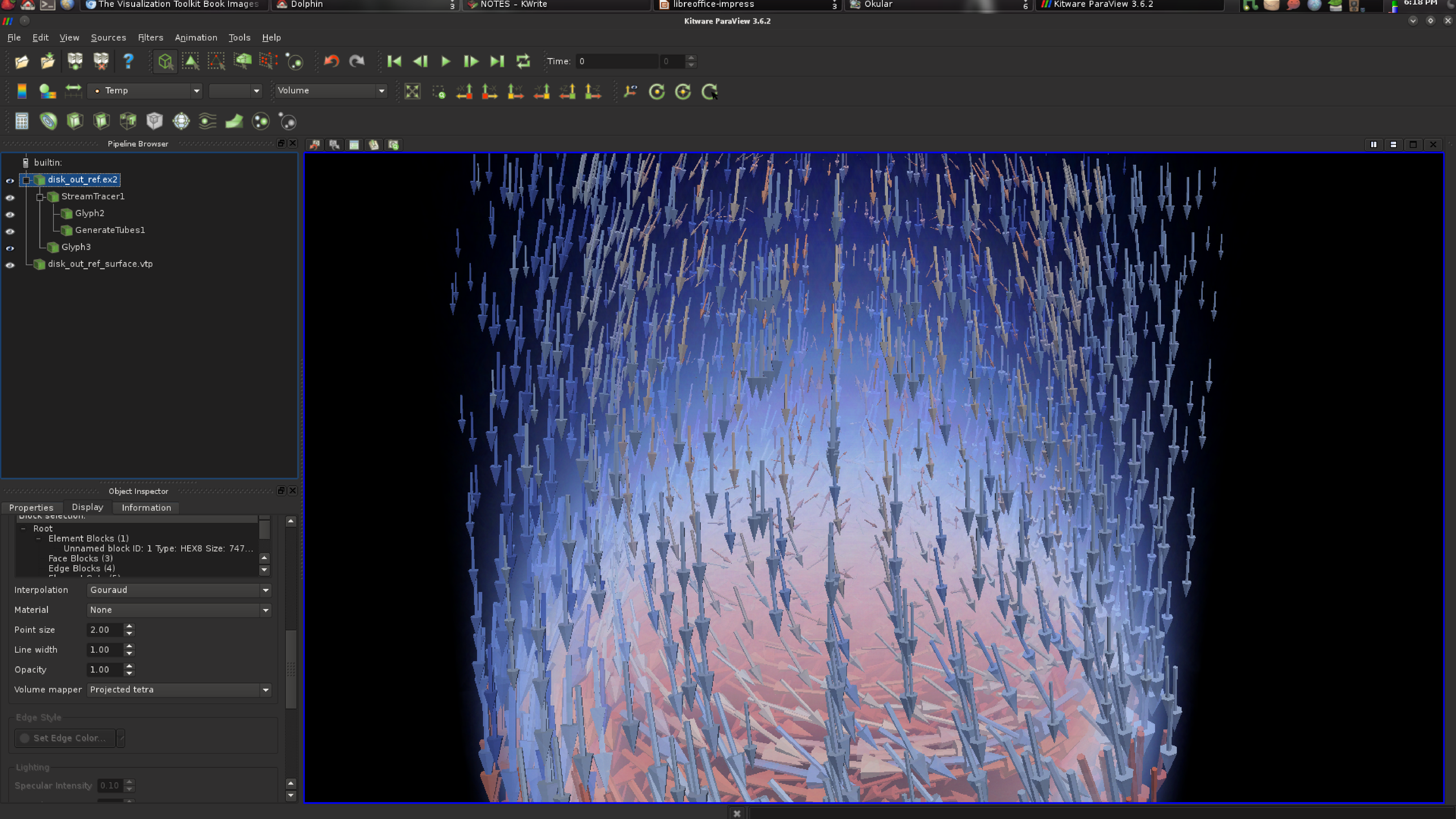
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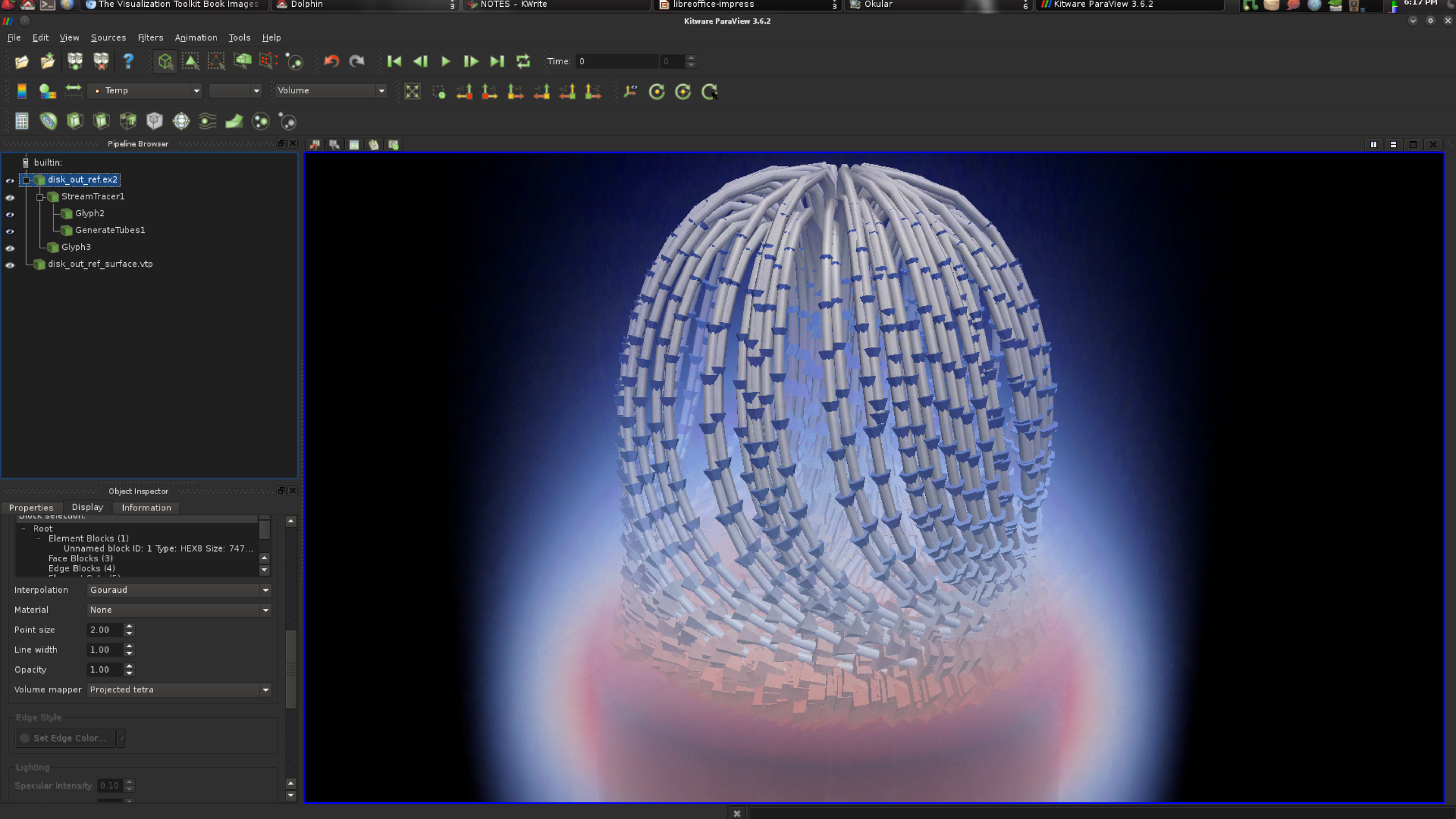
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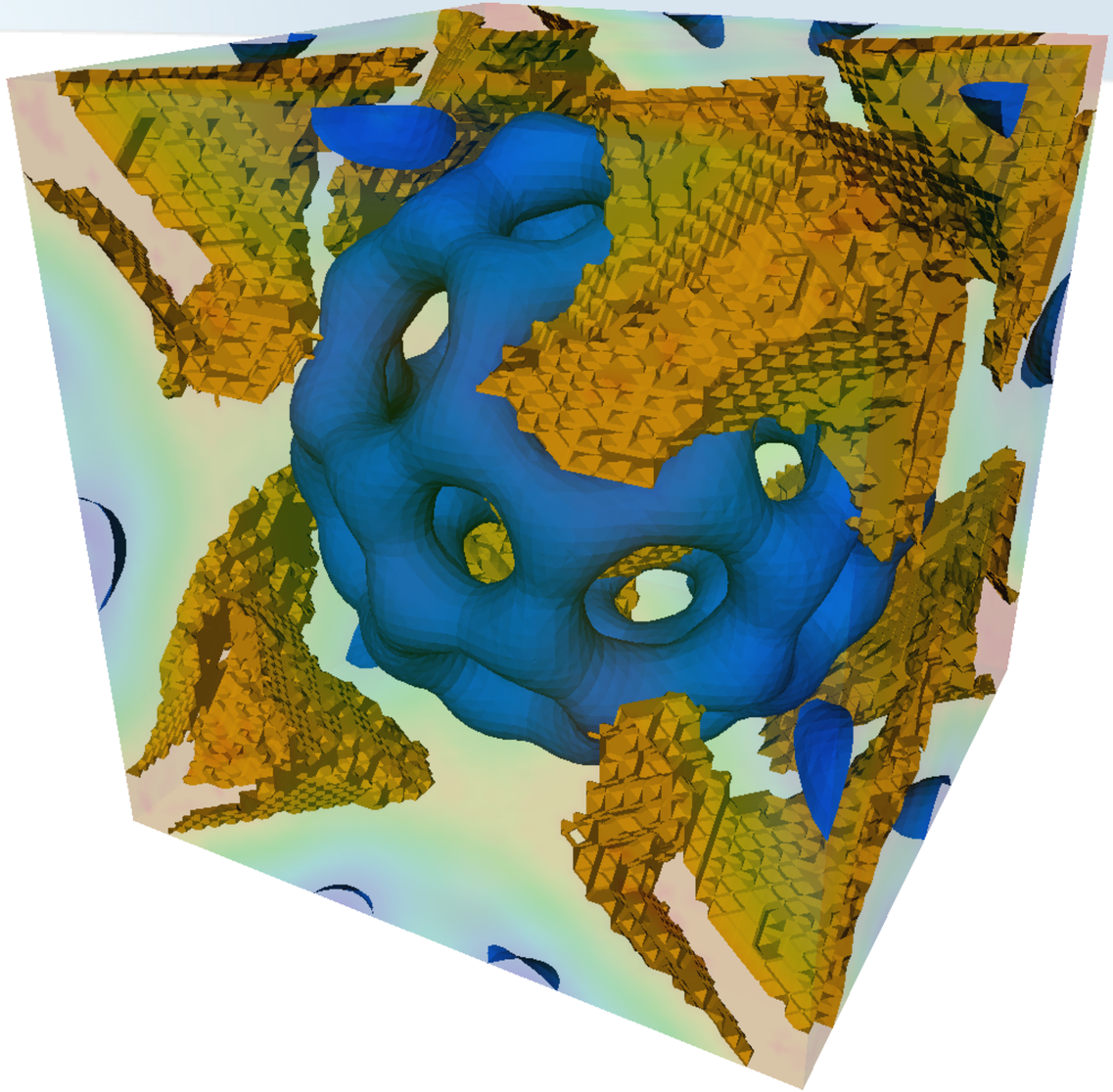

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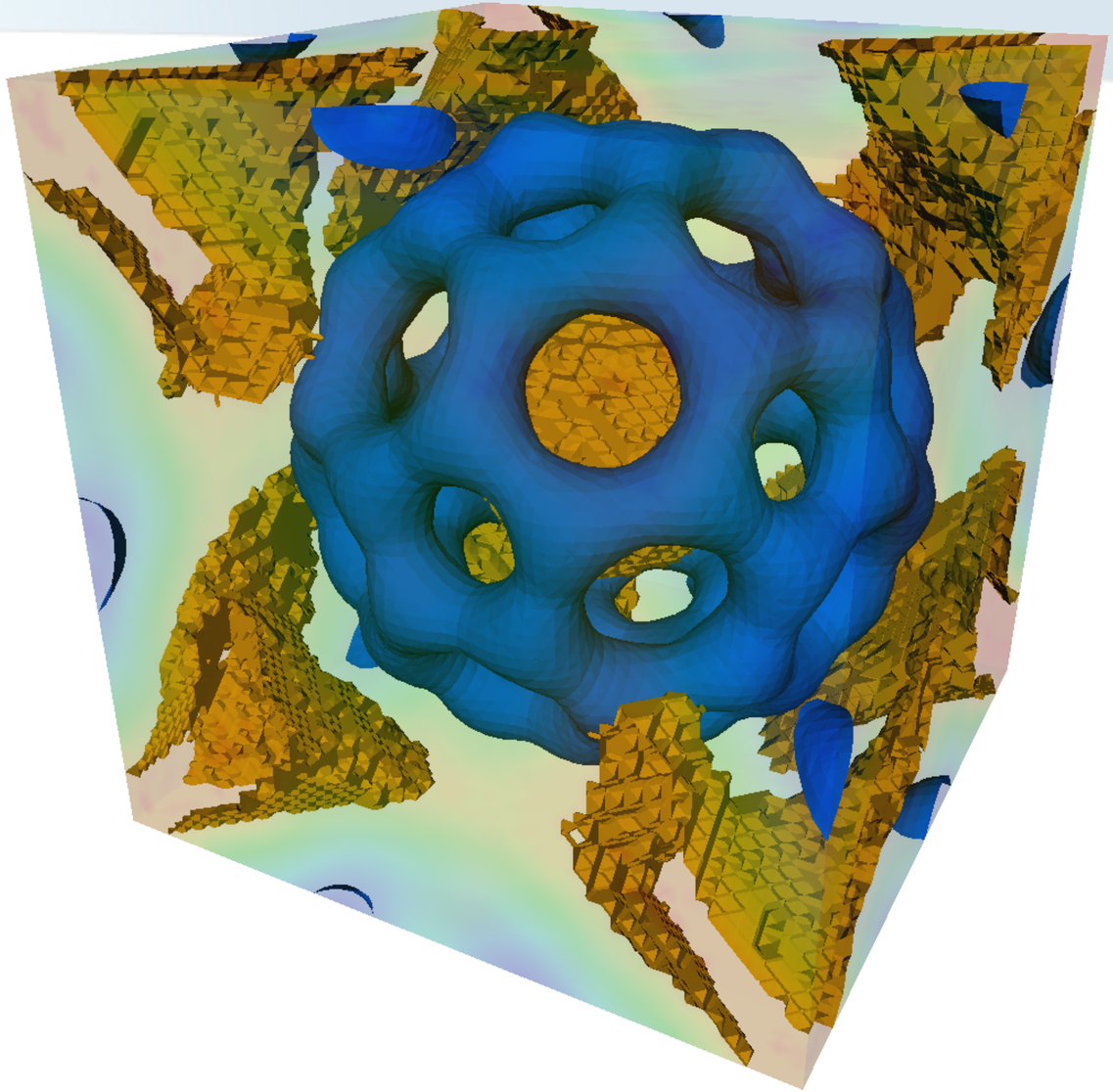

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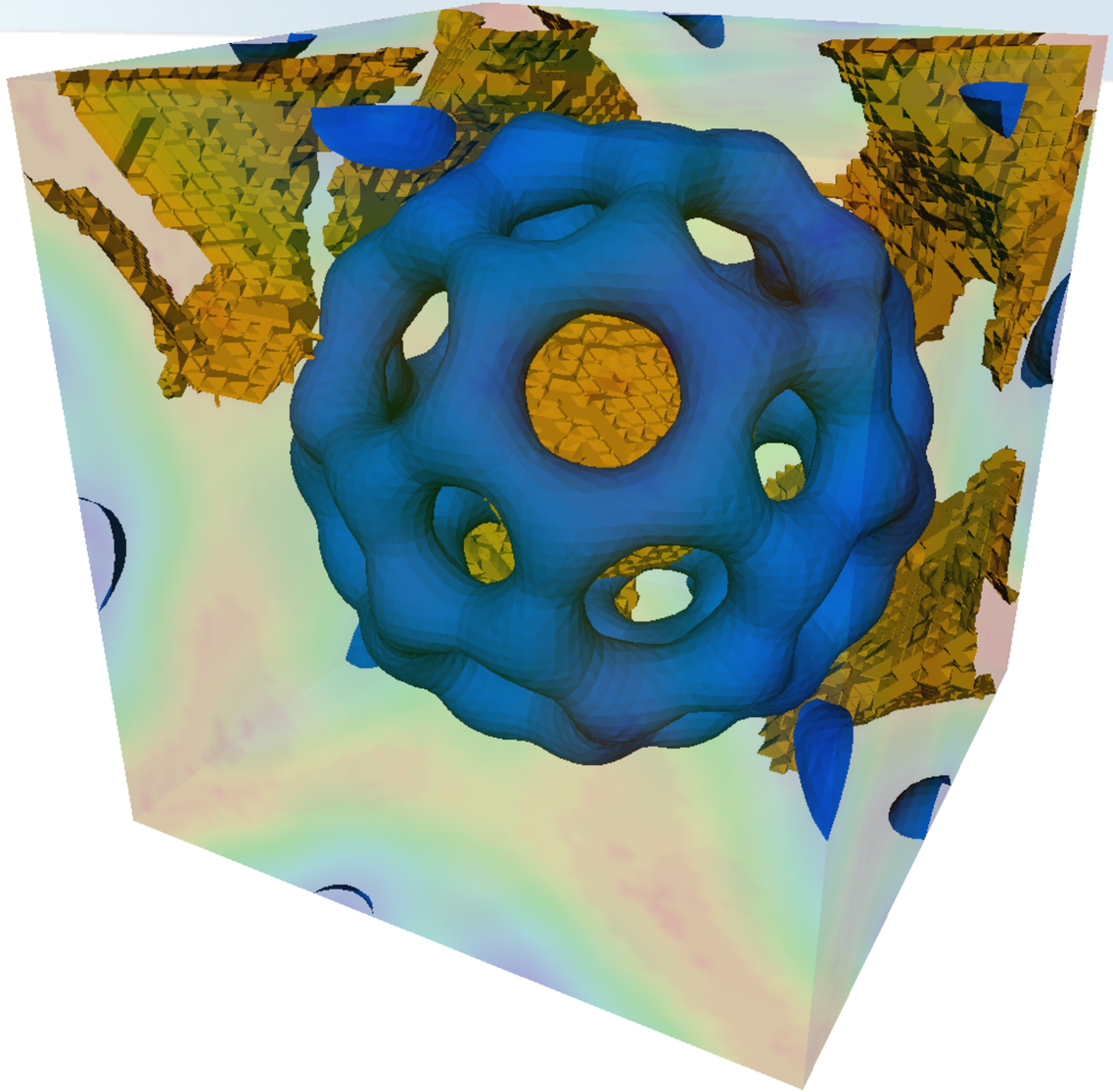

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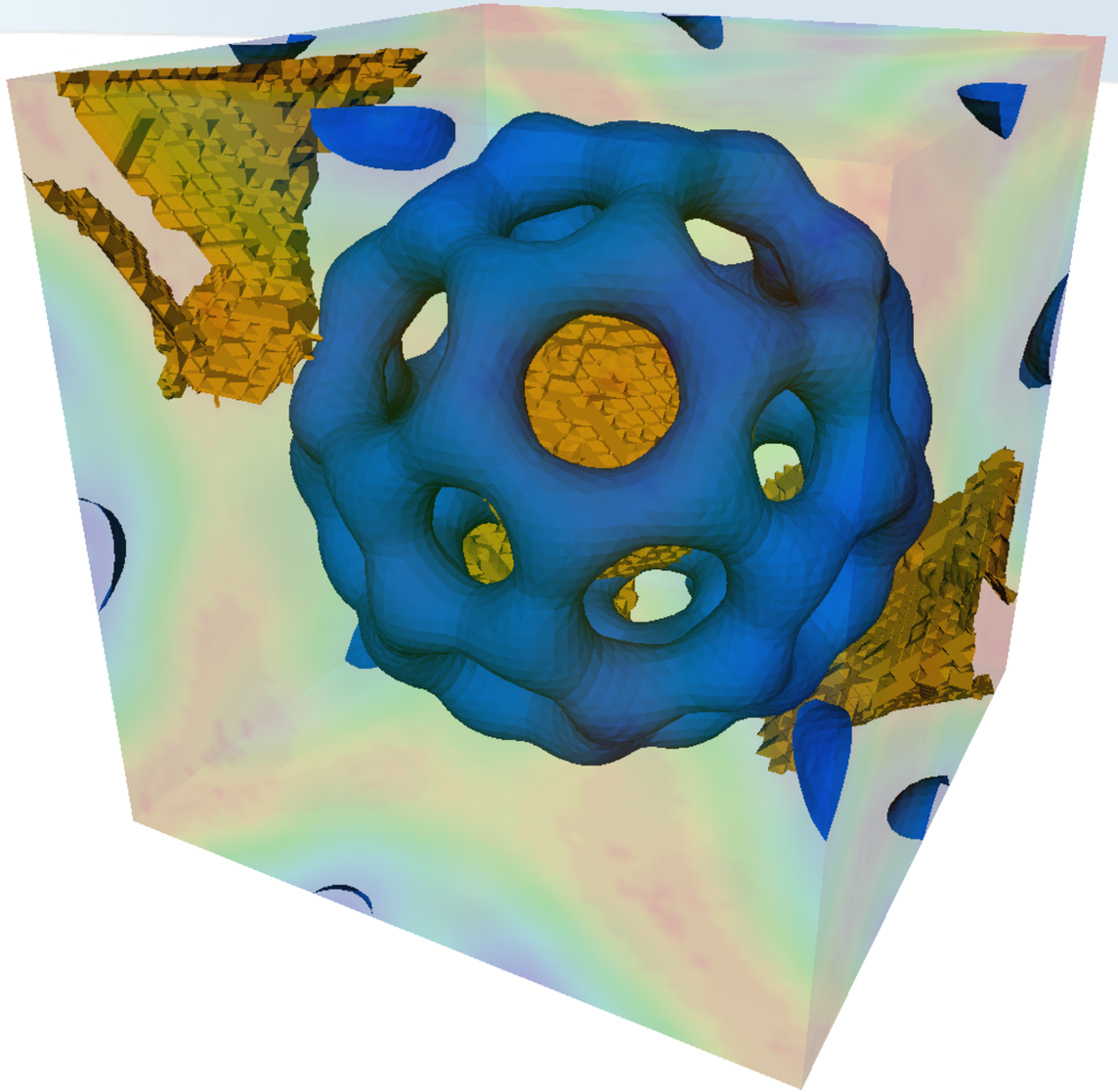

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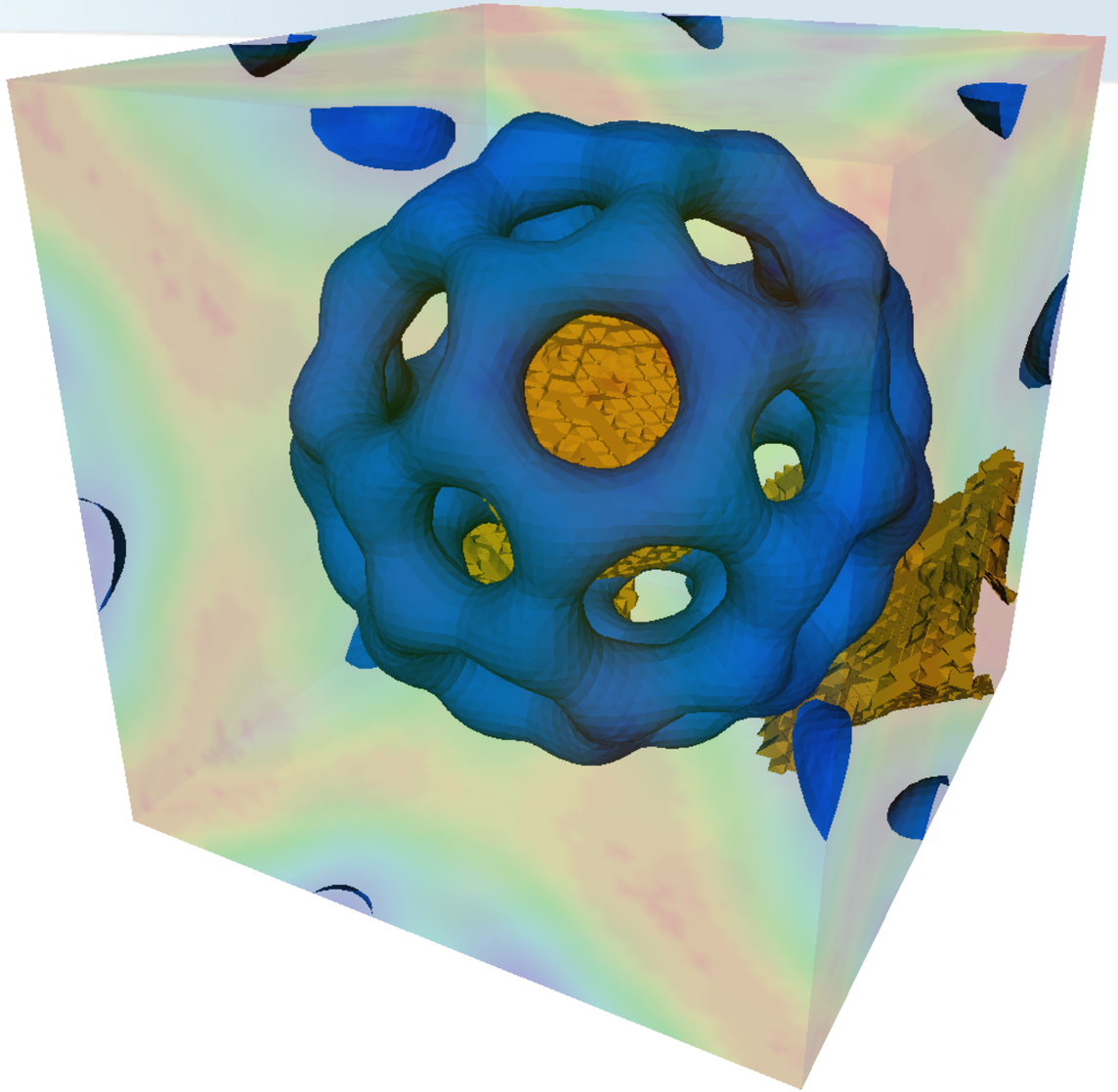

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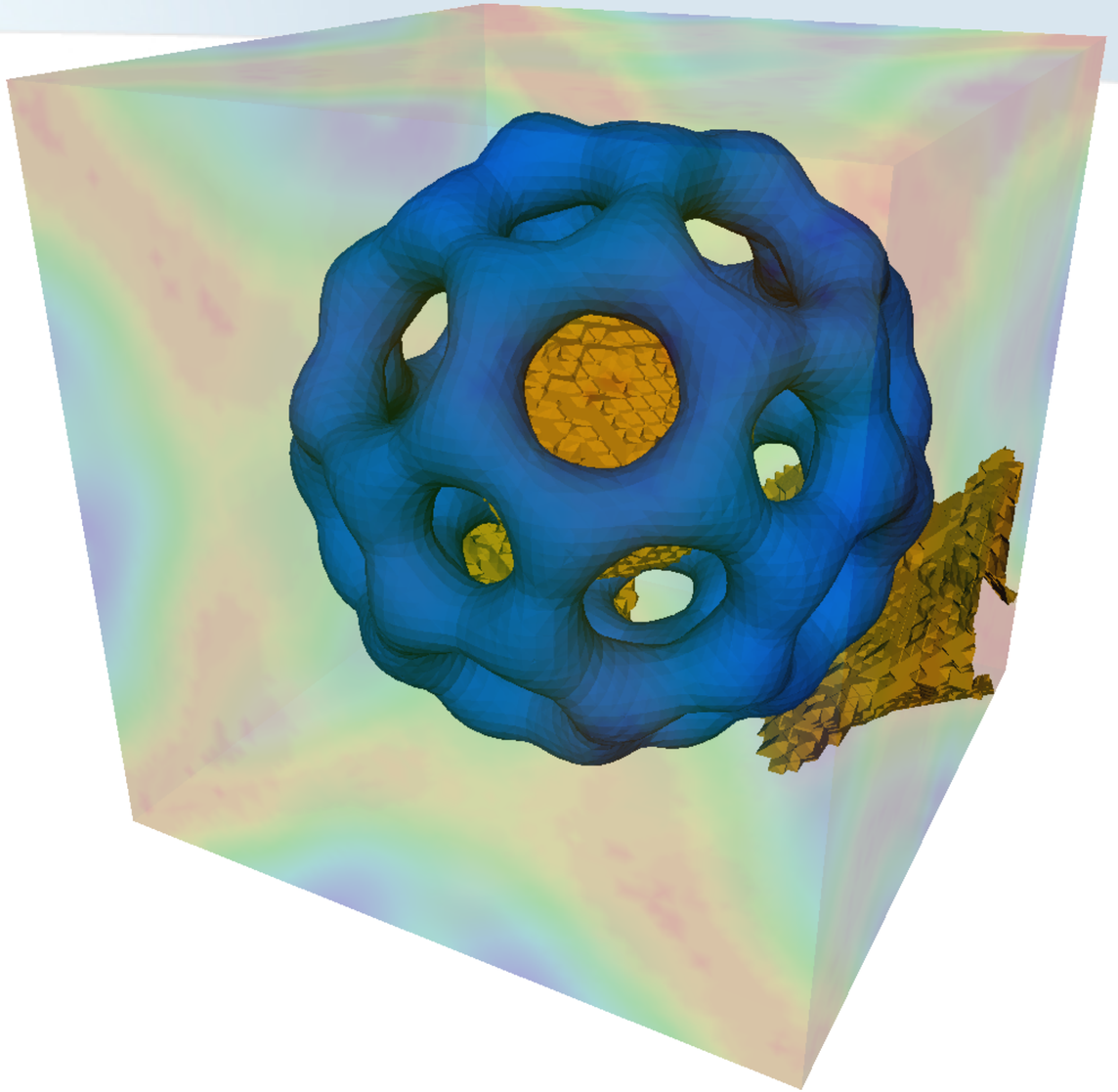

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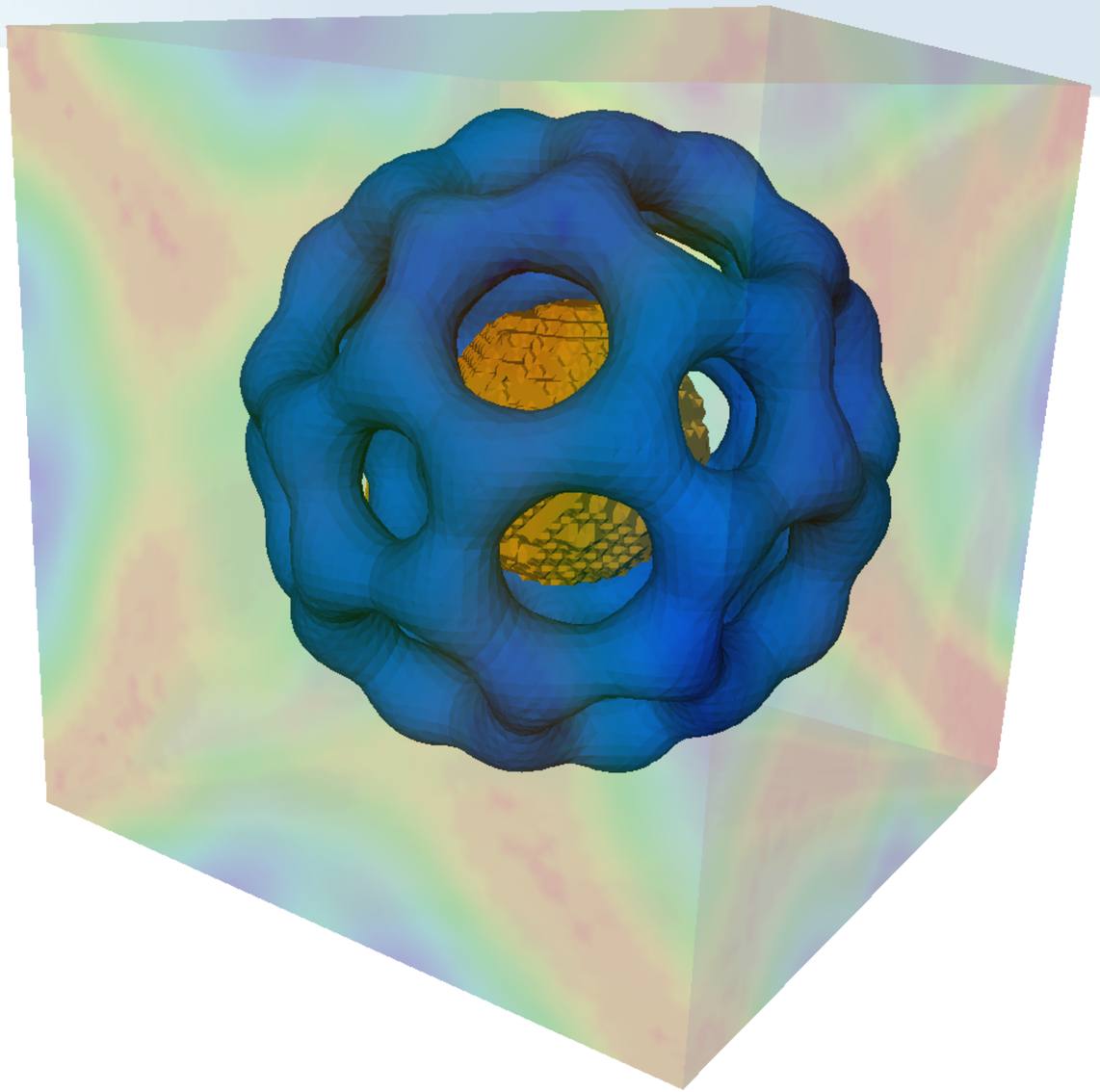

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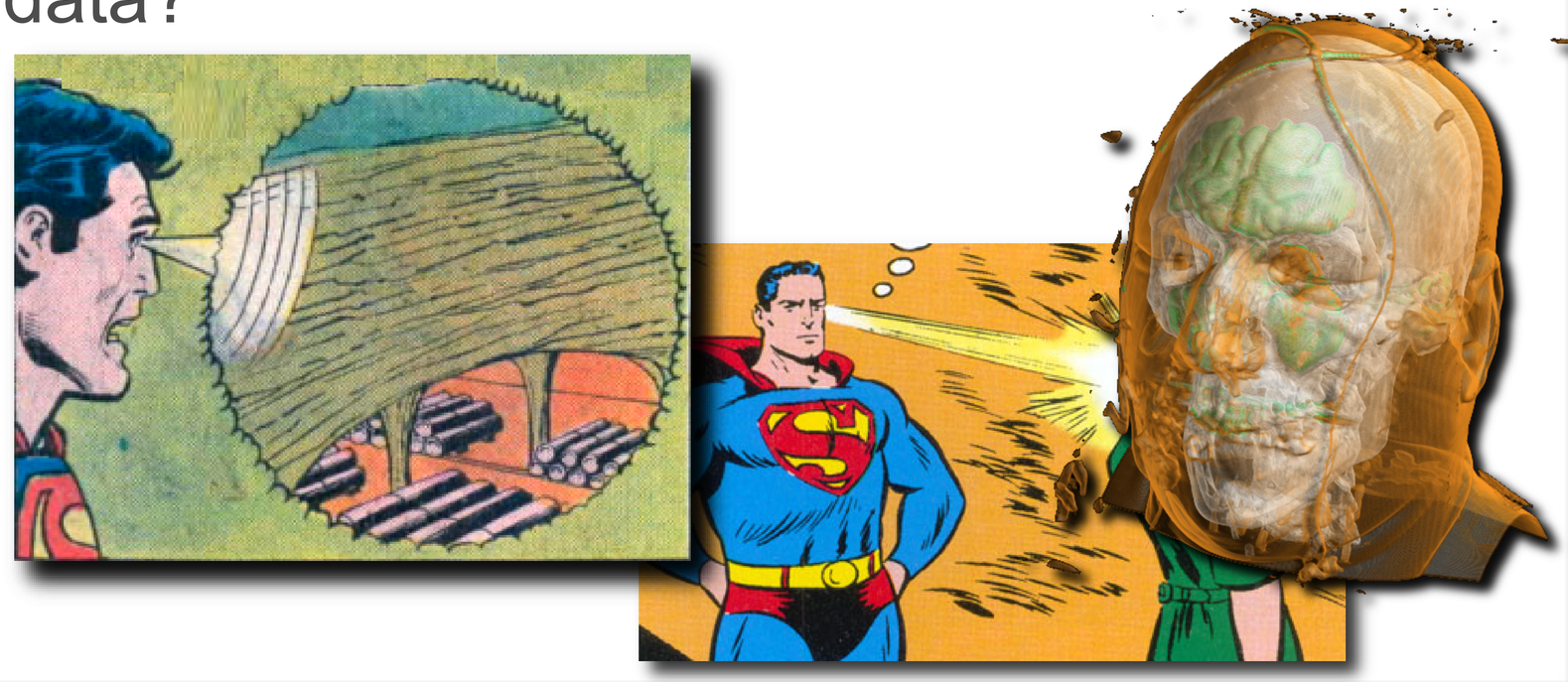

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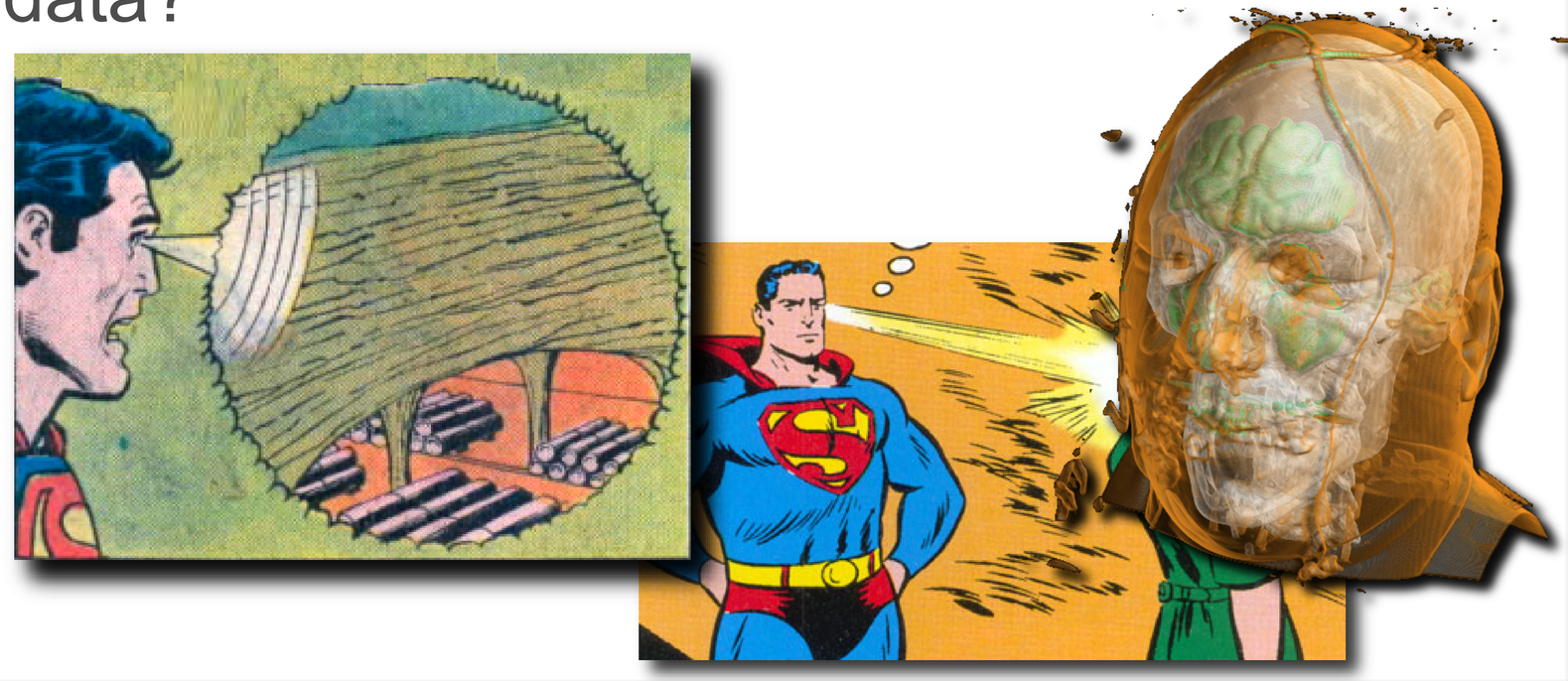
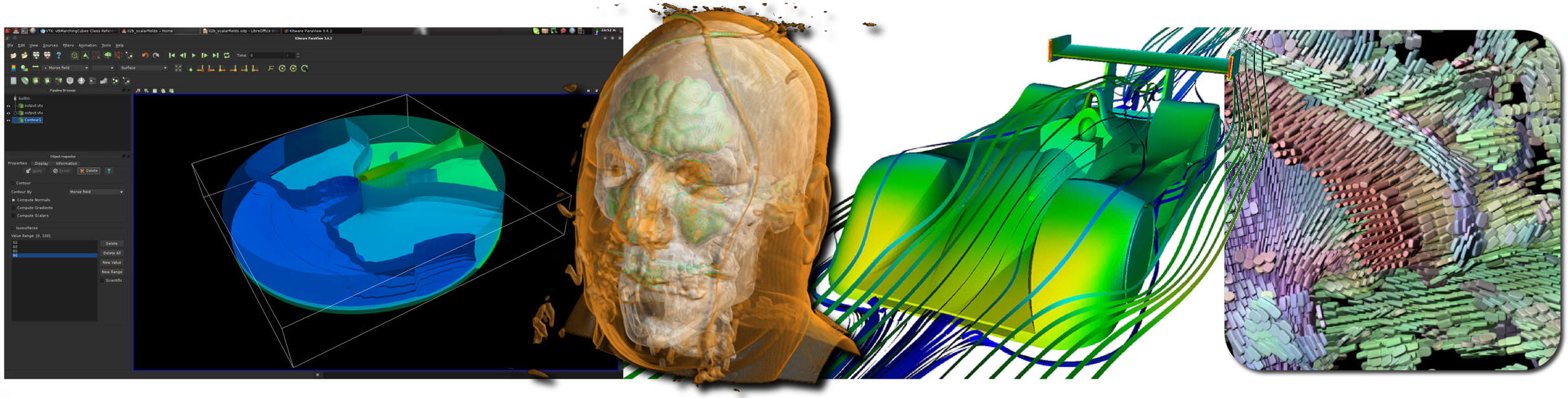


Challenges

- Be creative
- Be efficient



Summary

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- Numerical domain representations

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- Scalar field visualization
  - Level set extraction

Summary

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Summary

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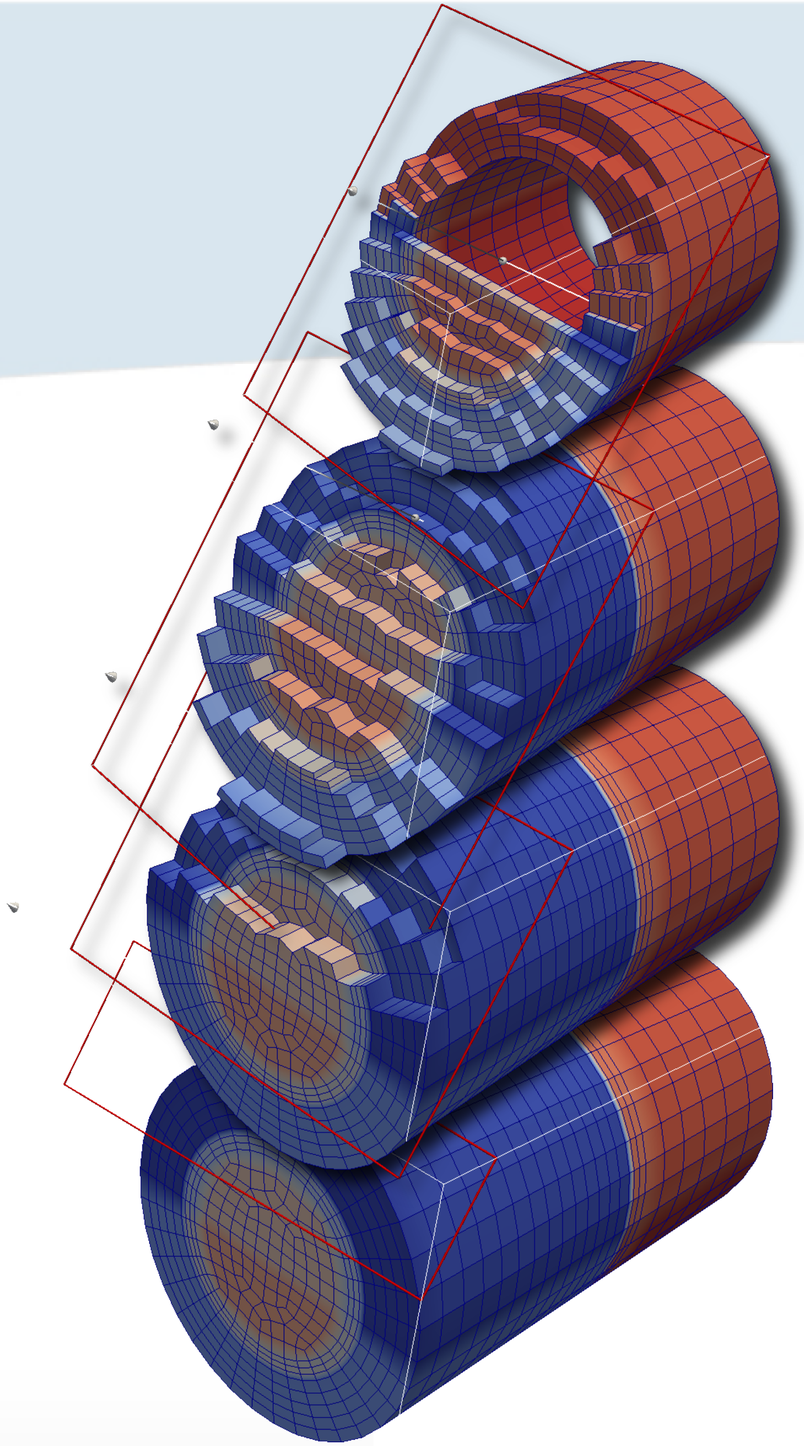

Domain Representations

What do we mean by domain?

What do we mean by domain?

- Continuous discrete representation of a space $\mathbb{X}$

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- *Continuous discrete* representation of a space $\mathbb{X}$

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- *Continuous discrete* representation of a space $\mathbb{X}$
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  - **Connectivity of the samples**
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Example

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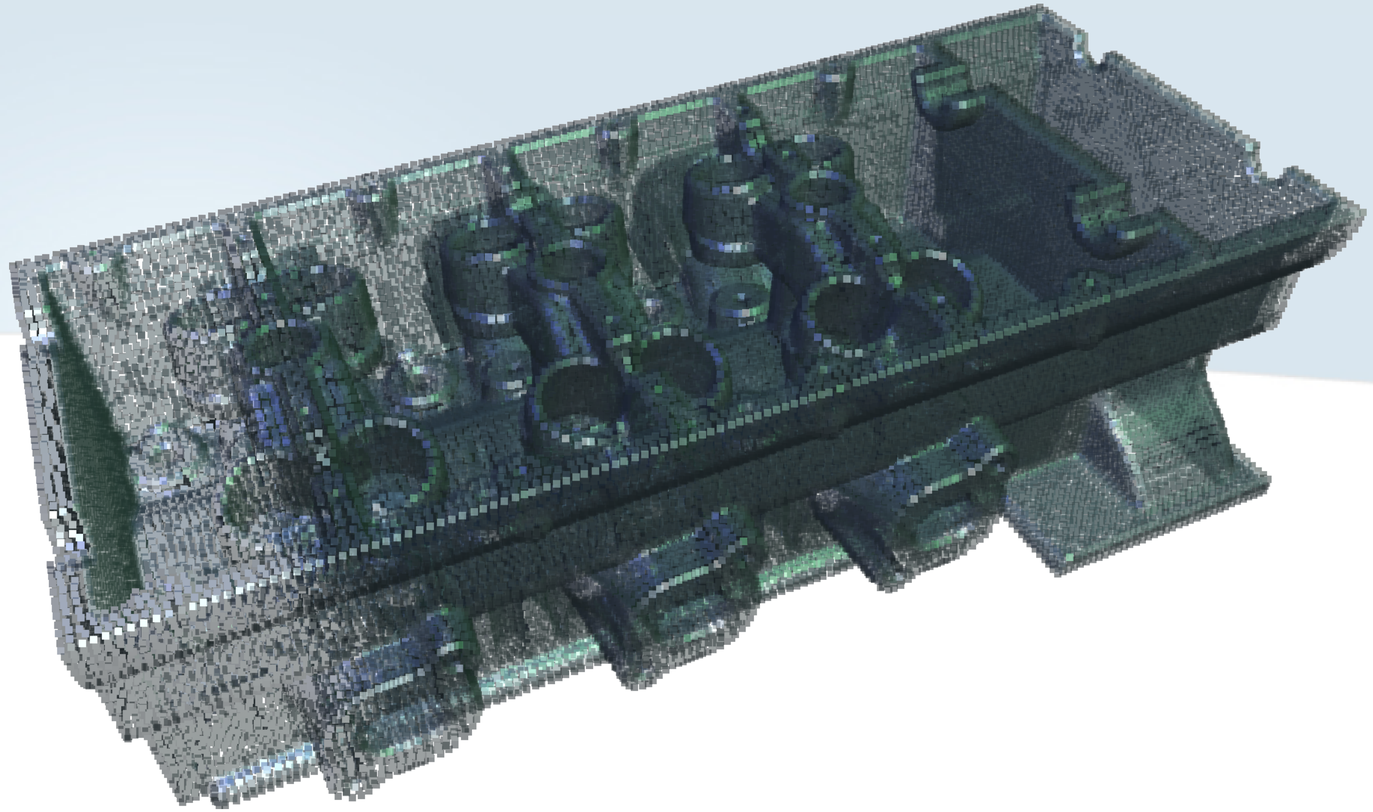

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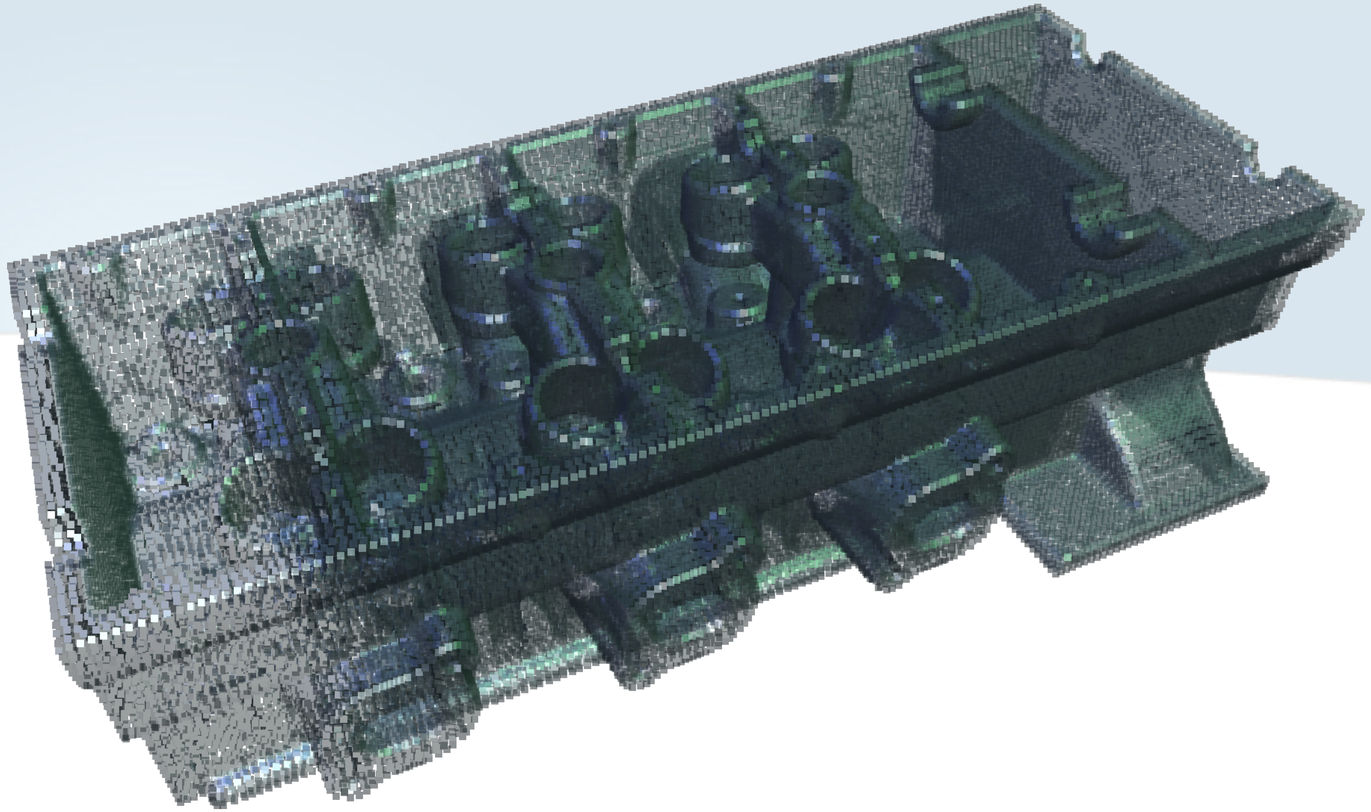
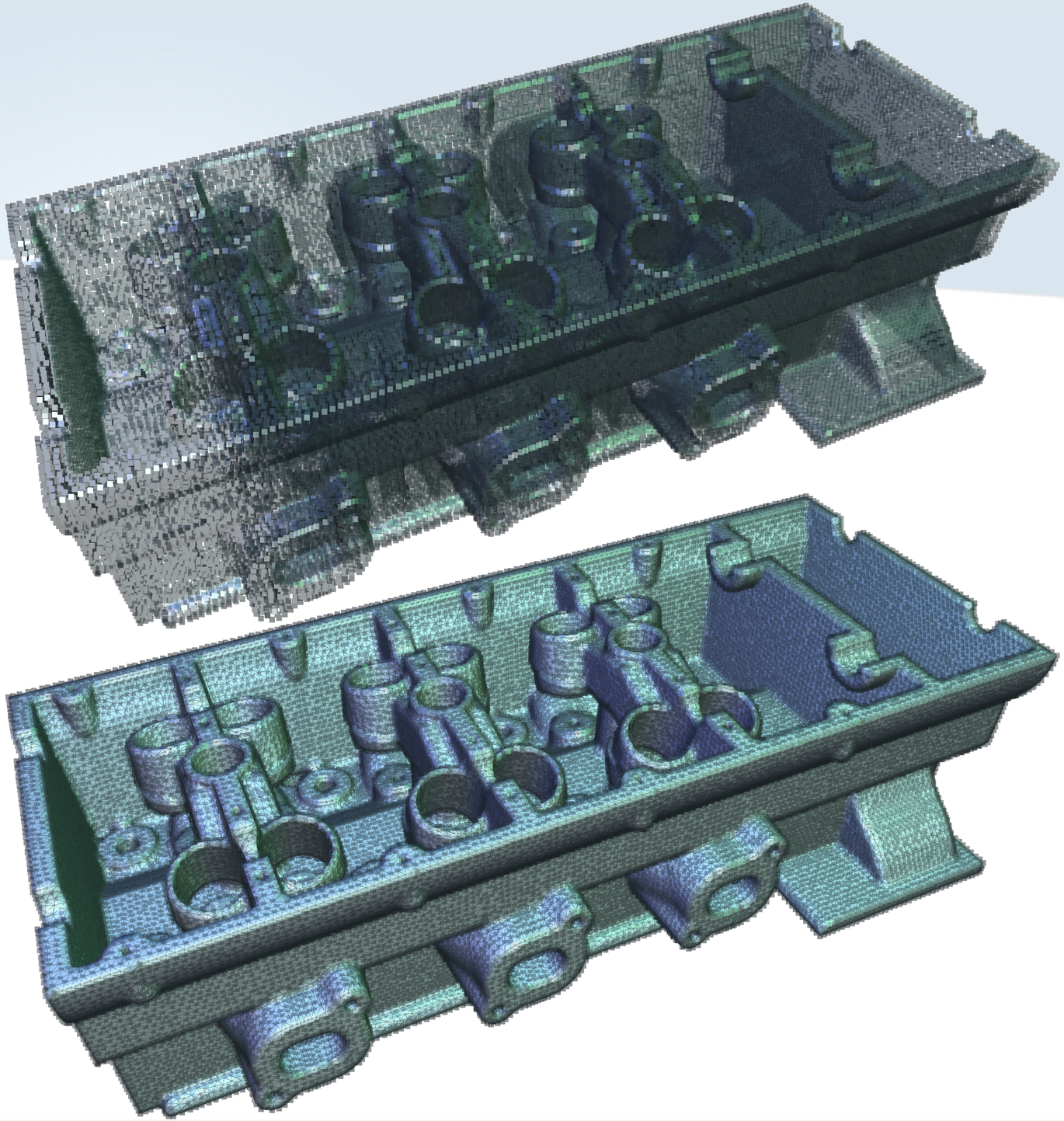
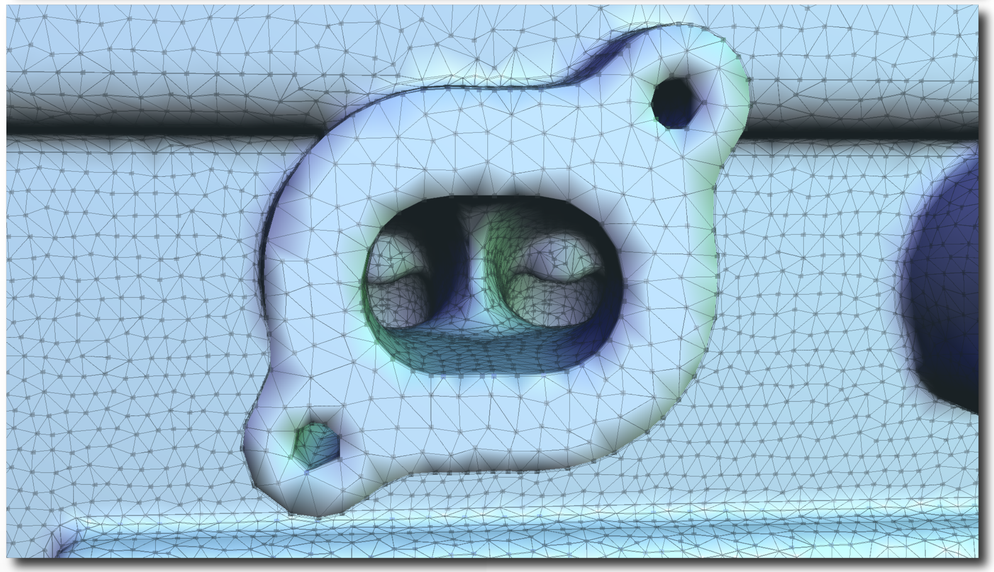
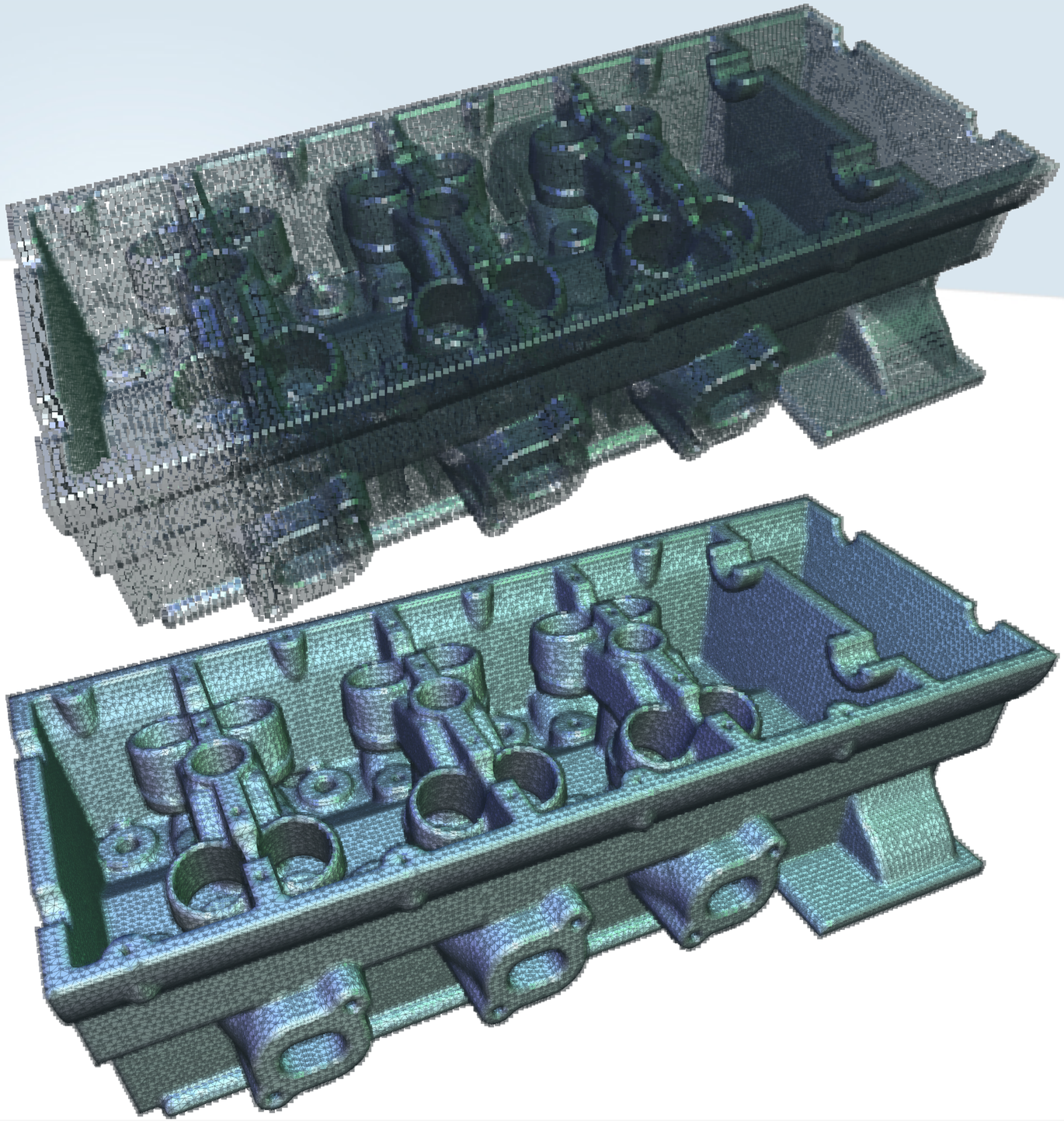


Example



Example





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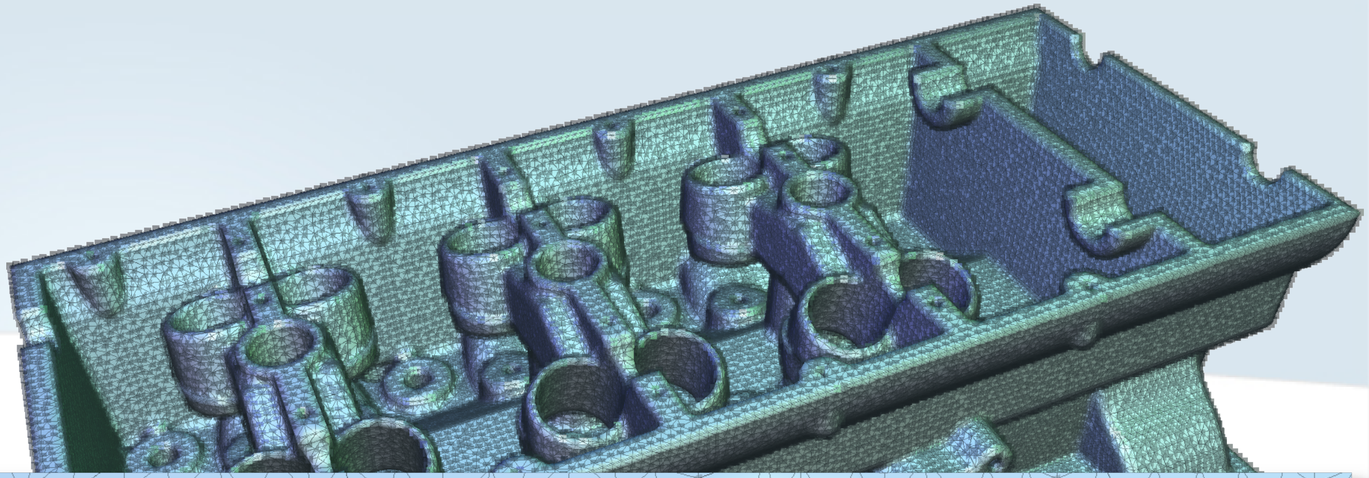
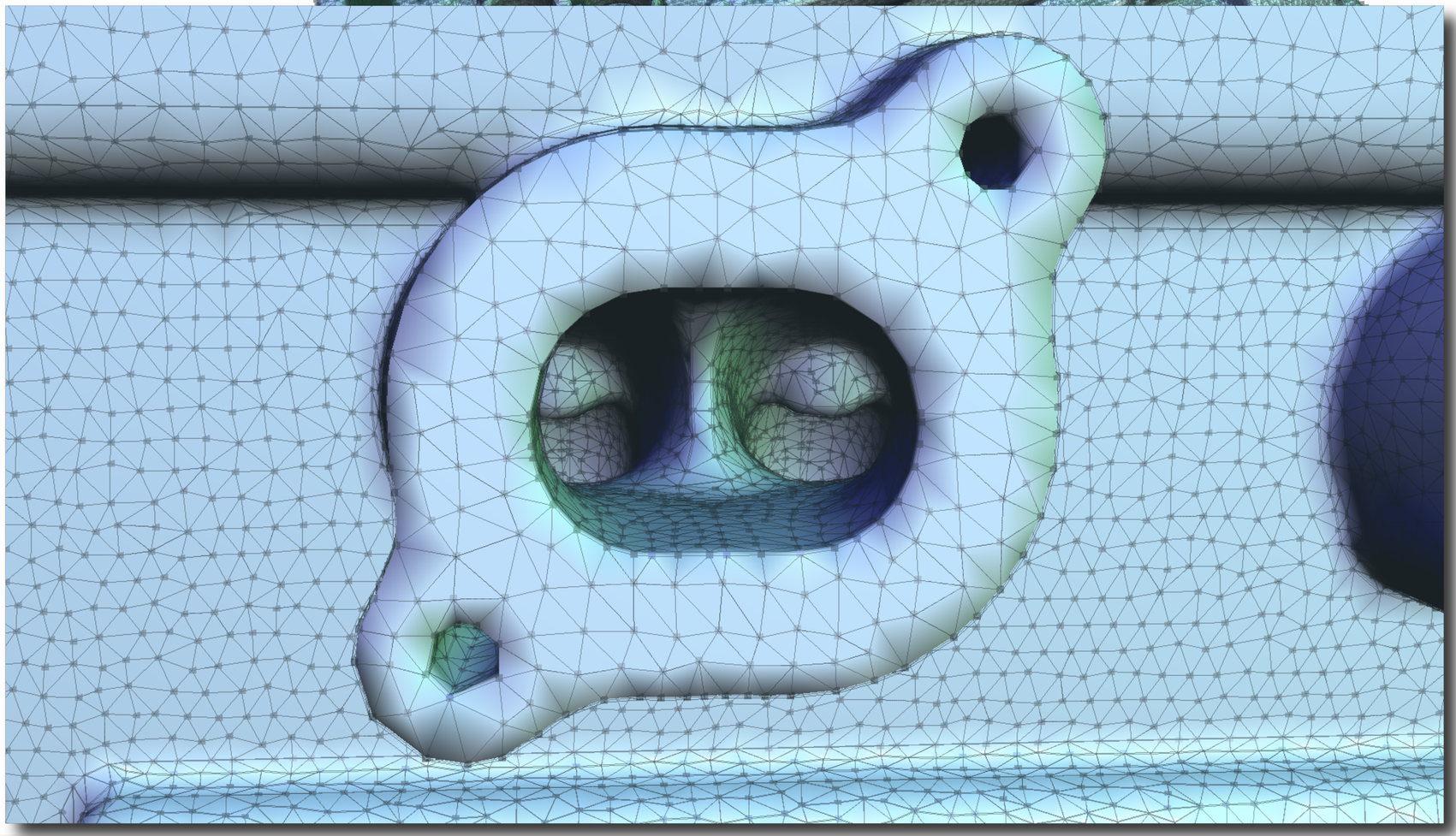
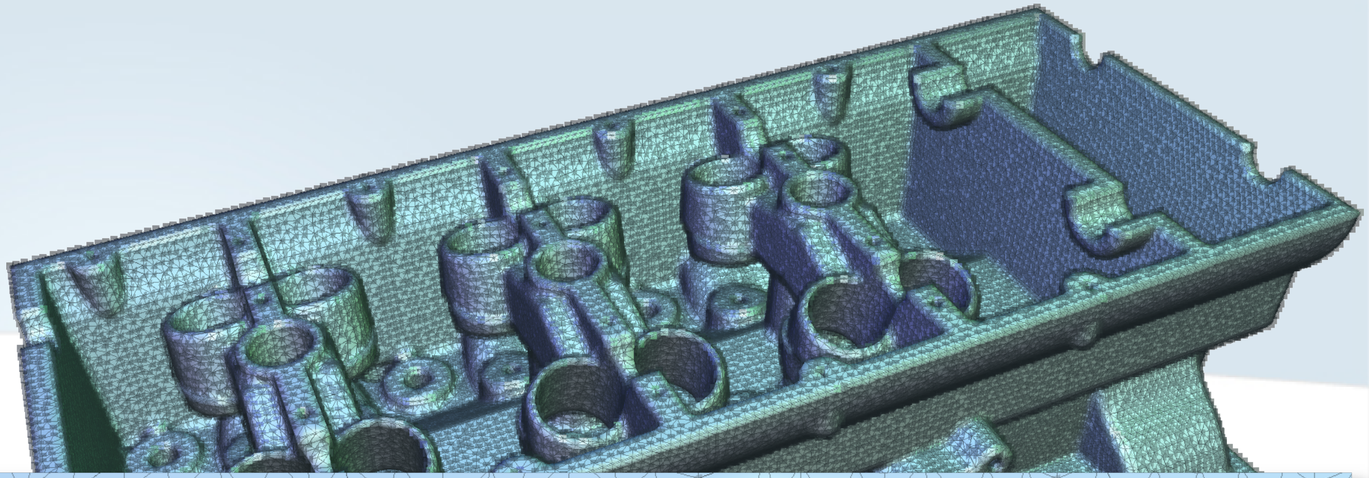
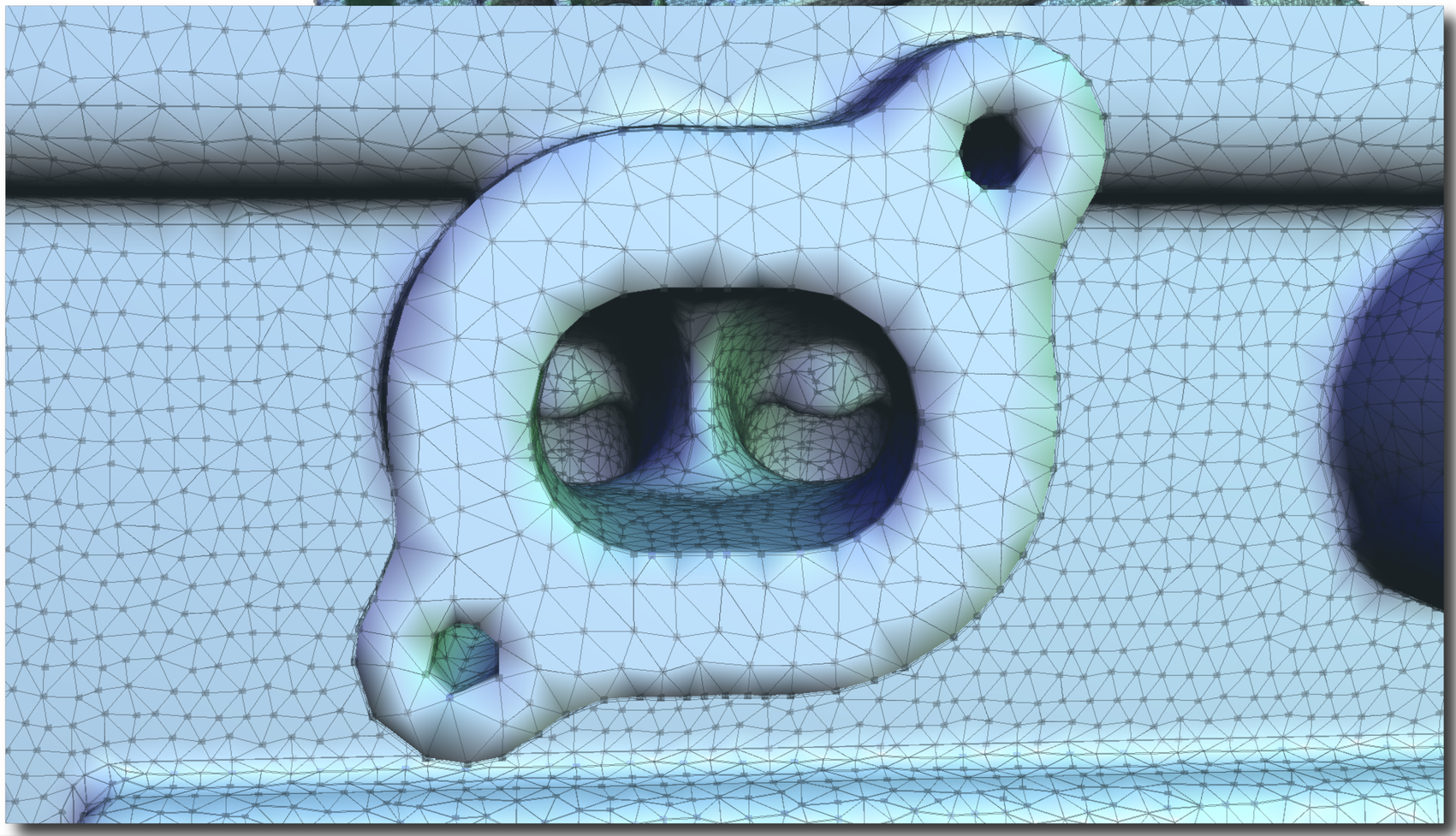




Example



- Domain
- Embedding space
- Cellular elements



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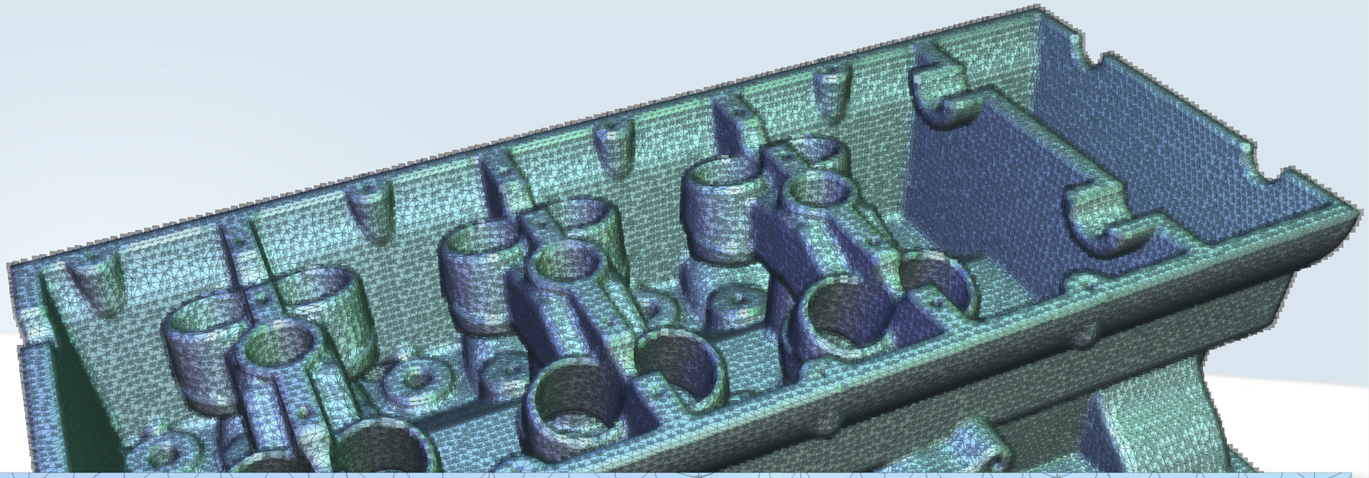
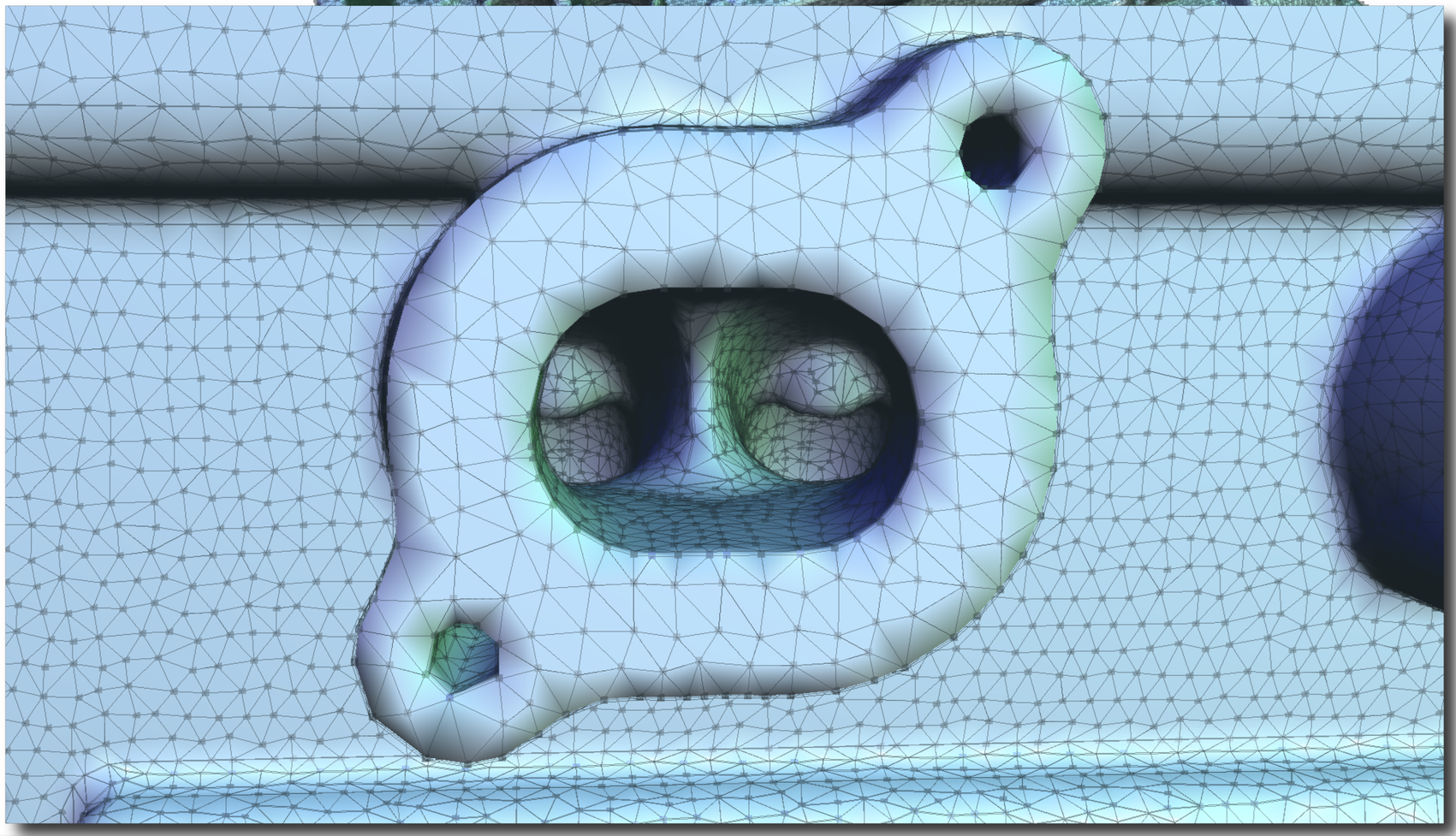


- Domain
  - 2-manifold $\mathcal{S}$
- Embedding space

- Cellular elements



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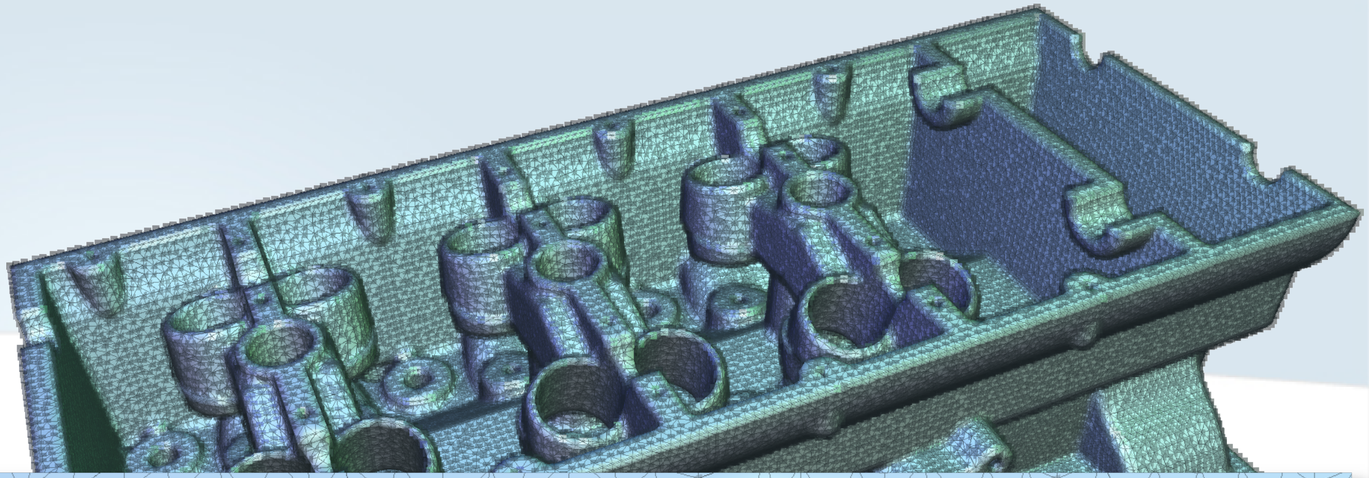
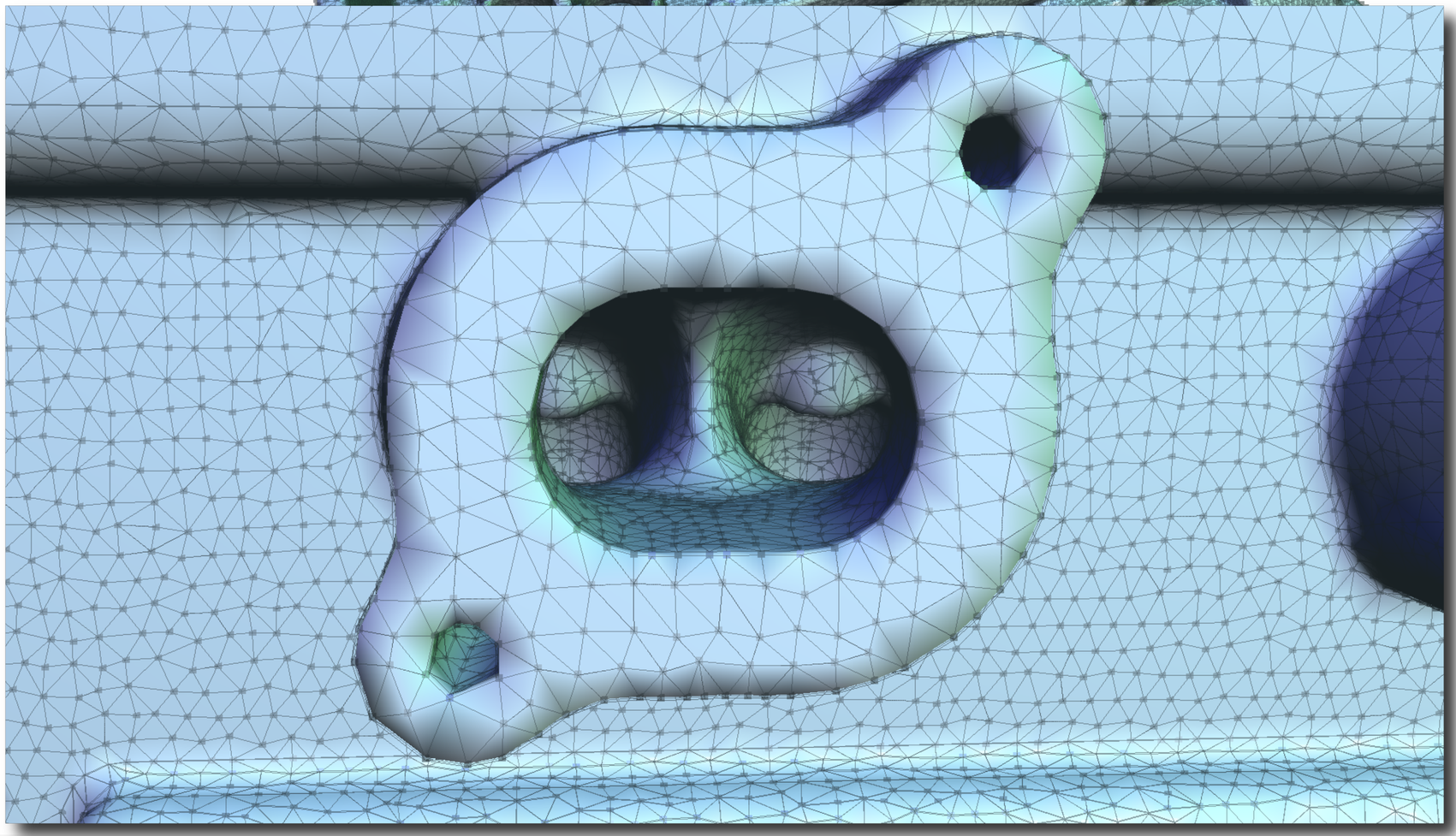


- Domain
  - 2-manifold $\mathcal{S}$
- Embedding space
  - $\mathbb{R}^3$
- Cellular elements



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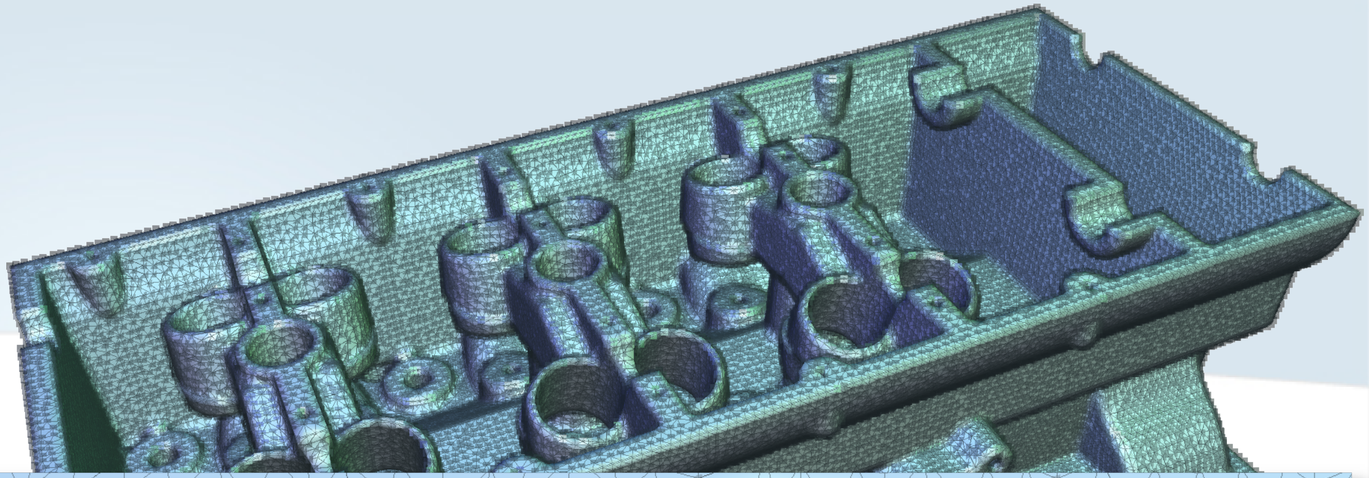
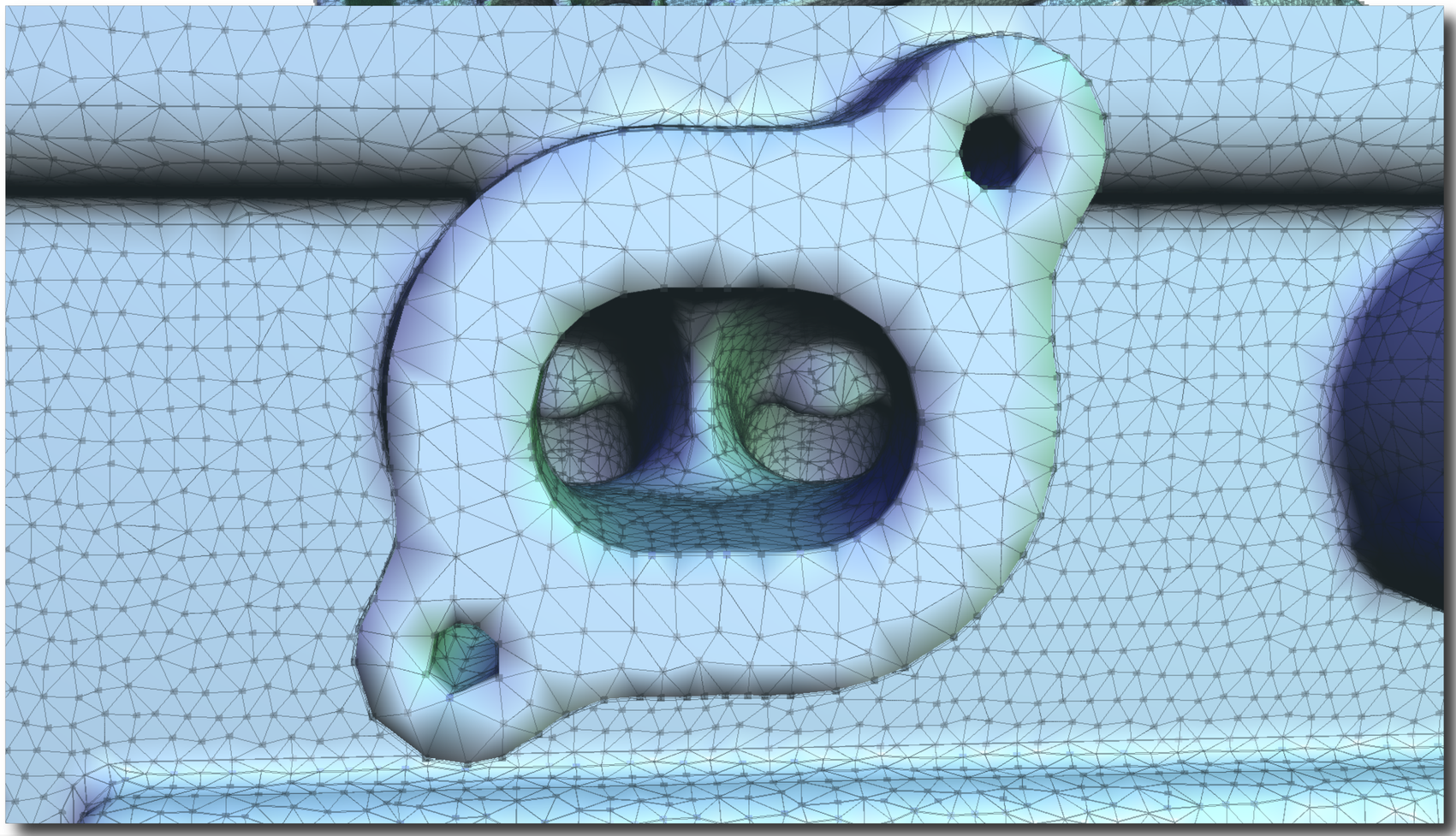


- Domain
  - 2-manifold $\mathcal{S}$
- Embedding space
  - $\mathbb{R}^3$
- Cellular elements
  - Triangles



Domains of interest: Manifolds

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- d-manifold with boundary
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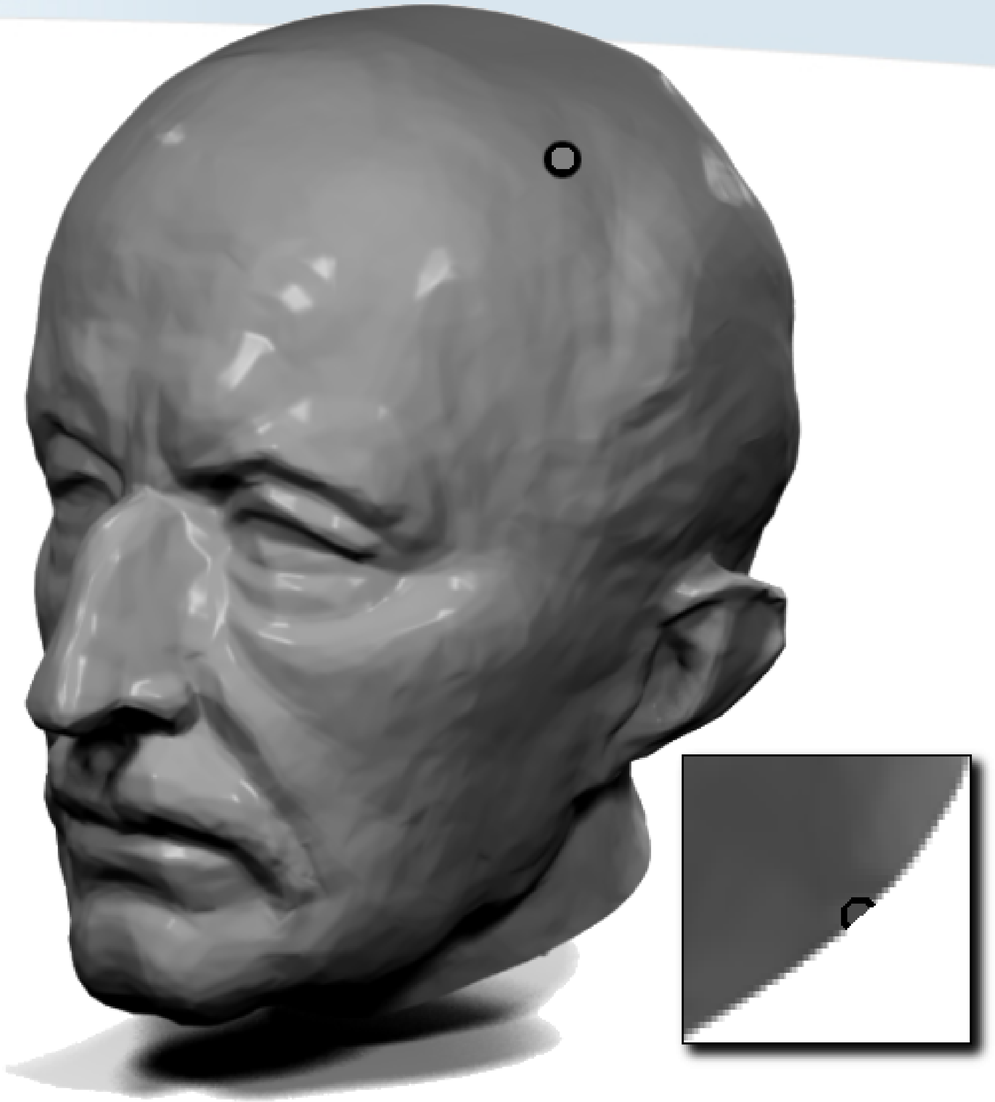

Domains of interest: Manifolds

- d-manifold with boundary
  - Topological space such that:
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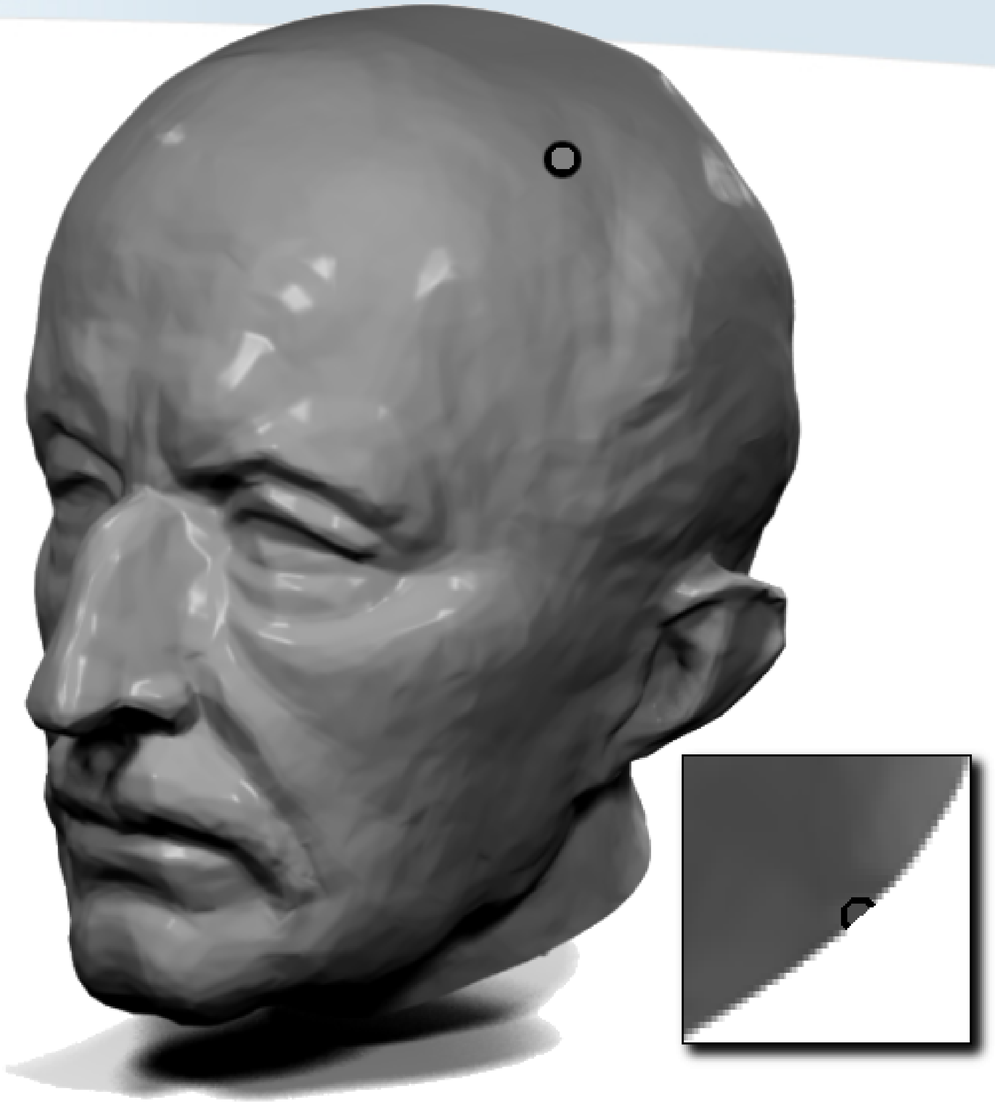
- Boundary: closed (d-1)-manifold



Manifold examples

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- Dimension?
- Embedding space?
- Boundary?

Manifold examples

$\mathbb{R}$

- Dimension?
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- Boundary?

Manifold examples

$\mathbb{R}$

$[0, 1]$

- Dimension?
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Manifold examples

$$\mathbb{R}$$

$$[0, 1]$$

$$\{p \in \mathbb{R}^2 \ / \ \sqrt{p_x^2 + p_y^2} = r\}$$

- Dimension?
- Embedding space?
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Manifold examples

$$\mathbb{R}^2$$

- Dimension?
- Embedding space?
- Boundary?

$$[0, 1]$$

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Manifold examples

$$\mathbb{R}^2$$

$$[0,1] \times [0,1]$$

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Manifold examples

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Manifold examples

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Manifold examples

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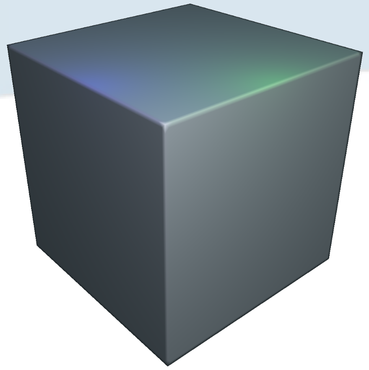
Manifold examples



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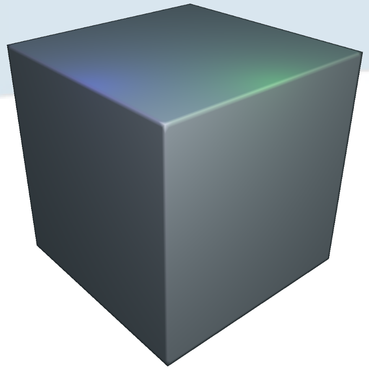
Manifold examples



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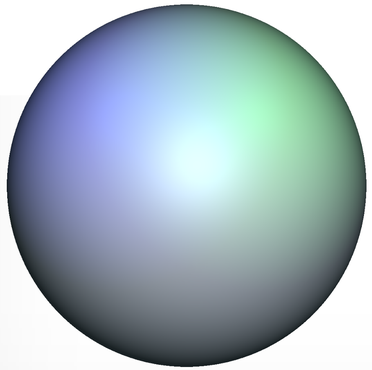

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Euclidean spaces on a computer

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- Given an origin

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  - $o = (0, 0, \ldots, 0) \in \mathbb{R}^n$

Euclidean spaces on a computer

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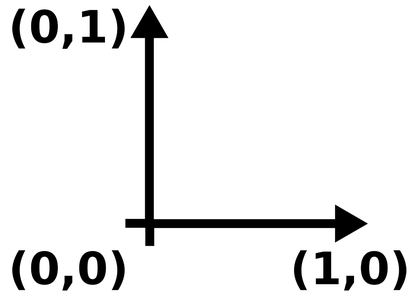

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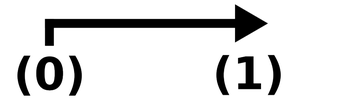
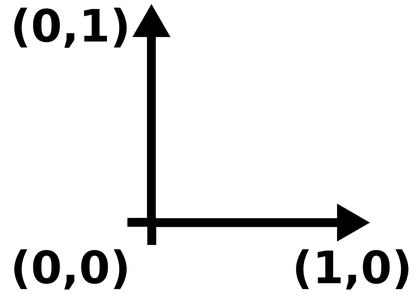
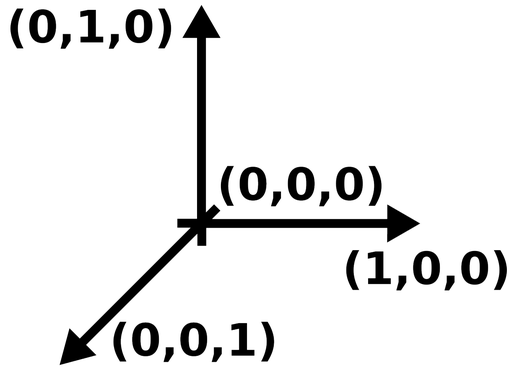
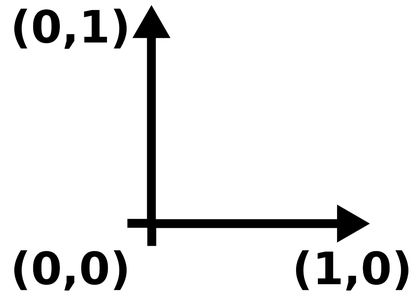


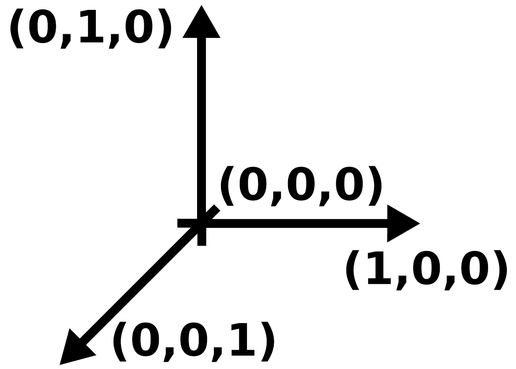




Euclidean spaces on a computer

- Direct product of closed unit intervals





Euclidean spaces on a computer

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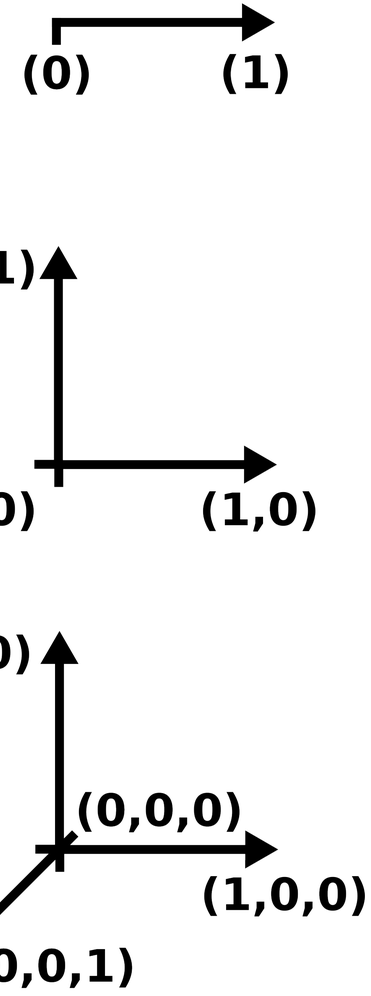
  - $[0,1] \times [0,1] \times \cdots \times [0,1]$



Euclidean spaces on a computer

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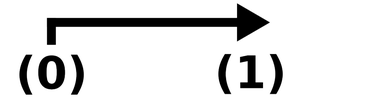
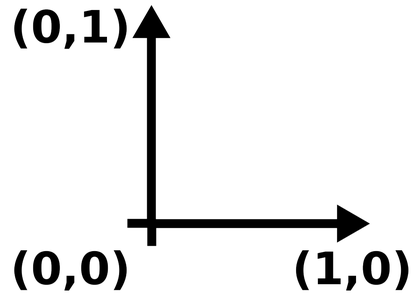
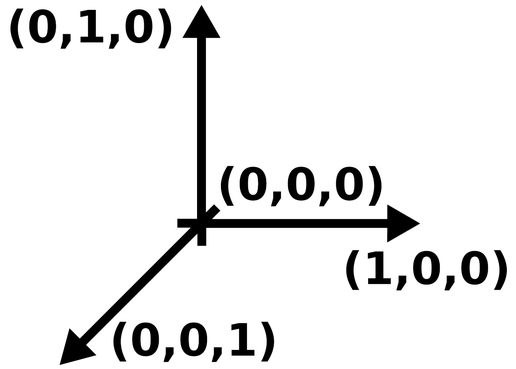
  - By construction







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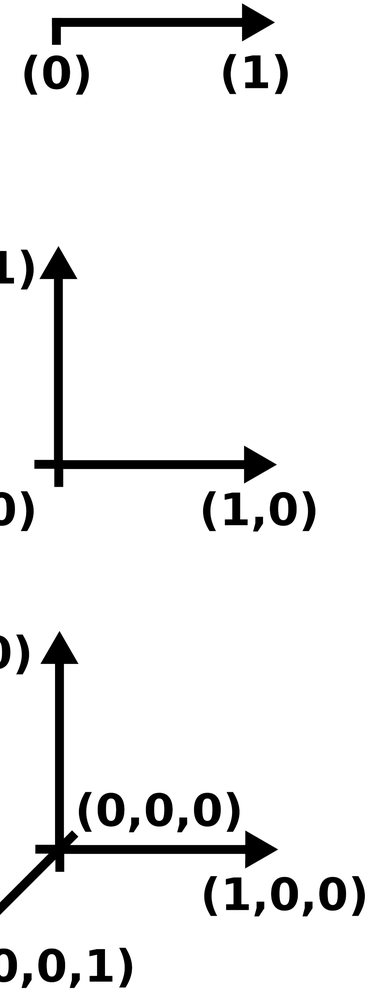
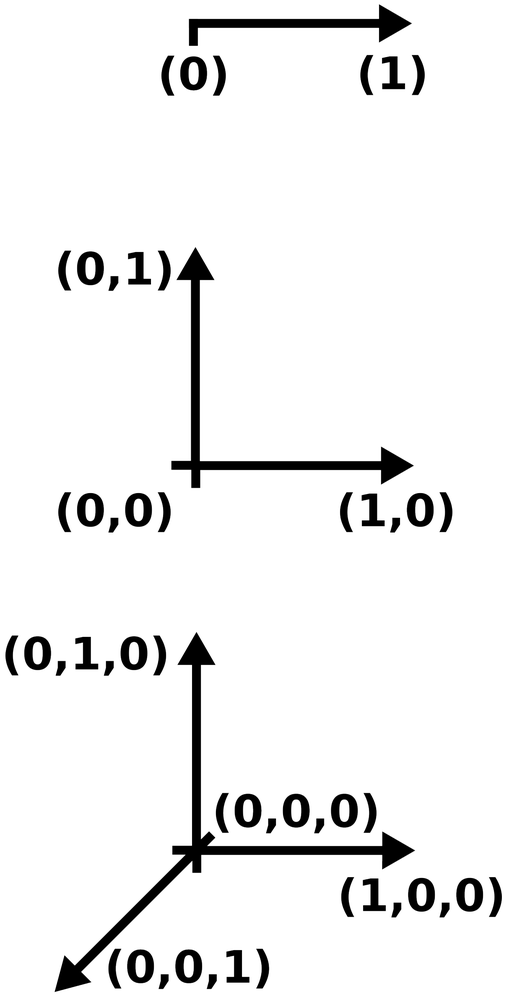
  - By construction
    - Unit translations along the vectors of the orthonormal basis



Euclidean spaces on a computer

- Direct product of closed unit intervals
  - $[0,1] \times [0,1] \times \cdots \times [0,1]$
  - By construction
    - Unit translations along the vectors of the orthonormal basis
    - Covering a bounded, compact region of $\mathbb{R}^n$



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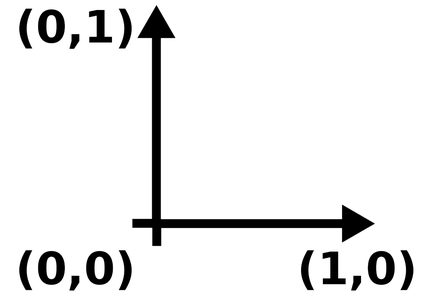

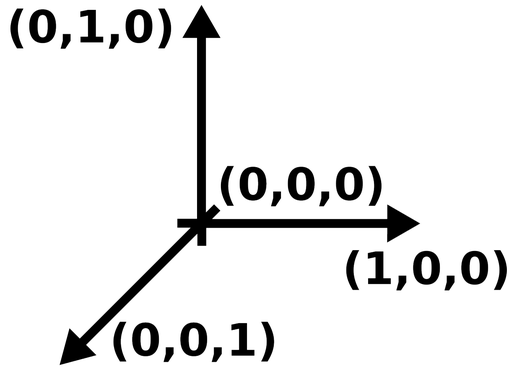

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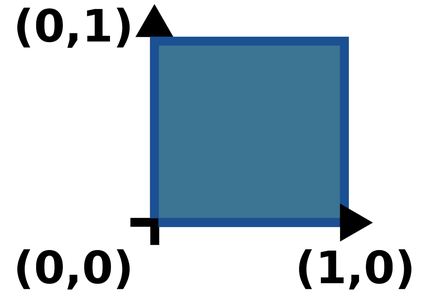

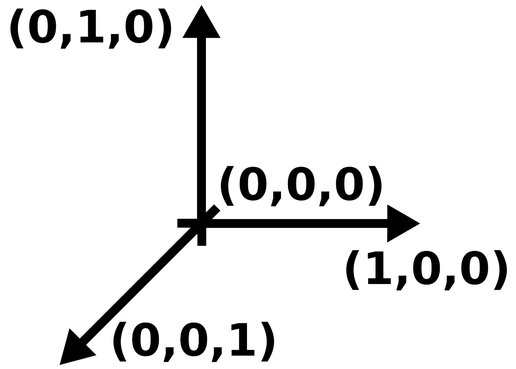

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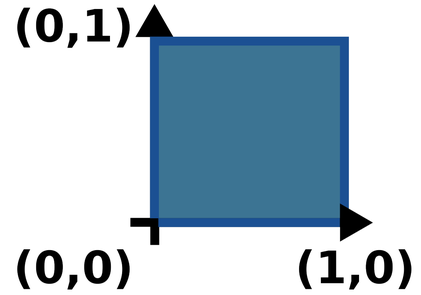

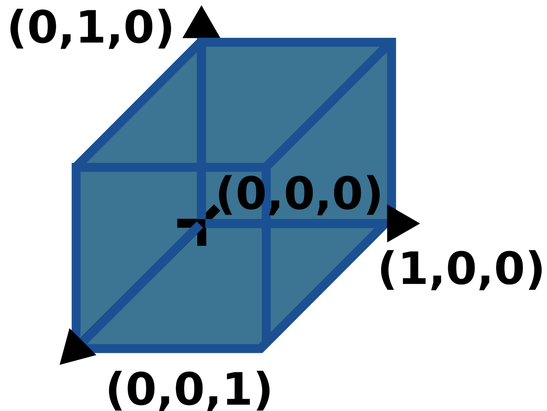

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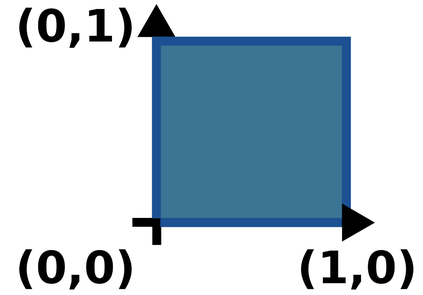

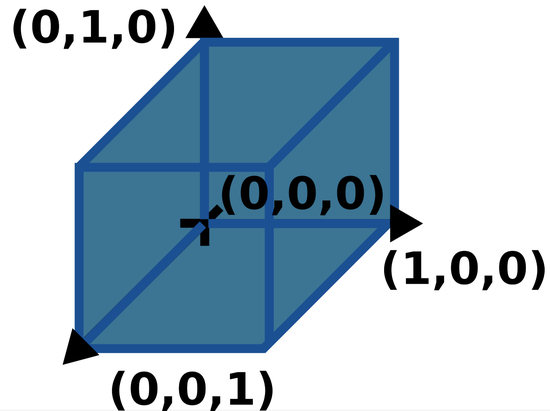

- **Unit cell**

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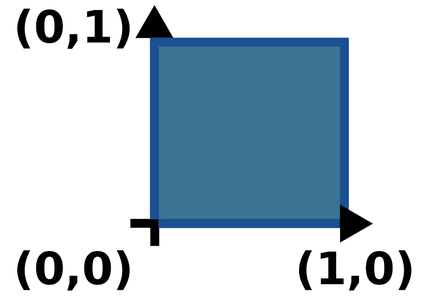

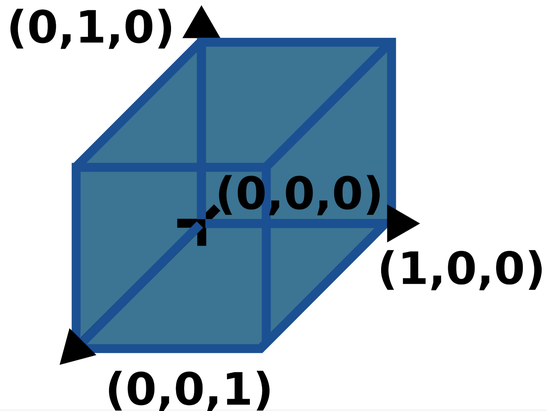

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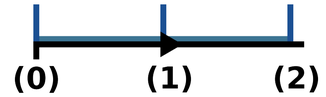

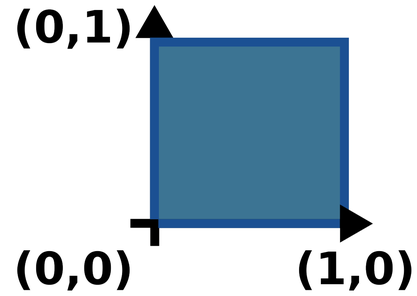

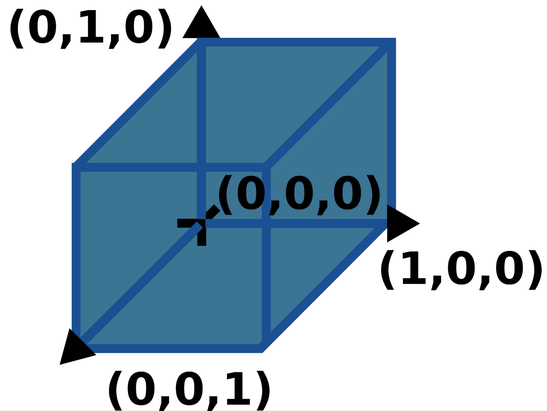

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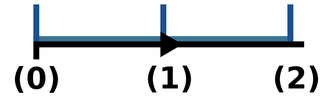

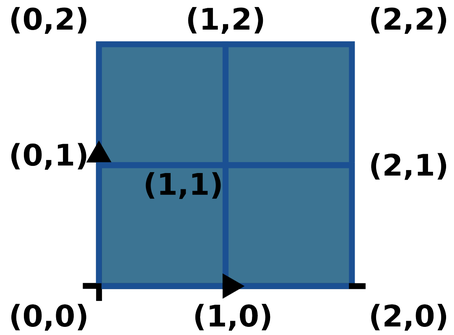

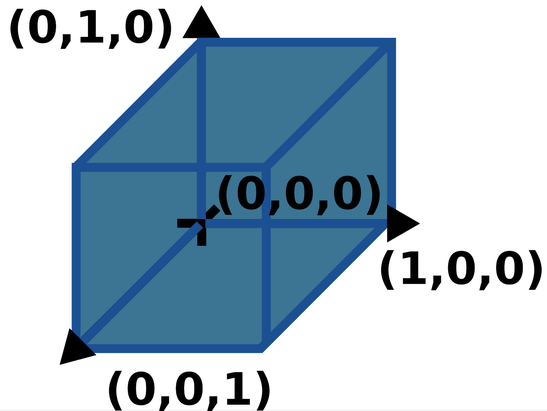

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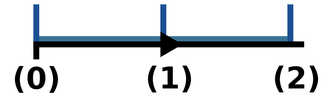

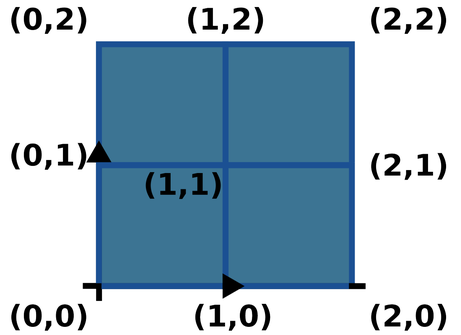

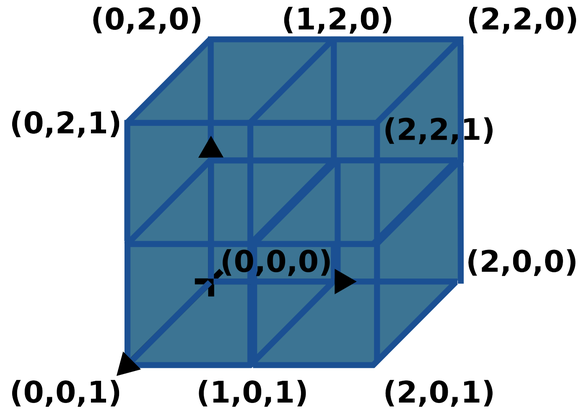

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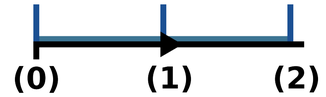

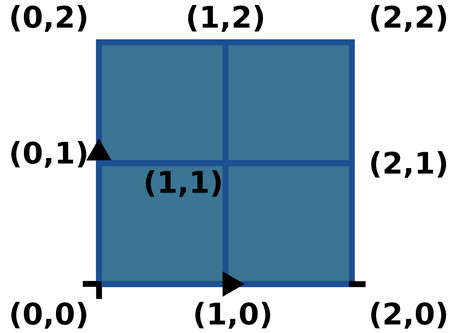

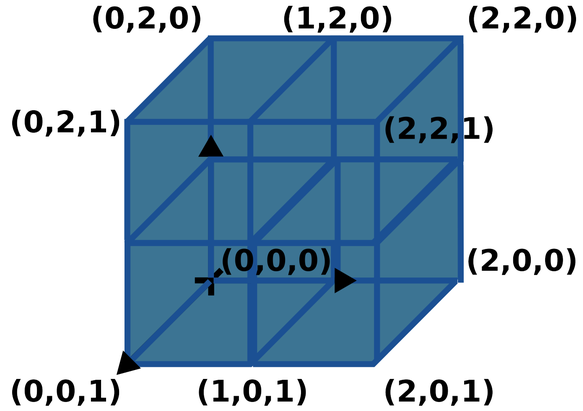

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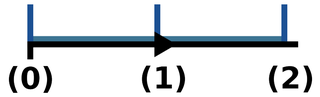

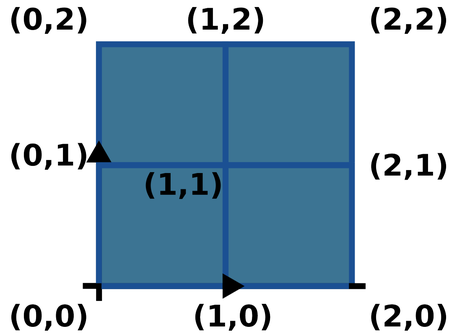

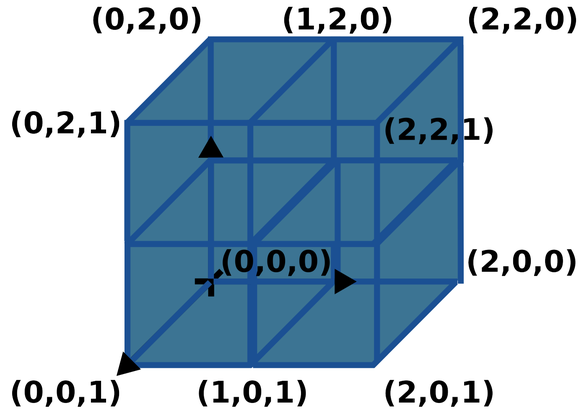

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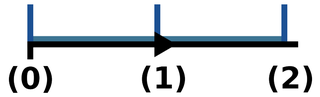

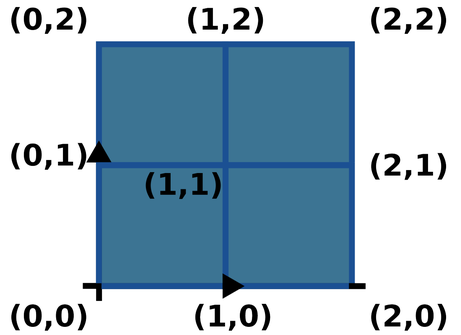

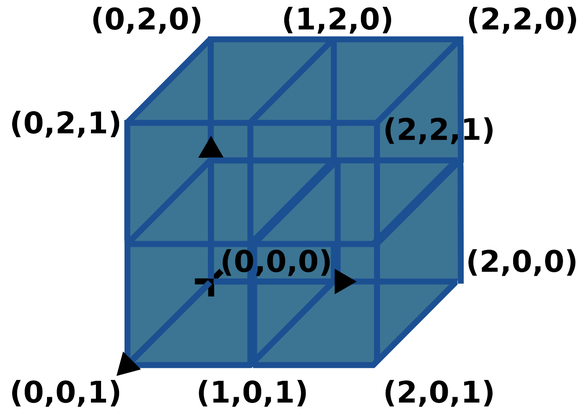

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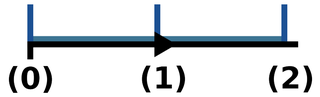

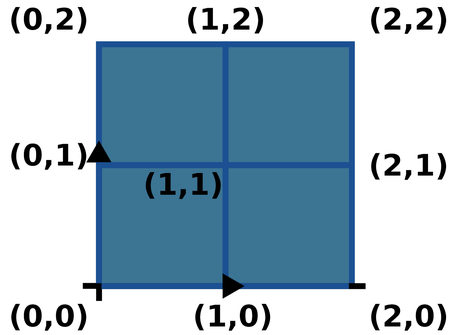

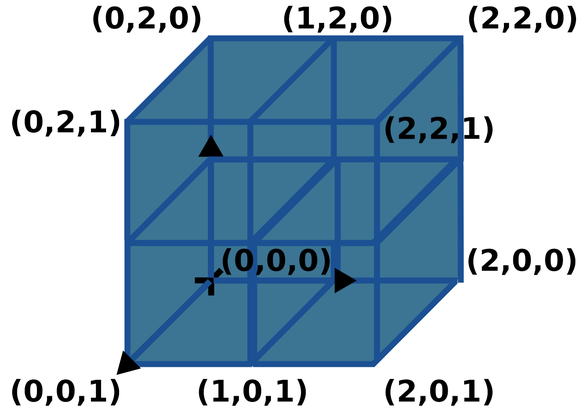

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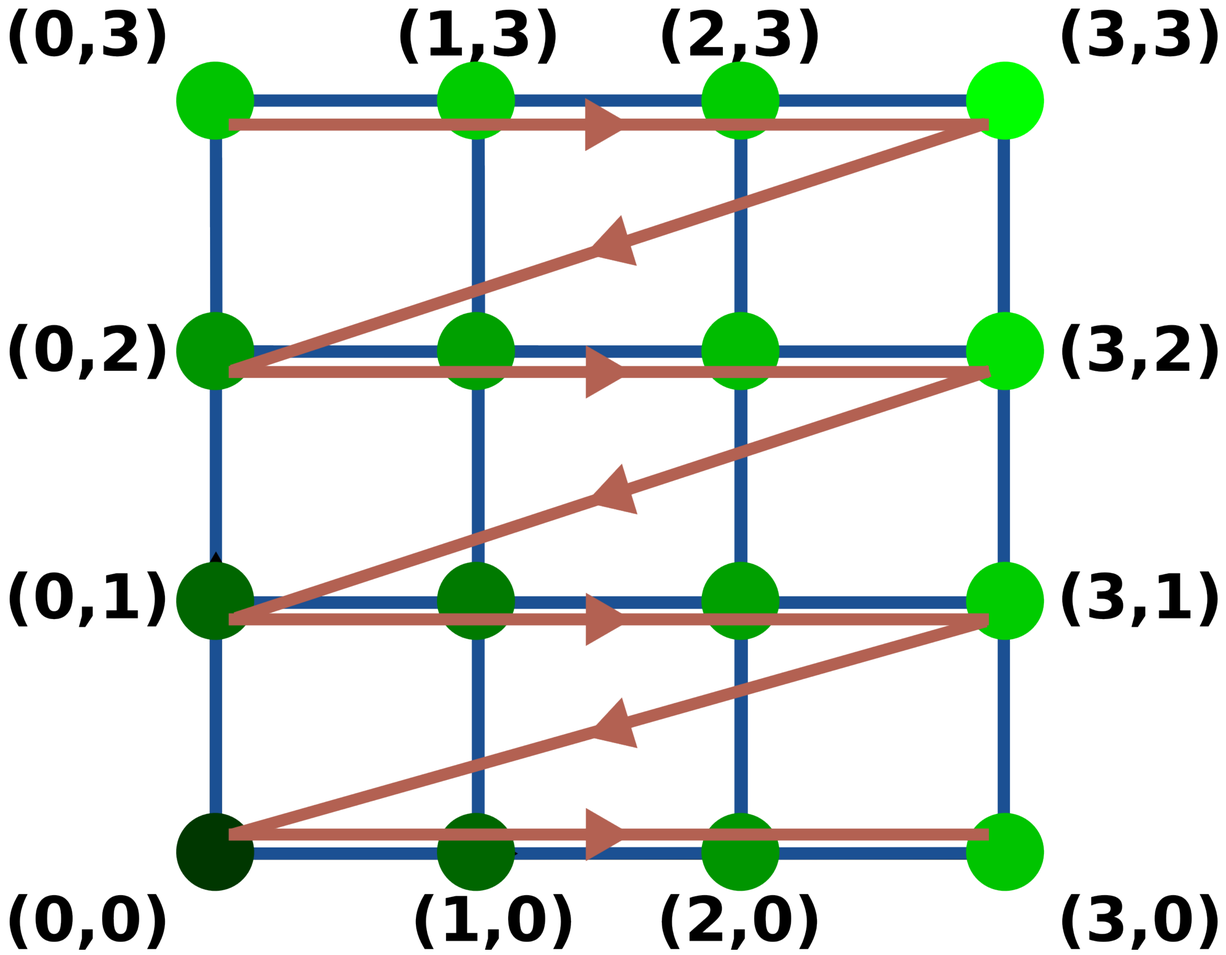

Regular grids

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- 3D regular grids
  - Unit cell: voxel
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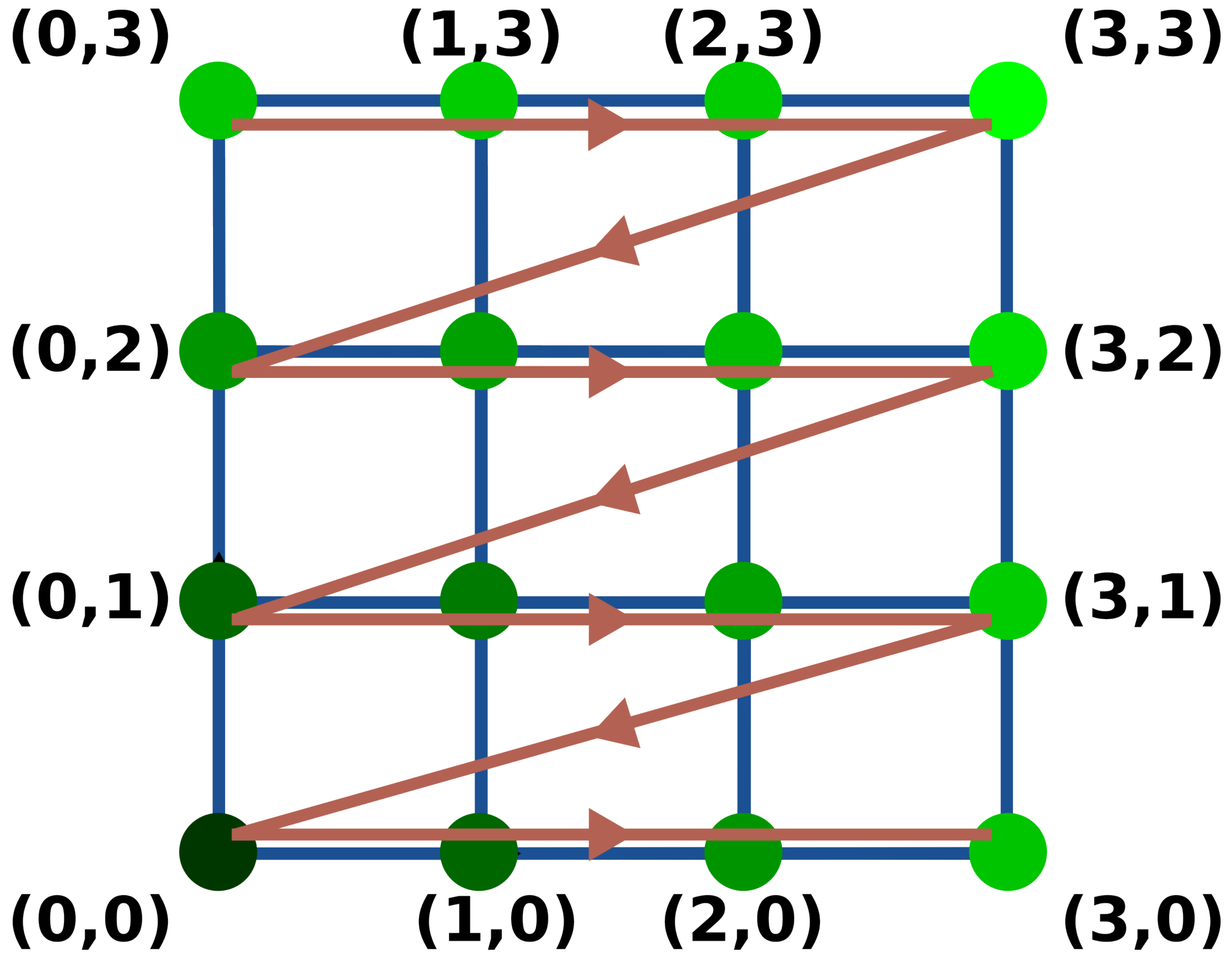

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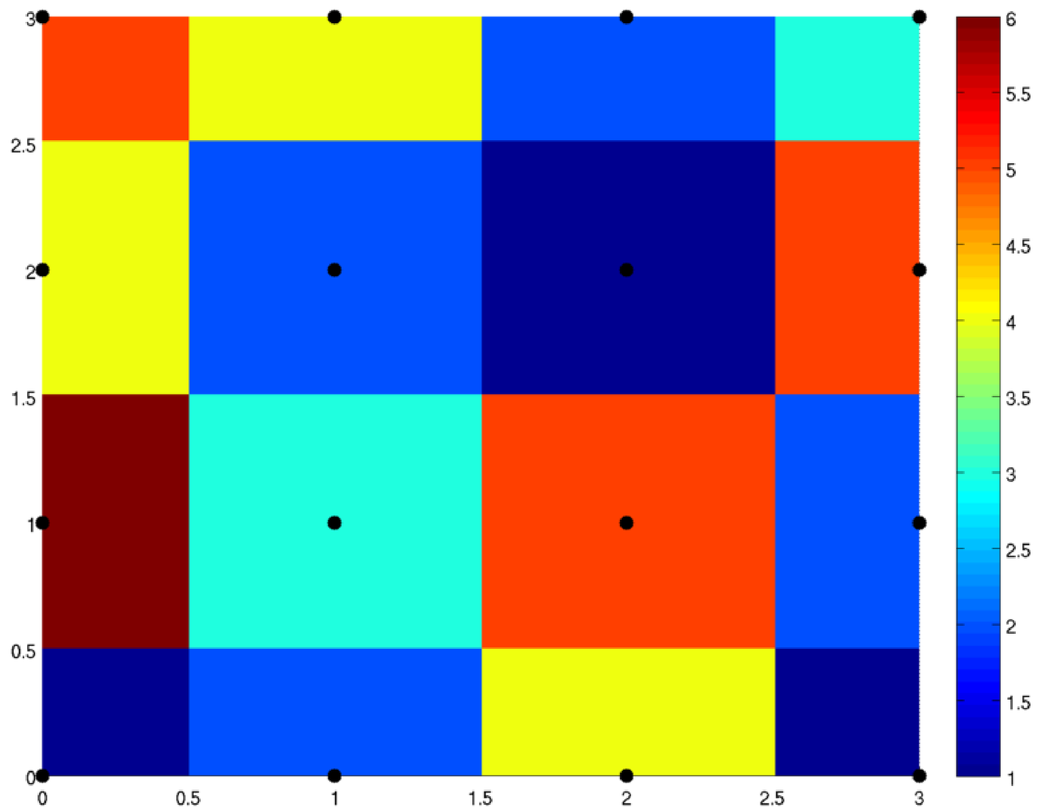

[Wikipedia]

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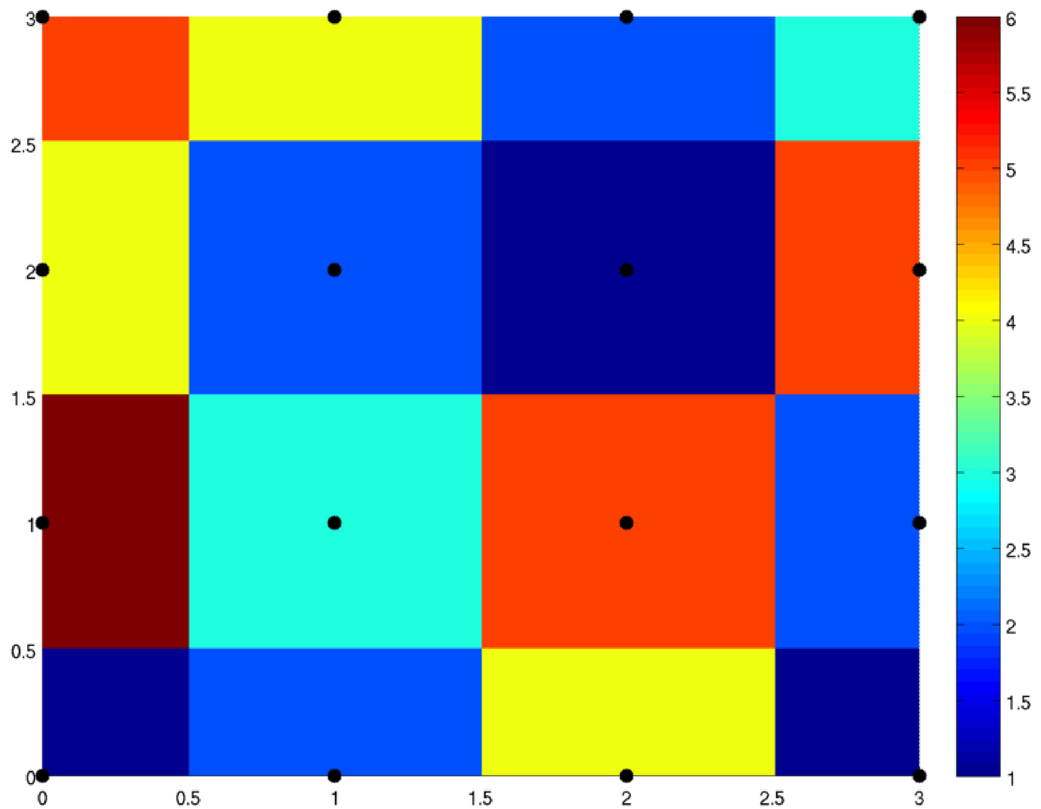

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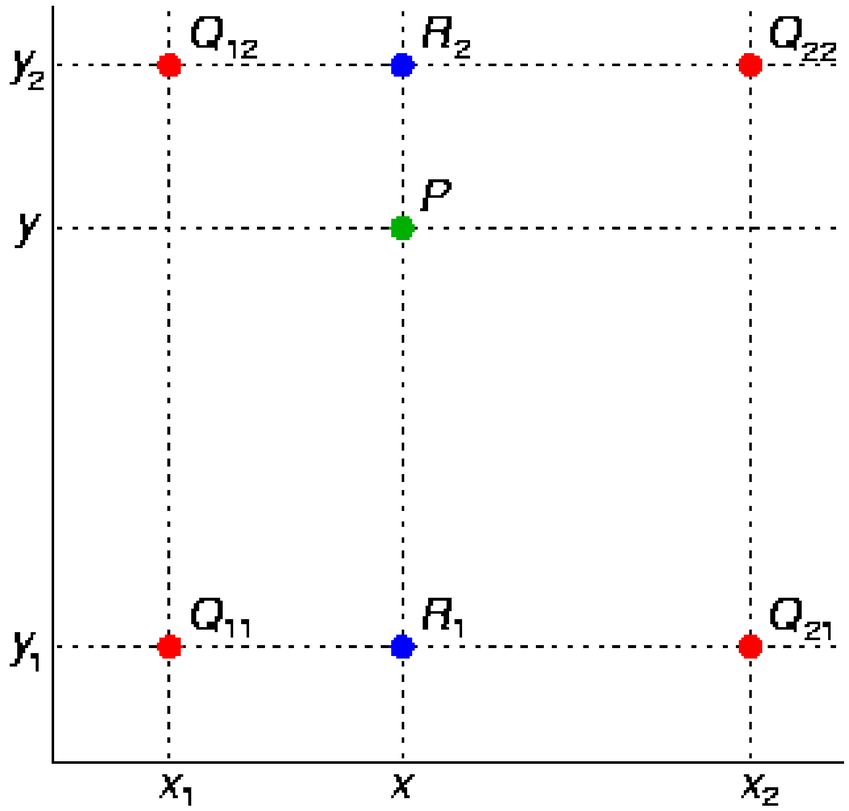

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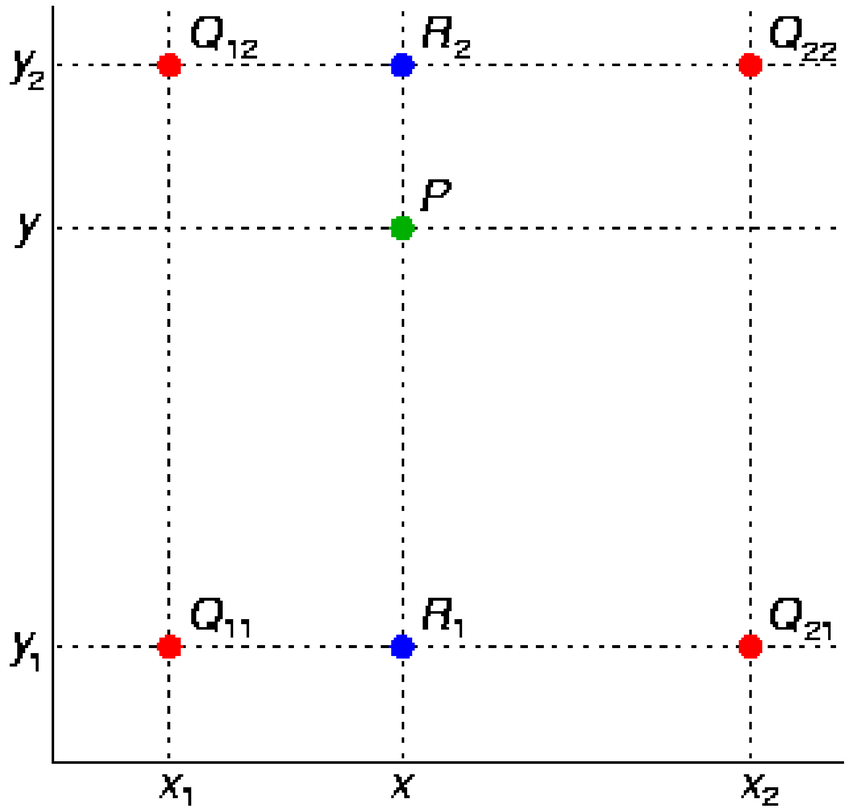

[Wikipedia]

Interpolants for regular grids

- For regular grids of $\mathbb{R}^2$
  - $f_{\mathbb{I}} : \ ]x_1, x_2[\ \times \ ]y_1, y_2[\ \rightarrow \mathbb{R}$
  - Examples
    - Bilinear interpolation
      - One dimension at a time

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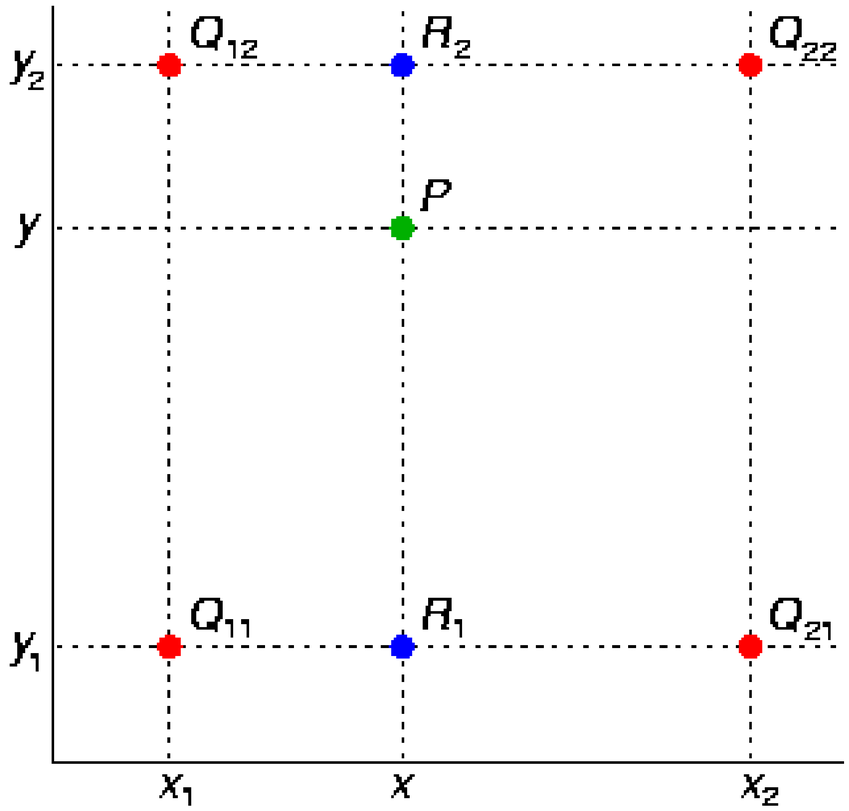

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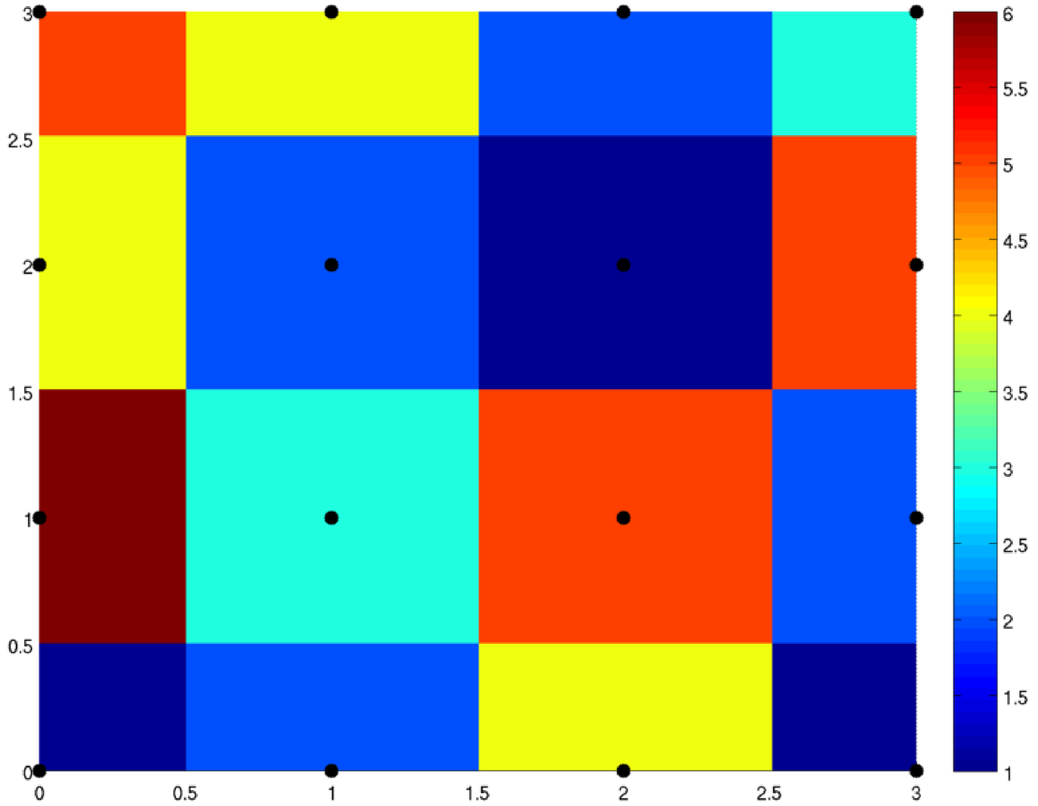

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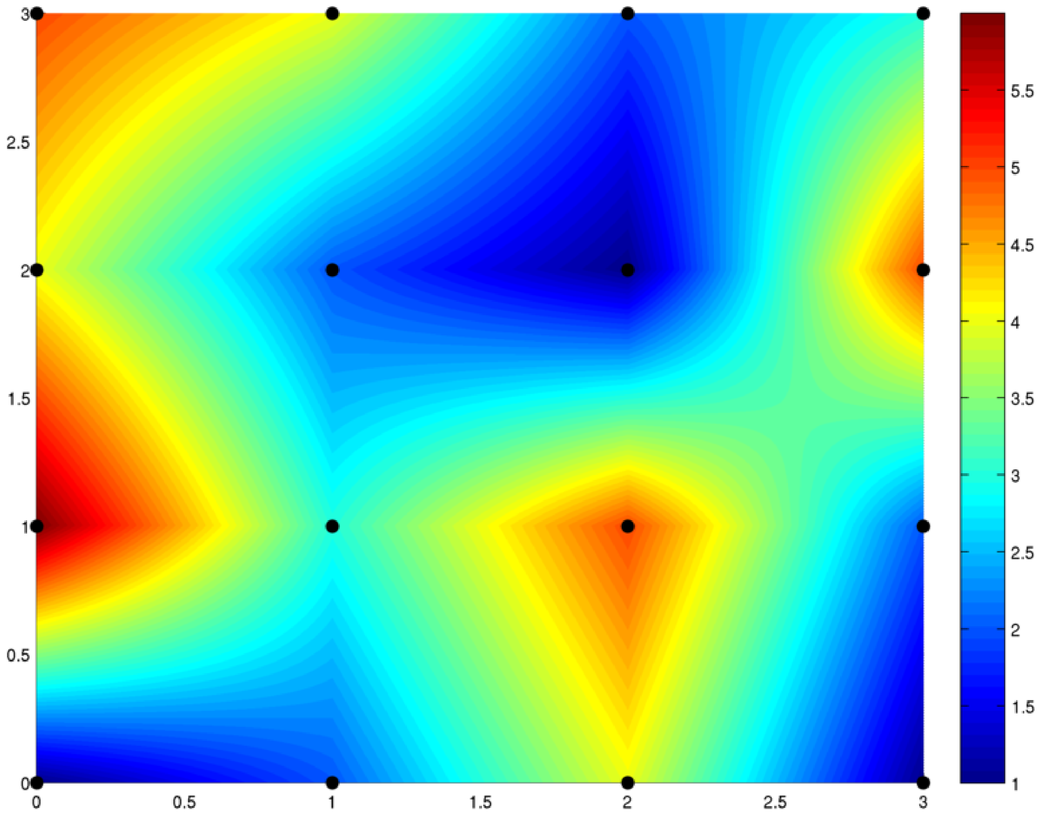

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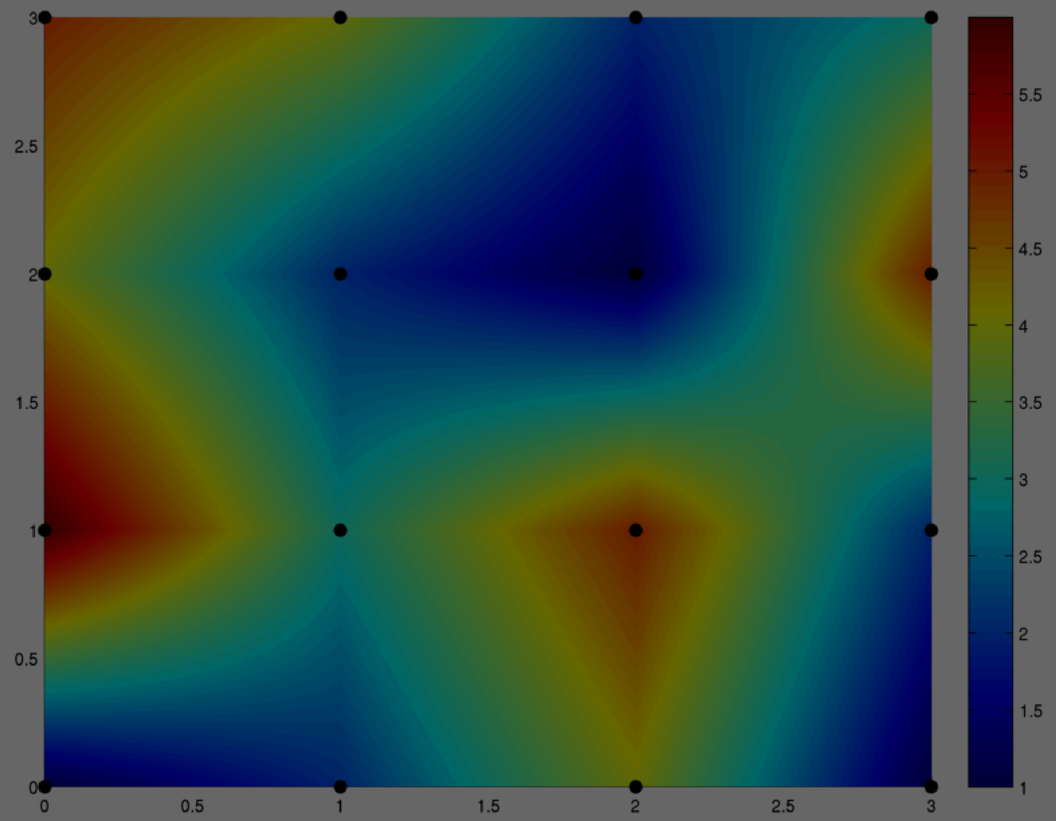

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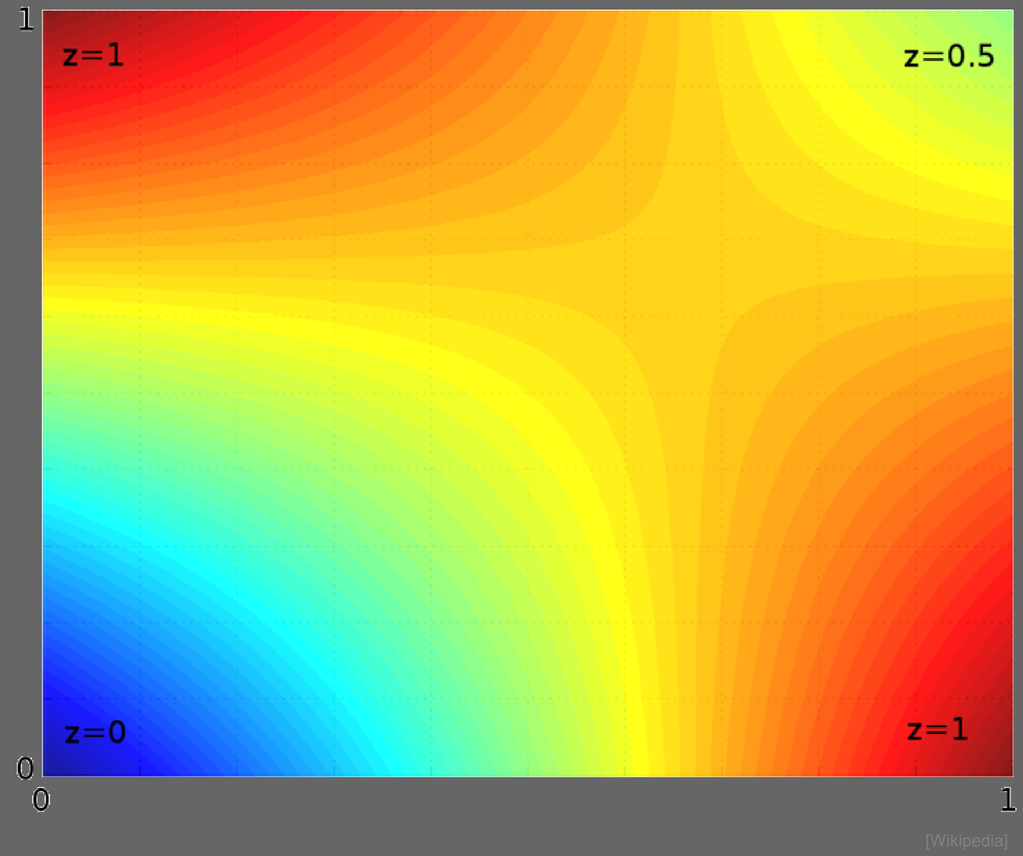

[Wikipedia]

Interpolants for regular grids

- For regular grids of $\mathbb{R}^2$
  - Problem

$$\frac{x_2 - x_1}{} (x_1, y_1)$$

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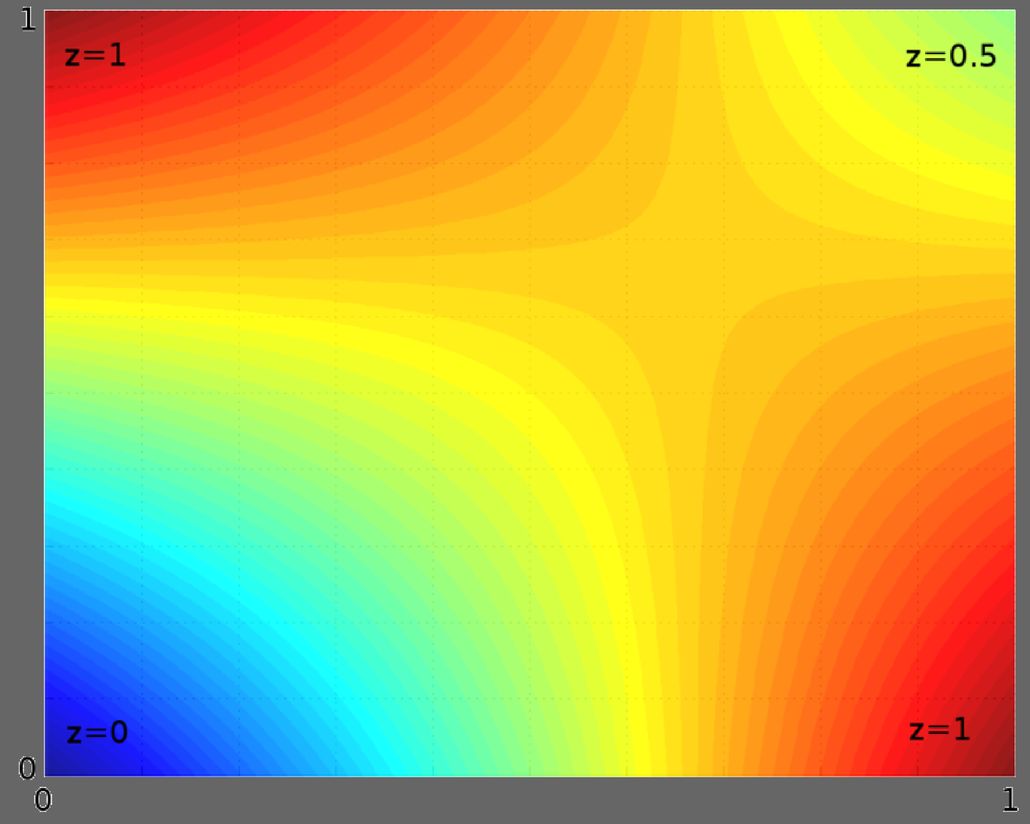

[Wikipedia]

Interpolants for regular grids

- For regular grids of $\mathbb{R}^2$
  - Problem
    - Critical points in the interior

$$x_2 - x_1 \qquad (x_1, y_1)$$

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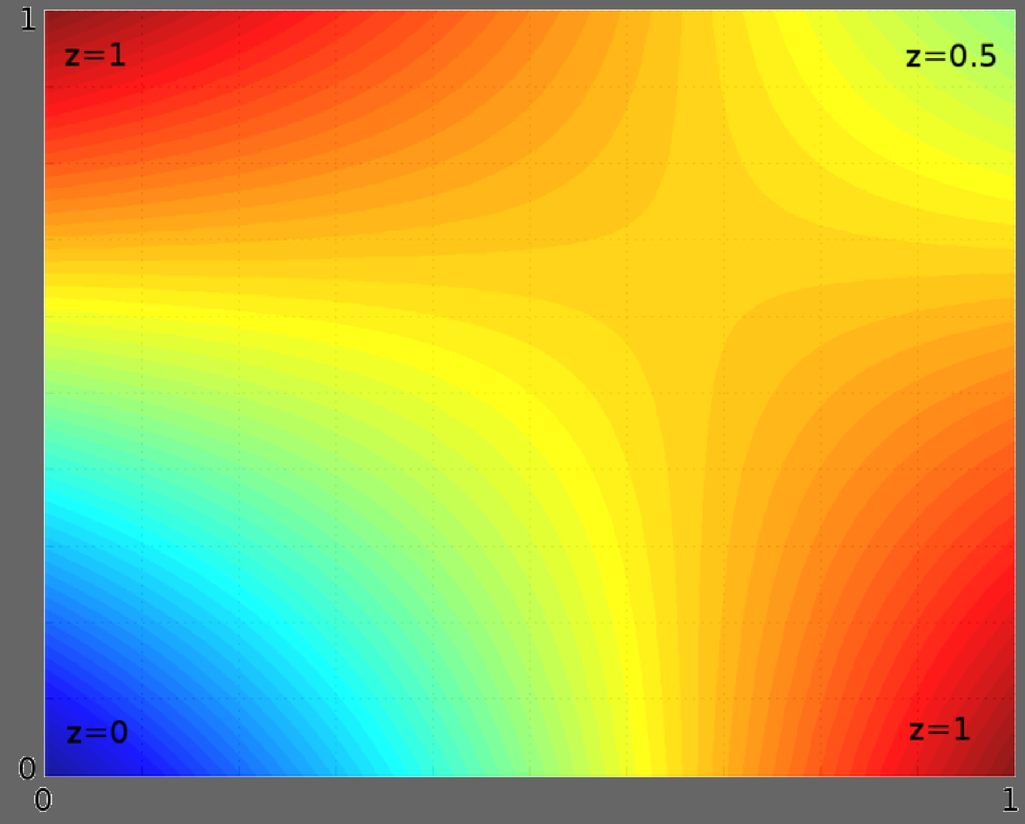

[Wikipedia]

Interpolants for regular grids

- For regular grids of $\mathbb{R}^2$
  - Problem
    - Critical points in the interior
    - Source of many ambiguities in visualization

$$\frac{x_2 - x_1}{} (x_1, y_1)$$

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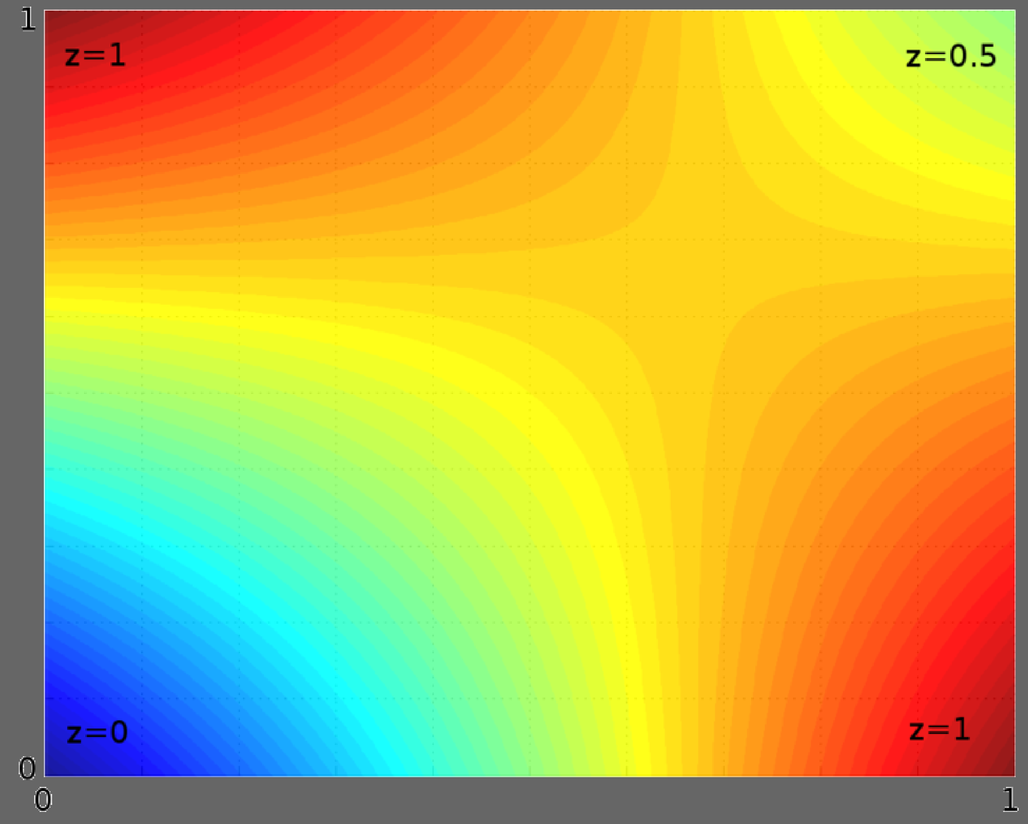

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Interpolants for regular grids

- For regular grids of $\mathbb{R}^2$
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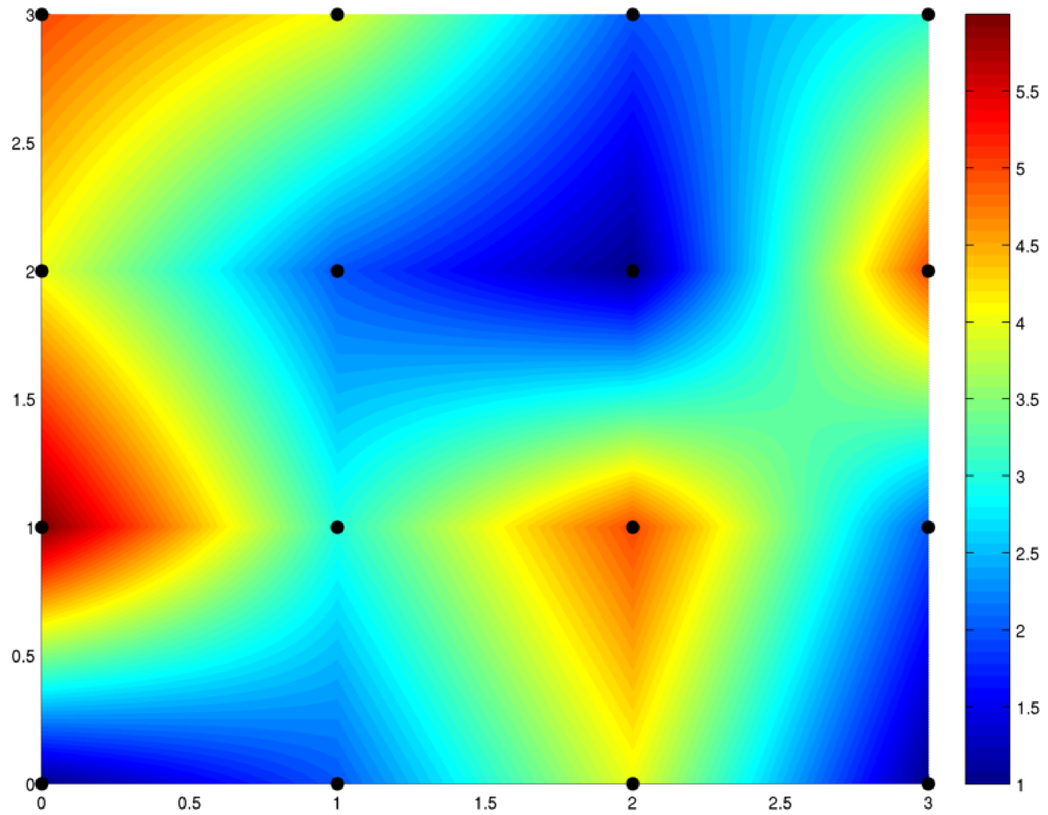

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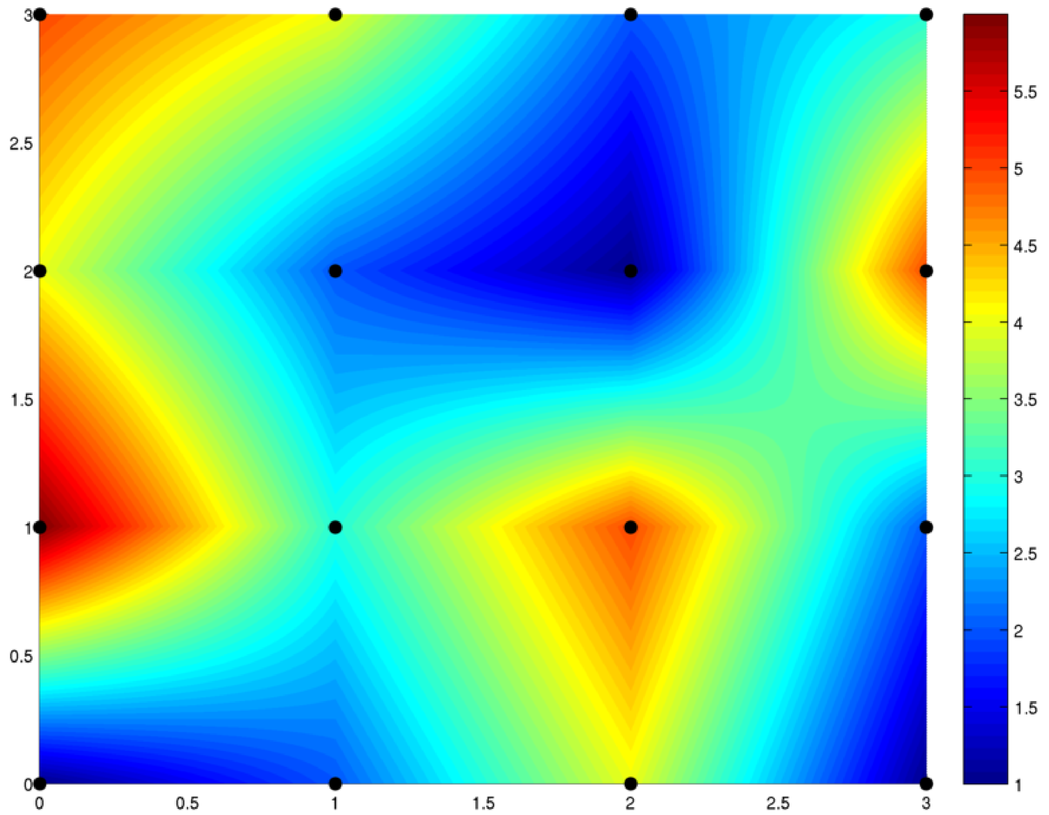

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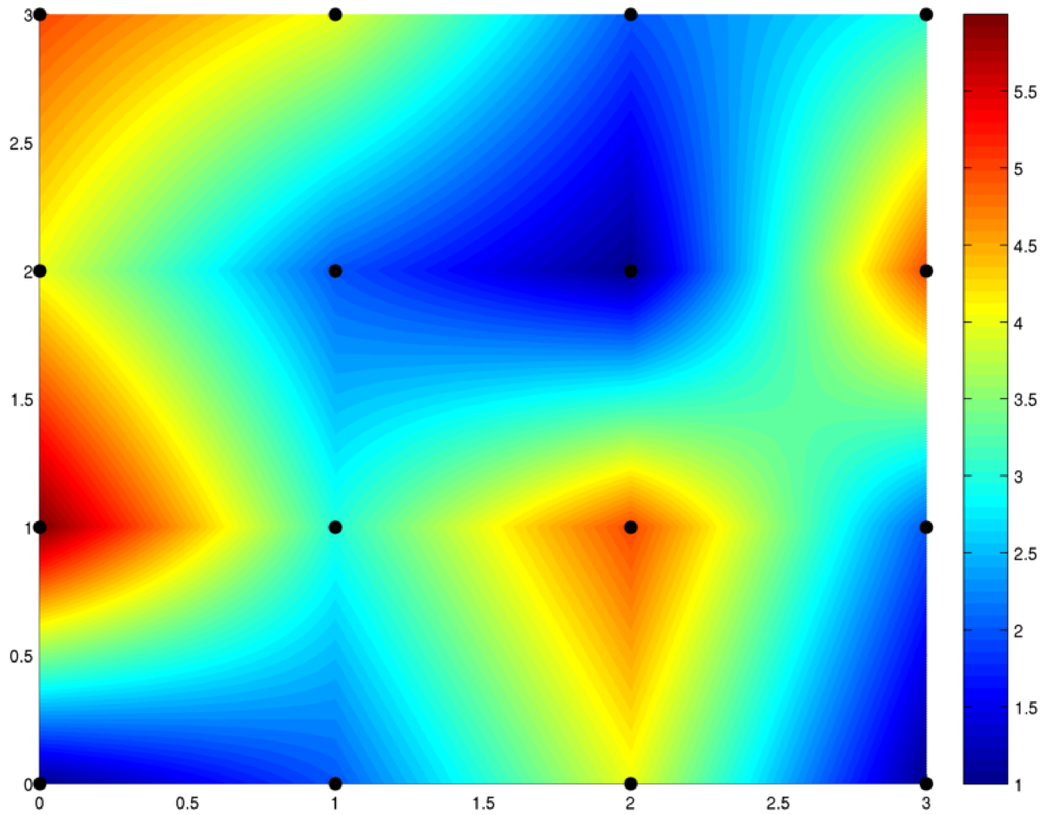

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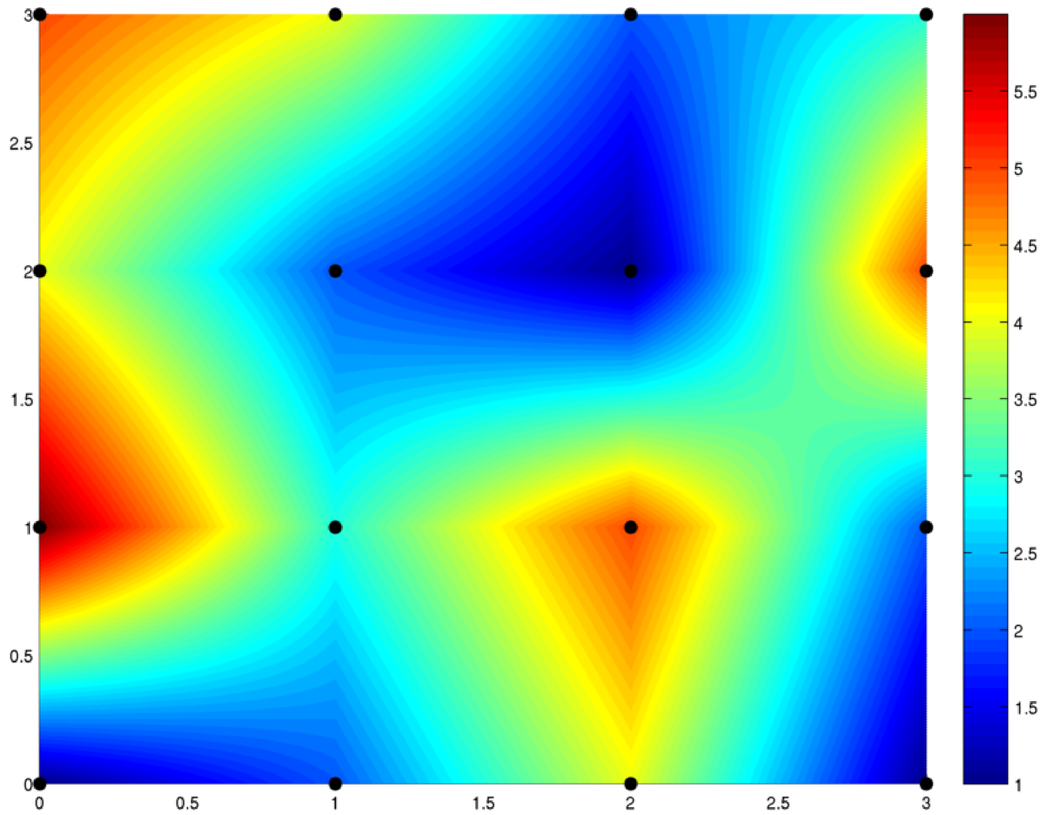

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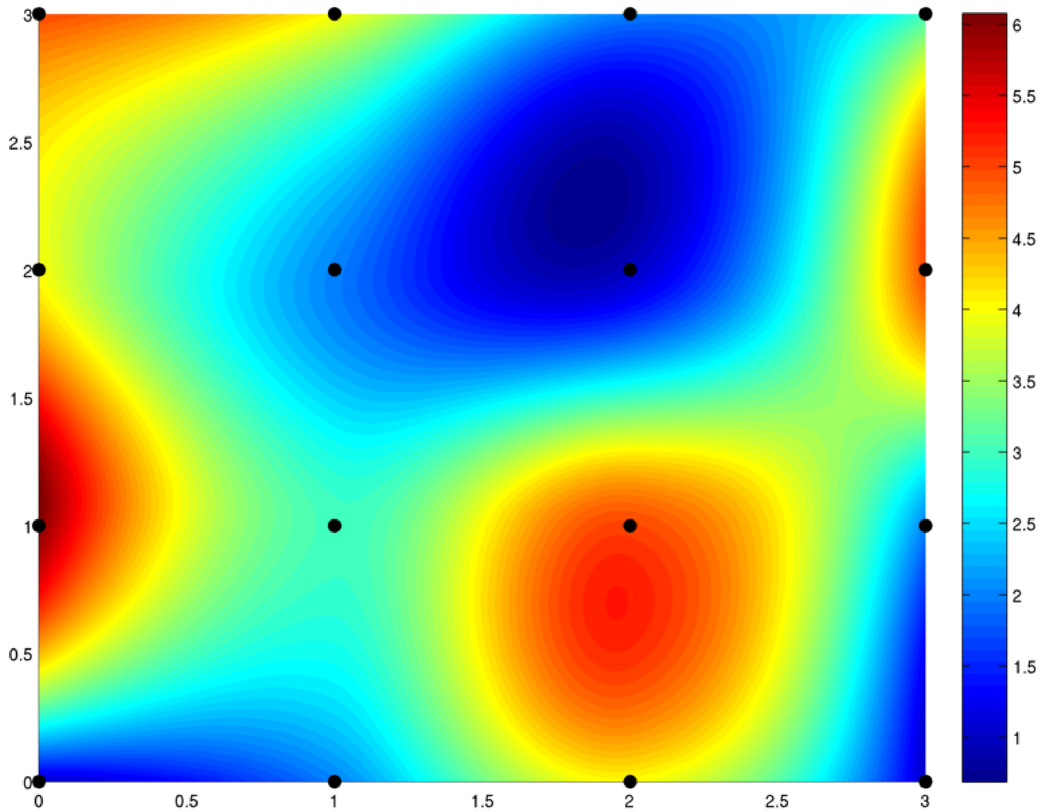

[Wikipedia]

Interpolants for regular grids

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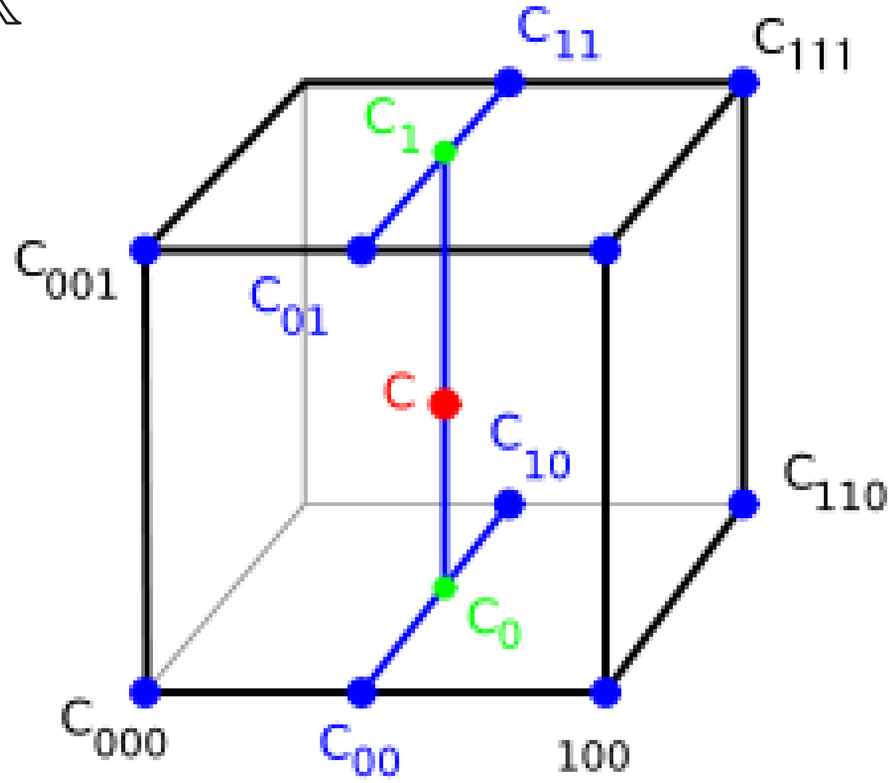

[Wikipedia]

Manifolds on a computer

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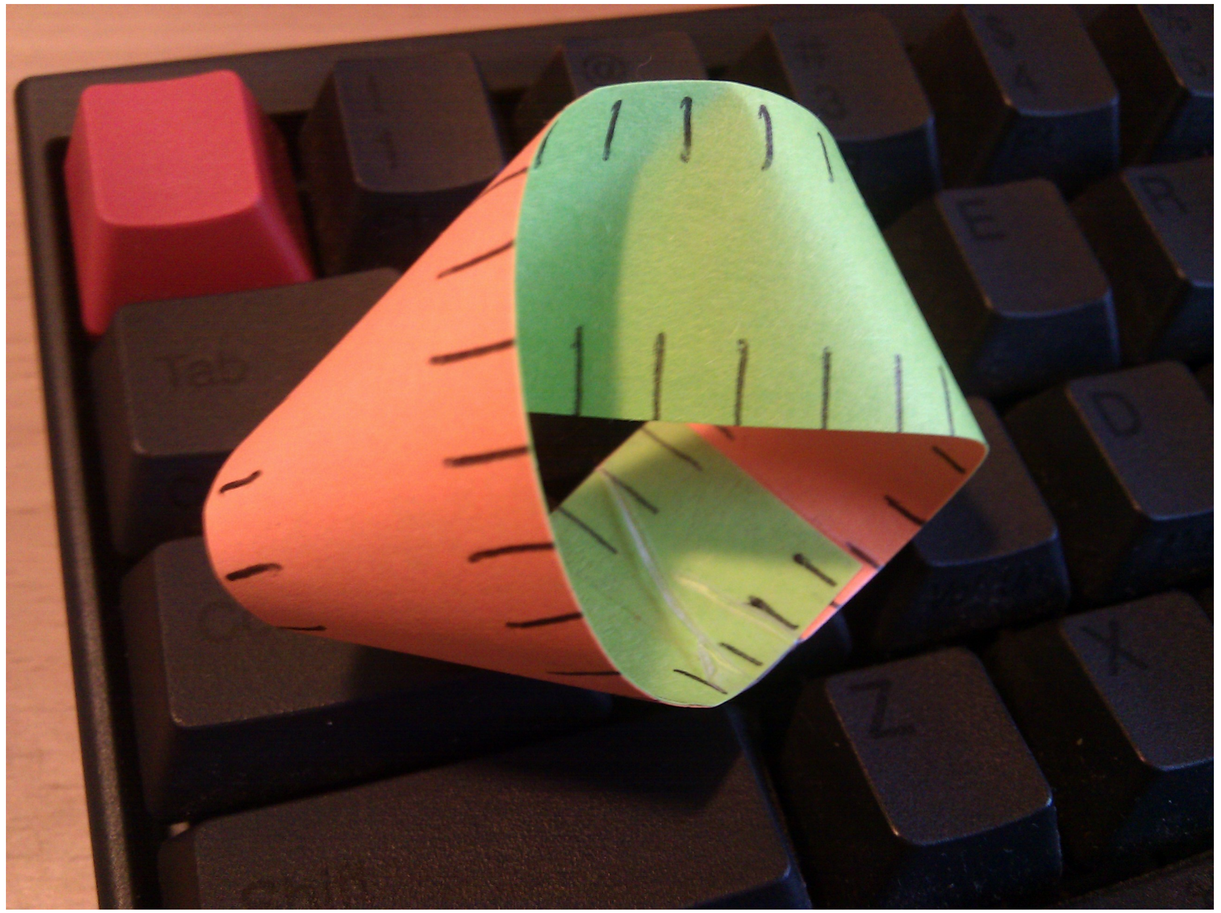
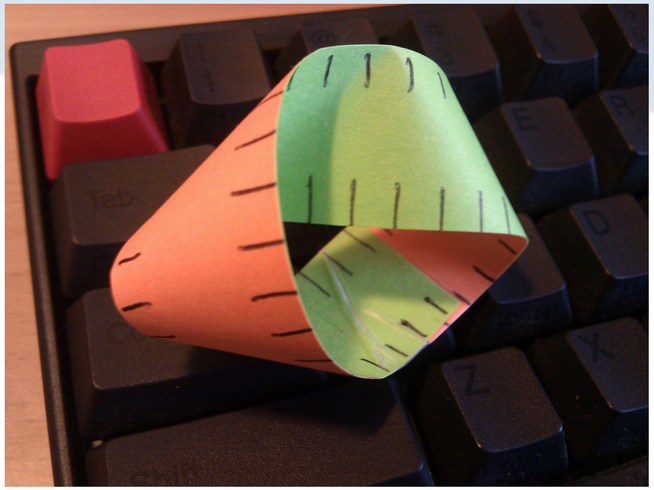


Manifolds on a computer

- Notion of simplex



Manifolds on a computer

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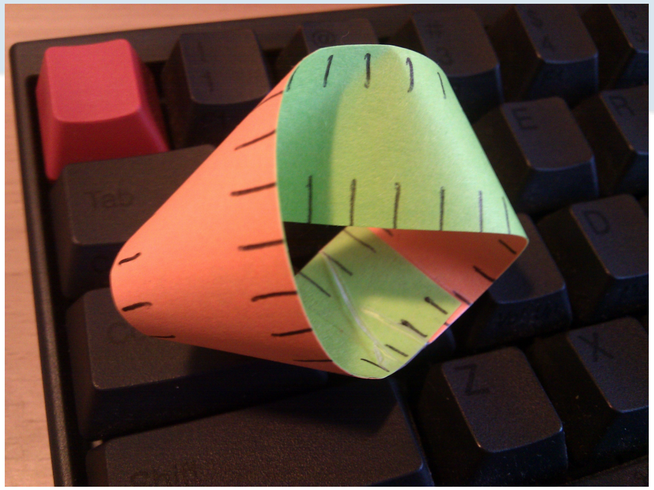
  - A d-simplex σ is the convex hull of (d + 1) affinely independent points of an euclidean space $\mathbb{R}^n$



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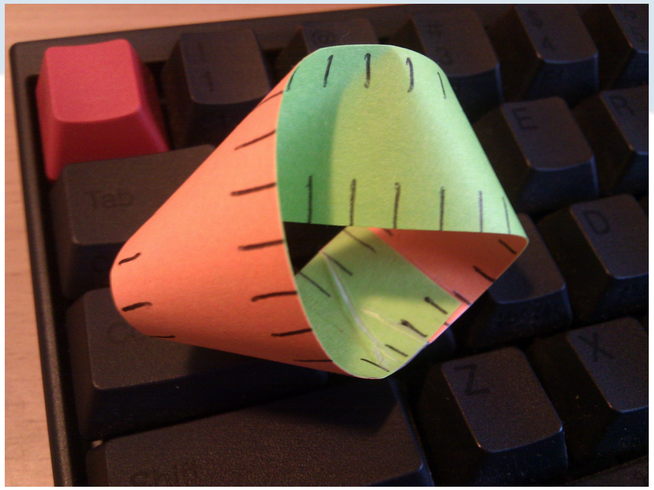

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  - $0 \leq d \leq n$

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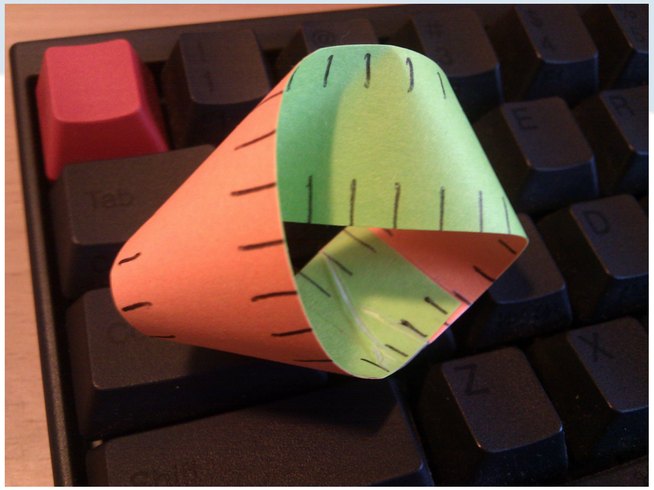

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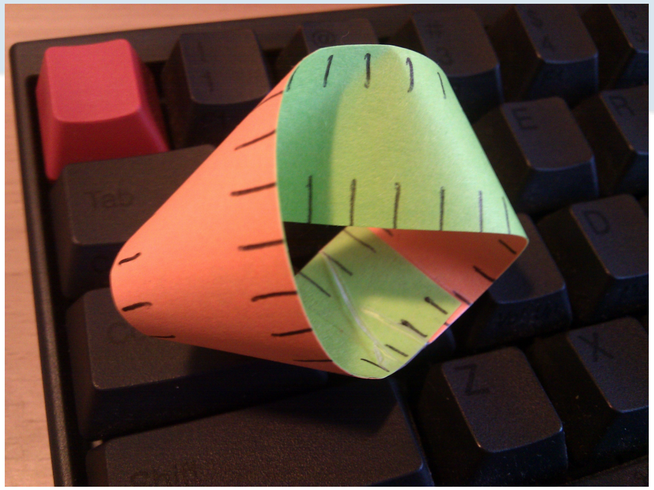

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  - Smallest combinatorial construction to represent a cell of dimension d

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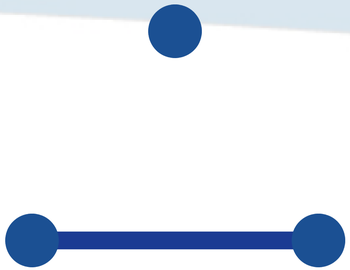
  - 1-simplex: *edge*



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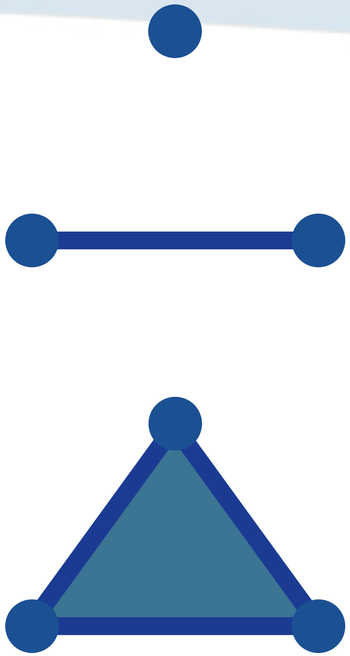
  - 2-simplex: *triangle*



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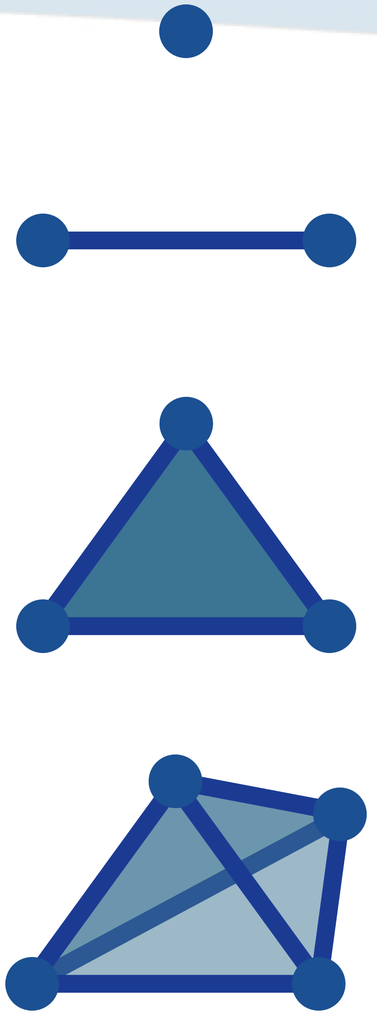
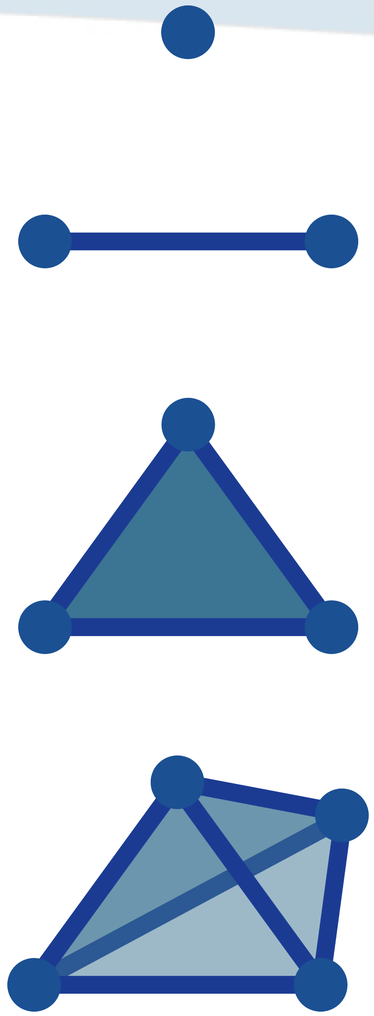
  - 3-simplex: *tetrahedron*



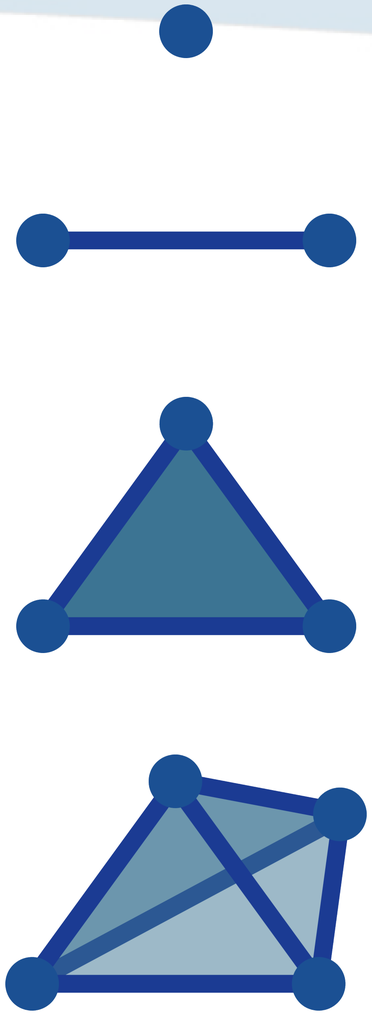
Manifolds on a computer

- Notion of face



Manifolds on a computer

- Notion of face
  - A face τ of a d-simplex σ is the simplex defined by a non-empty subset of the d+1 points of σ
  - Noted $\tau \leq \sigma$



Manifolds on a computer

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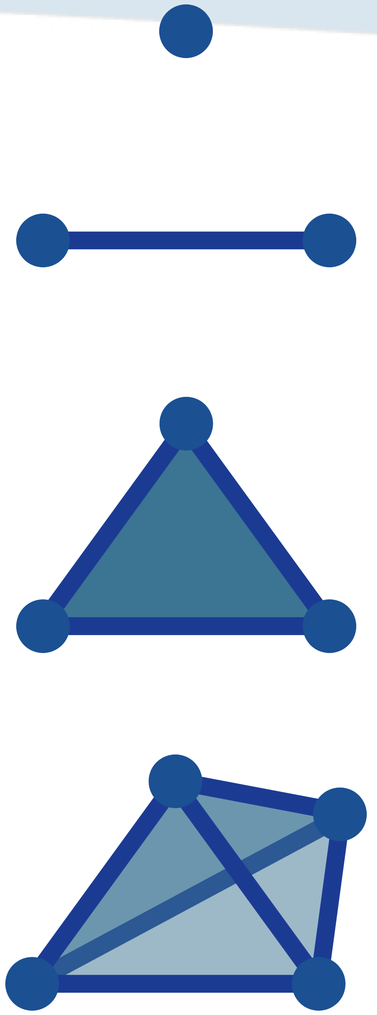
- Recursive construction!



Manifolds on a computer

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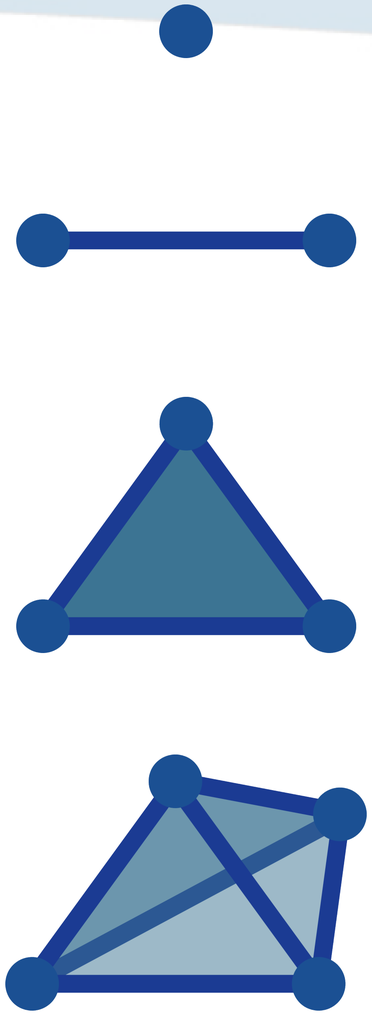

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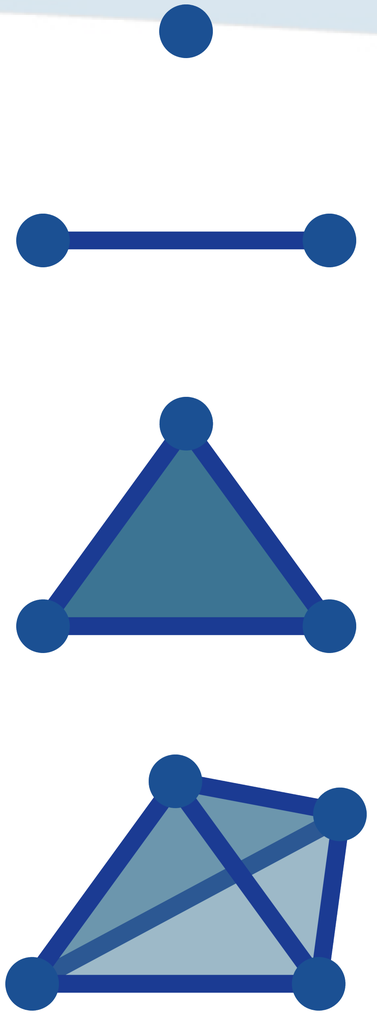

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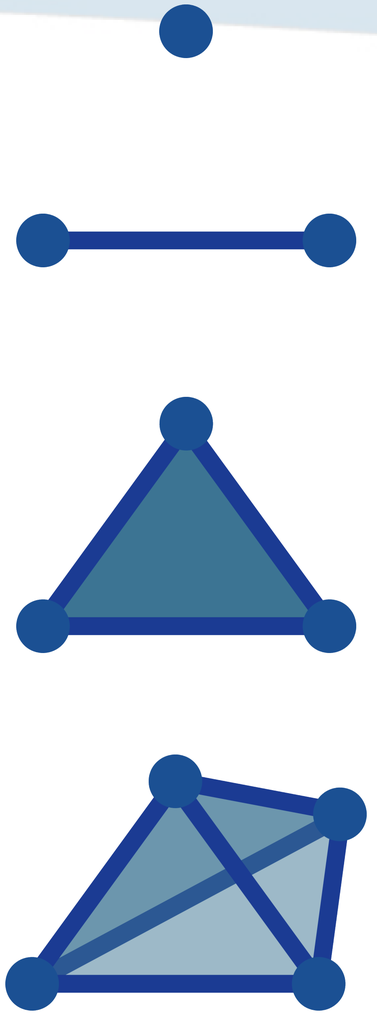

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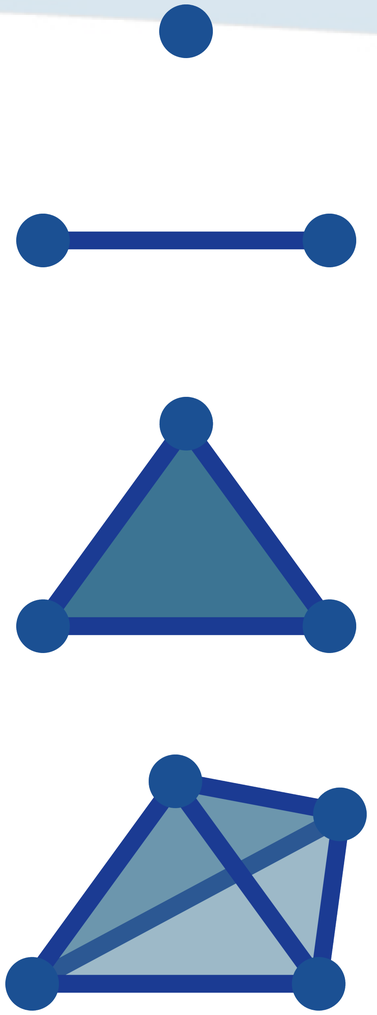

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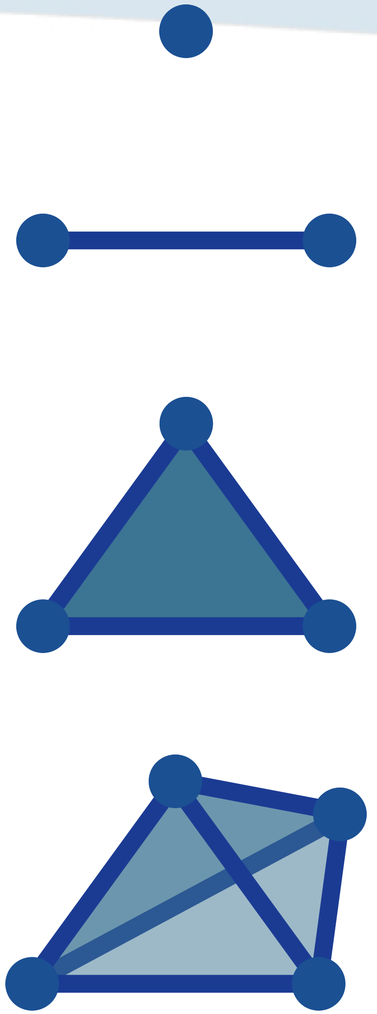

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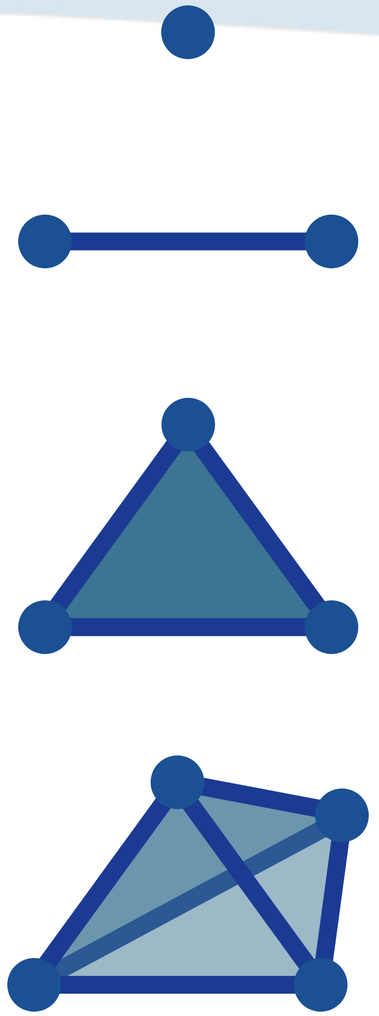

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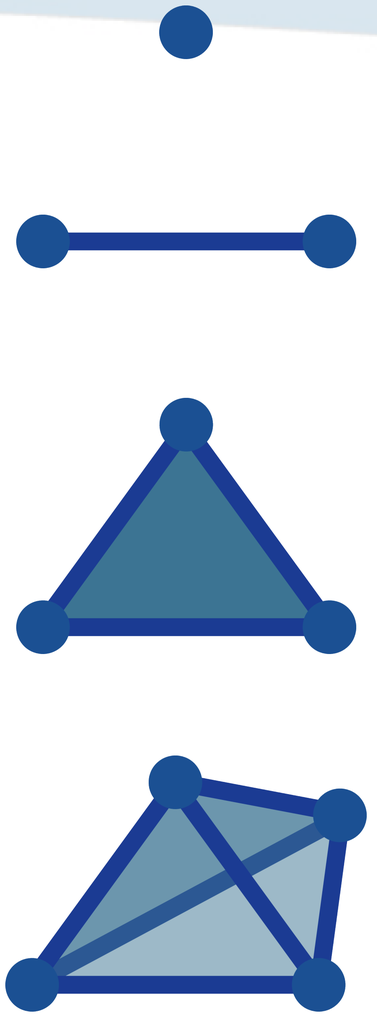

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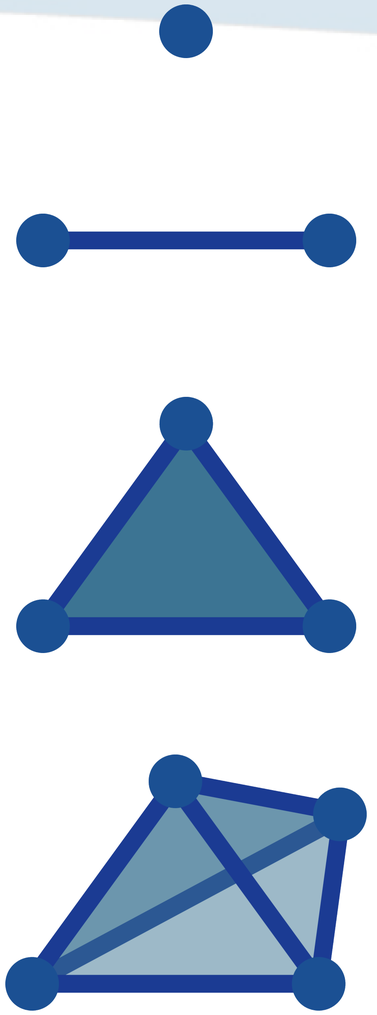

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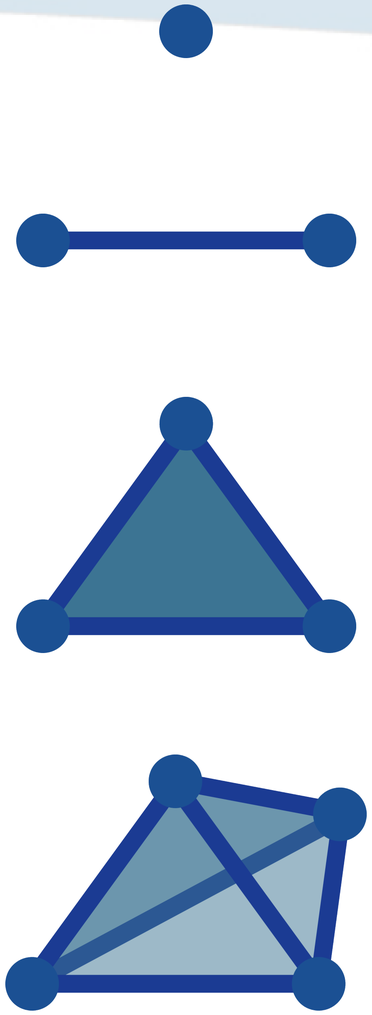

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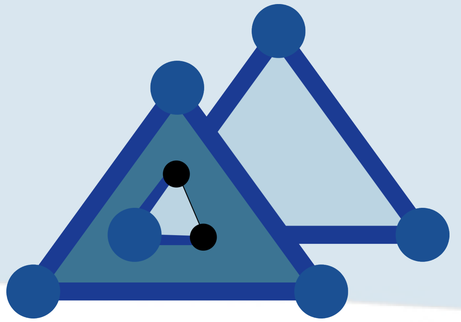

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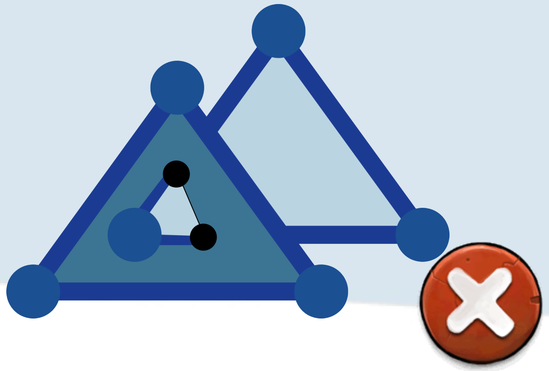

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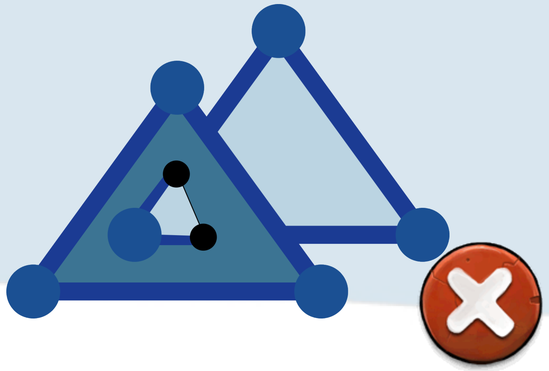

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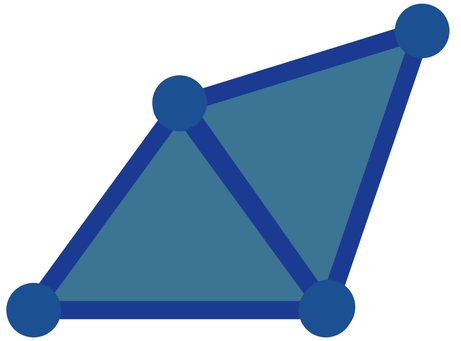

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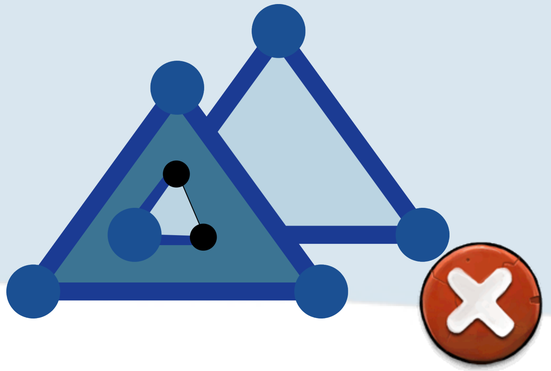

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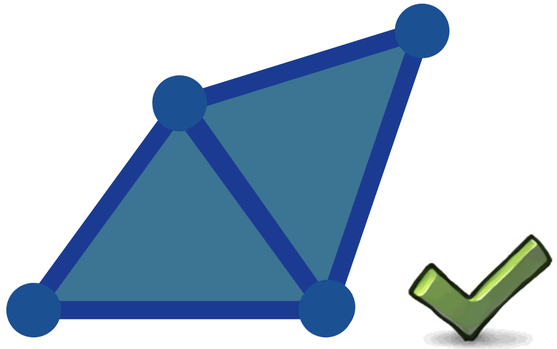

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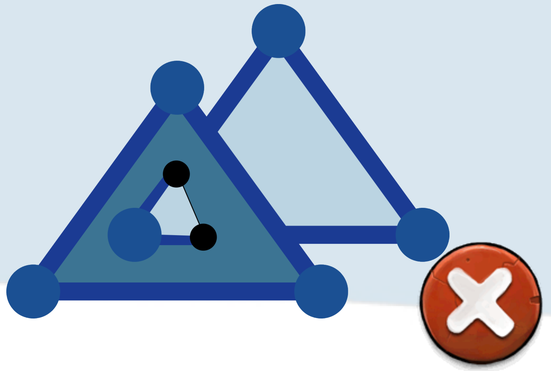

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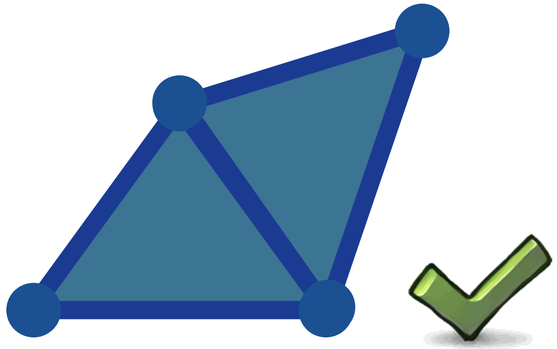

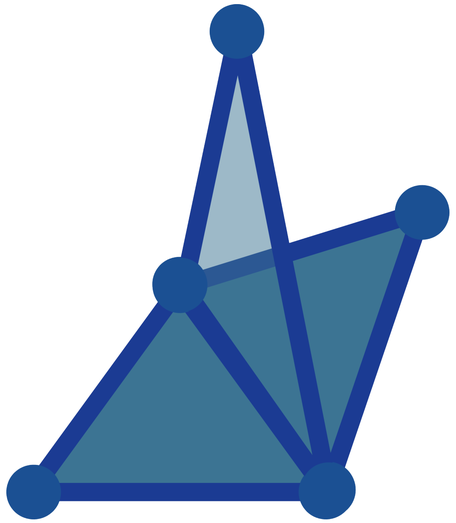

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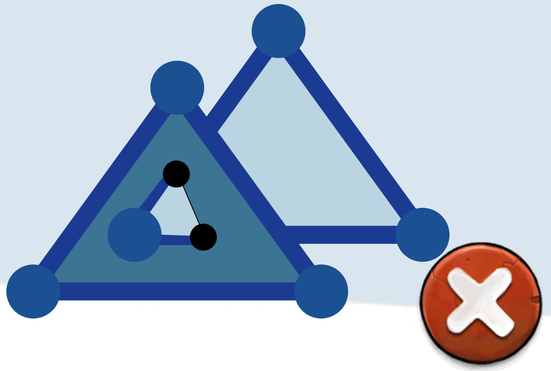

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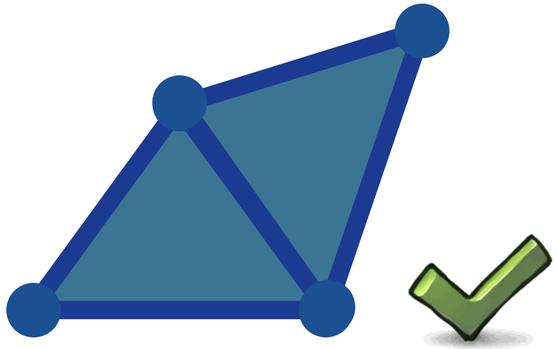

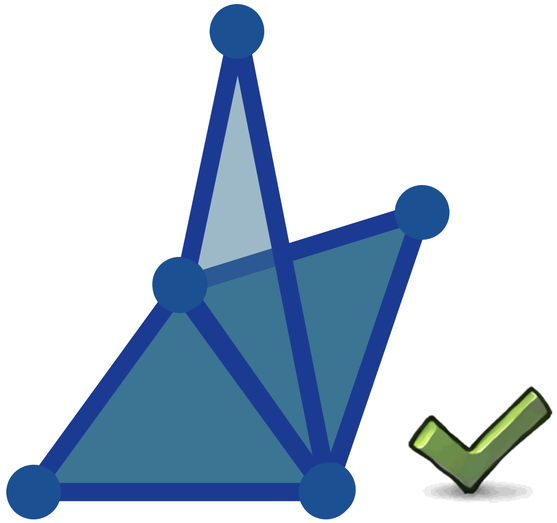

Manifolds on a computer

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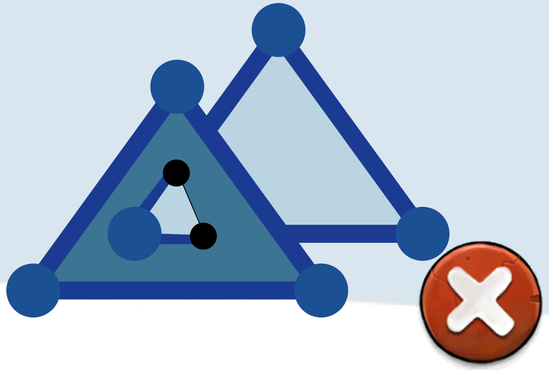

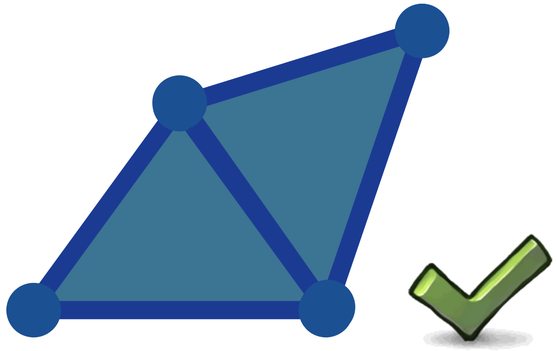

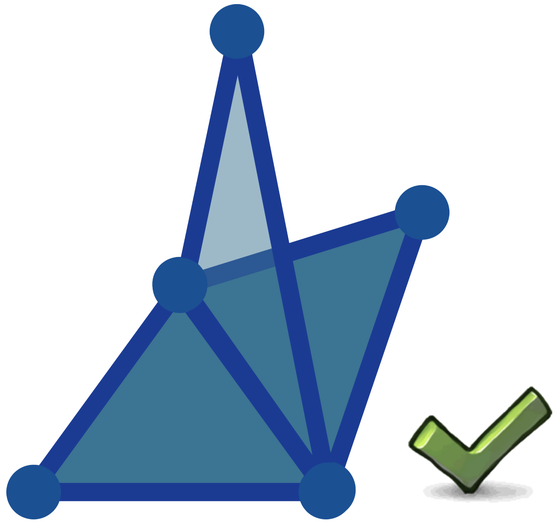

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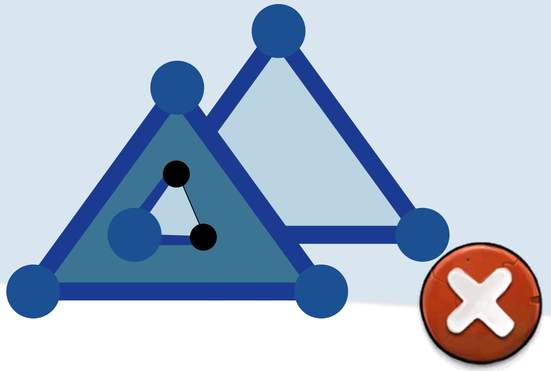

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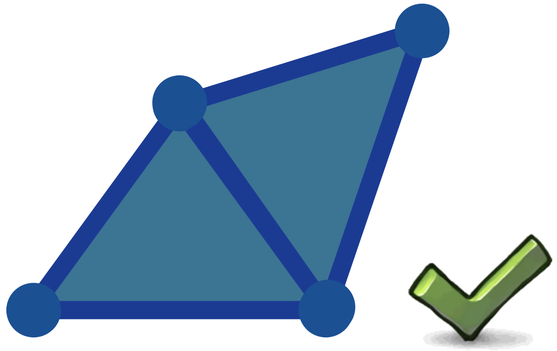

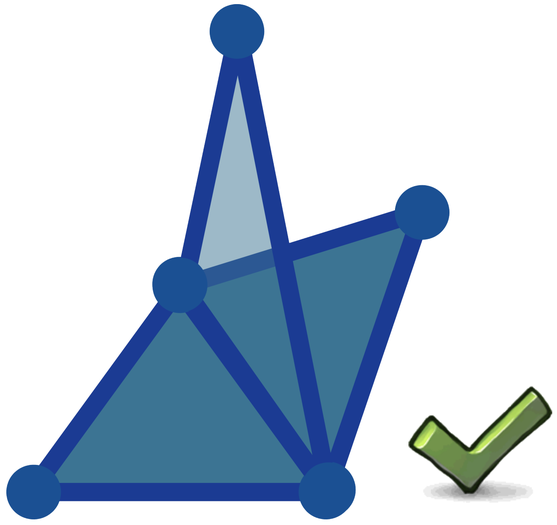

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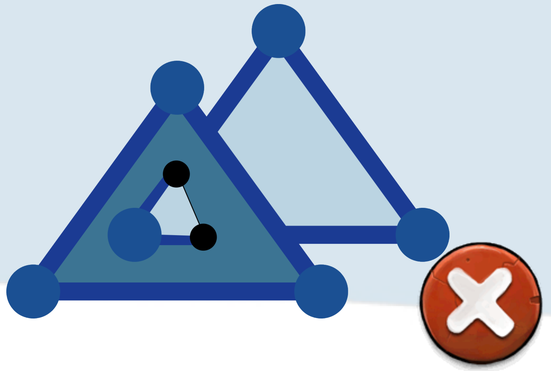

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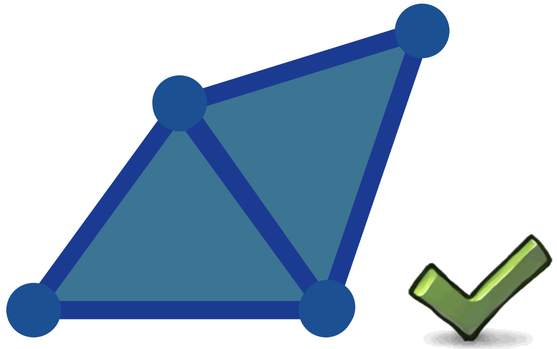

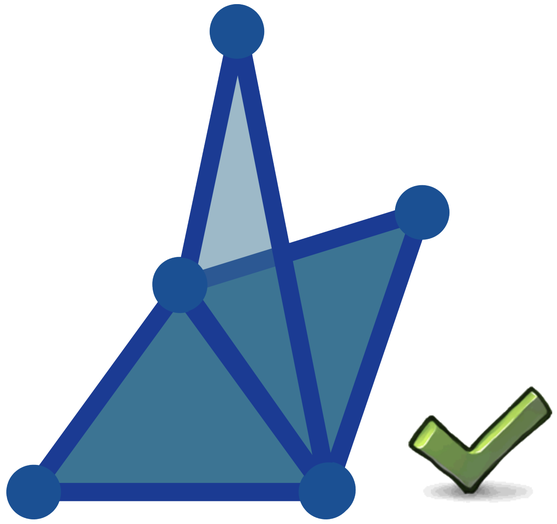

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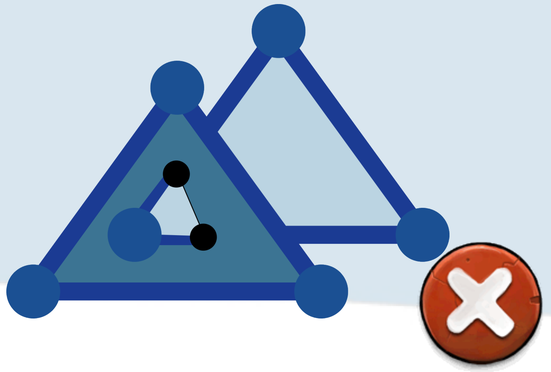

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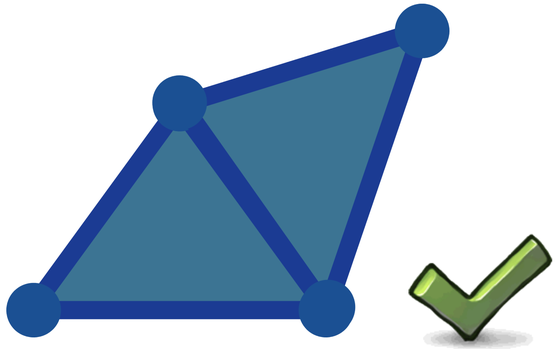

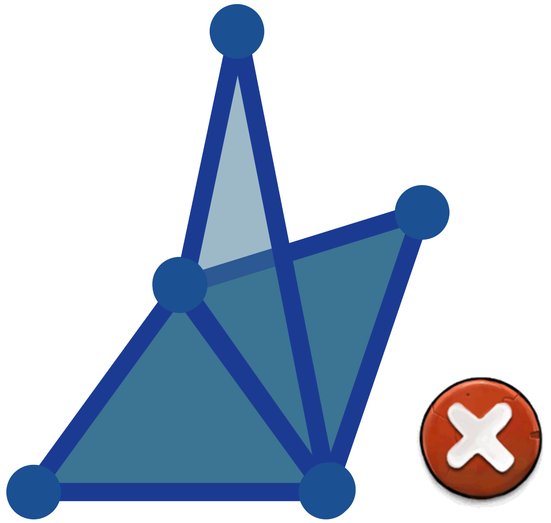

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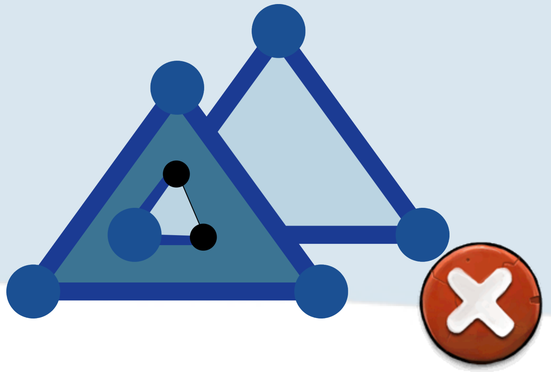

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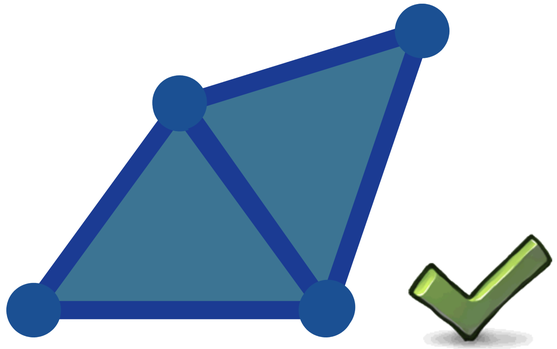

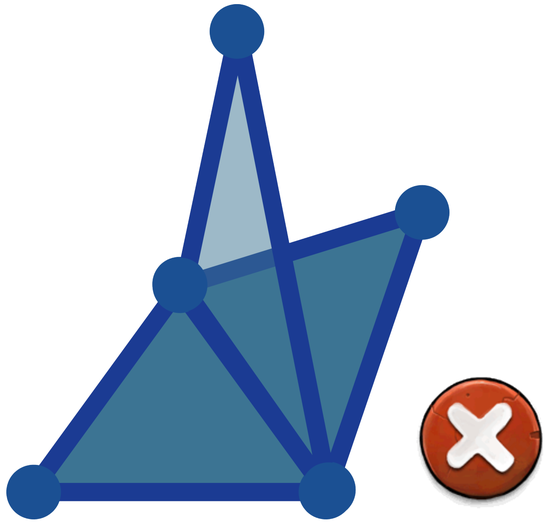

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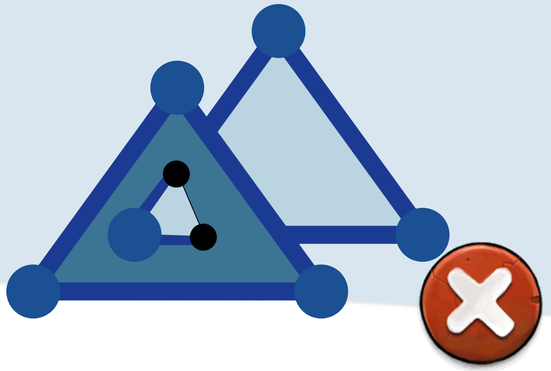

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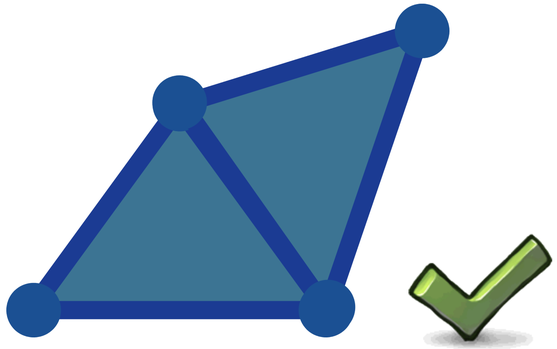
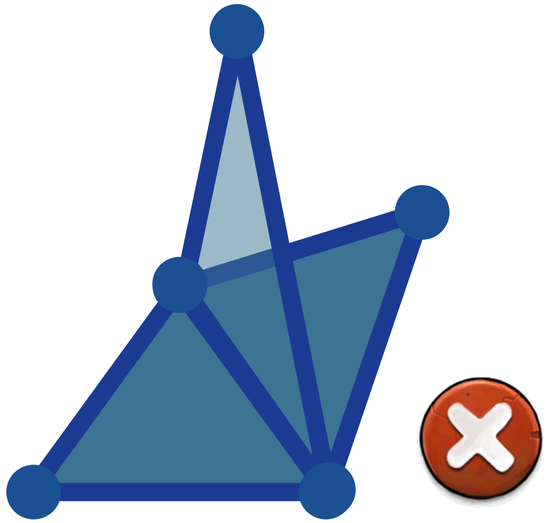
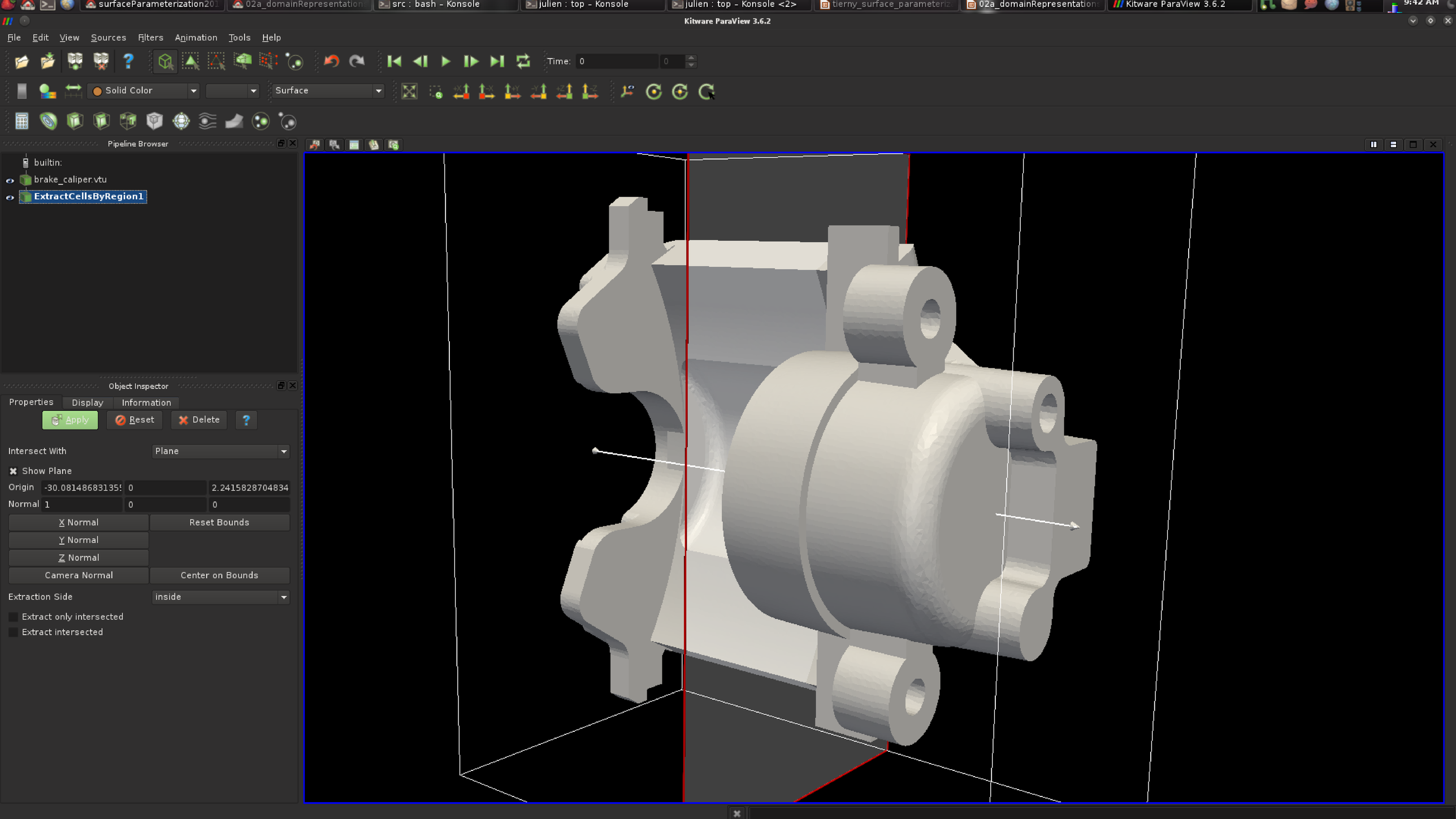


- 2-triangulation: *triangle mesh*
- 3-triangulation: *tetrahedral mesh*

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02a_domainRepresentation
src : bash - Konsole
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julien : top - Konsole <2>
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02a_domainRepresentation
Kitware ParaView 3.6.2
9:42 AM
Kitware ParaView 3.6.2
File Edit View Sources Filters Animation Tools Help
Time: 0
Solid Color
Surface
Pipeline Browser
builtin:
brake_caliper.vtu
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Object Inspector
Properties Display Information
Apply Reset Delete
Intersect With Plane
Show Plane
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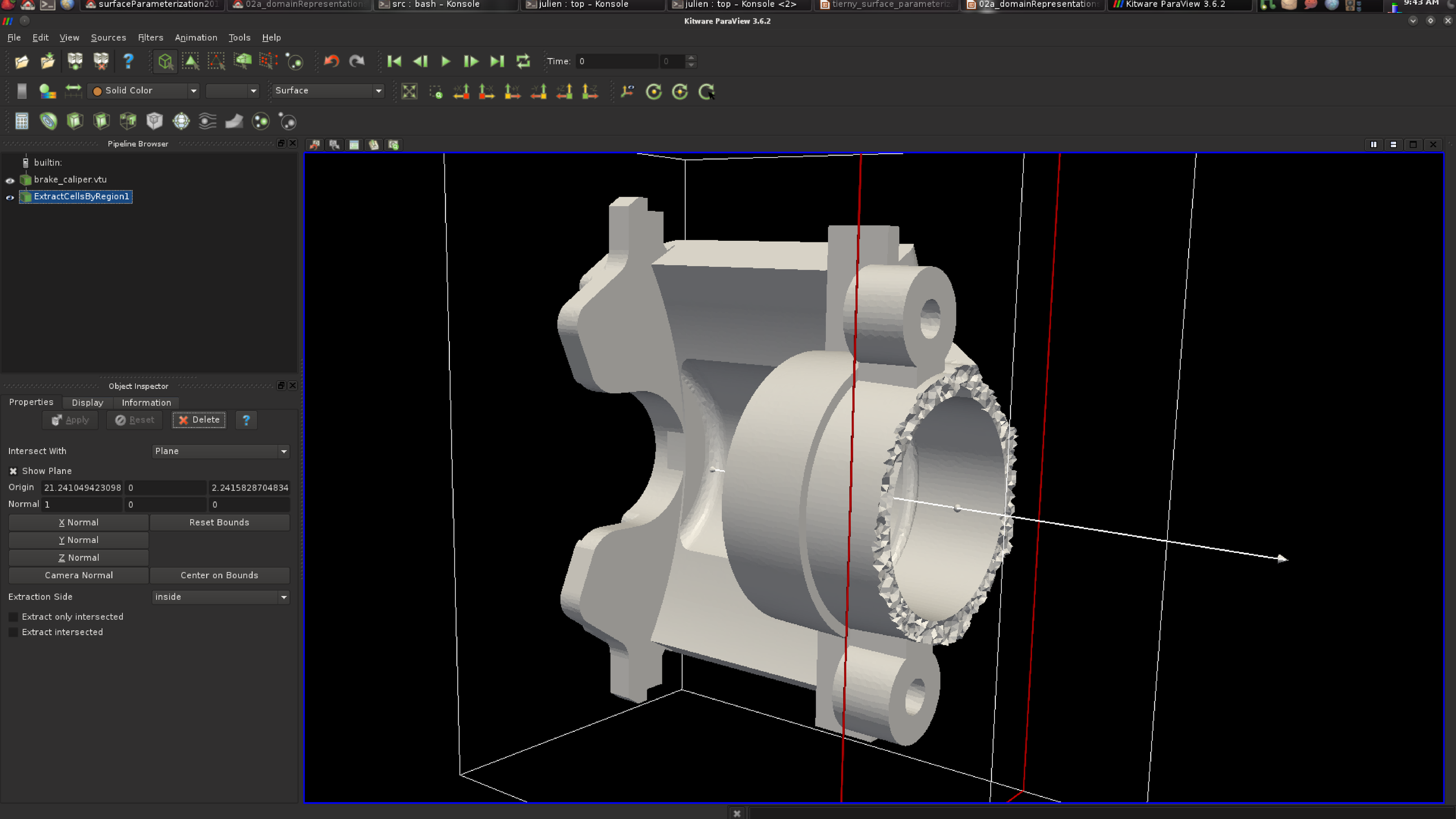
X Normal
Reset Bounds
Y Normal
Z Normal
Camera Normal
Center on Bounds
Extraction Side inside
Extract only intersected
Extract intersected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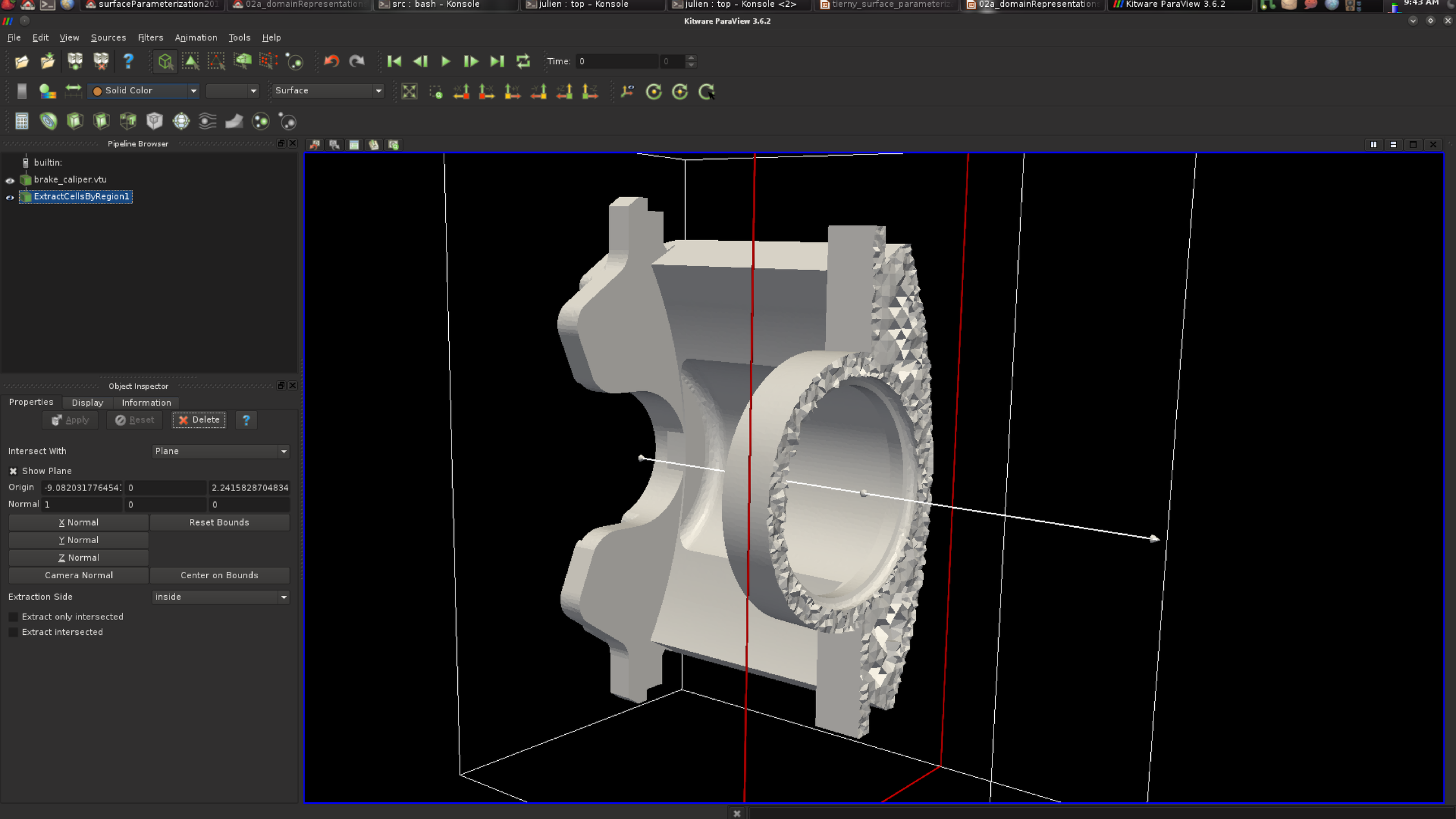
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Properties Display Information
Apply Reset Delete
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Y Normal
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Center on Bounds
Extraction Side inside
Extract only intersected
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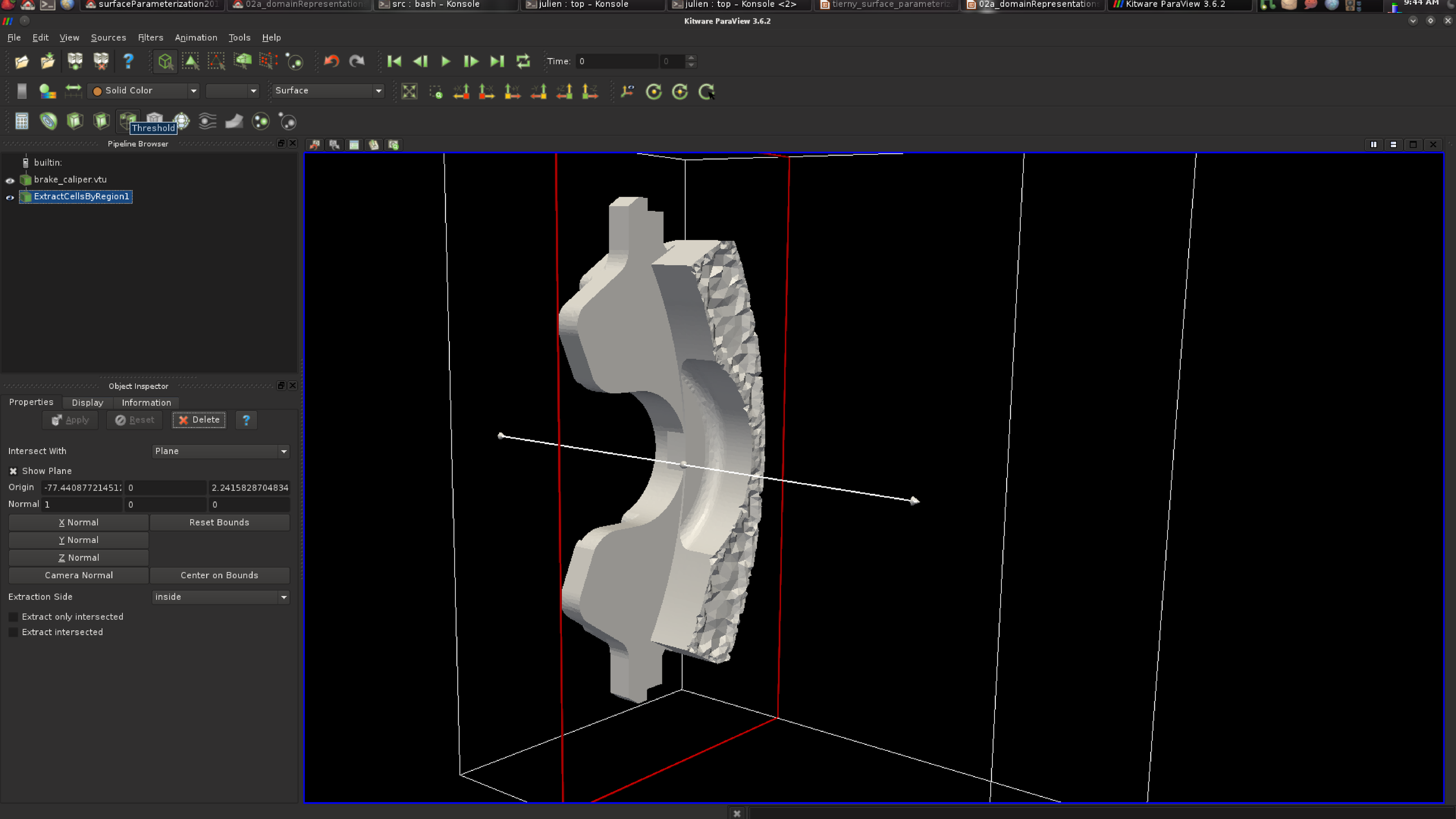
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X Normal
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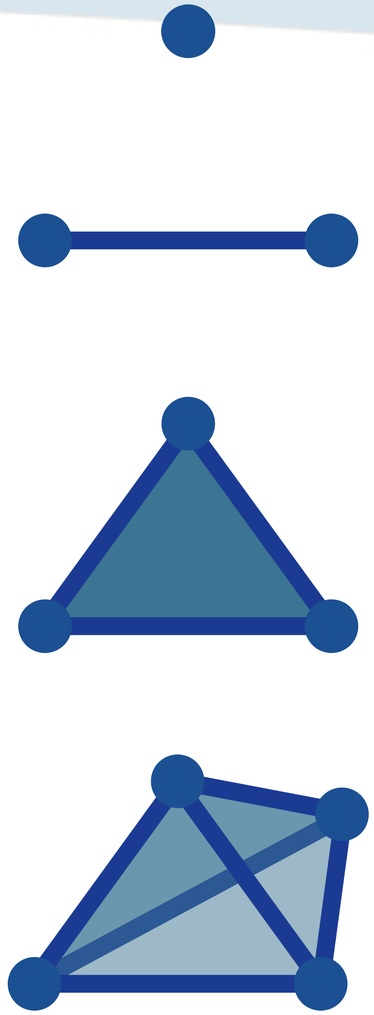
Camera Normal
Center on Bounds
Extraction Side inside
Extract only intersected
Extract intersected

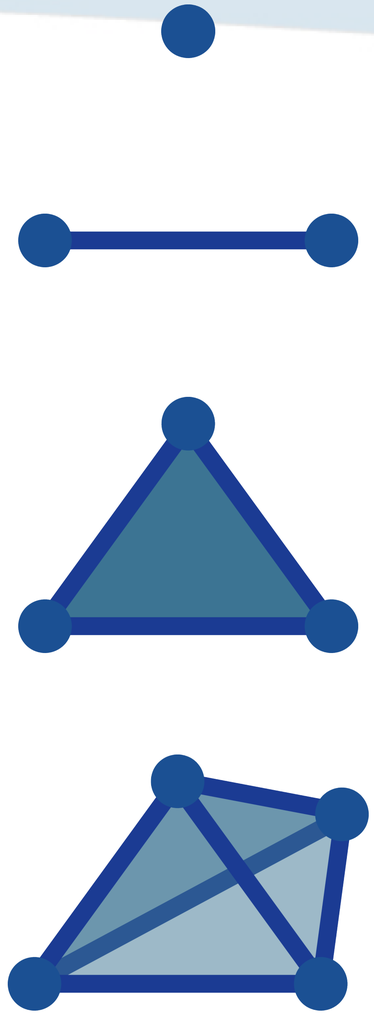
Interpolants for triangulations

- Like regular grids, several options



Interpolants for triangulations

- Like regular grids, several options
  - Piecewise constant
  - Piecewise linear
  - Piecewise polynomials,
  - etc.



Interpolants for triangulations

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  - Piecewise constant
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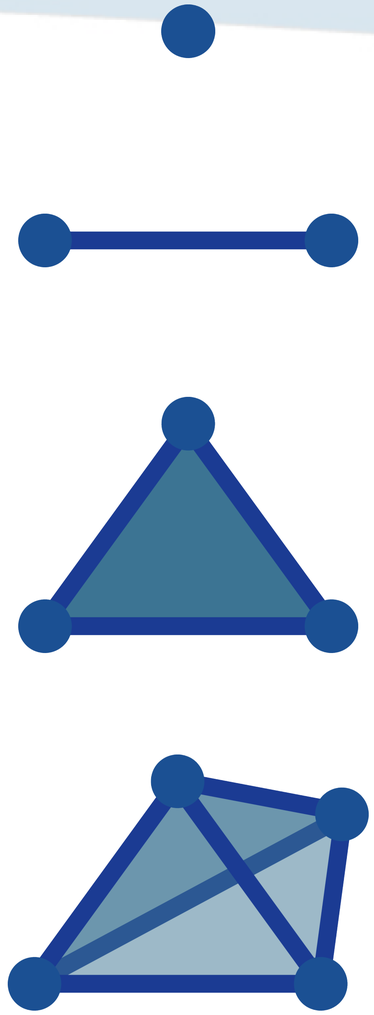
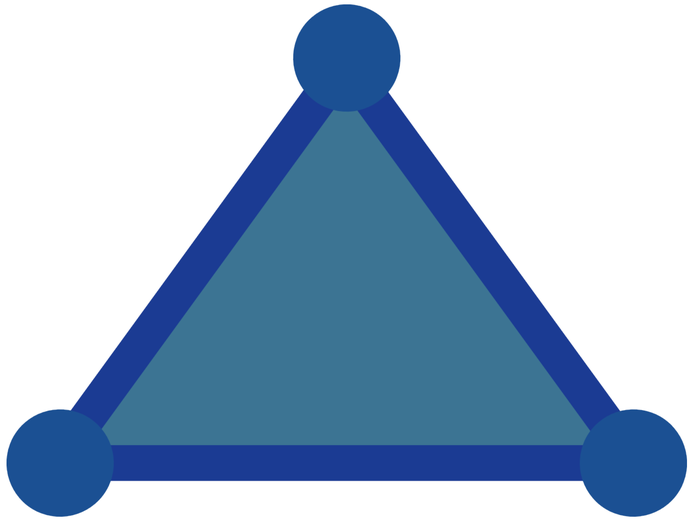


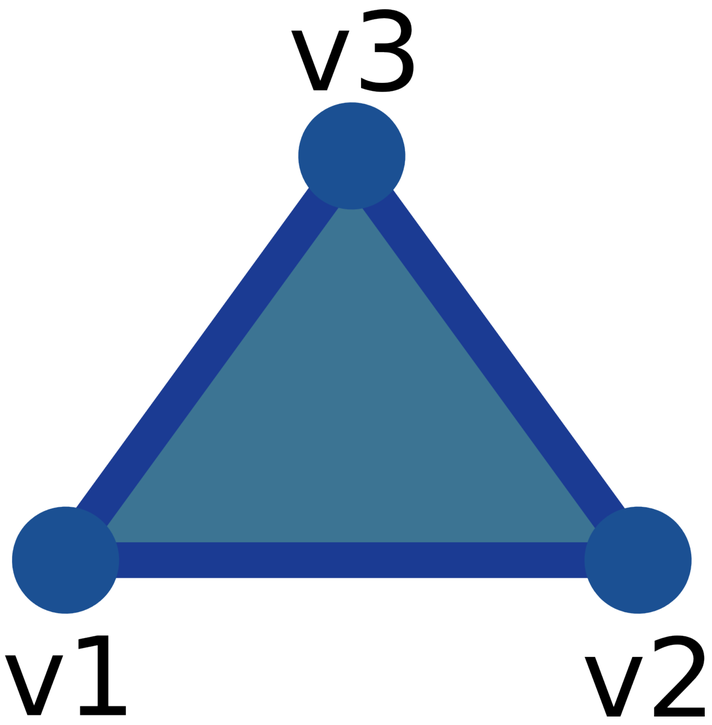
Piecewise linear interpolant on triangulations

- Interpolation as a boundary value problem



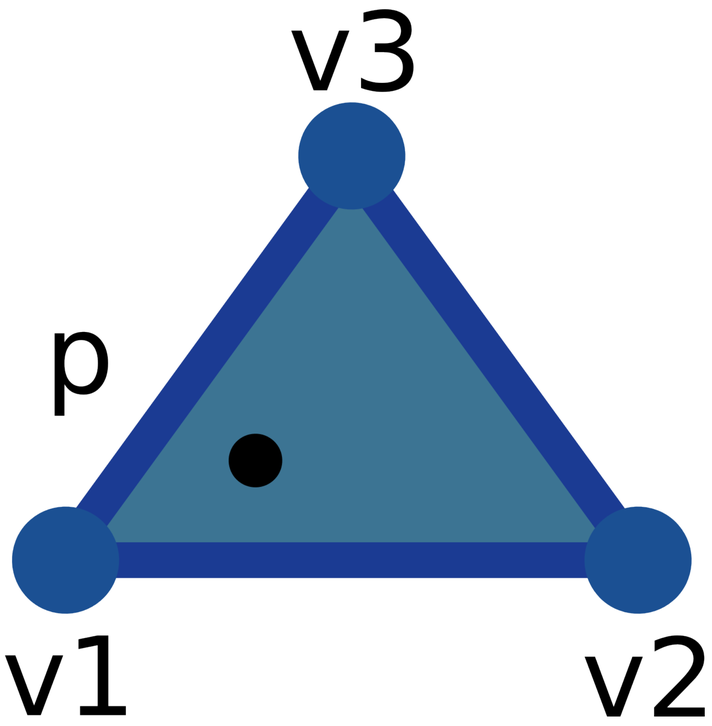
Piecewise linear interpolant on triangulations

- Interpolation as a boundary value problem
  - Given a simplex and its vertices



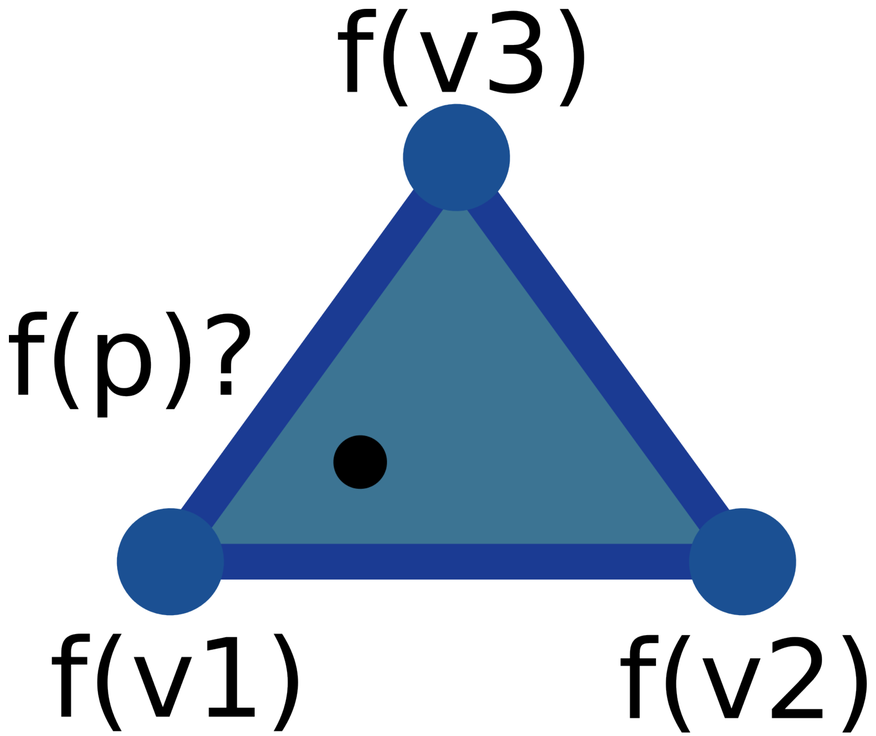
Piecewise linear interpolant on triangulations

- Interpolation as a boundary value problem
  - Given a simplex and its vertices
  - Express the value of any point



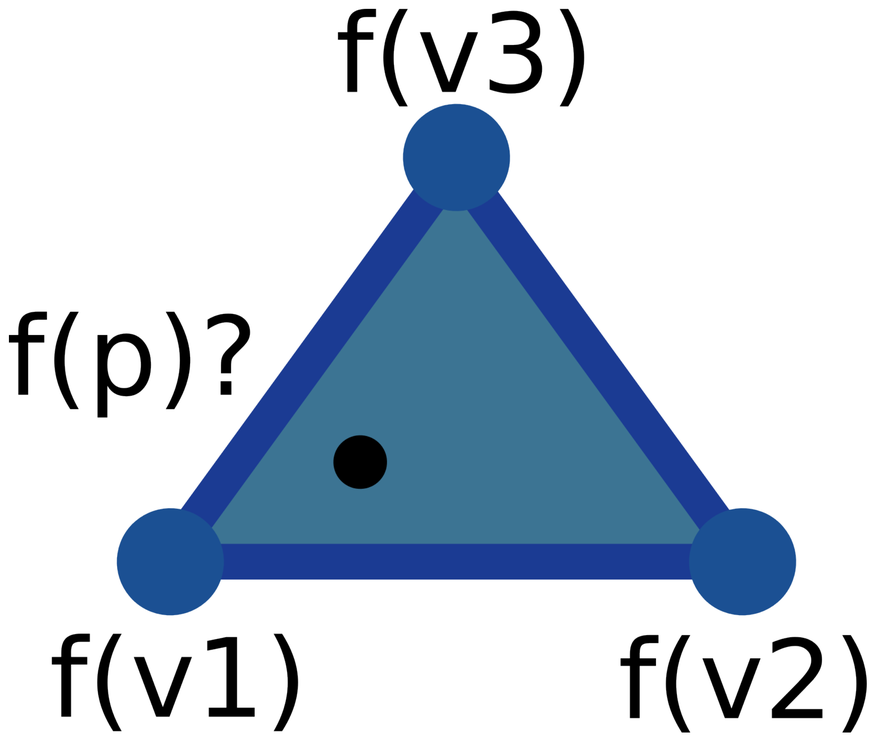
Piecewise linear interpolant on triangulations

- Interpolation as a boundary value problem
  - Given a simplex and its vertices
  - Express the value of any point
  - As a linear combination of those of the vertices



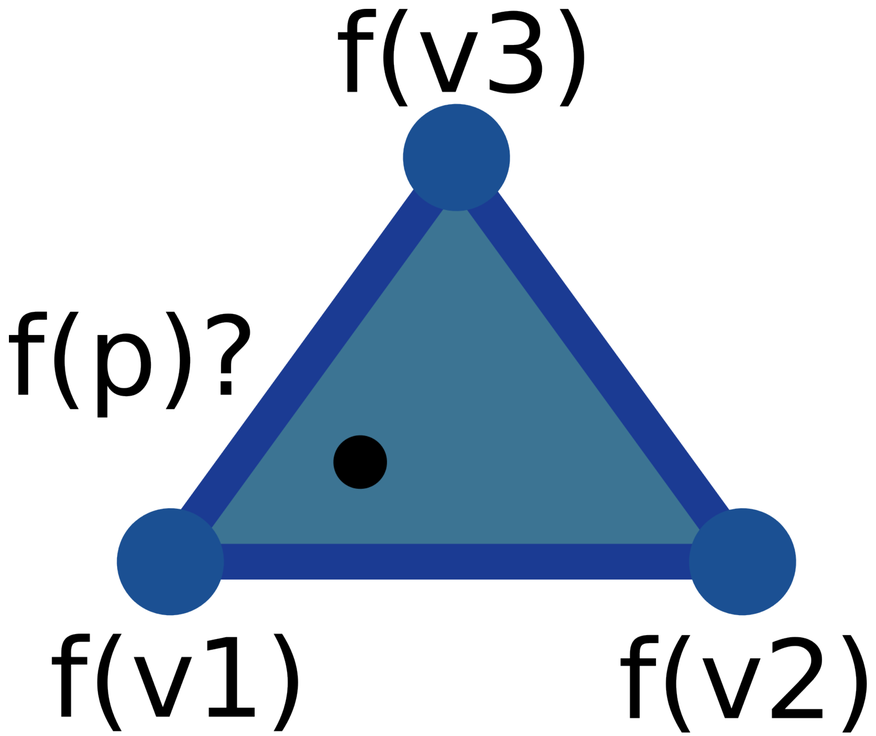
Piecewise linear interpolant on triangulations

- Interpolation as a boundary value problem
  - Given a simplex and its vertices
  - Express the value of any point
  - As a linear combination of those of the vertices
- Notion of barycentric coordinates:
  - $f(p) = \alpha_1 f(v_1) + \alpha_2 f(v_2) + \alpha_3 f(v_3)$



Barycentric coordinates

- There exists many forms
  - With specific properties



Barycentric coordinates

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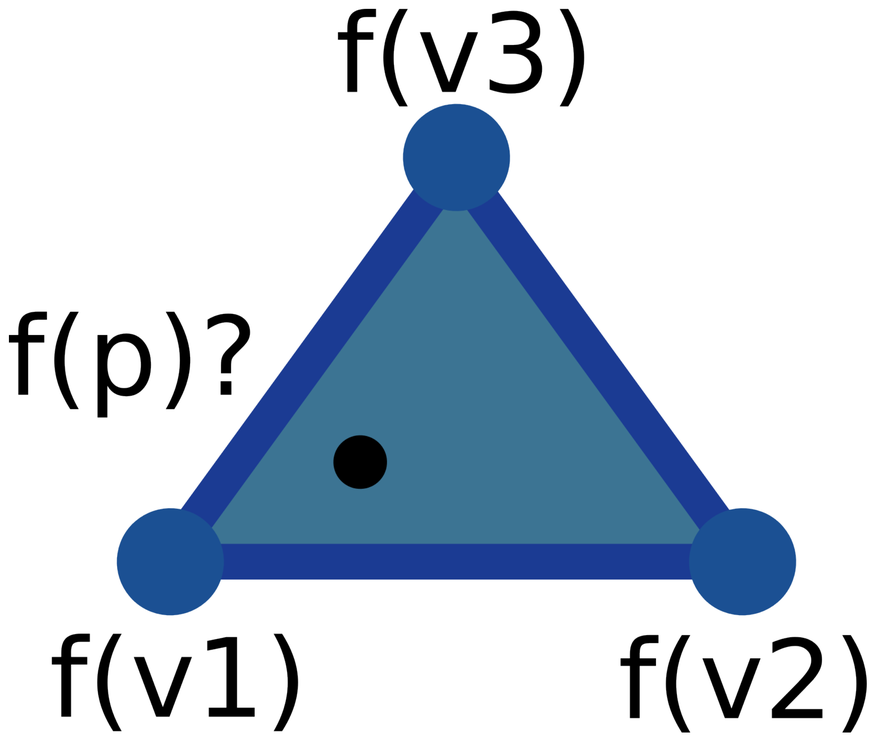

Barycentric coordinates

- There exists many forms
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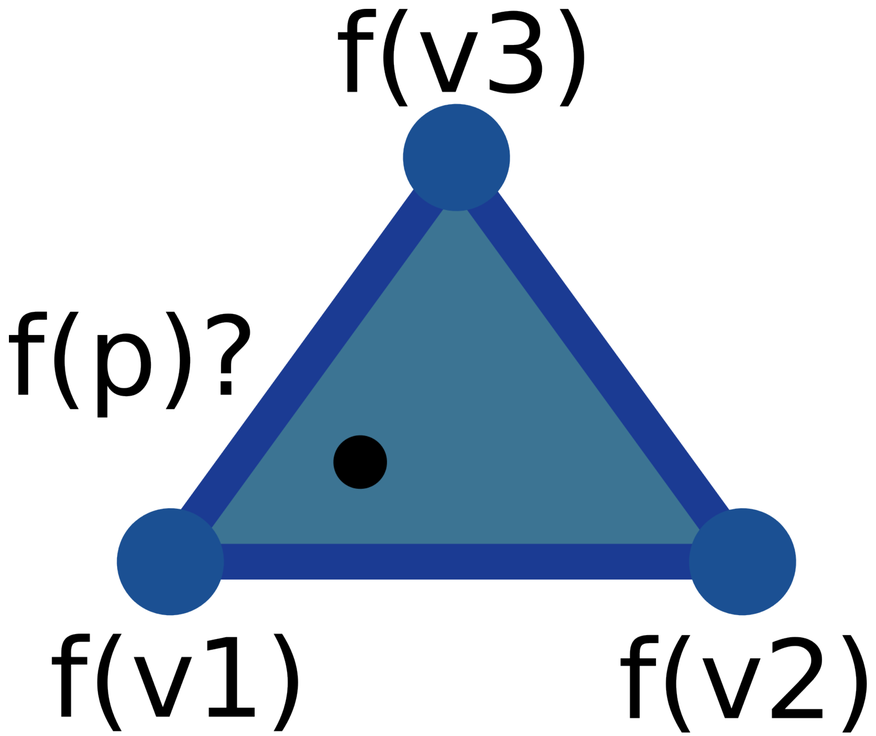
- We want to produce linear interpolations
  - $\alpha_1 + \alpha_2 + \alpha_3 = 1$



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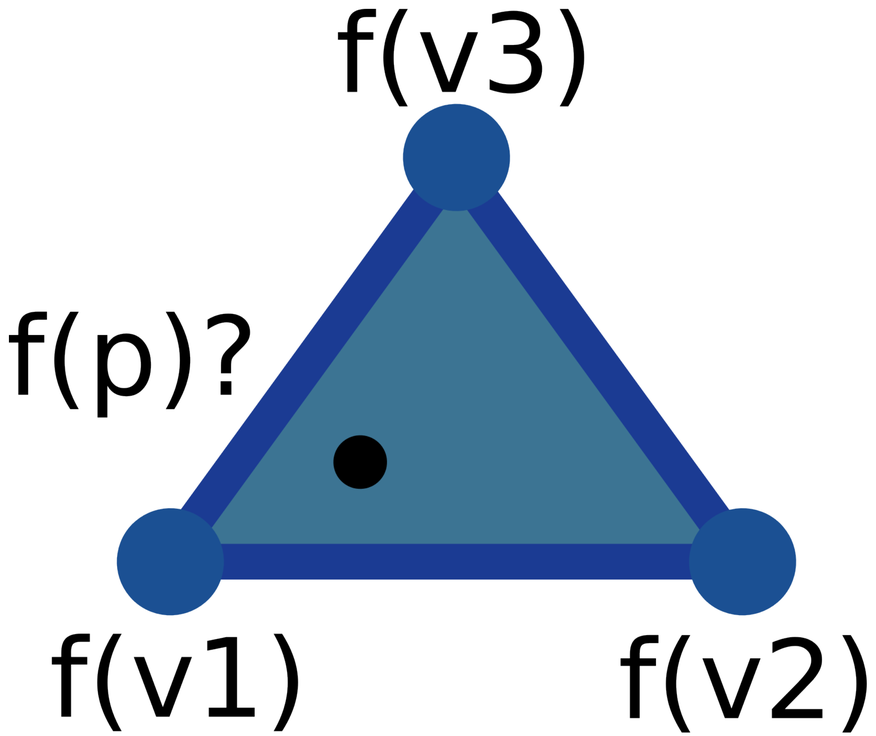

Barycentric coordinates

- There exists many forms
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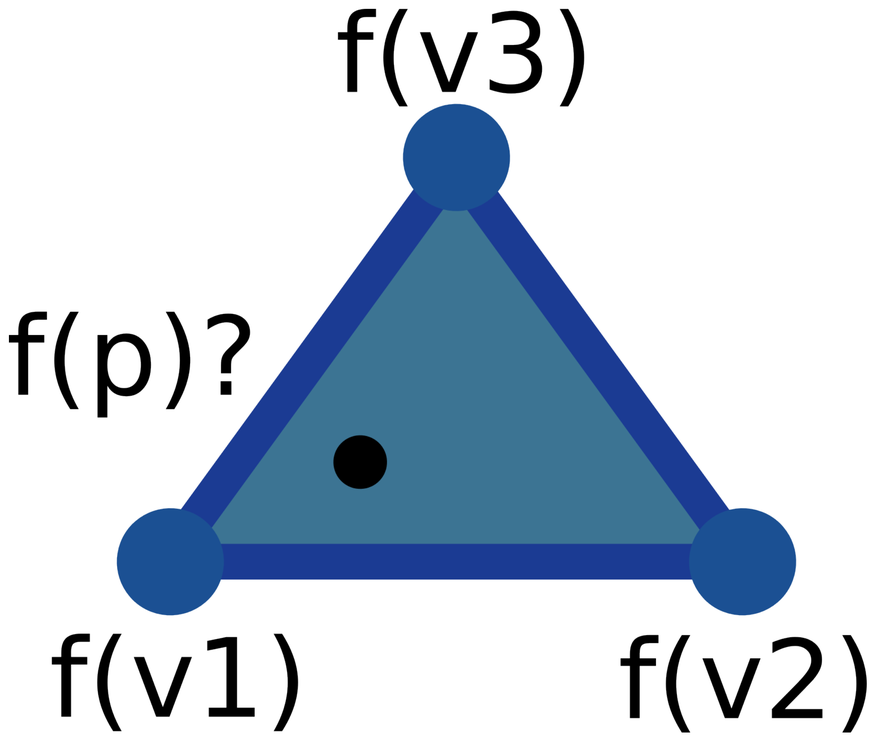
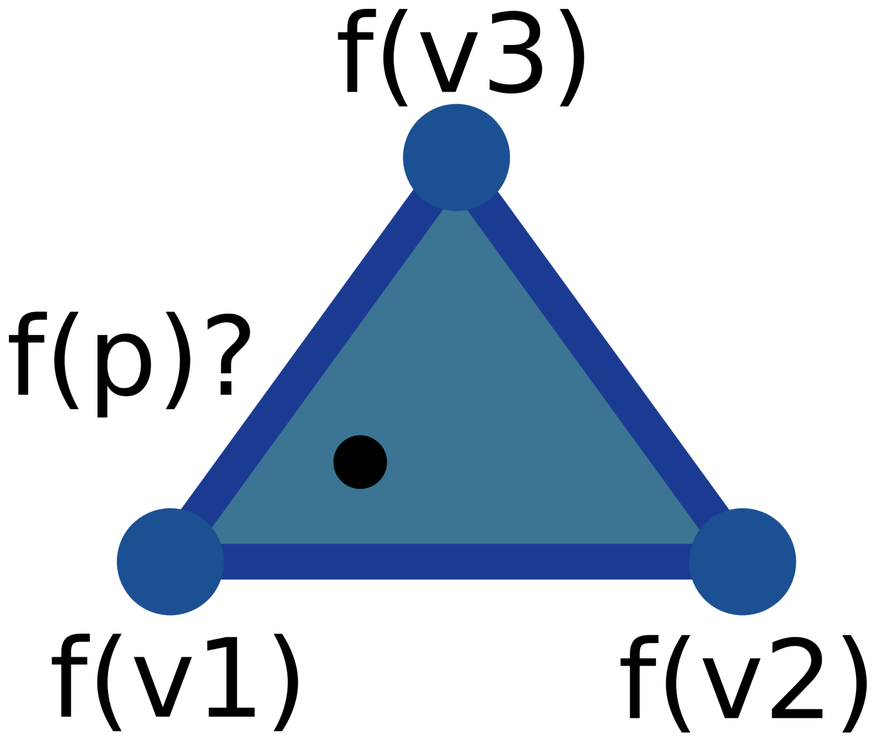
  - $f(p) = \alpha_1 f(v_1) + \alpha_2 f(v_2) + \alpha_3 f(v_3)$
  - Must hold for any $f : \mathcal{T} \rightarrow \mathbb{R}$



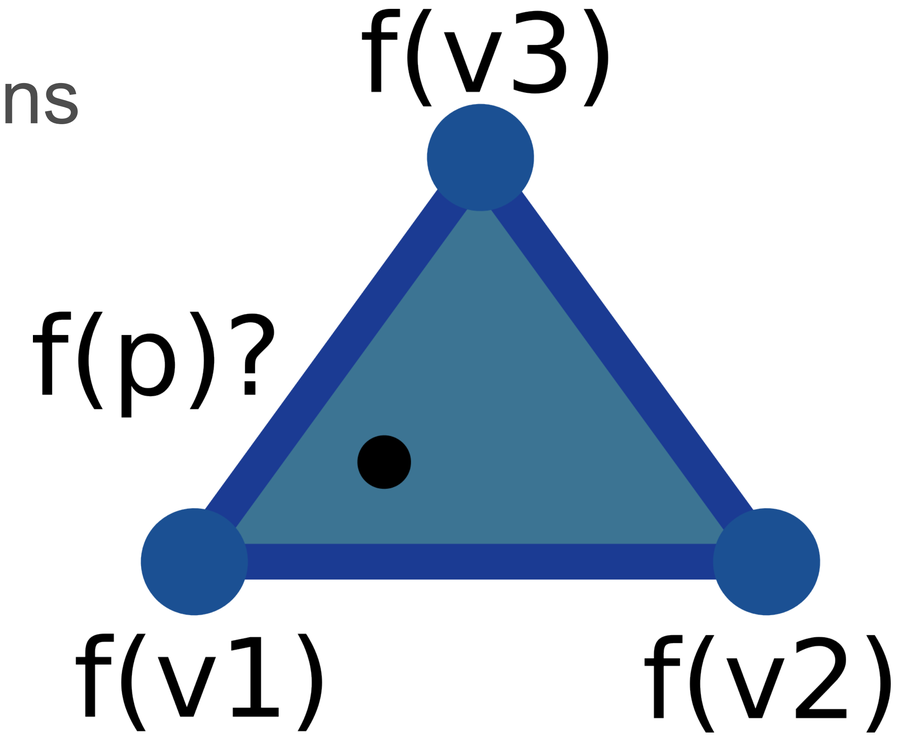
Barycentric coordinates

- In particular



Barycentric coordinates

- In particular
  - It must also hold for the embedding functions



Barycentric coordinates

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  - It must also hold for the embedding functions
  - $f(p) = \alpha_1 f(v_1) + \alpha_2 f(v_2) + \alpha_3 f(v_3)$
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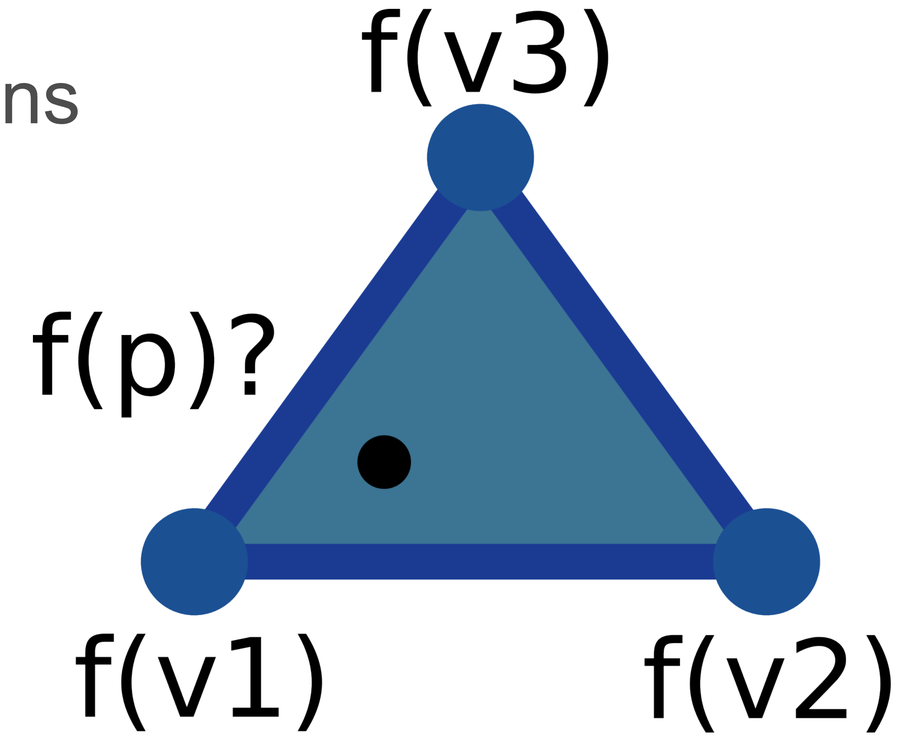

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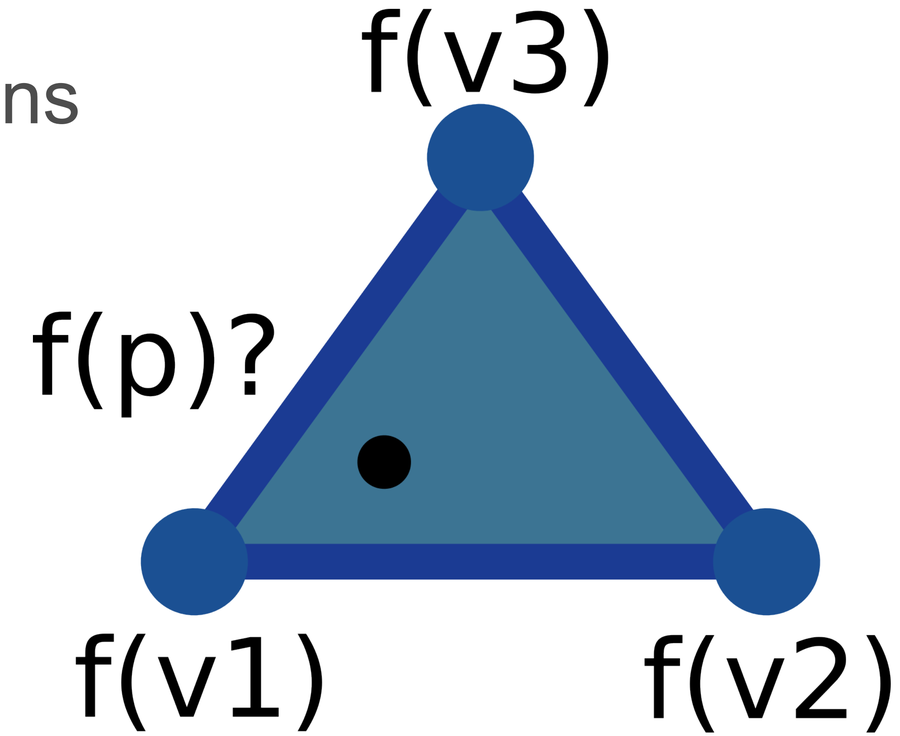
  - $y(p) = \alpha_1 y(v_1) + \alpha_2 y(v_2) + \alpha_3 y(v_3)$
- 3 linear equations with 3 unknowns



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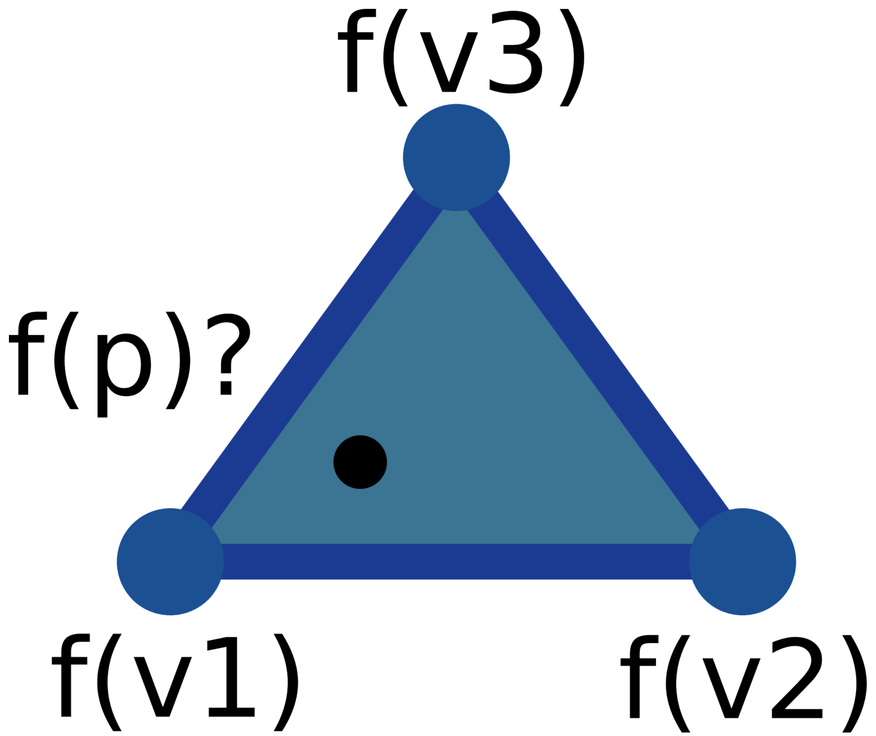

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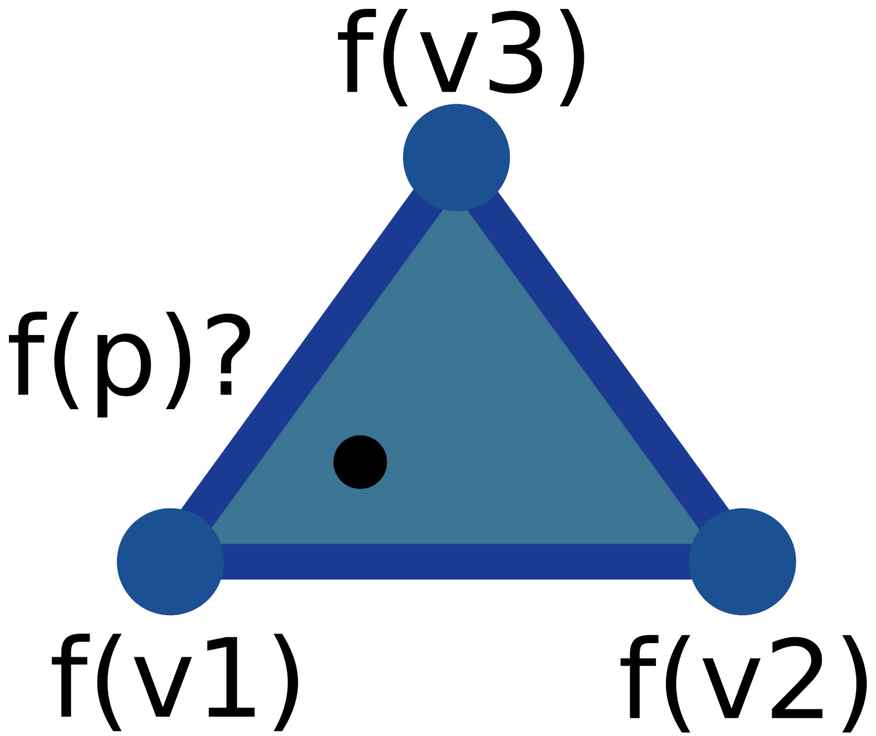
- Automatically interpolates on the edges



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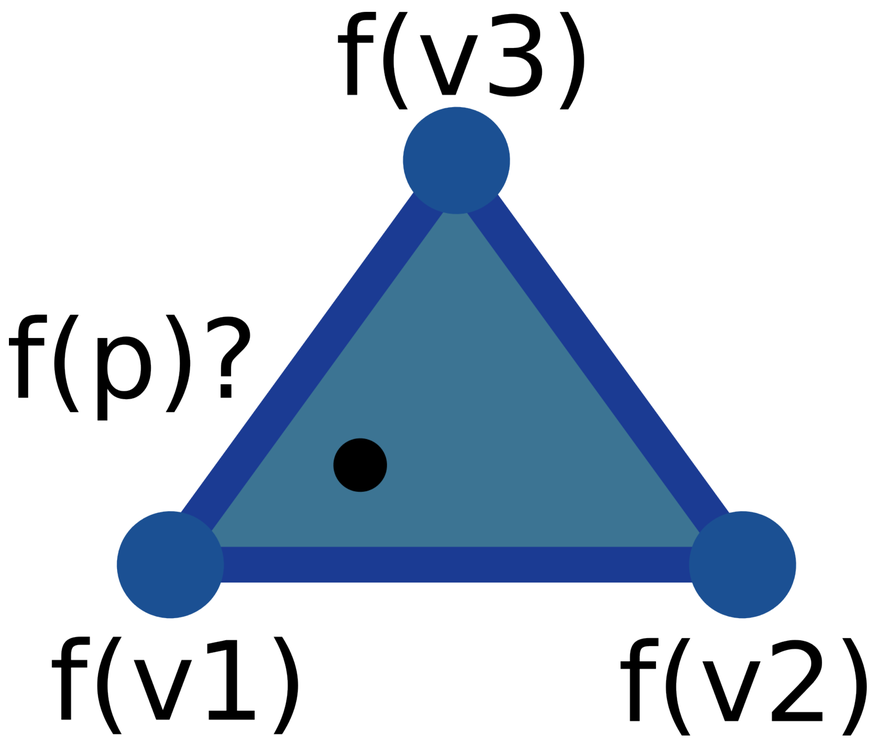
- Automatically interpolates on the edges
- Similar reasoning for d-simplex



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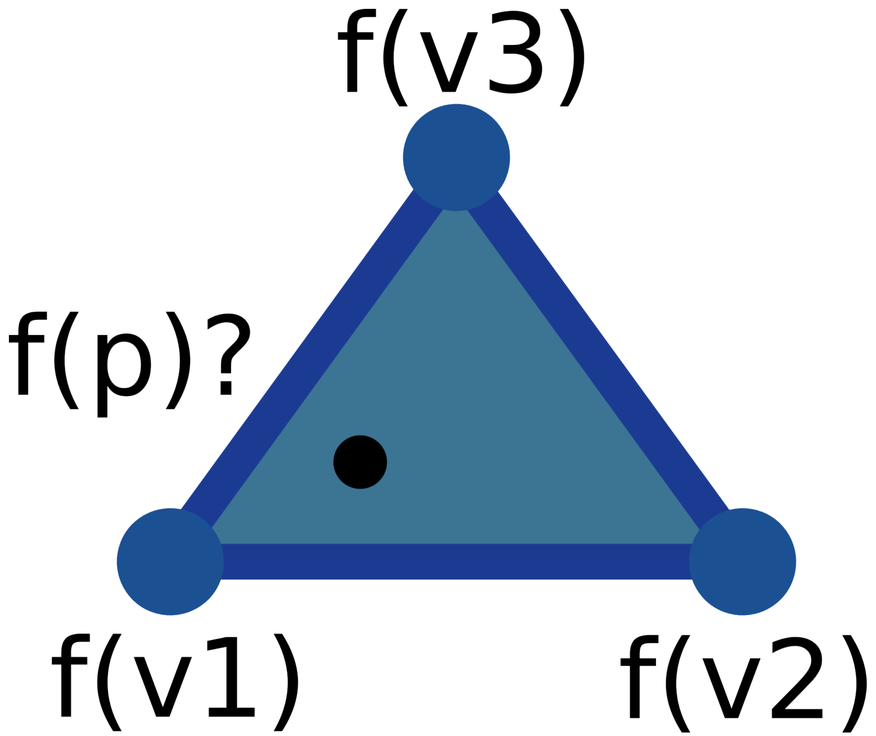

- Automatically interpolates on the edges
- Similar reasoning for d-simplex
- Can be used to determine if a point lies within a simplex

Barycentric coordinates

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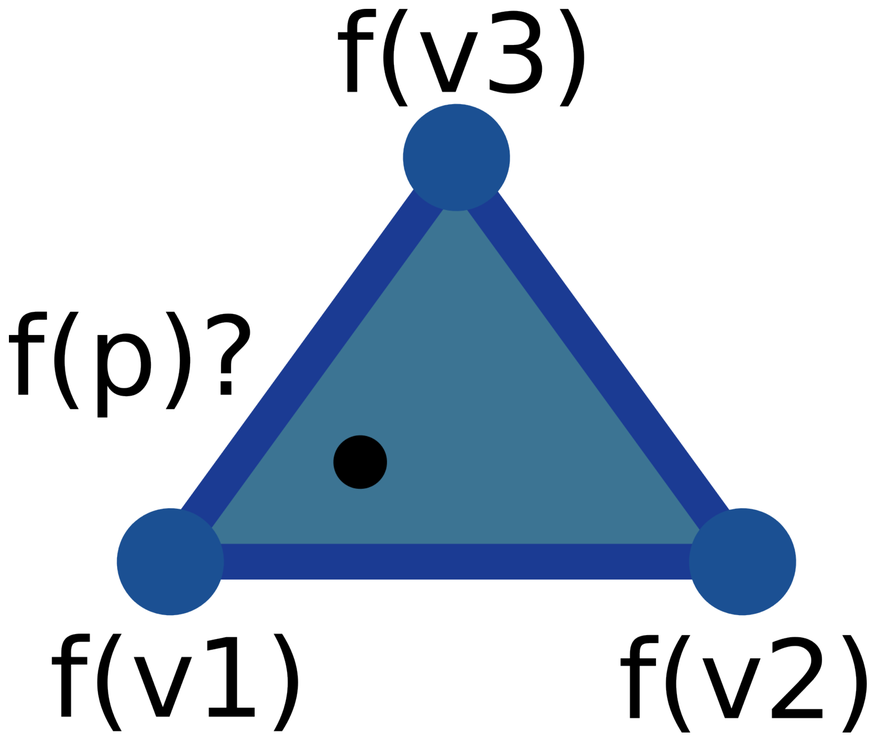

- Automatically interpolates on the edges
- Similar reasoning for d-simplex
- Can be used to determine if a point lies within a simplex $(\alpha_1, \alpha_2, \alpha_3) \in [0,1] \times [0,1] \times [0,1]$

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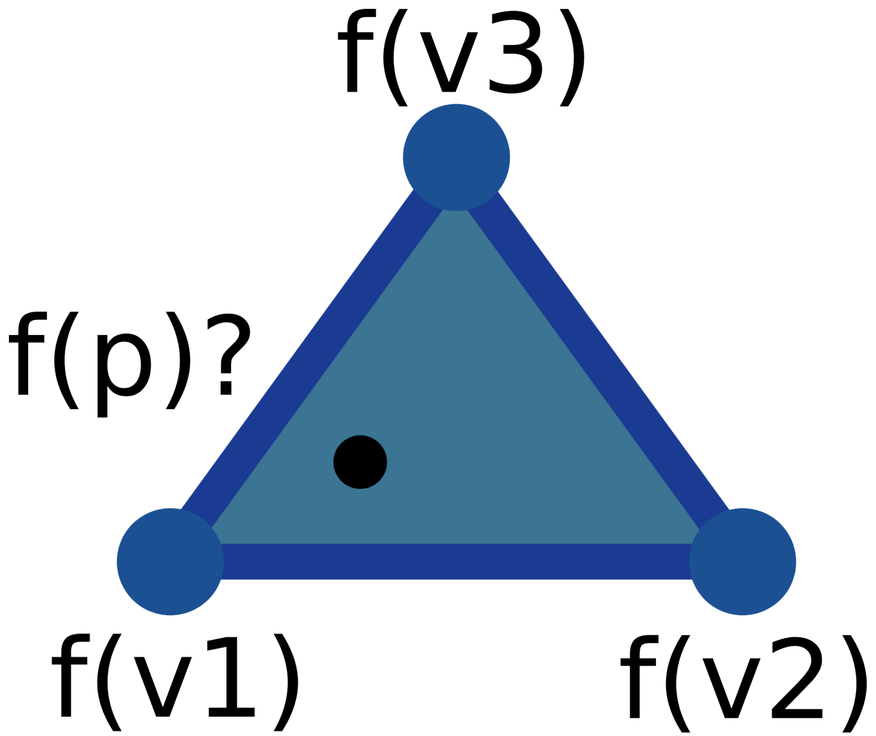
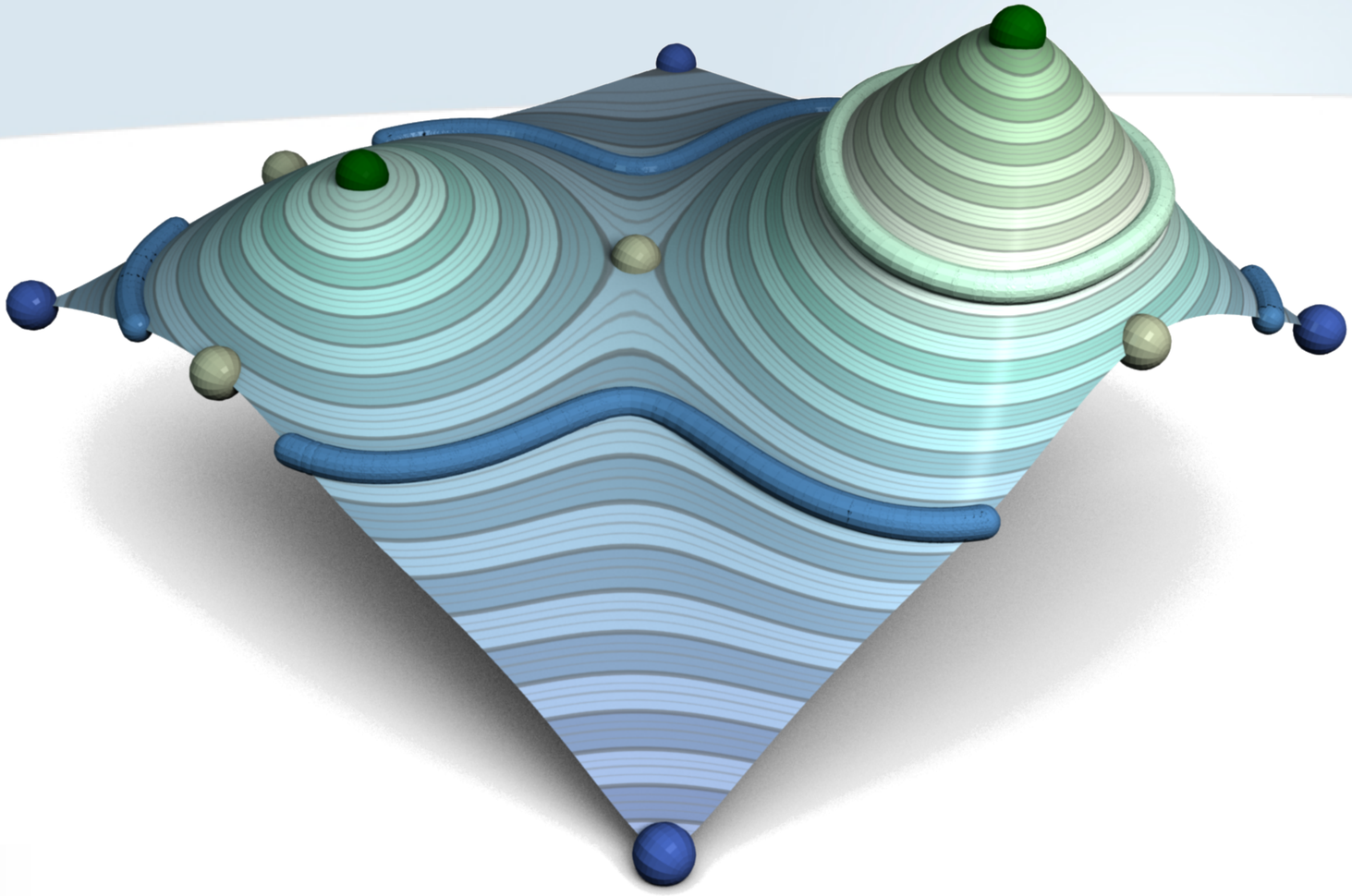


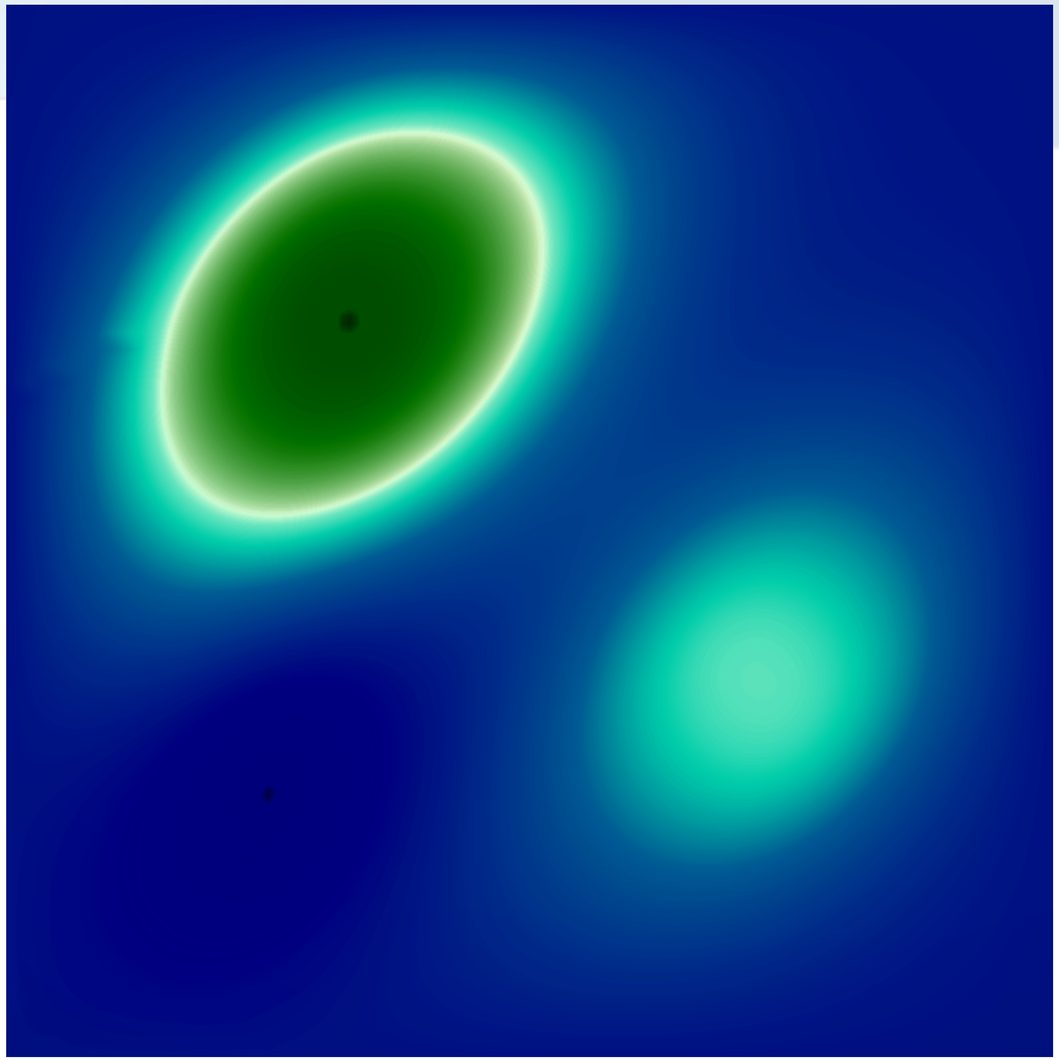
- Automatically interpolates on the edges
- Similar reasoning for d-simplex
- Can be used to determine if a point lies within a simplex $(\alpha_1, \alpha_2, \alpha_3) \in [0,1] \times [0,1] \times [0,1]$
- No critical point in the interior!



Scalar Field Visualization

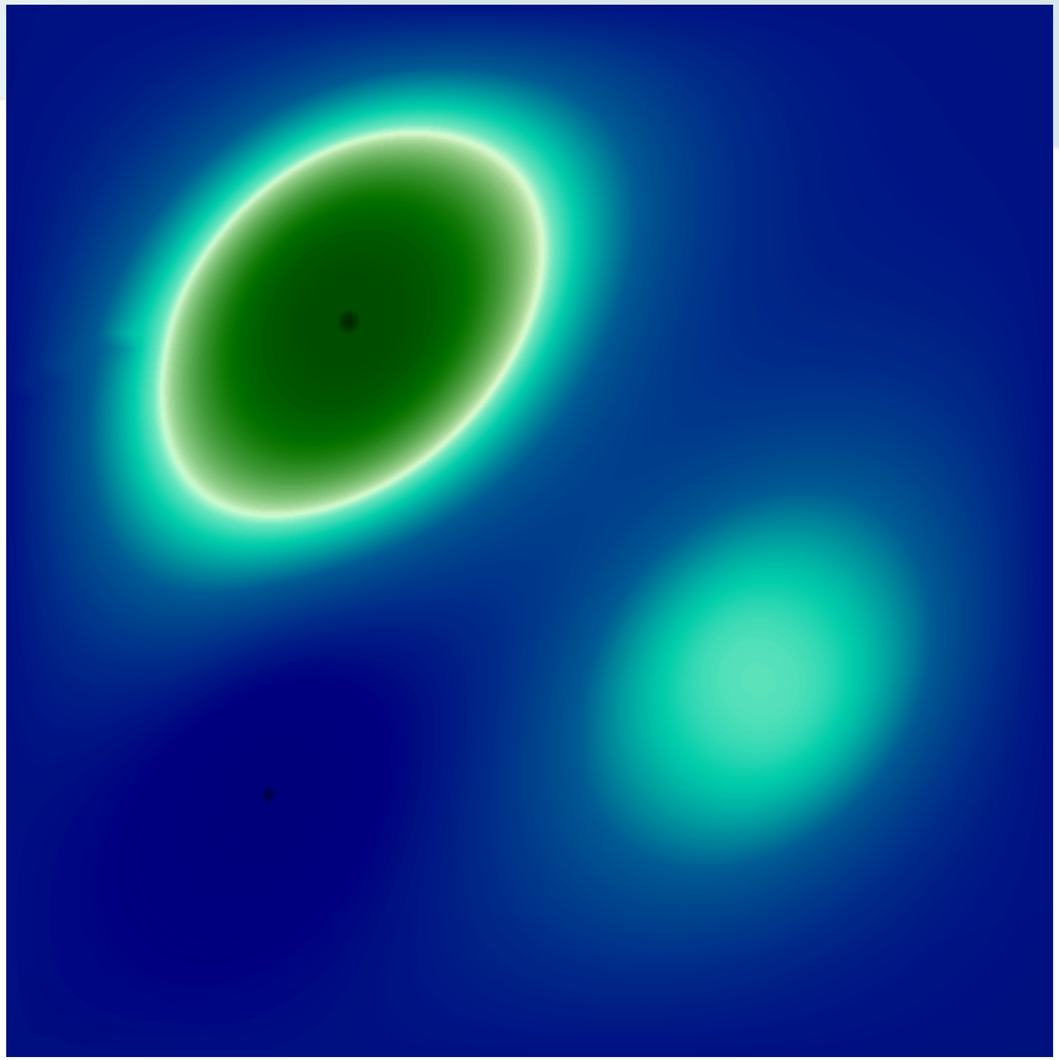
Scalar field geometry

- Continuous color maps



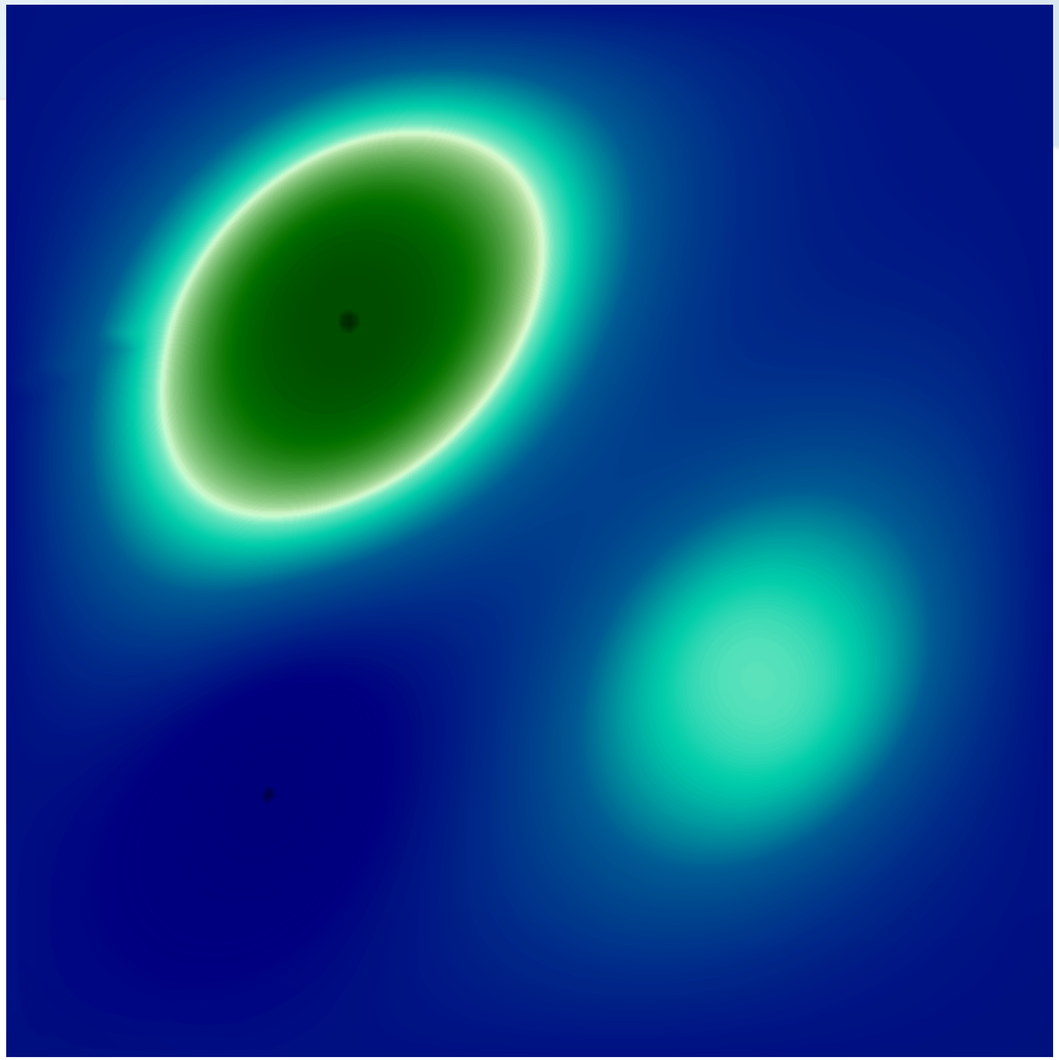
Scalar field geometry

- Continuous color maps
  - Difficulty to estimate the geometry



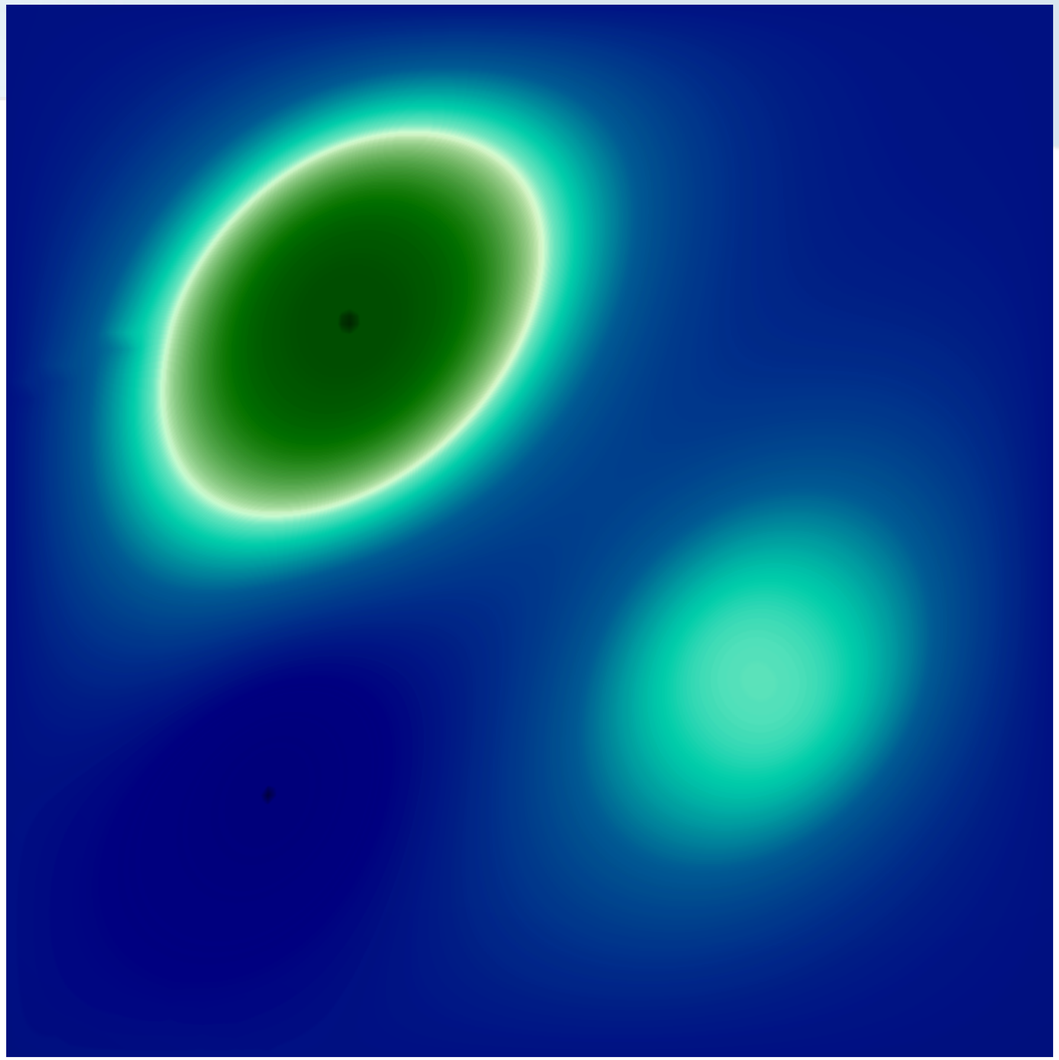
Scalar field geometry

- Continuous color maps
  - Difficulty to estimate the geometry
    - Locally (gradient)
    - Globally (level sets)



Scalar field geometry

- Continuous color maps
  - Difficulty to estimate the geometry
    - Locally (gradient)
    - Globally (level sets)
- Level sets



Scalar field geometry

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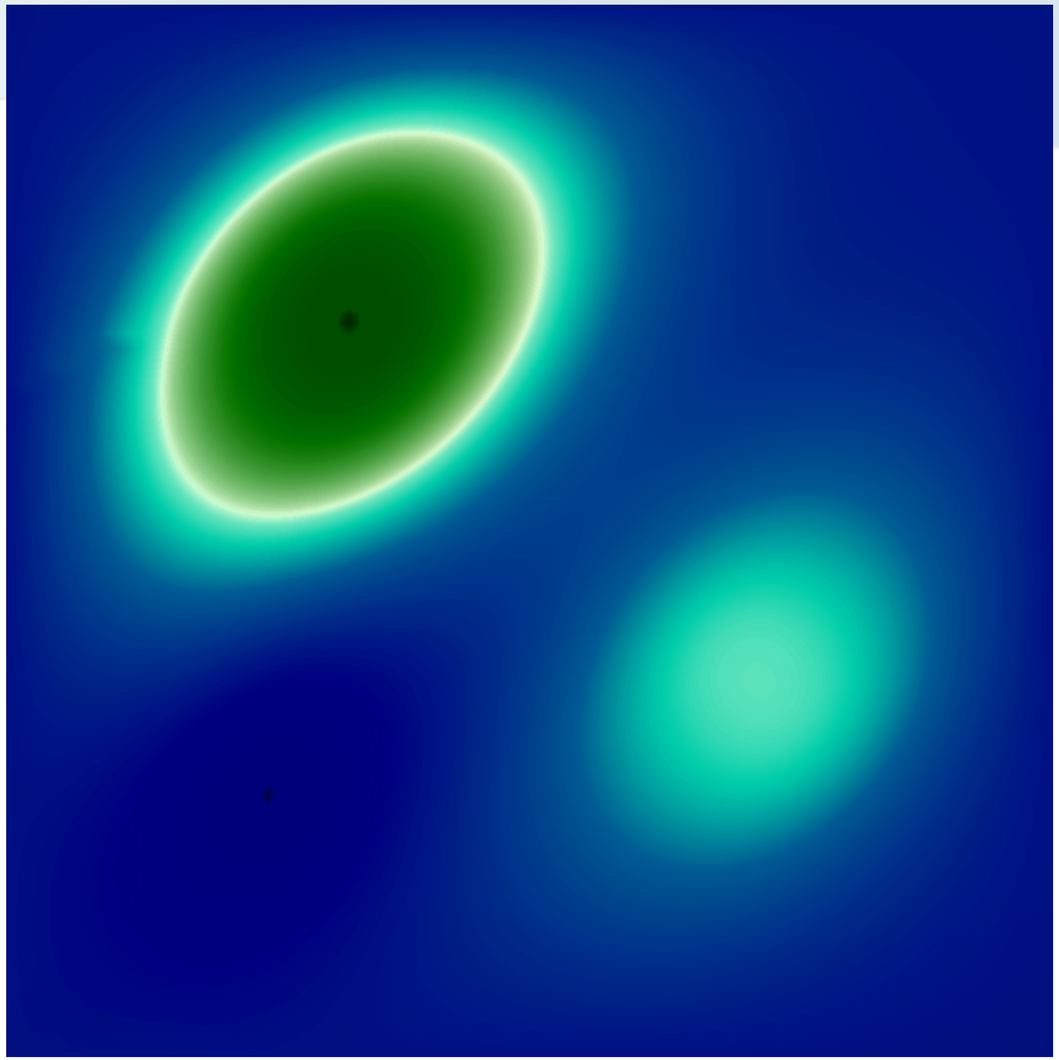

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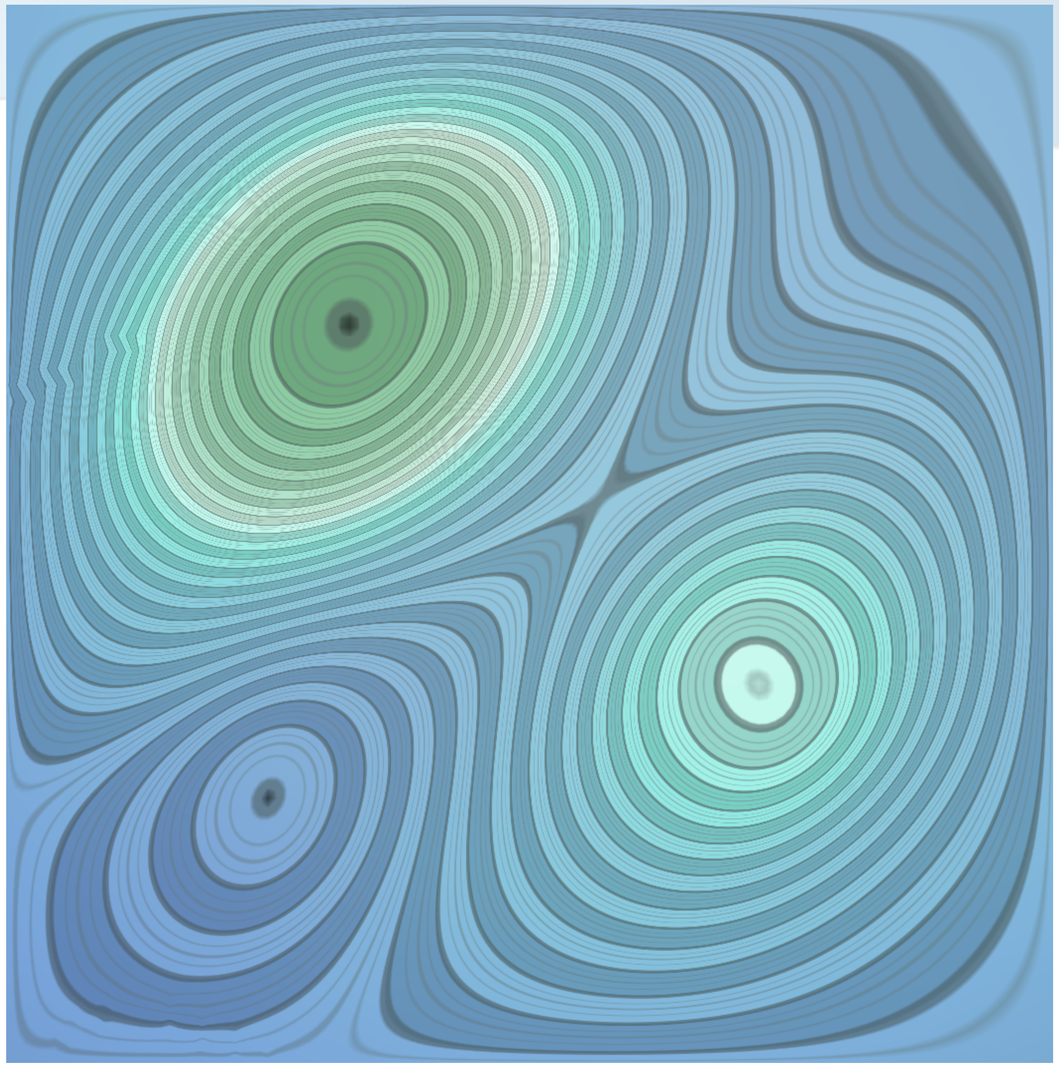
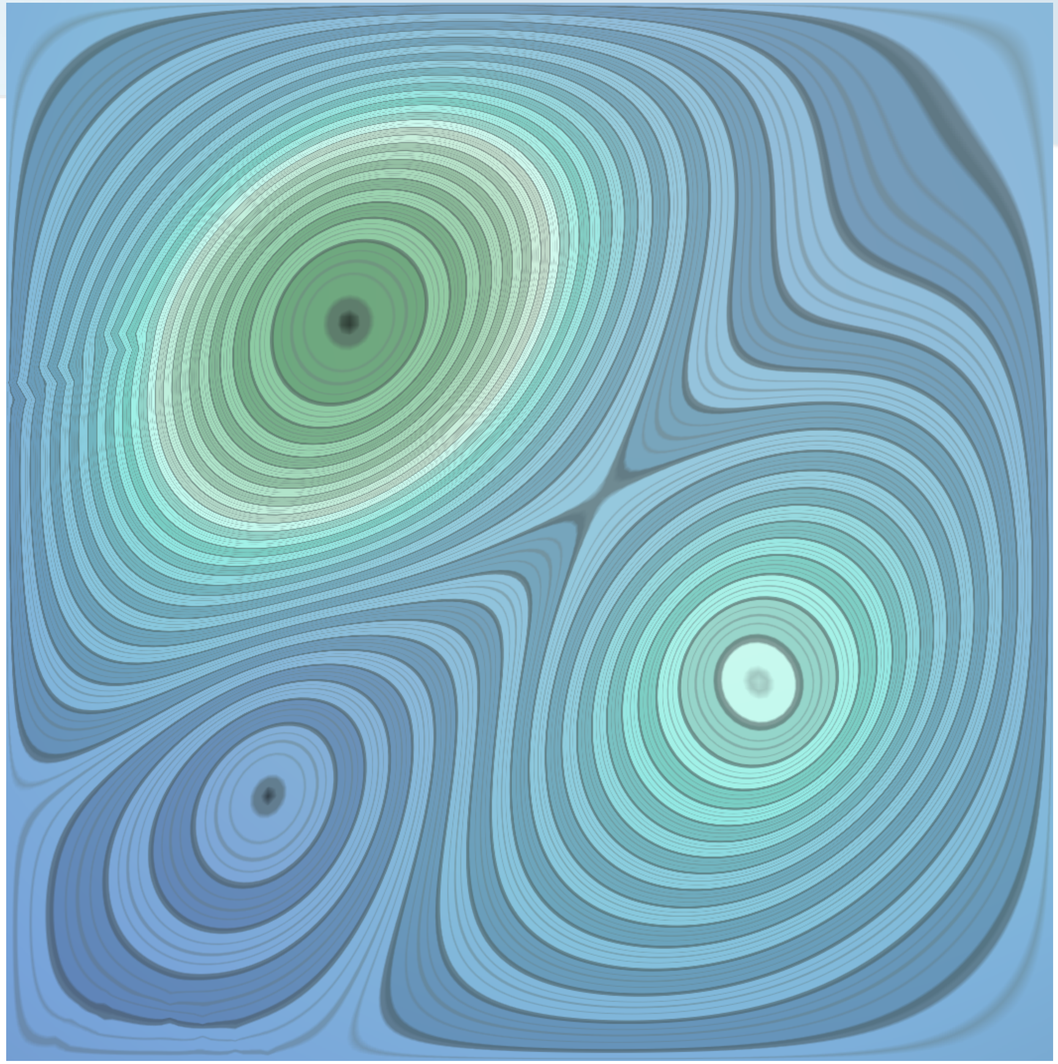


Scalar field geometry

- Continuous color maps
  - Difficulty to estimate the geometry
    - Locally (gradient)
    - Globally (level sets)
- Level sets
  - $f^{-1}(i) = \{p \in \mathcal{D} \ / \ f(p) = i\}$
  - Critical points
    - Where ∇f vanishes



Scalar field geometry

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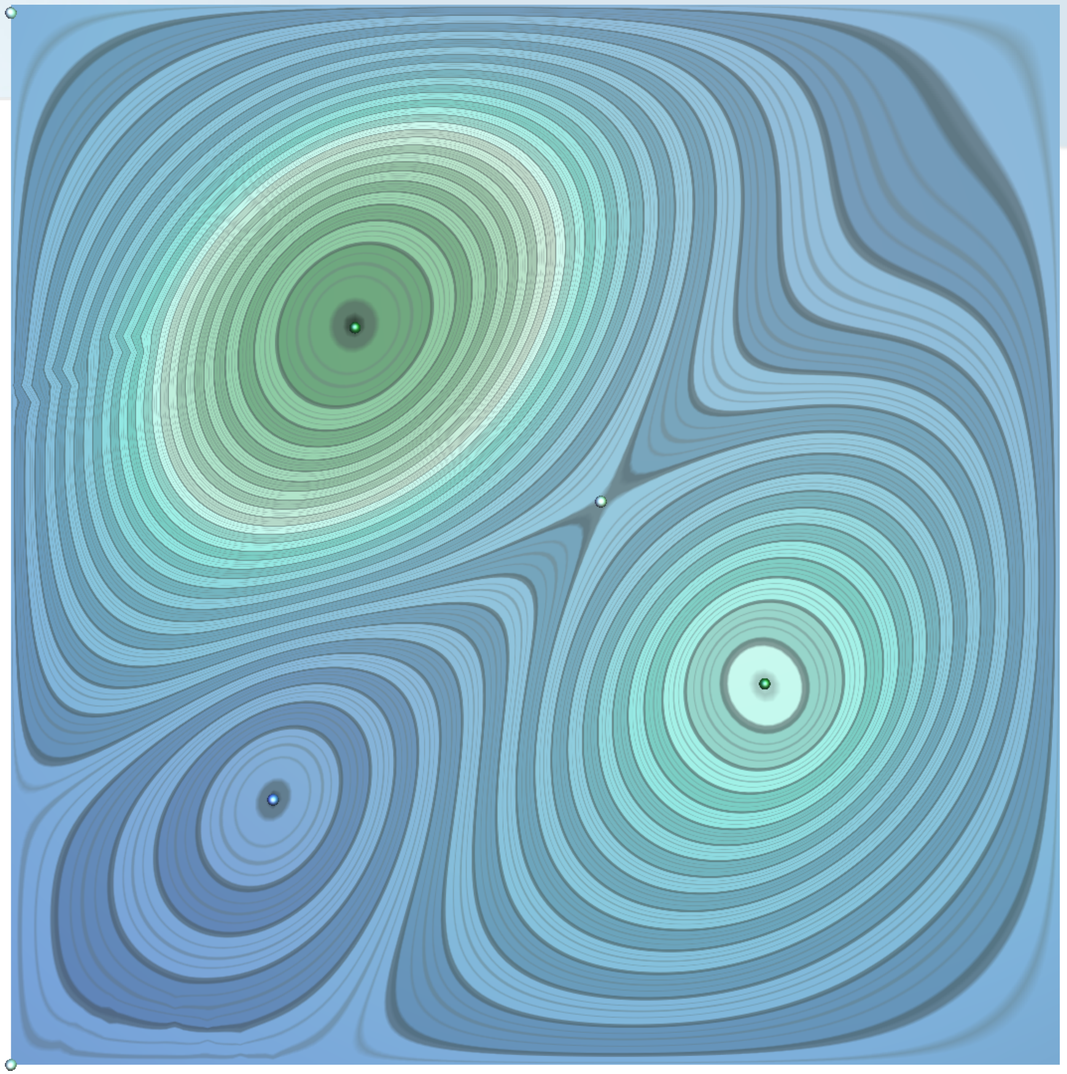
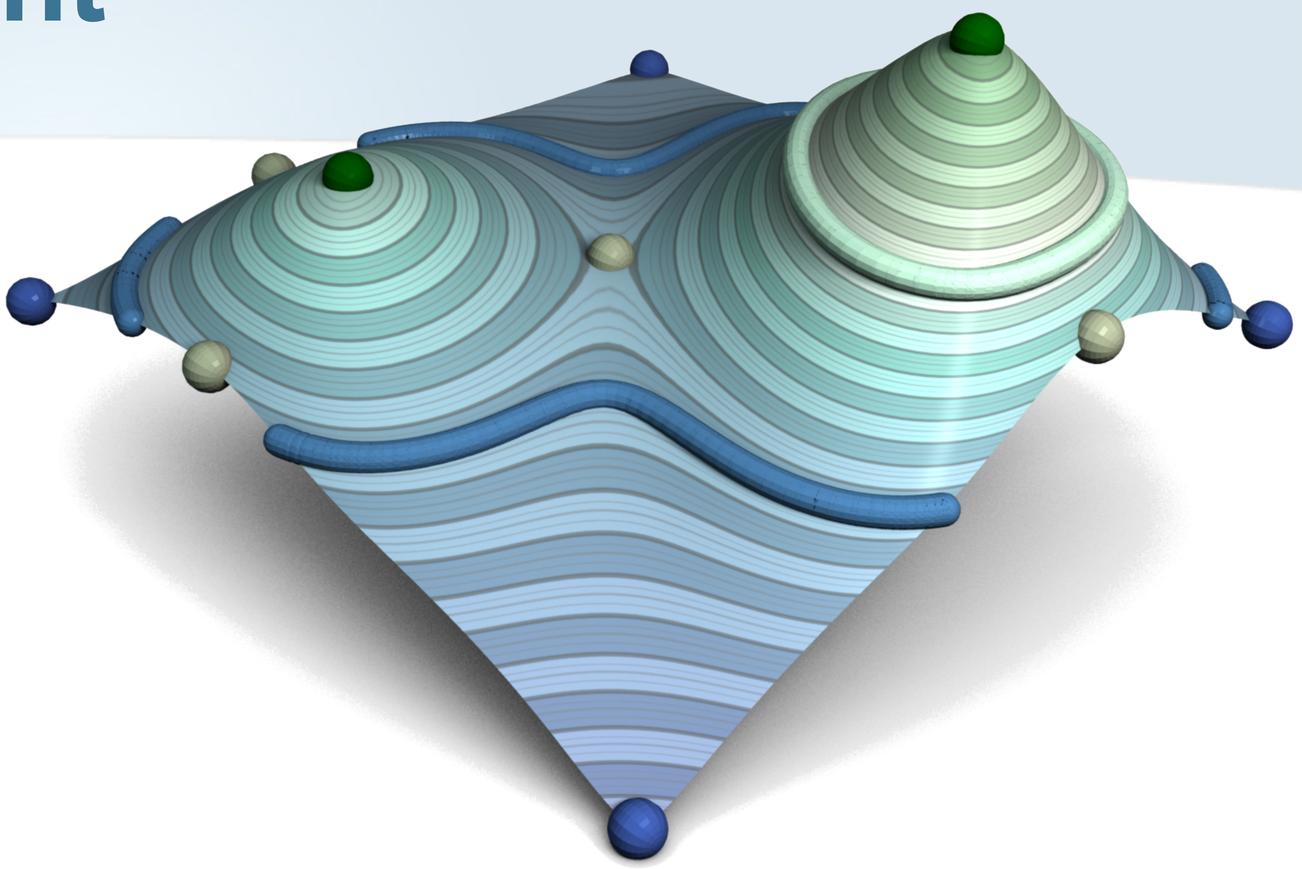
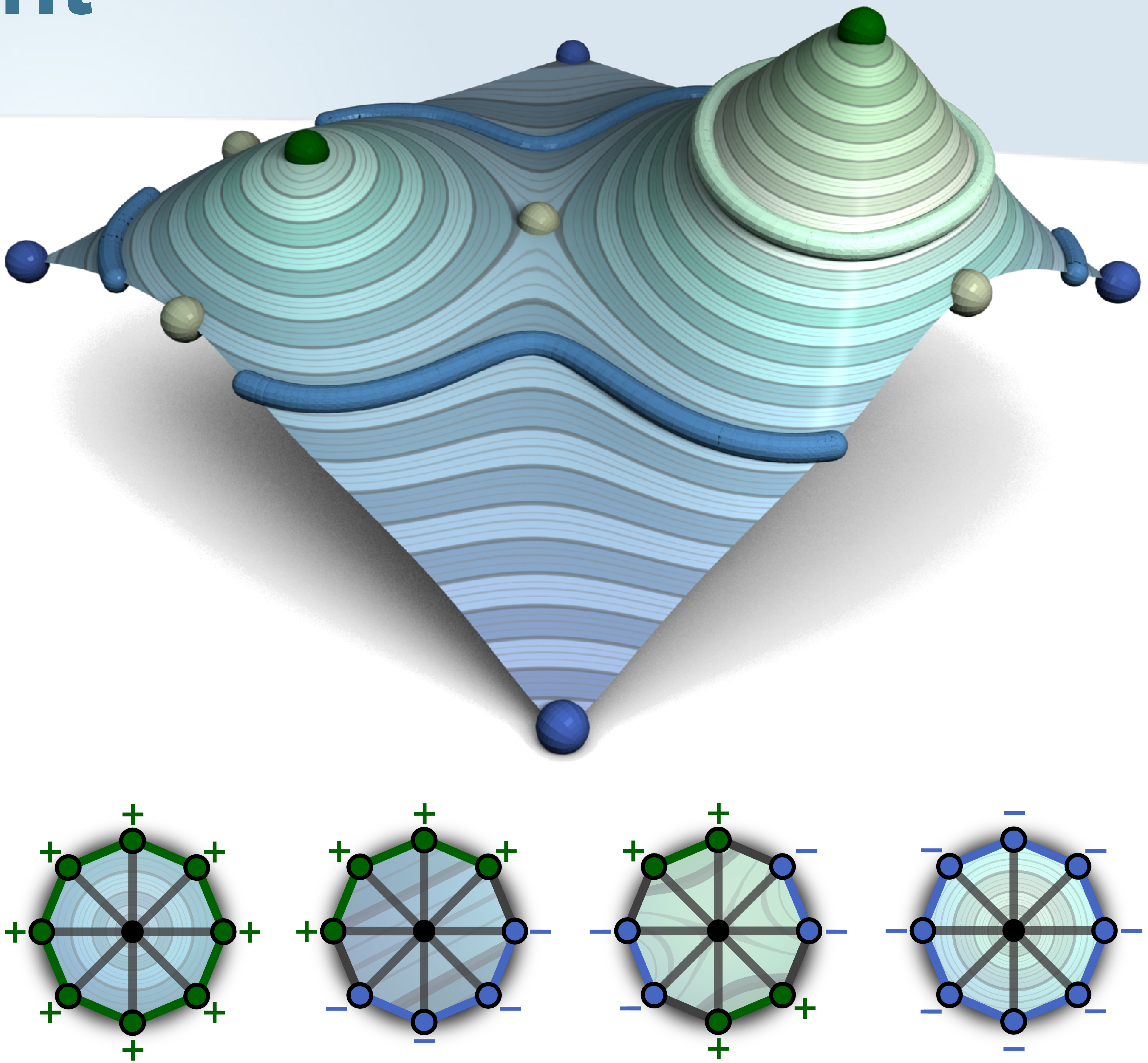


Notion of critical point



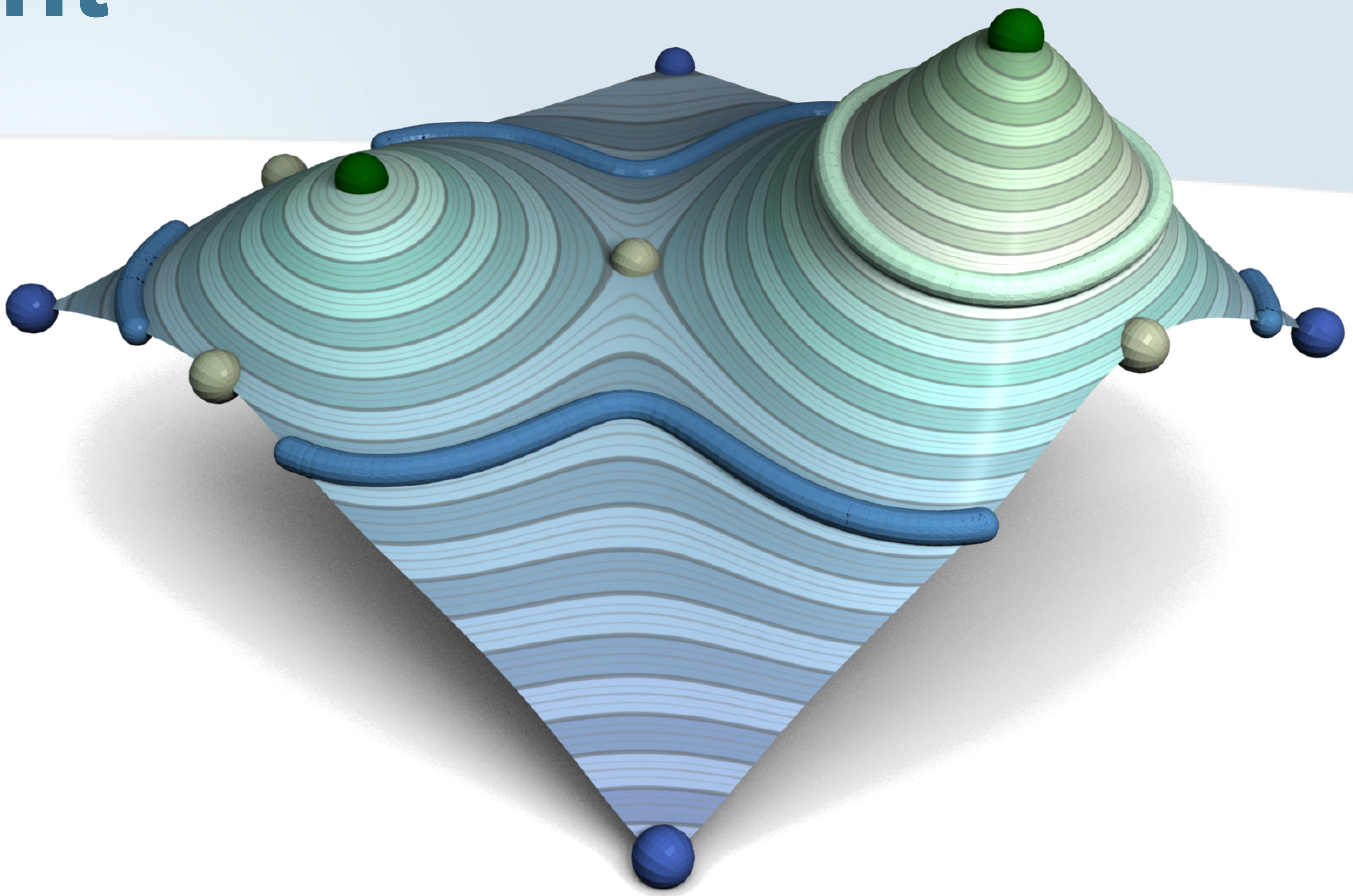
Notion of critical point

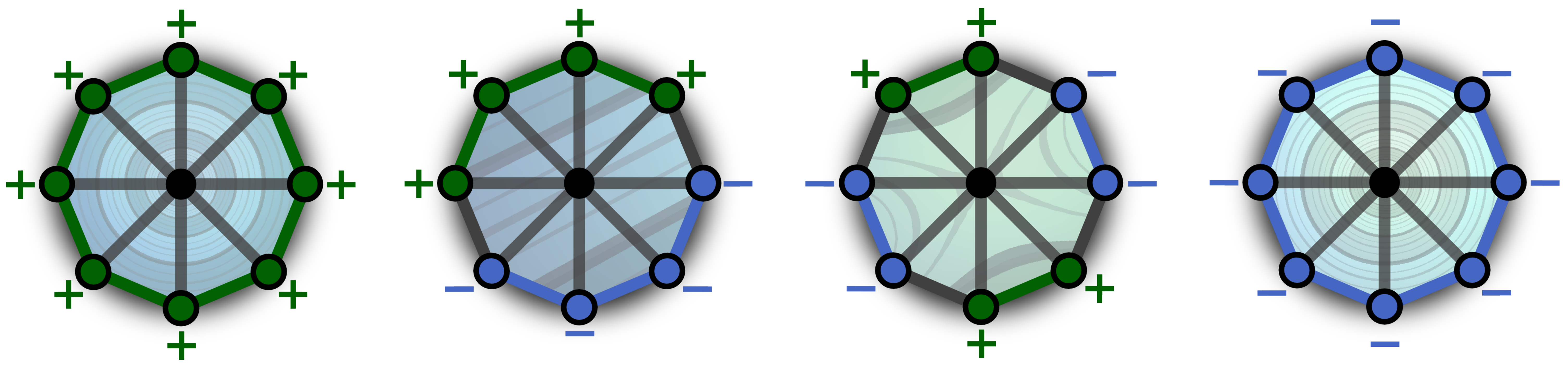
- Combinatorial identification



Notion of critical point

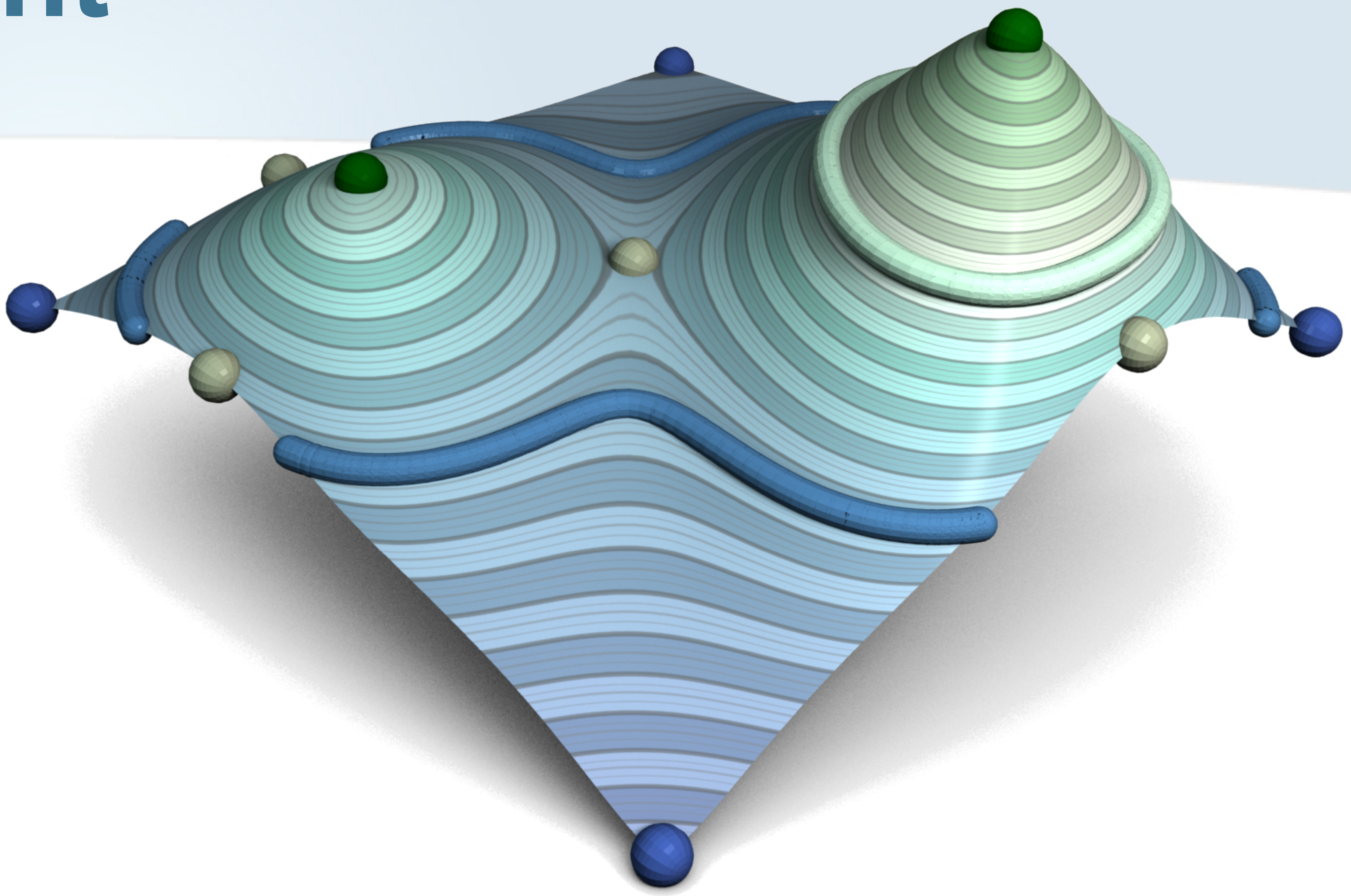
- Combinatorial identification
  - Star of a simplex σ

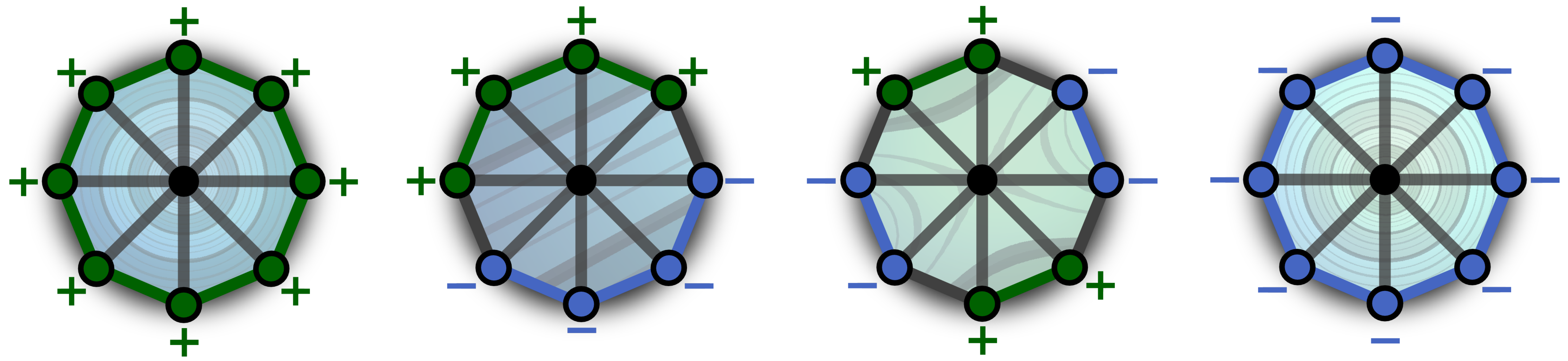




Notion of critical point

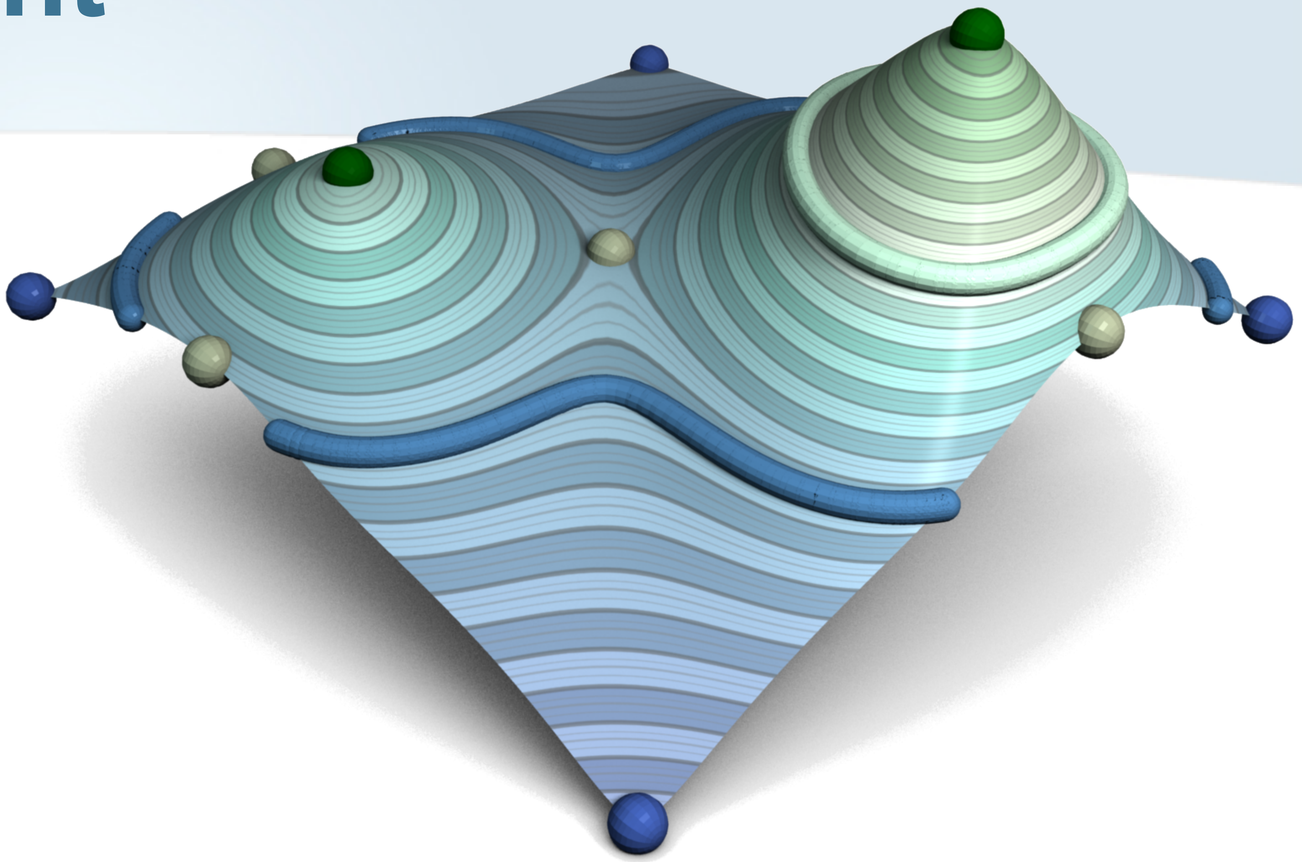
- Combinatorial identification
  - Star of a simplex σ
    - Simplices that contain σ as a face

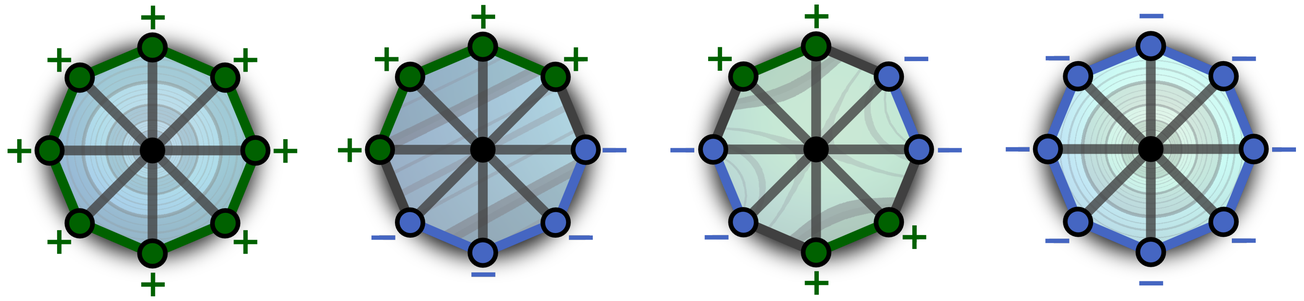




Notion of critical point

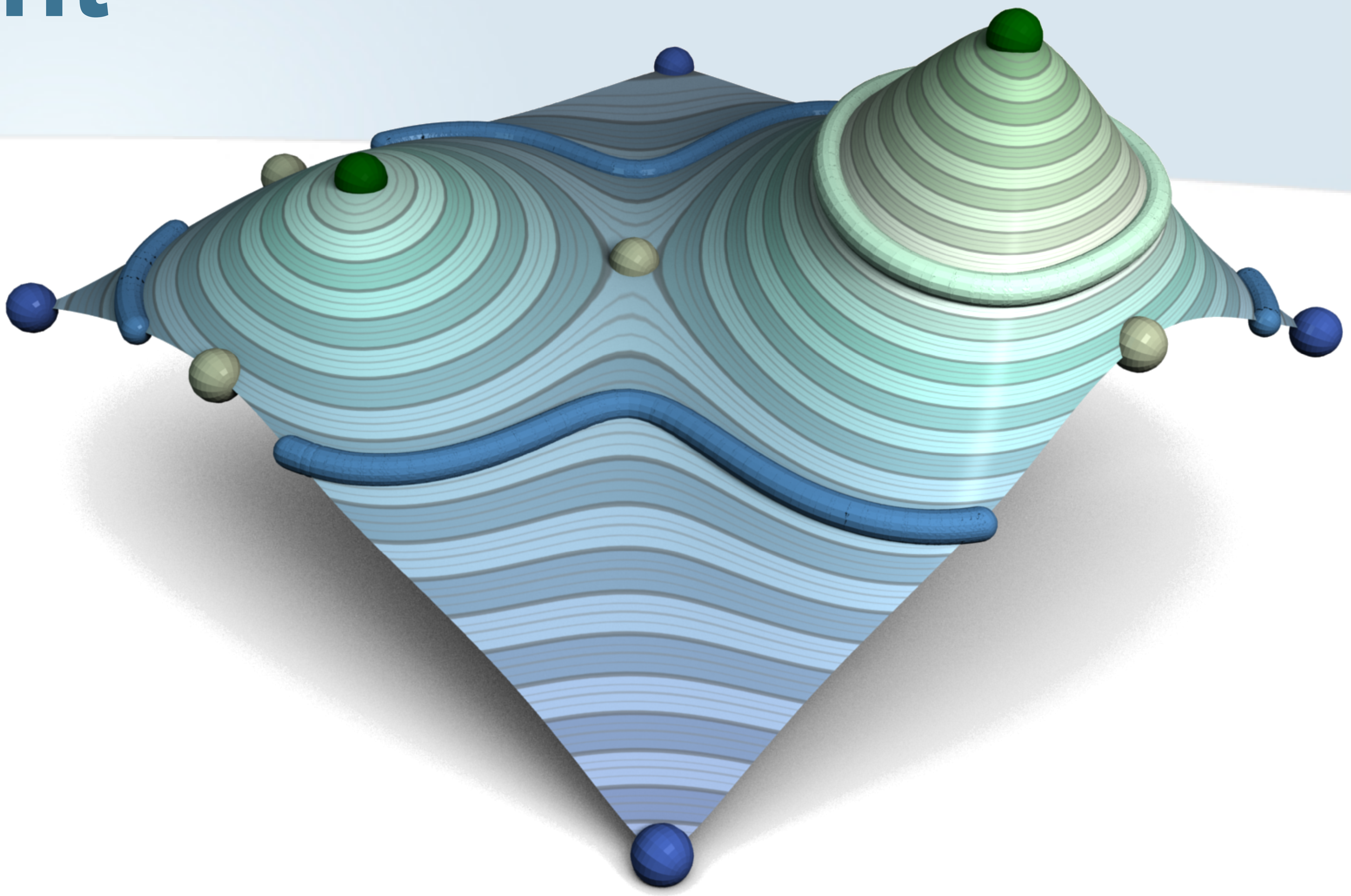
- Combinatorial identification
  - Star of a simplex σ
    - Simplices that contain σ as a face
  - Link of a simplex

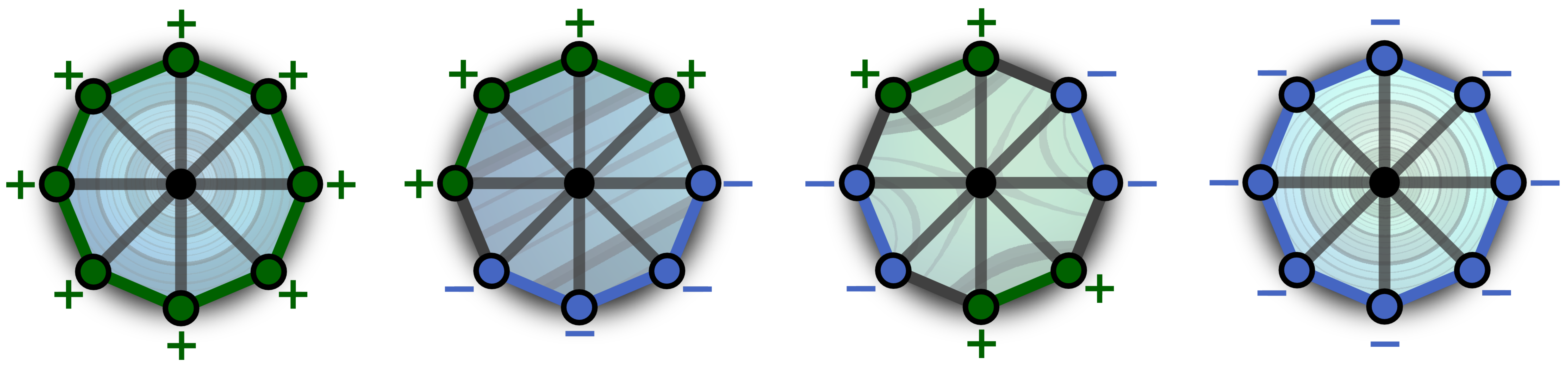




Notion of critical point

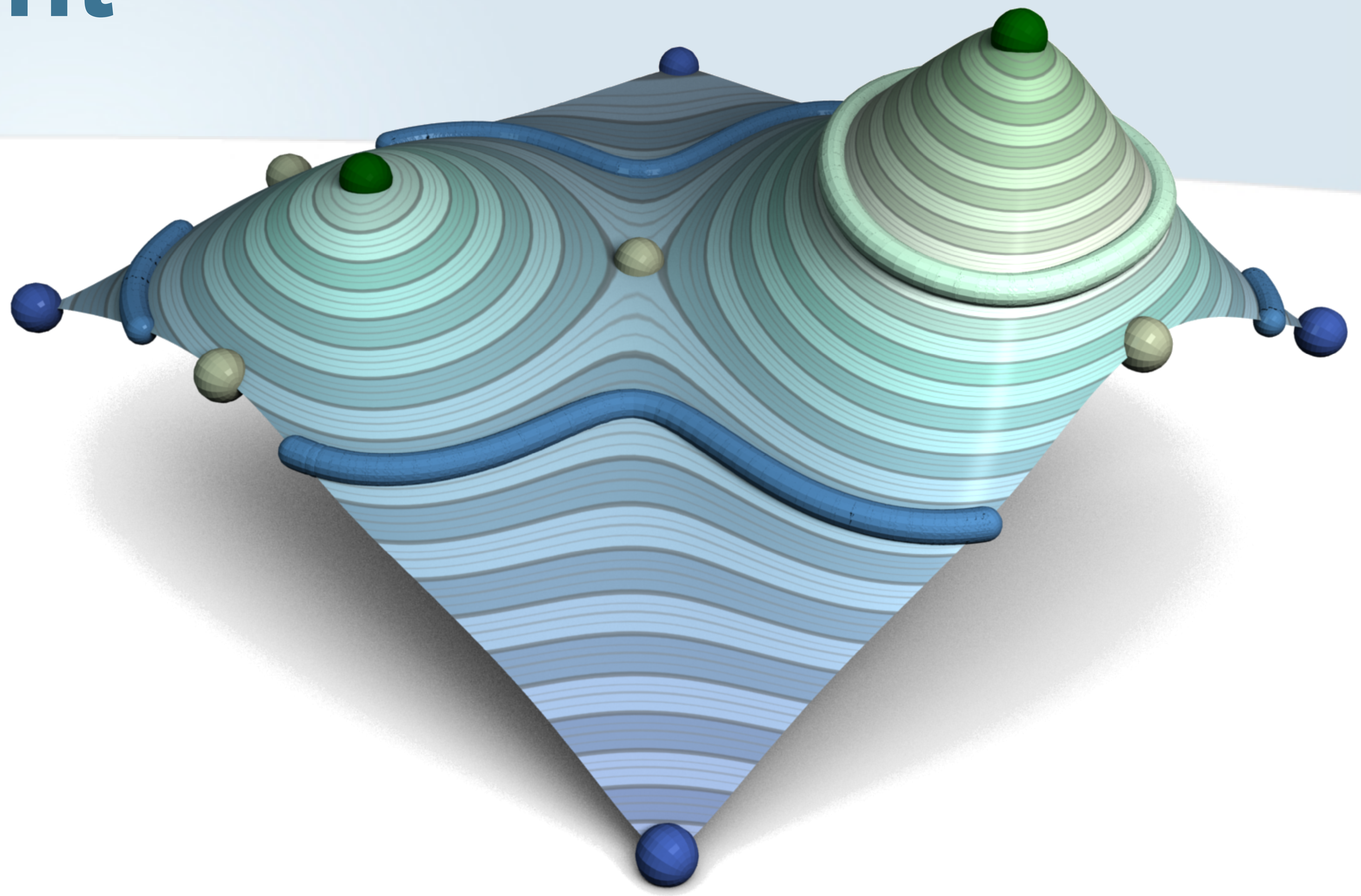
- Combinatorial identification
  - Star of a simplex σ
    - Simplices that contain σ as a face
  - Link of a simplex
    - Simplices of the closure of the star that do not contain σ as a face

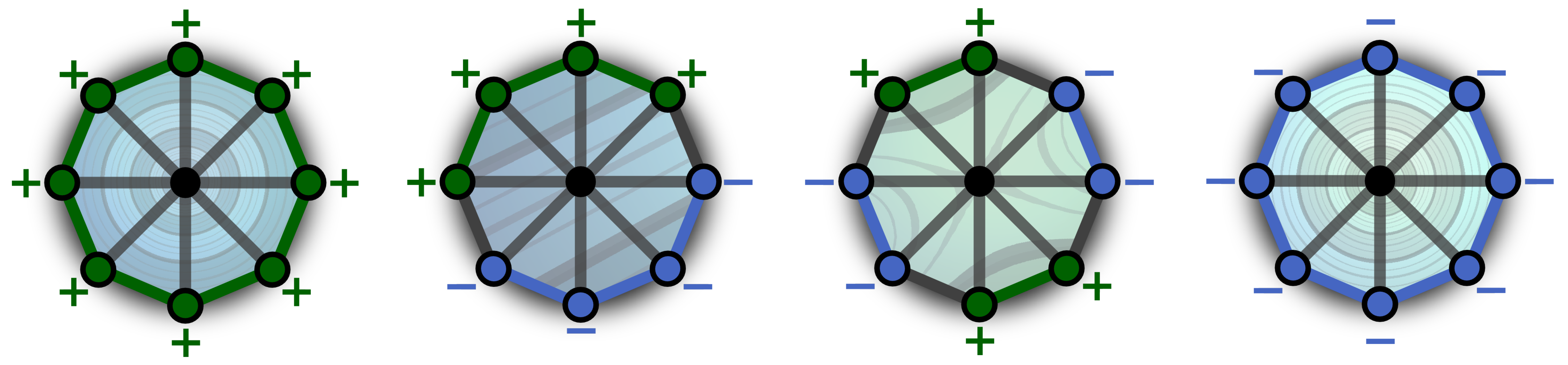




Notion of critical point

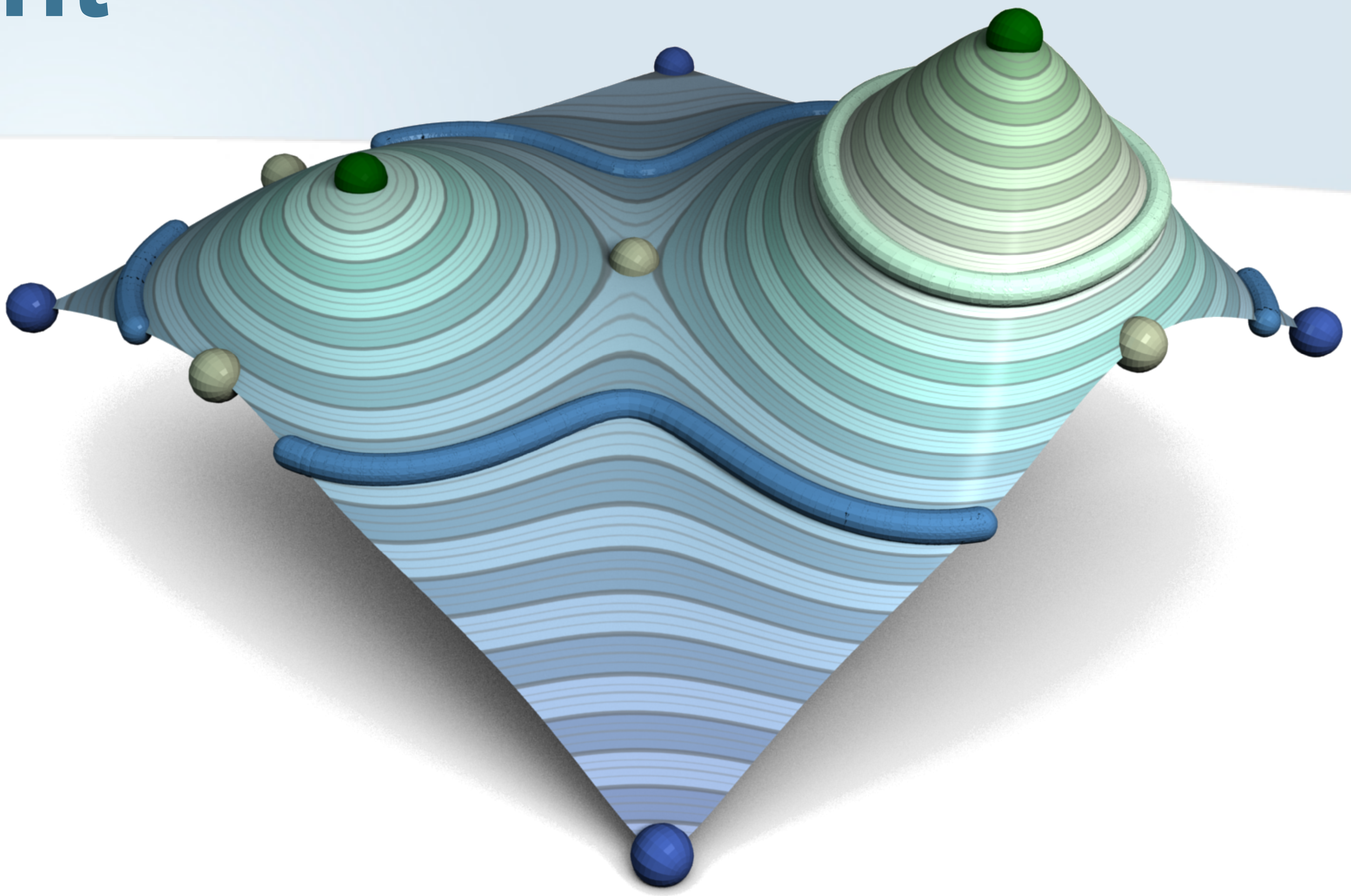
- Combinatorial identification
  - Lower link of v

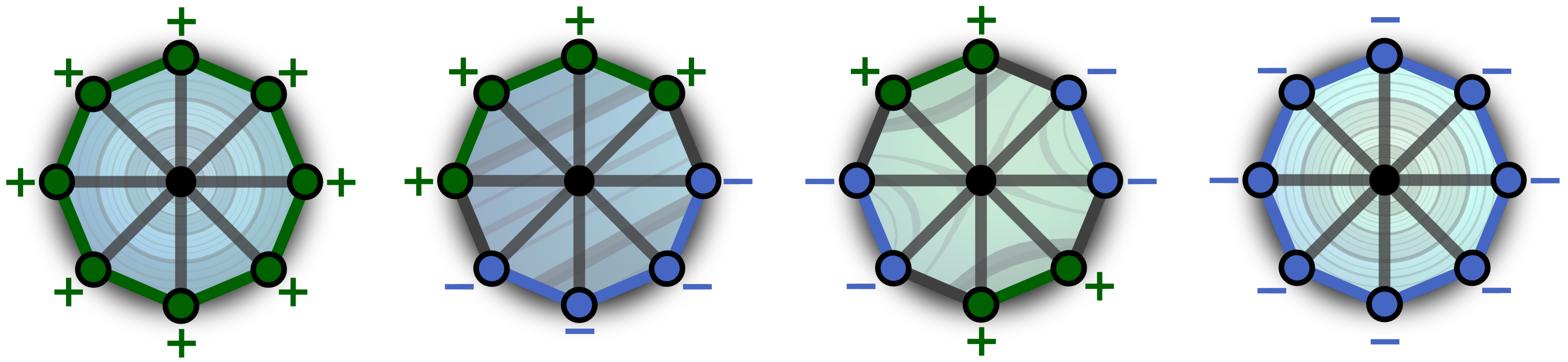




Notion of critical point

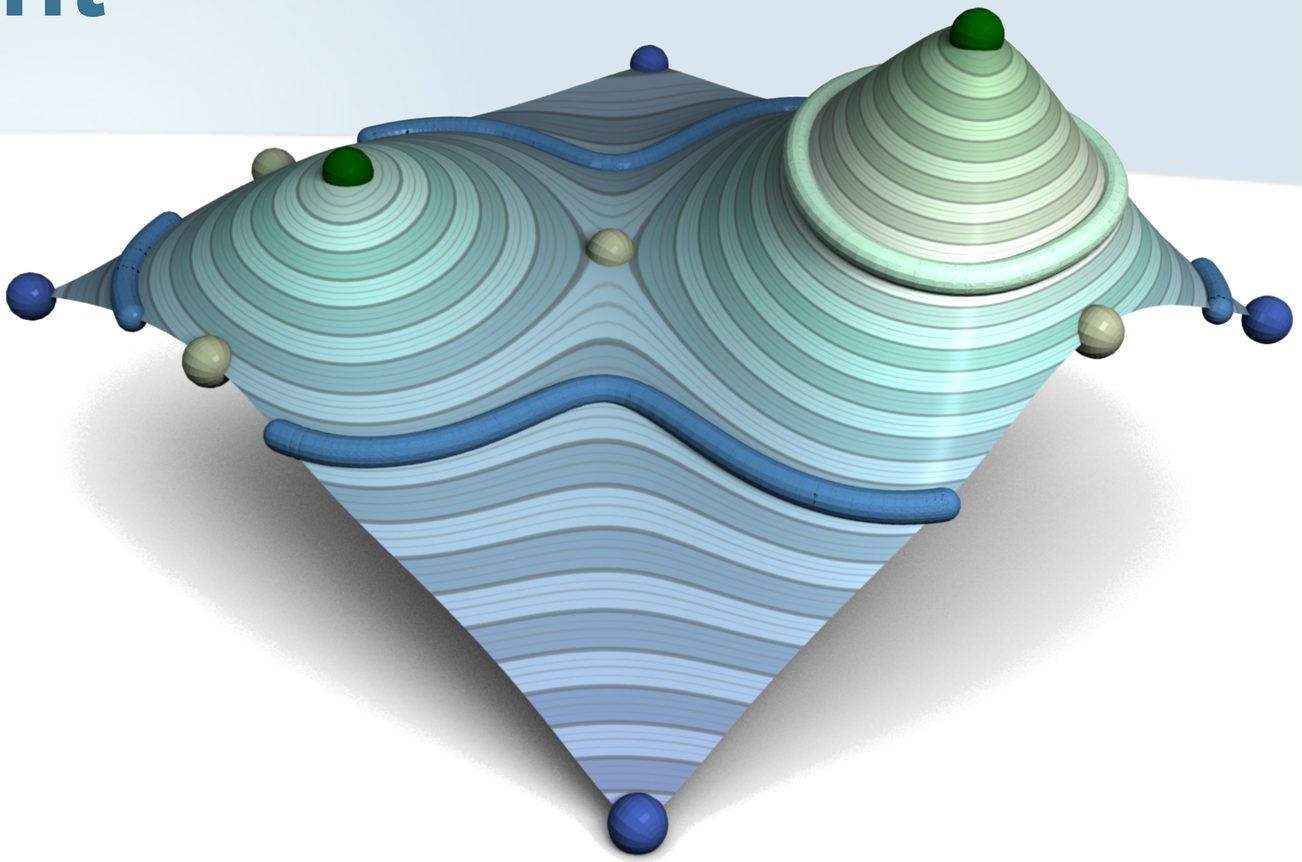
- Combinatorial identification
  - Lower link of v
    - Simplices whose function values are strictly below $f(v)$

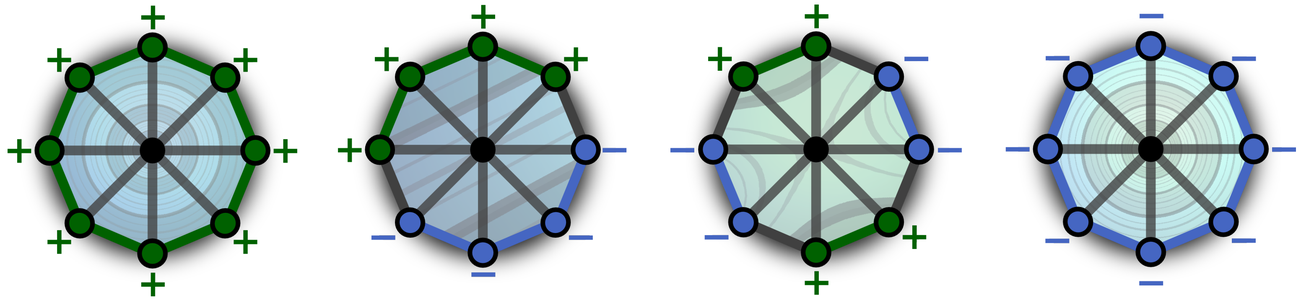




Notion of critical point

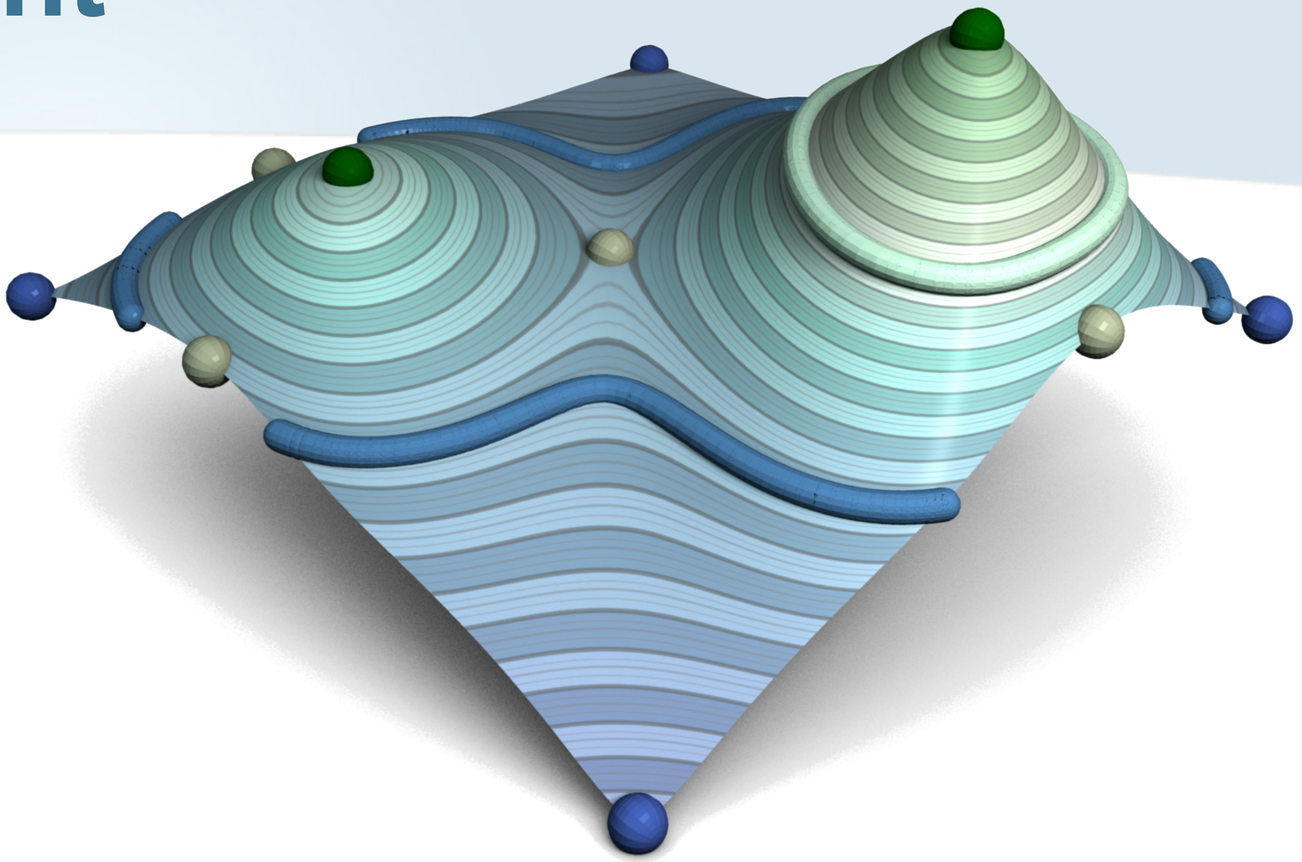
- Combinatorial identification
  - Lower link of v
    - Simplices whose function values are strictly below $f(v)$
  - Upper link of v

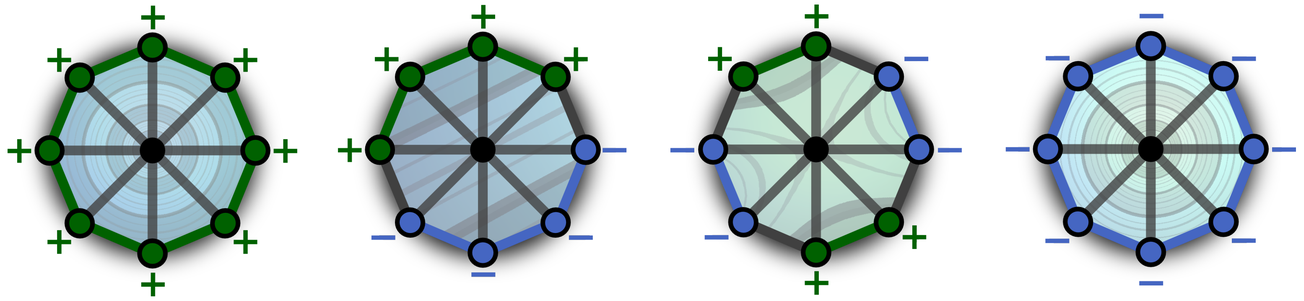




Notion of critical point

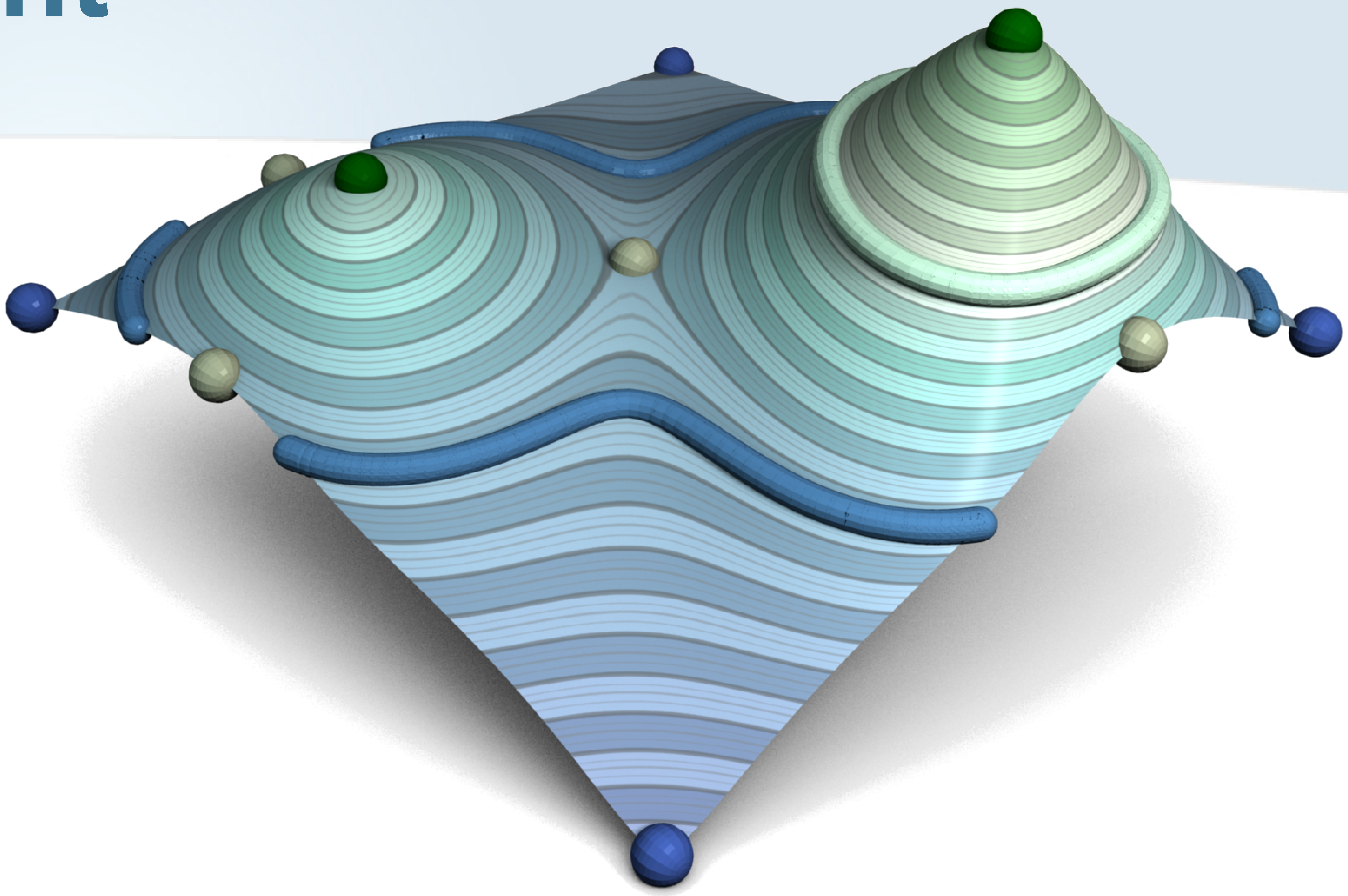
- Combinatorial identification
  - Lower link of v
    - Simplices whose function values are strictly below $f(v)$
  - Upper link of v
    - Simplices whose function values are strictly above $f(v)$

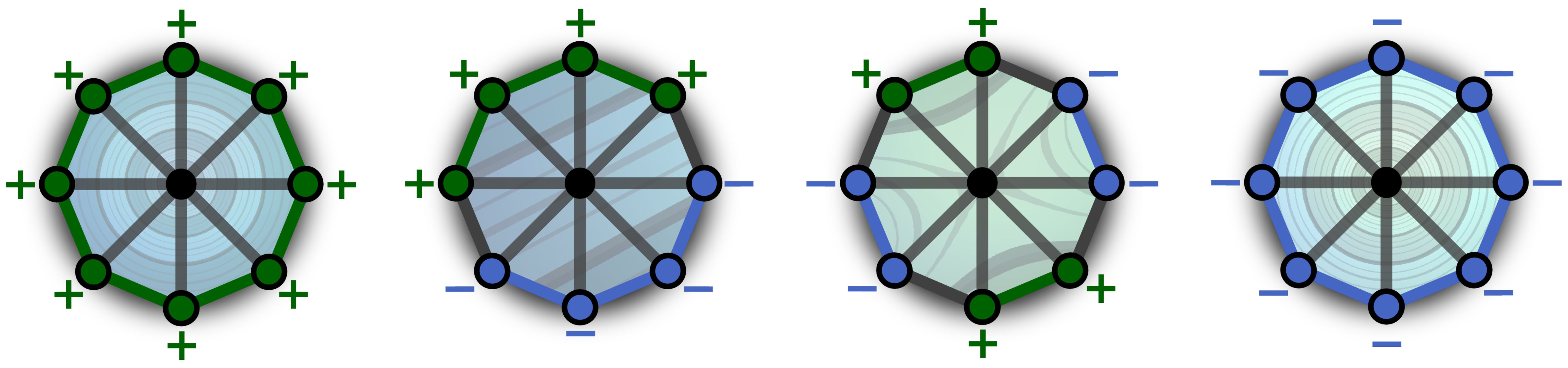




Notion of critical point

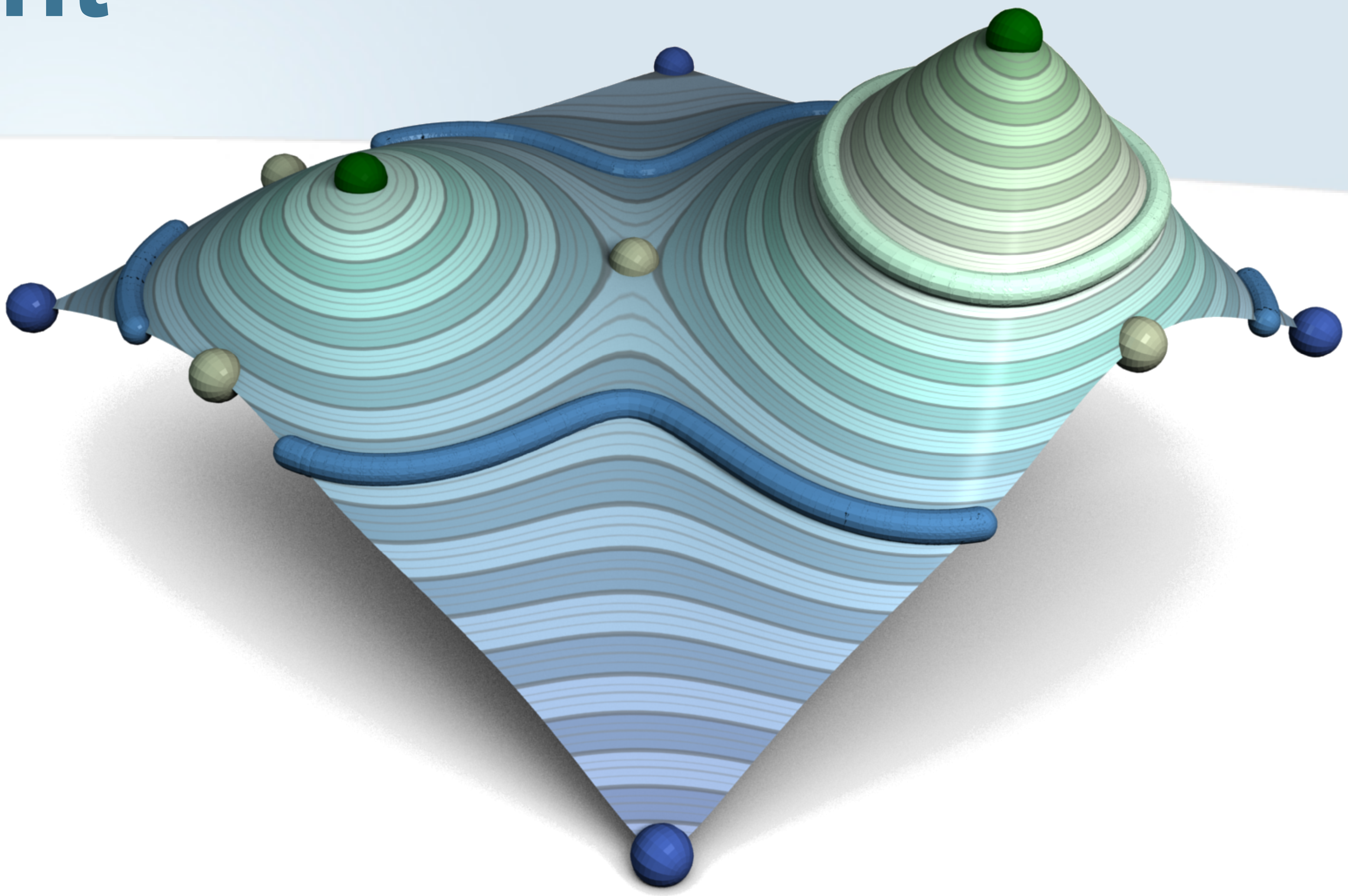
- Combinatorial identification

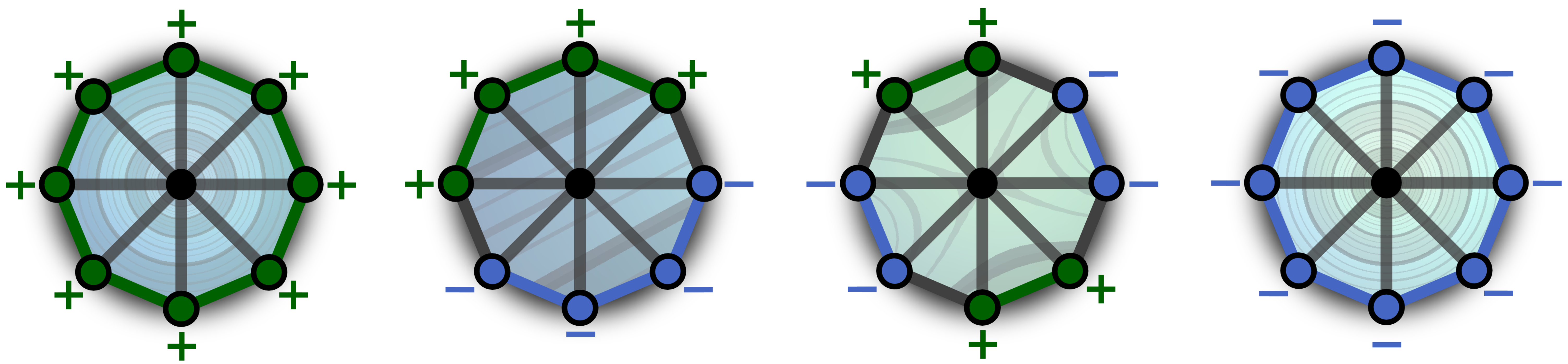




Notion of critical point

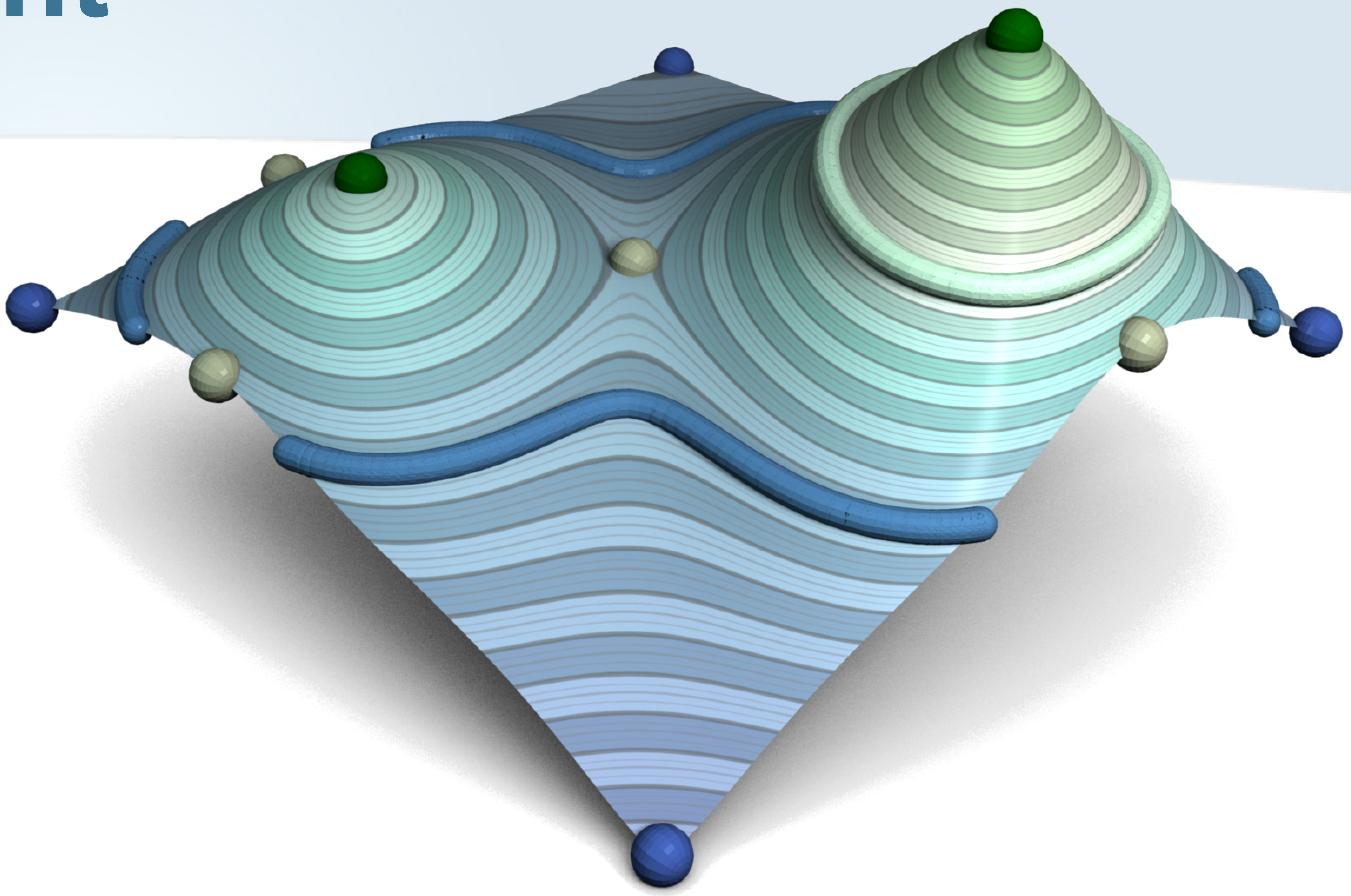
- Combinatorial identification
  - Minimum

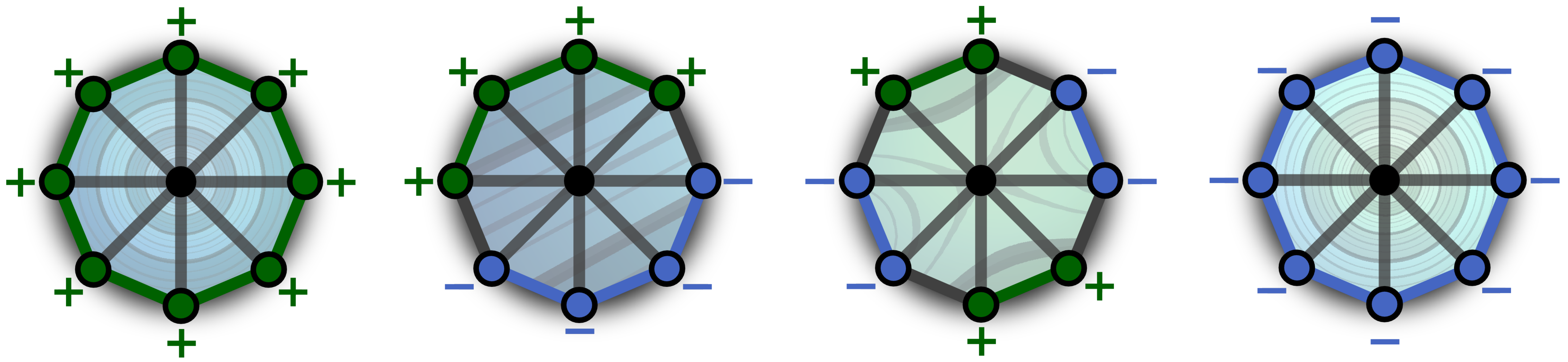




Notion of critical point

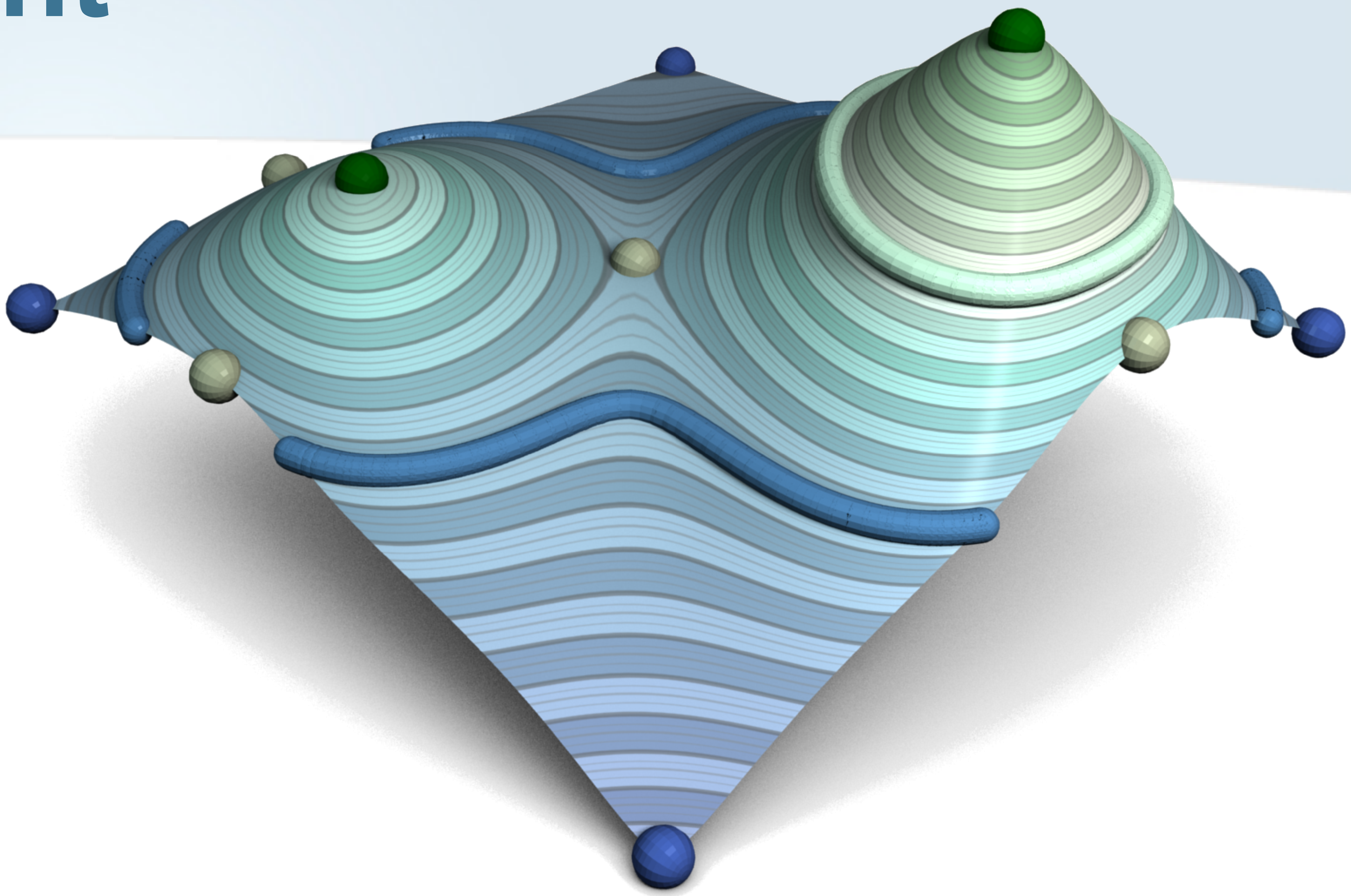
- Combinatorial identification
  - Minimum
    - Empty lower link

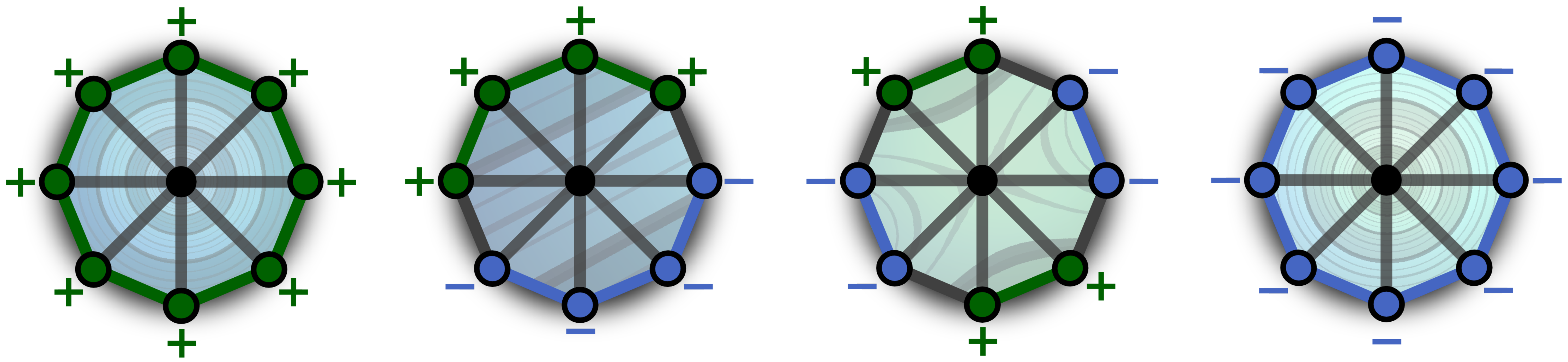




Notion of critical point

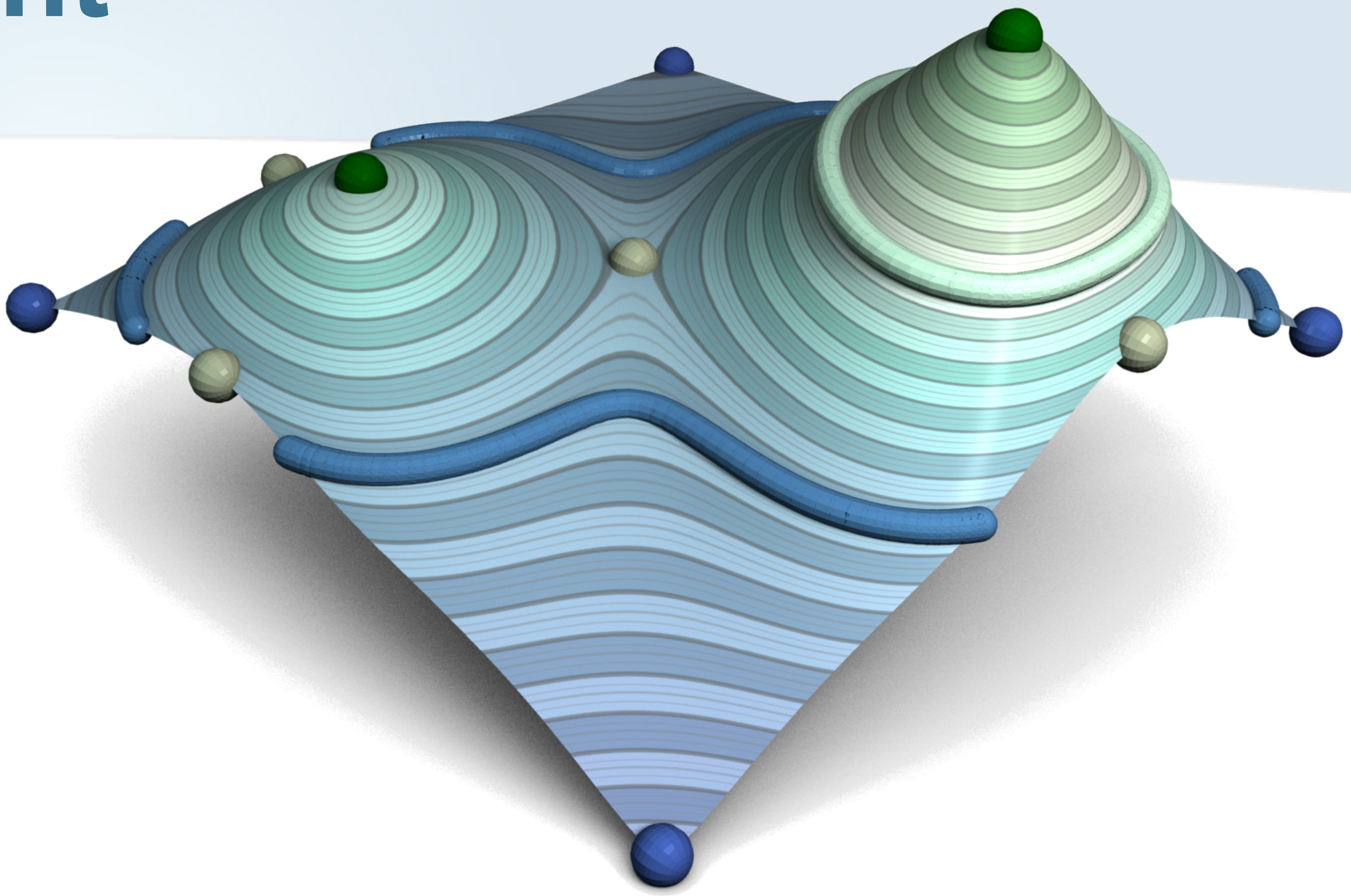
- Combinatorial identification
  - Minimum
    - Empty lower link
  - Maximum

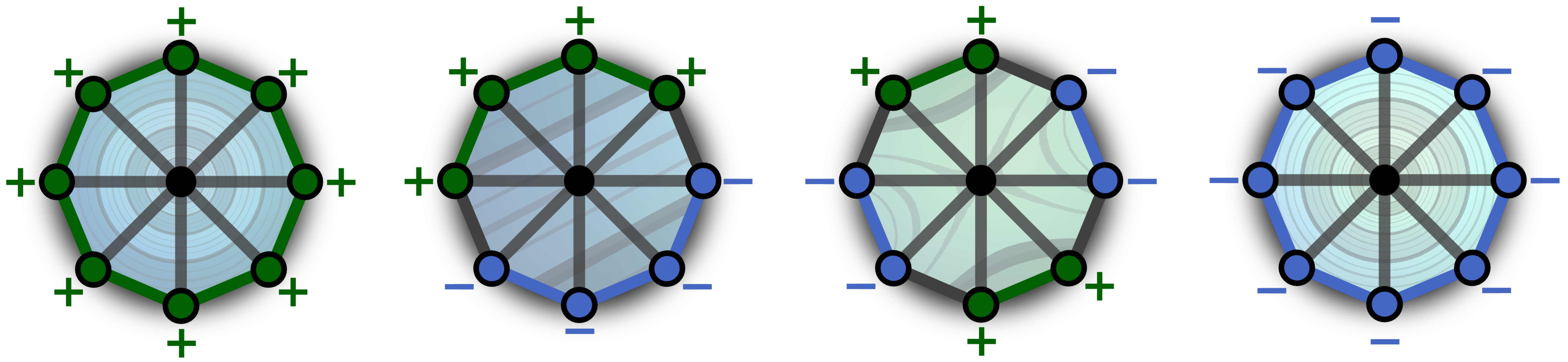




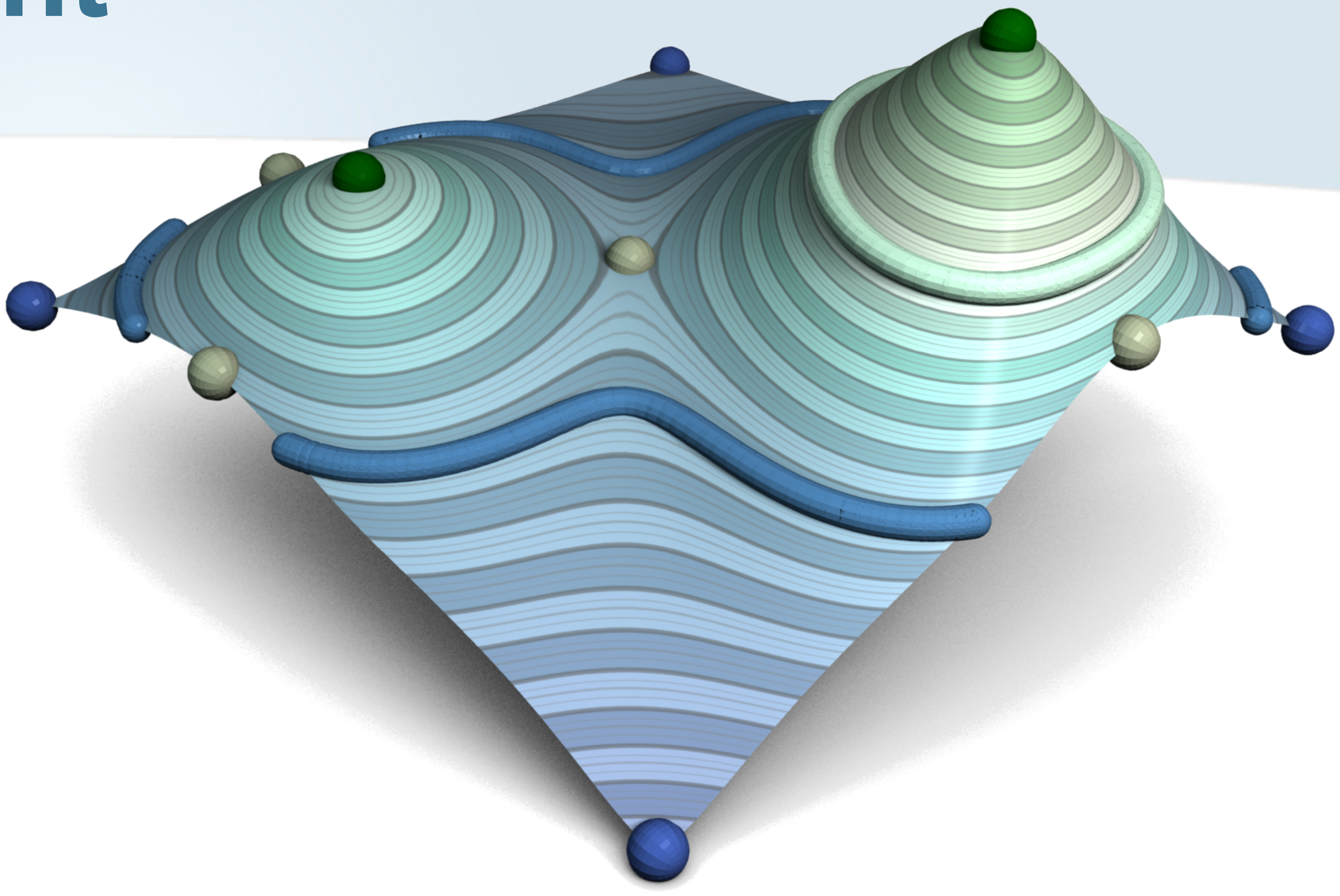
Notion of critical point

- Combinatorial identification
  - Minimum
    - Empty lower link
  - Maximum
    - Empty upper link

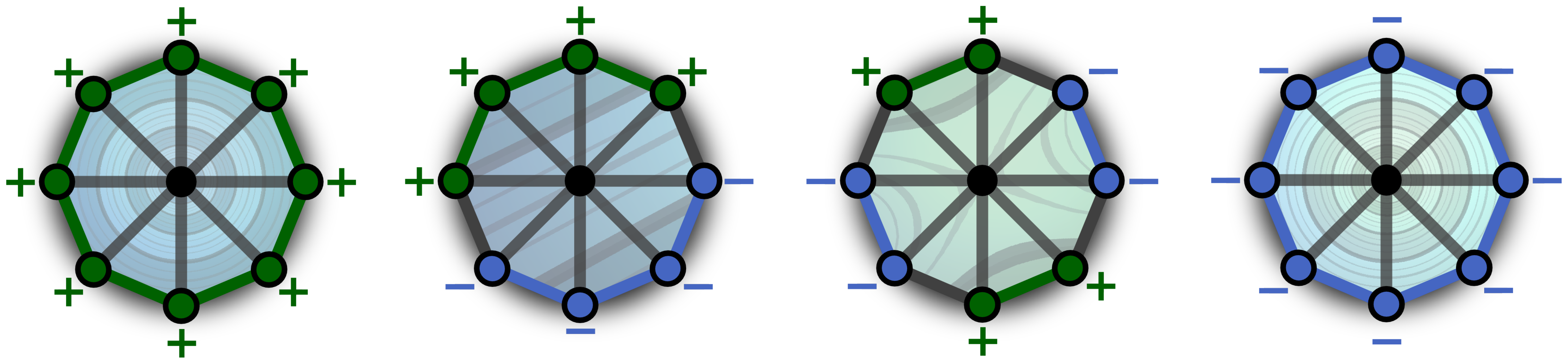




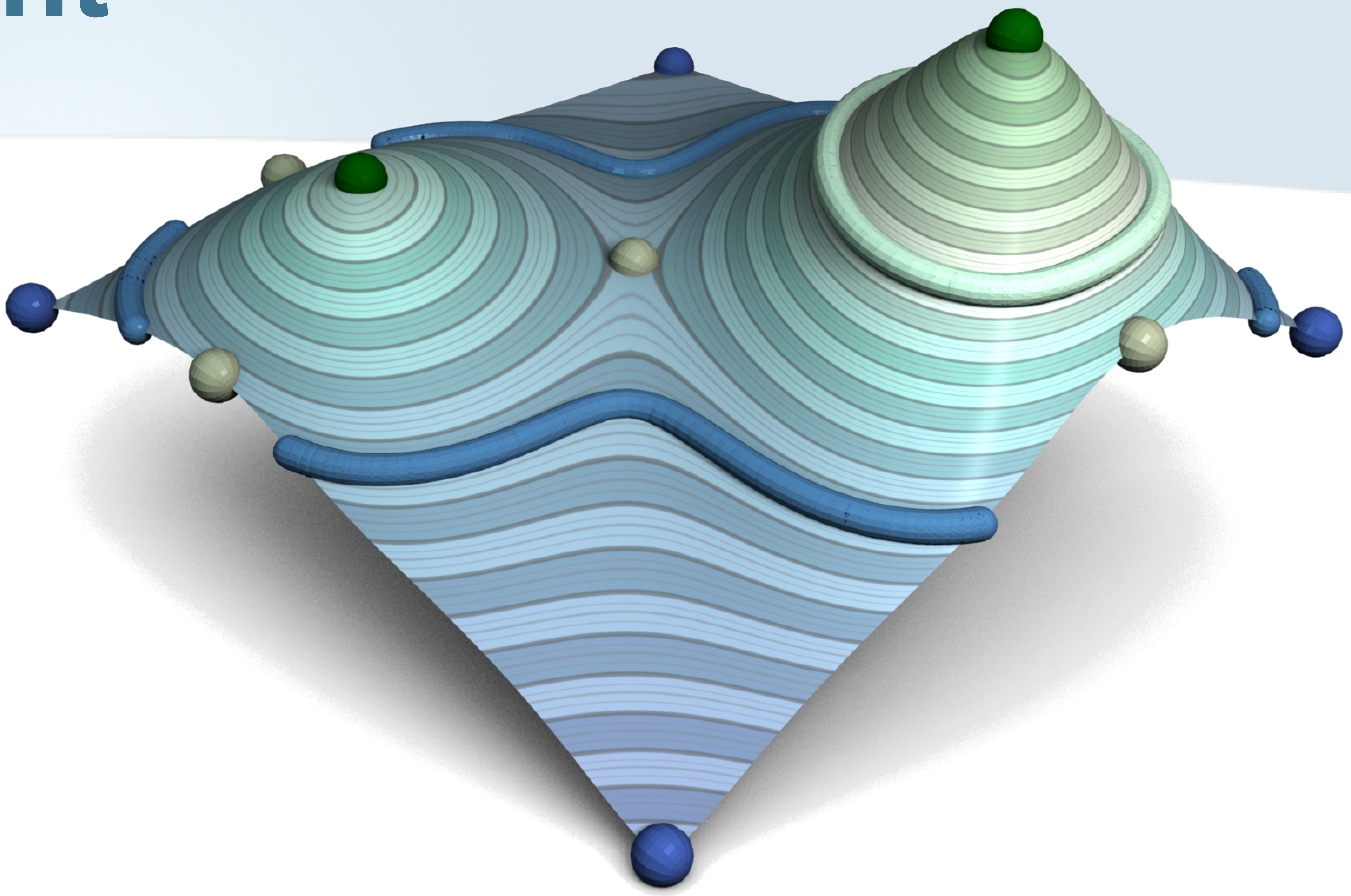
Notion of critical point



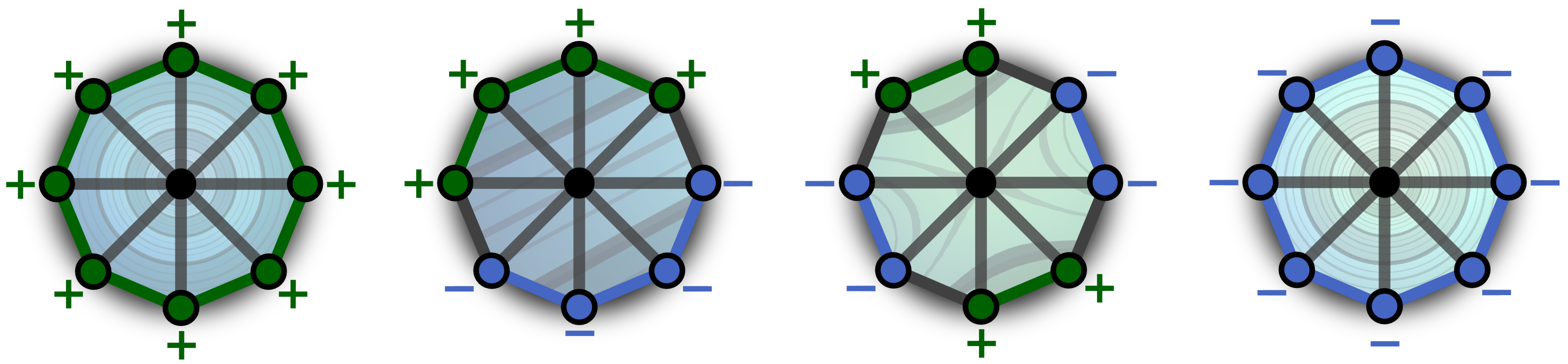
- Combinatorial identification
  - Minimum
    - Empty lower link
  - Maximum
    - Empty upper link
  - Regular point



Notion of critical point

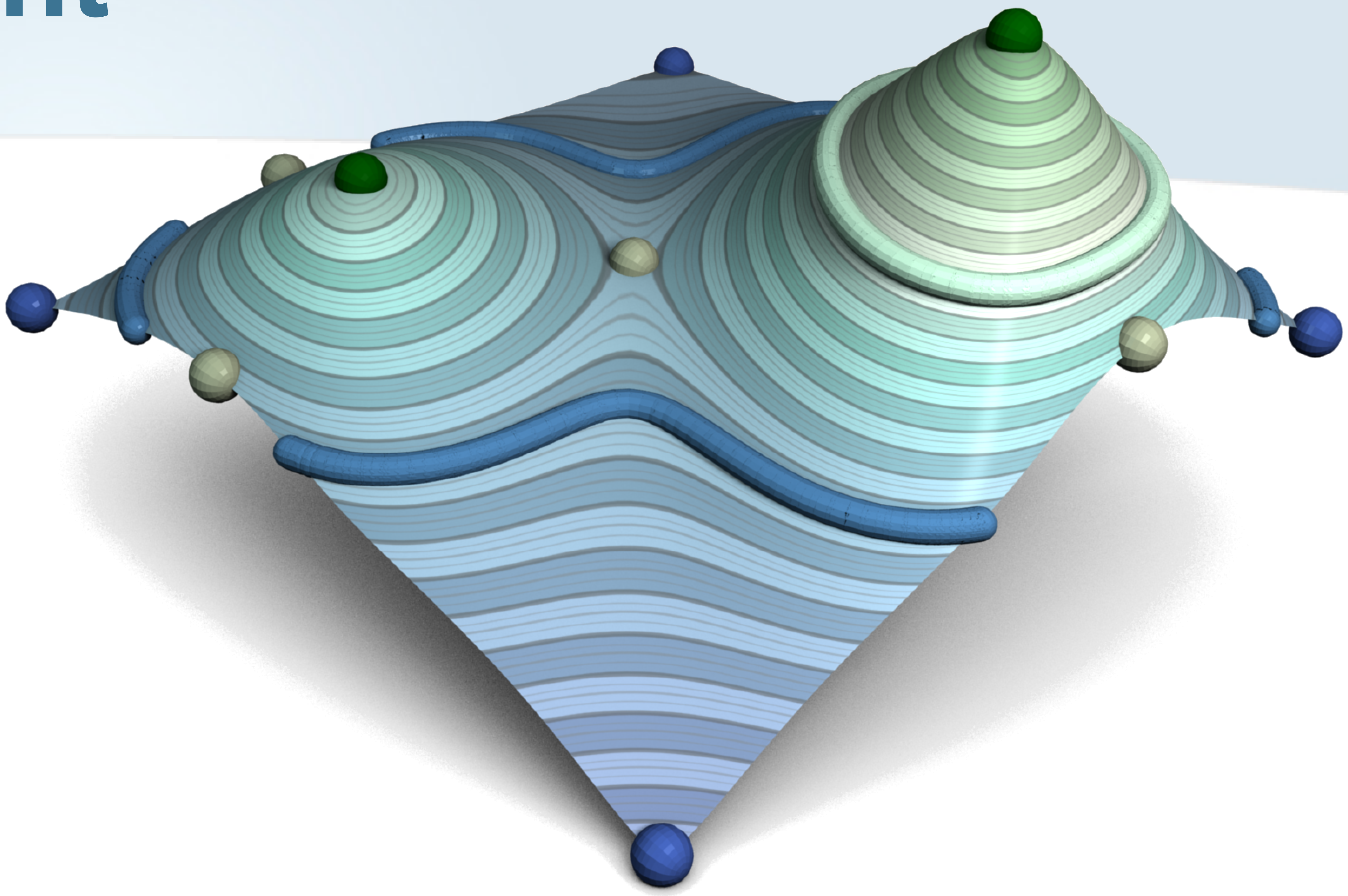


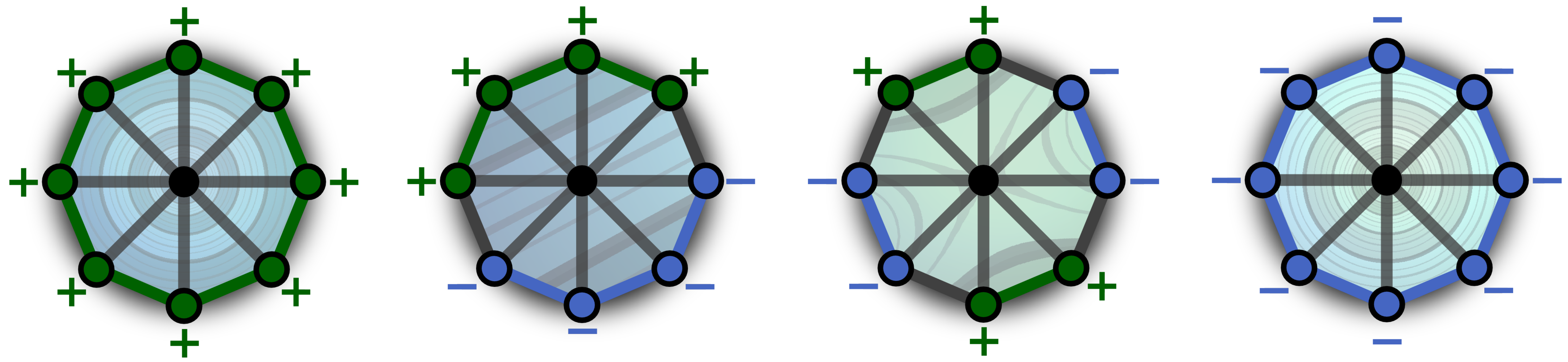
- Combinatorial identification
  - Minimum
    - Empty lower link
  - Maximum
    - Empty upper link
  - Regular point
    - Lower and upper links both *simply connected*



Notion of critical point

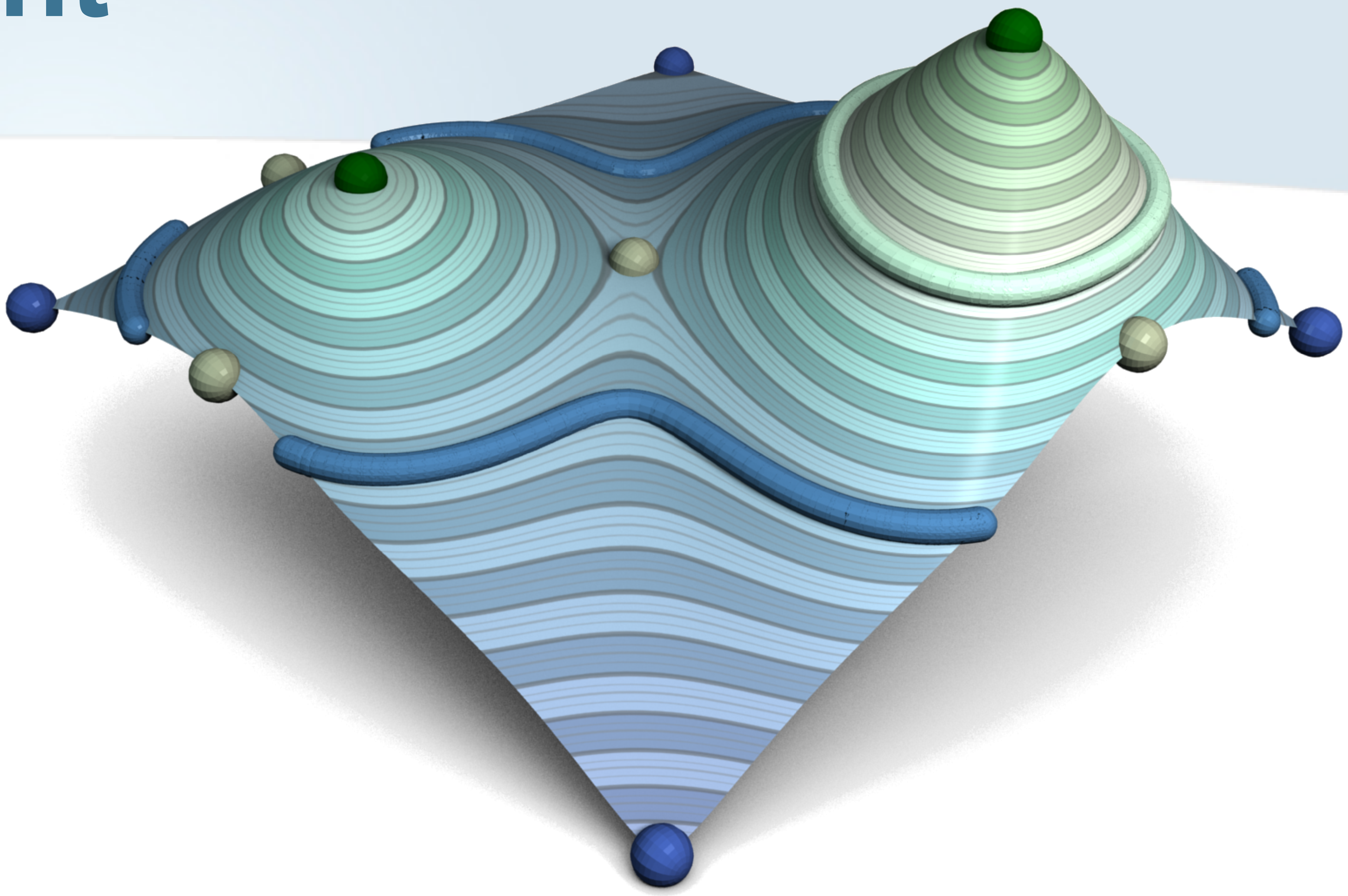
- Combinatorial identification
  - Everything else

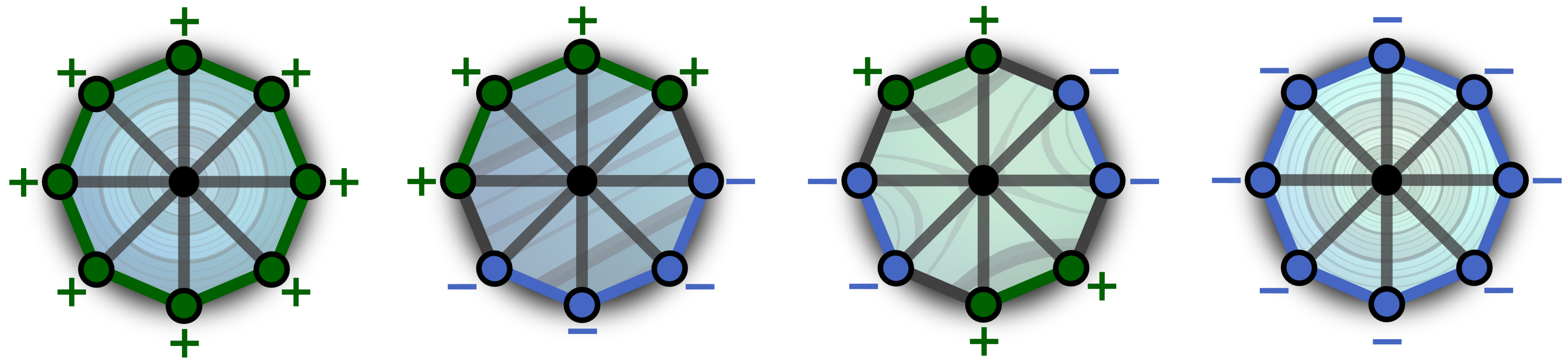




Notion of critical point

- Combinatorial identification
  - Everything else
    - Saddle

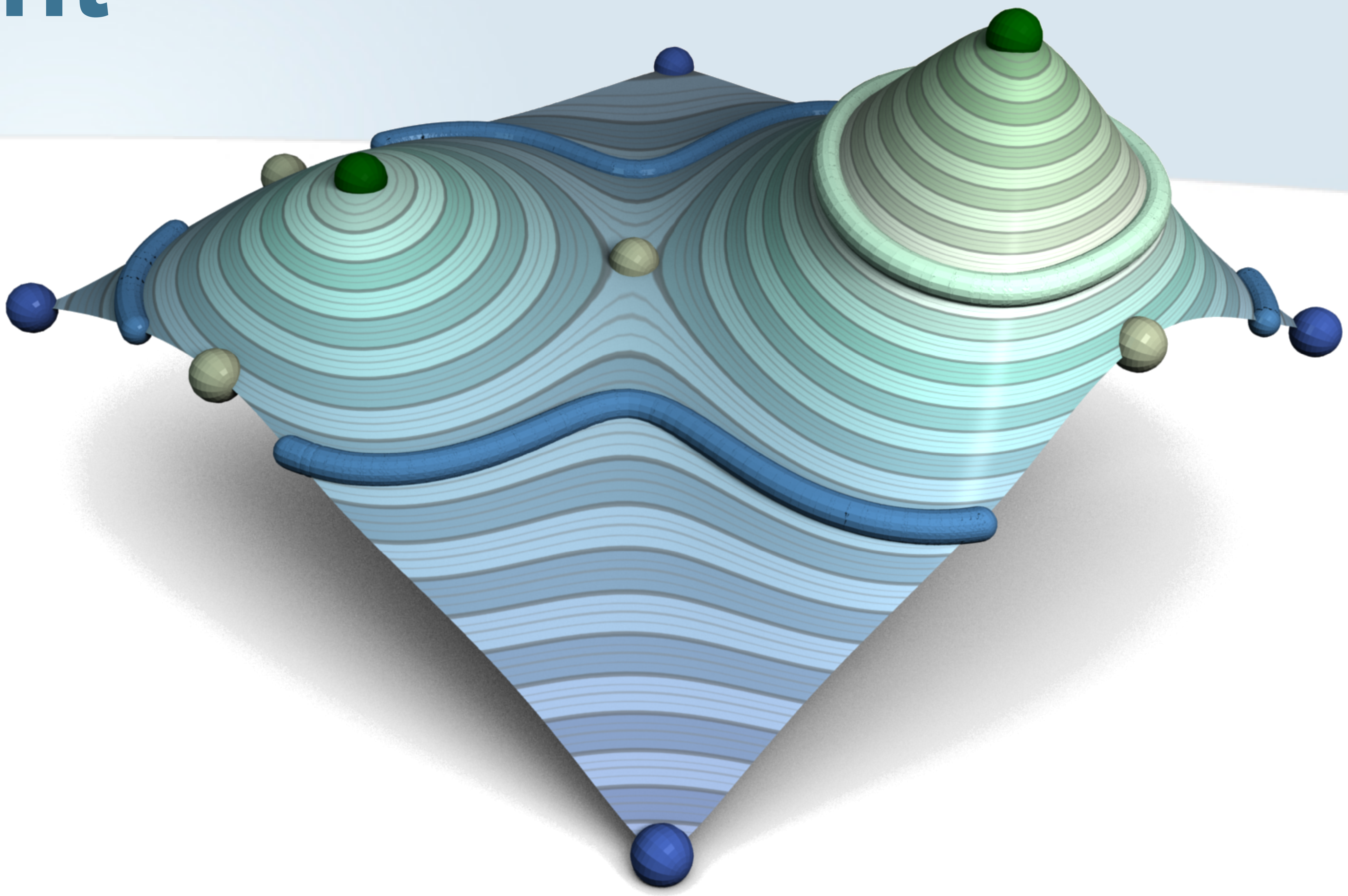


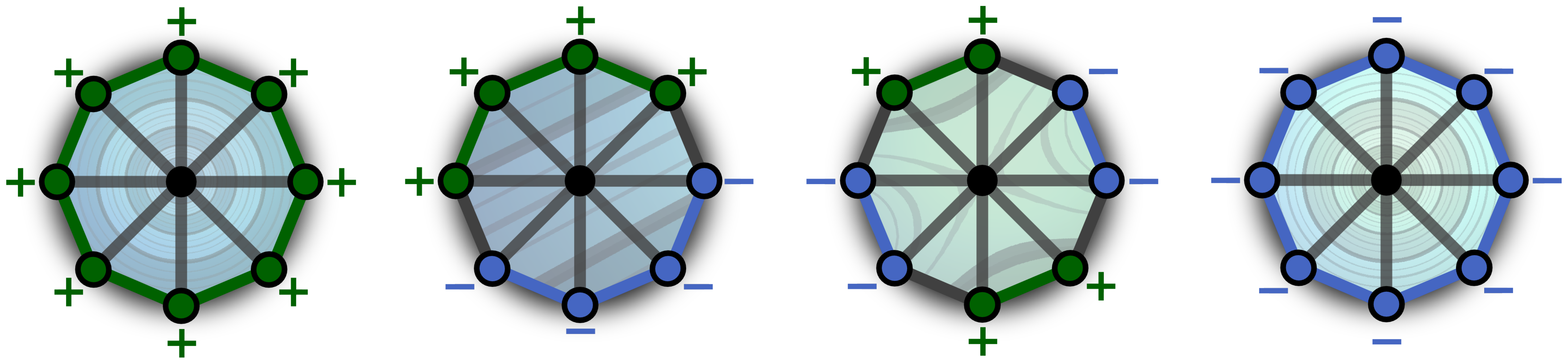


Notion of critical point

- Combinatorial identification
  - Everything else
    - Saddle

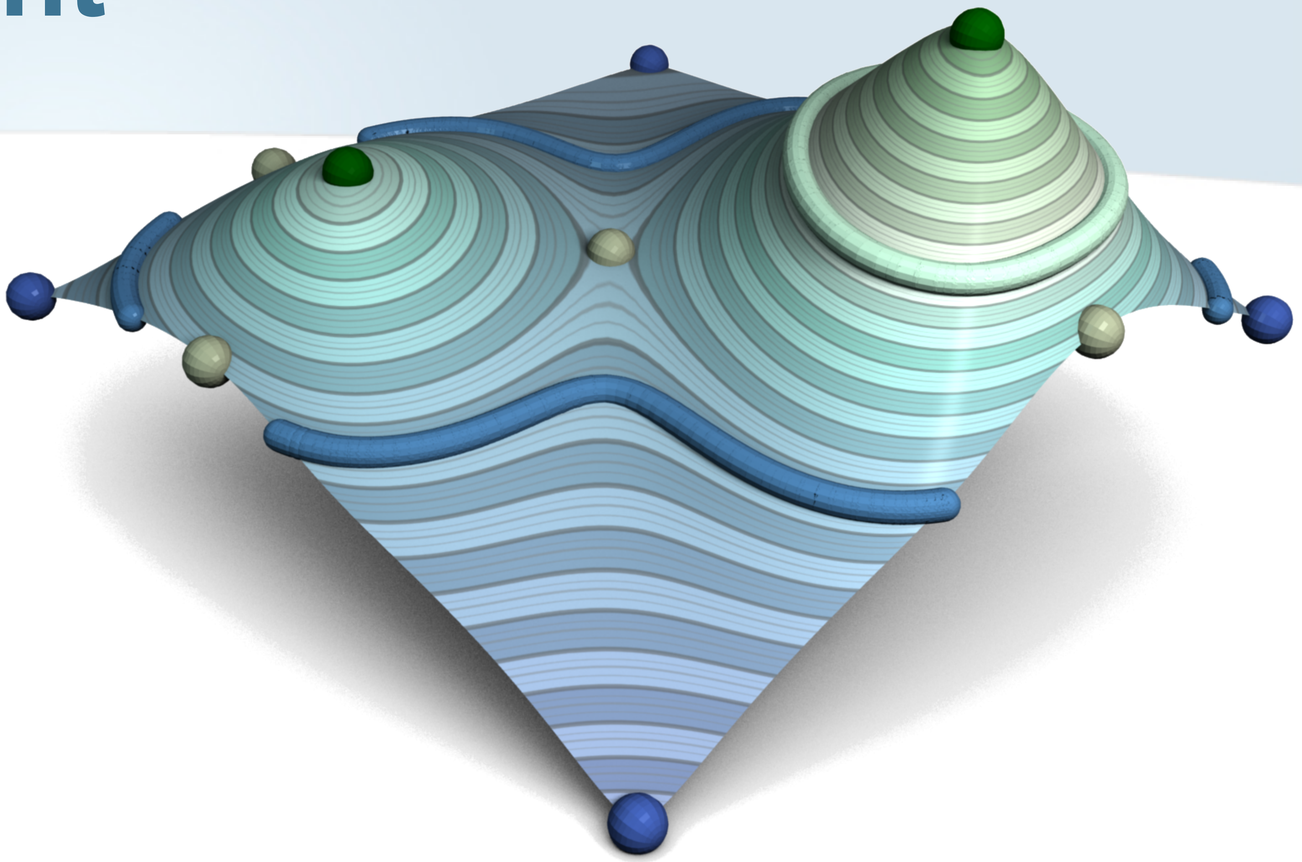
  - Works in arbitrary dimension

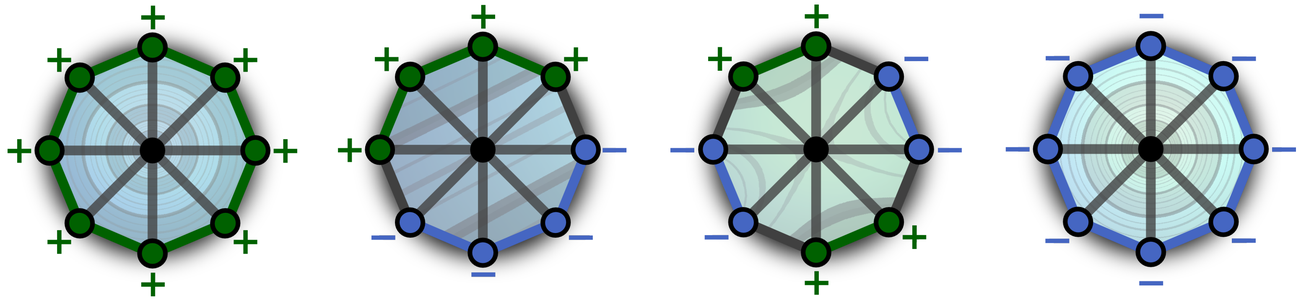




Notion of critical point

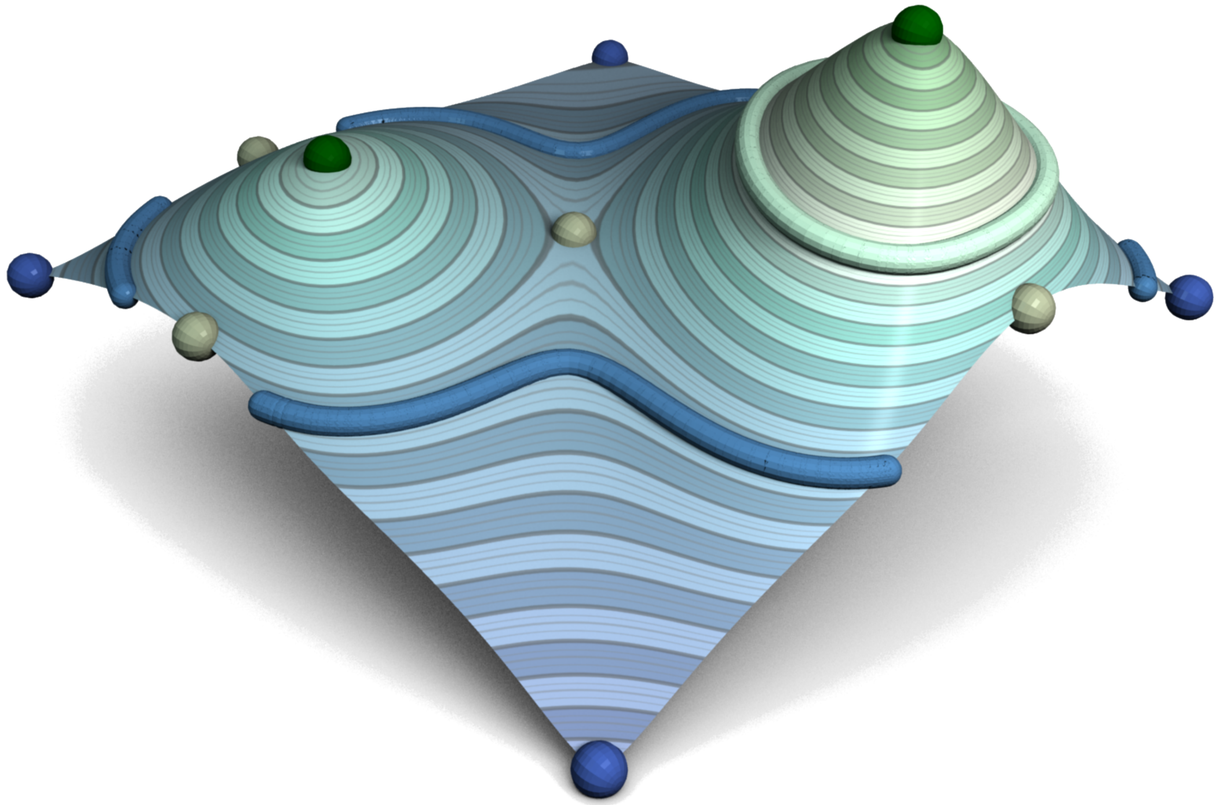
- Combinatorial identification
  - Everything else
    - Saddle
  - Works in arbitrary dimension
  - Value of a critical point
    - *Critical value*





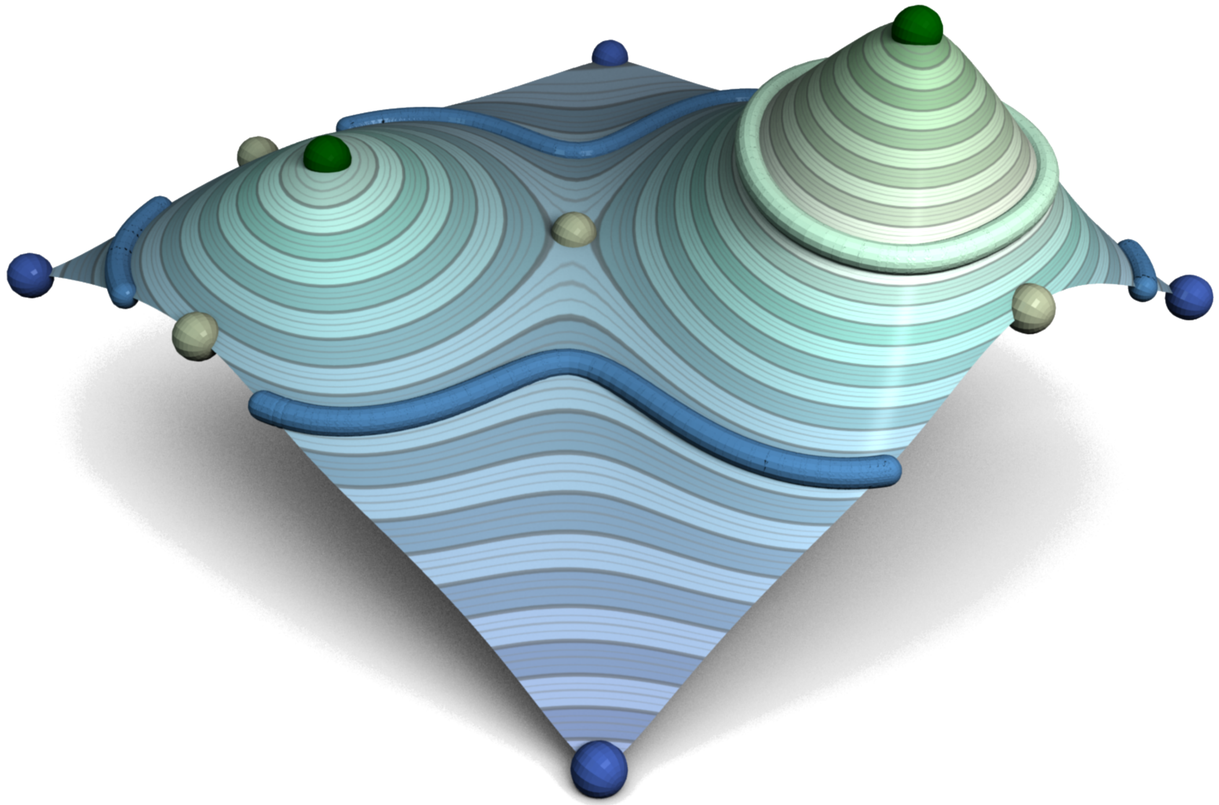
Level sets

- Given a domain $\mathcal{D}$ of dimension d
  - Level set
    - $f^{-1}(i) = \{p \in \mathcal{D} \ / \ f(p) = i\}$



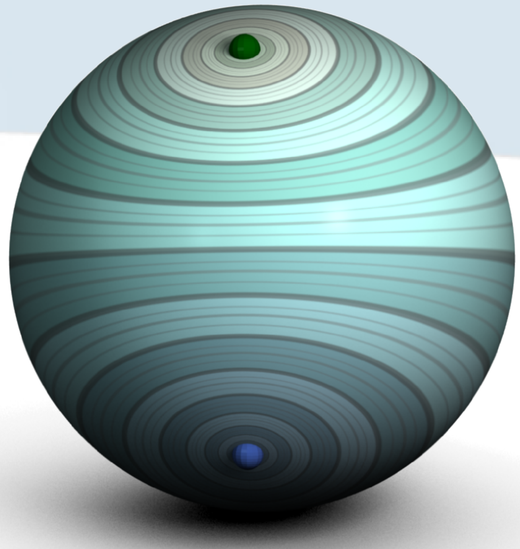
Level sets

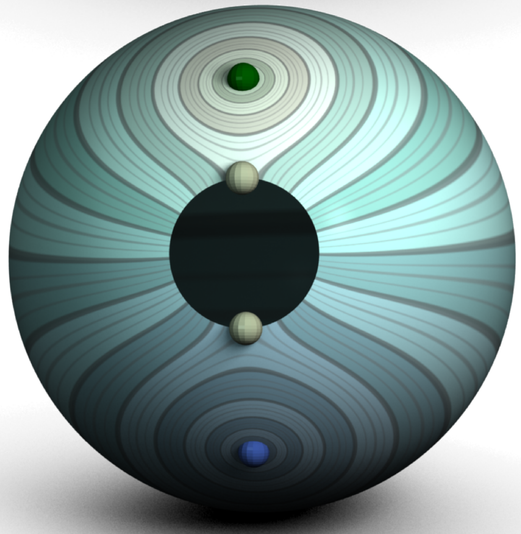
- Given a domain $\mathcal{D}$ of dimension d
  - Level set
    - $f^{-1}(i) = \{p \in \mathcal{D} \ / \ f(p) = i\}$
  - If i is not a critical value
    - $f^{-1}(i)$ is a (d-1) manifold



Level sets

- Given a domain $\mathcal{D}$ of dimension d
  - Level set
    - $f^{-1}(i) = \{p \in \mathcal{D} \ / \ f(p) = i\}$
  - If i is not a critical value
    - $f^{-1}(i)$ is a (d-1) manifold
  - If $\mathcal{D}$ is closed, $f^{-1}(i)$ is closed
    - Otherwise it may be open





Level set in a 1-simplex

- Let $\mathcal{D}$ be a single edge
  - $f(v_0)$
  - $f(v_1)$
  - $f(v_0) < f(v_1)$

Level set in a 1-simplex

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Level set in a 1-simplex

- Let $\mathcal{D}$ be a single edge
  - $f(v_0)$
  - $f(v_1)$
  - $f(v_0) < f(v_1)$

- Let i be an isovalue
  - $f^{-1}(i) = \{p \in \mathcal{D} \ / \ f(p) = i\}$
    - 0-manifold

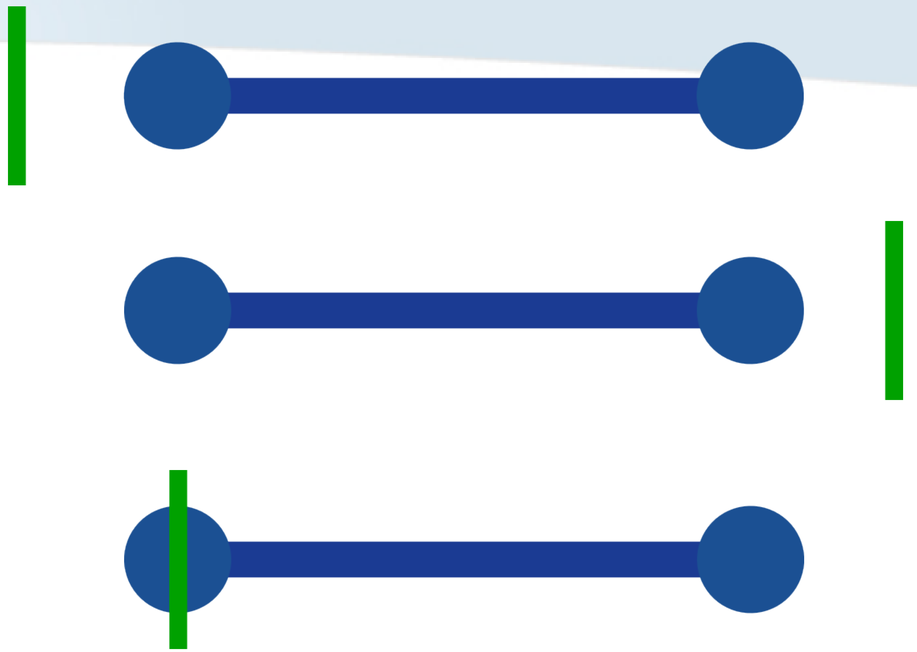
Level set in a 1-simplex

- Let $\mathcal{D}$ be a single edge
  - $f(v_0)$
  - $f(v_1)$
  - $f(v_0) < f(v_1)$

- Let i be an isovalue
  - $f^{-1}(i) = \{p \in \mathcal{D} \ / \ f(p) = i\}$
    - 0-manifold
  - If intersection, **3 cases only**

Level set in a 1-simplex

- Let $\mathcal{D}$ be a single edge
  - $f(v_0)$
  - $f(v_1)$
  - $f(v_0) < f(v_1)$



- Let i be an isovalue
  - $f^{-1}(i) = \{p \in \mathcal{D} \ / \ f(p) = i\}$
    - 0-manifold
  - If intersection, **3 cases only**

Level set in a 1-simplex

- Let $\mathcal{D}$ be a single edge
  - $f(v_0)$
  - $f(v_1)$
  - $f(v_0) < f(v_1)$

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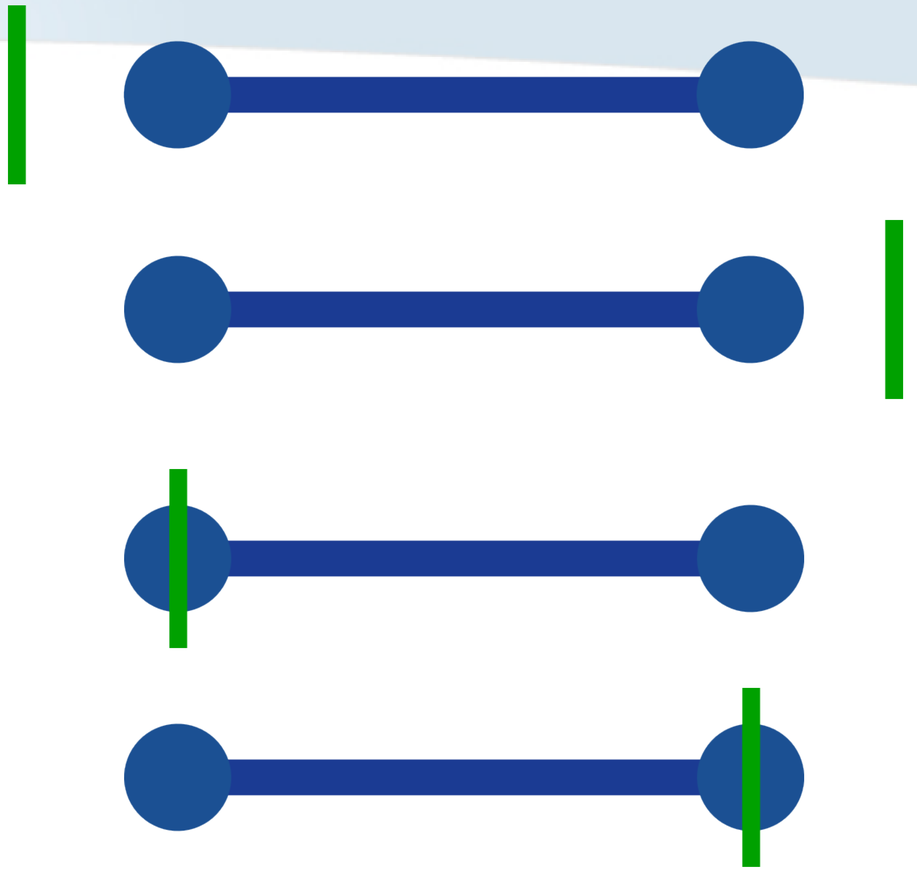

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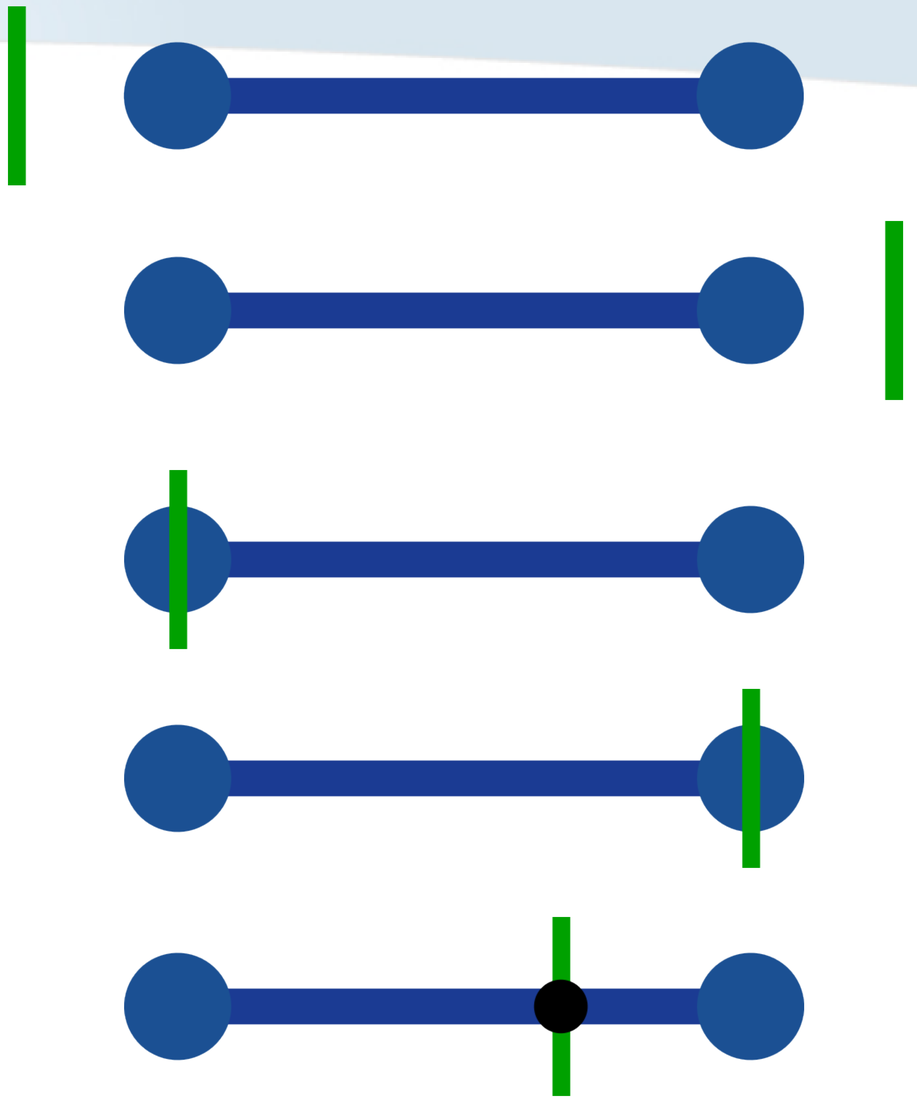

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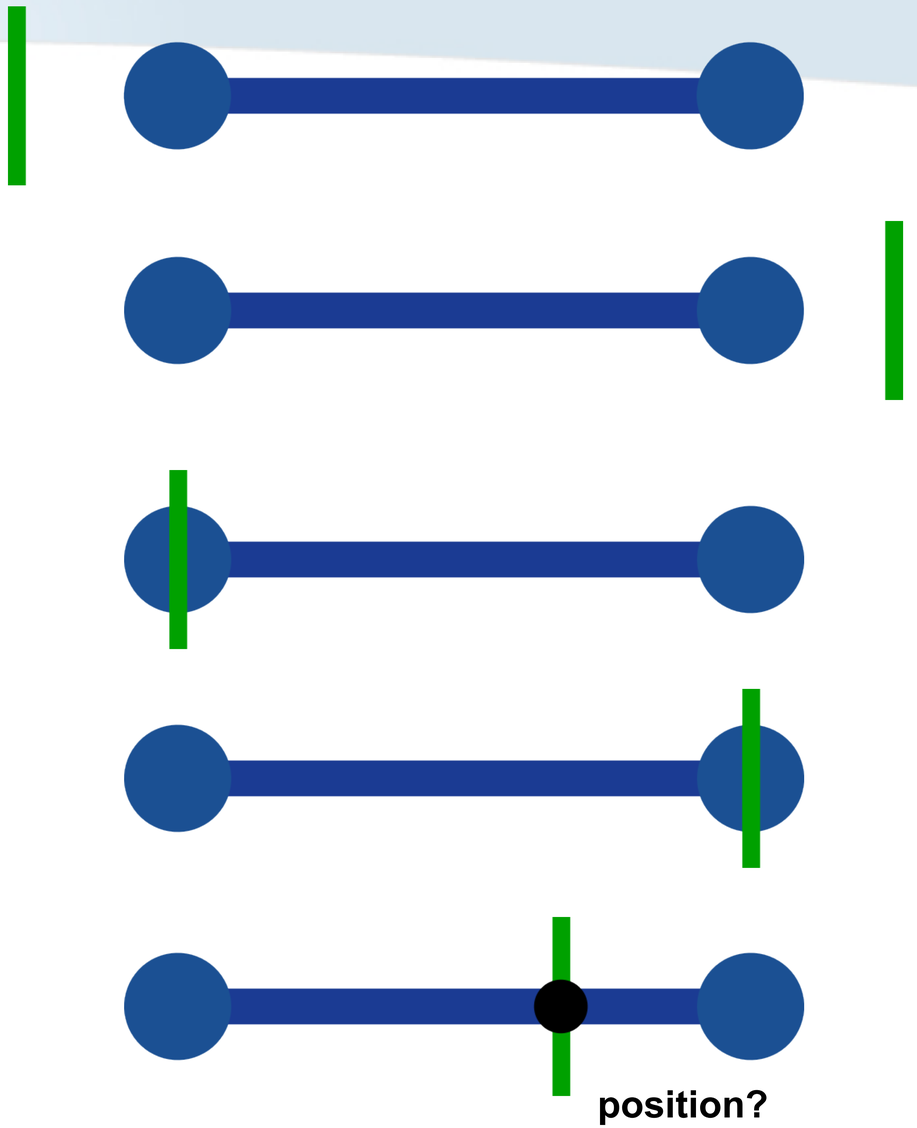
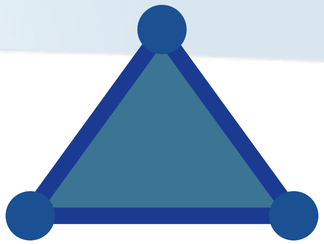


Level set in a 2-simplex



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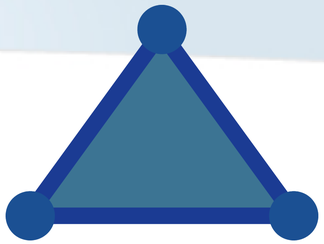
Level set in a 2-simplex



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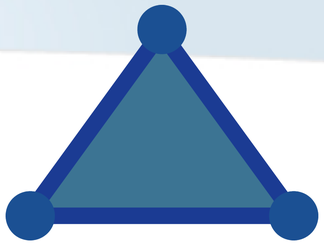
Level set in a 2-simplex



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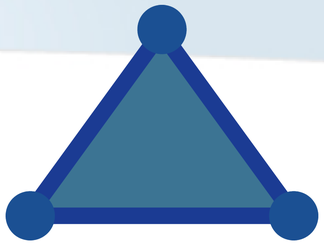

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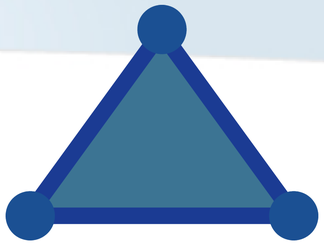

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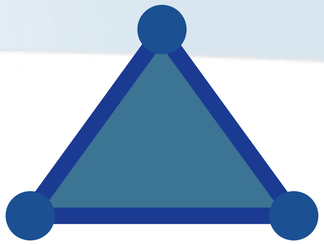

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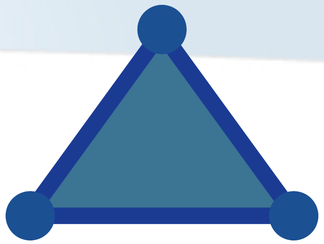

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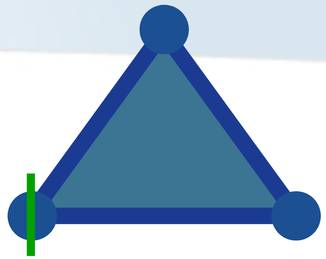

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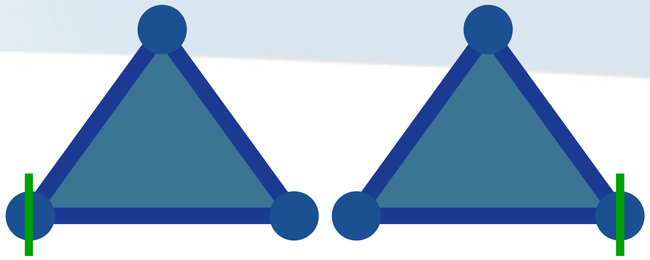

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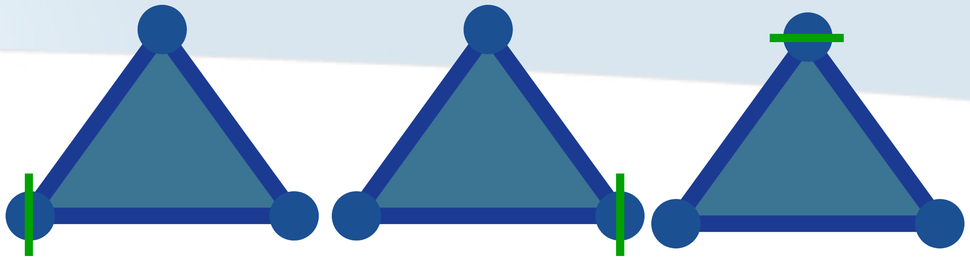

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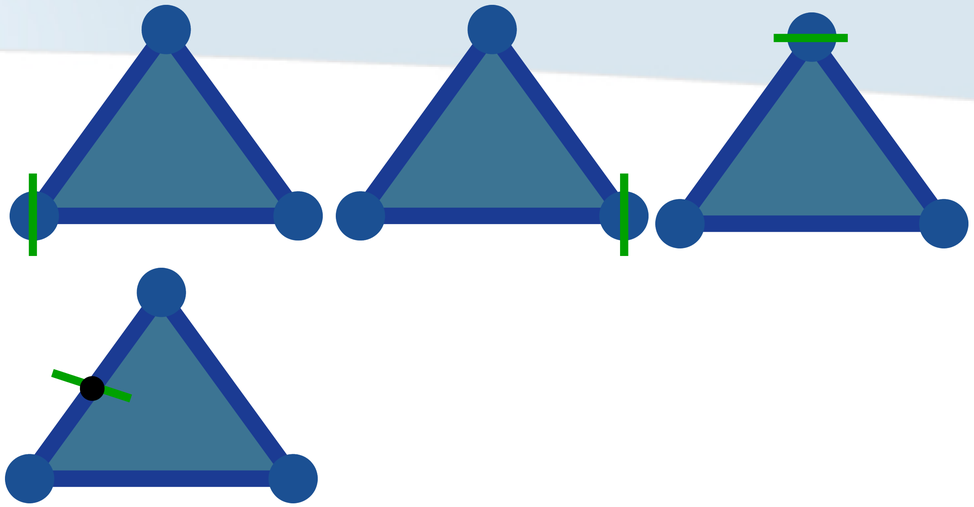

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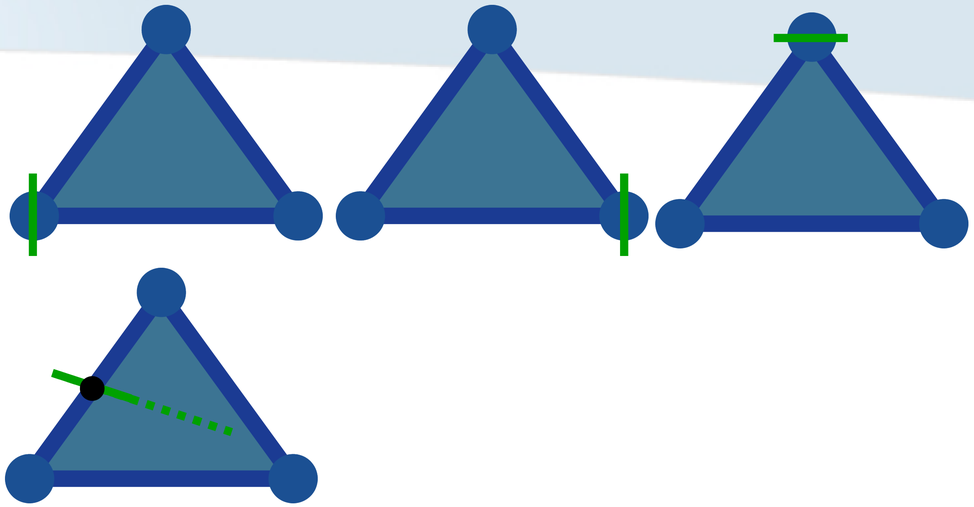

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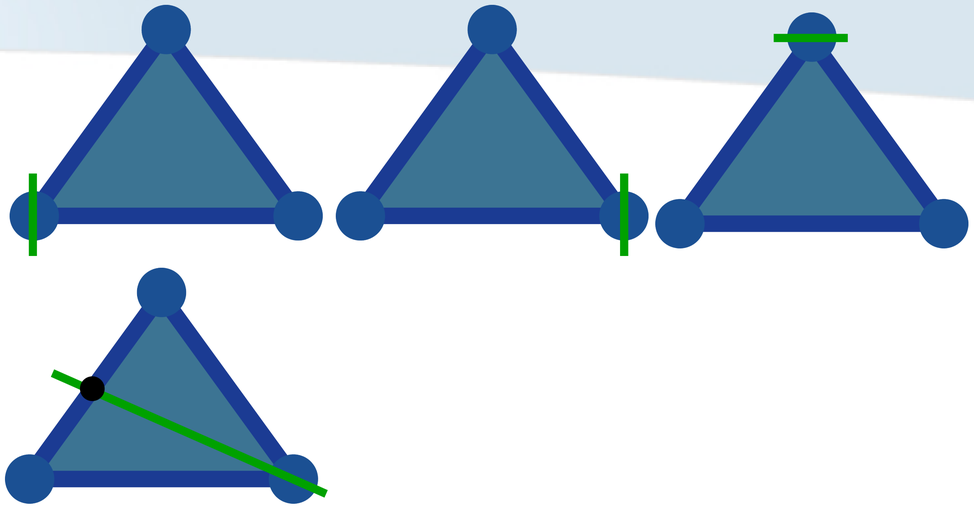

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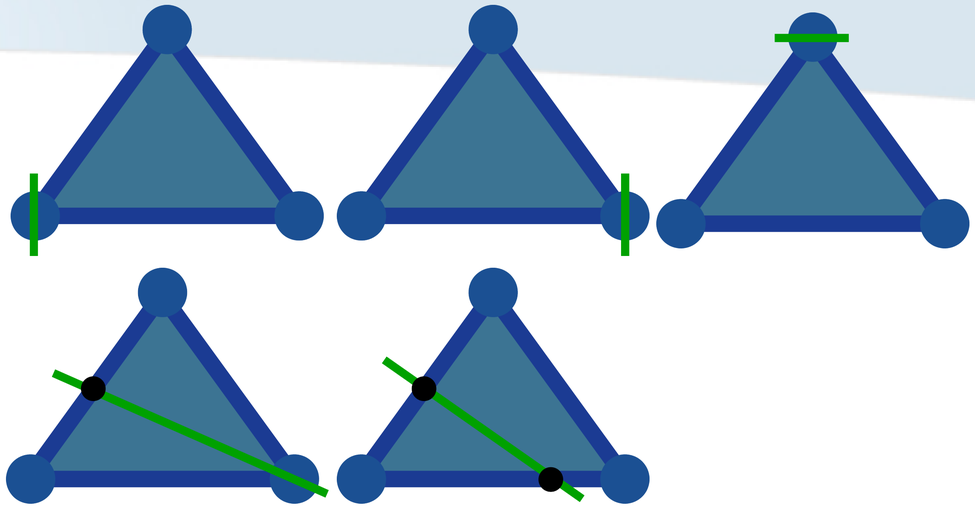

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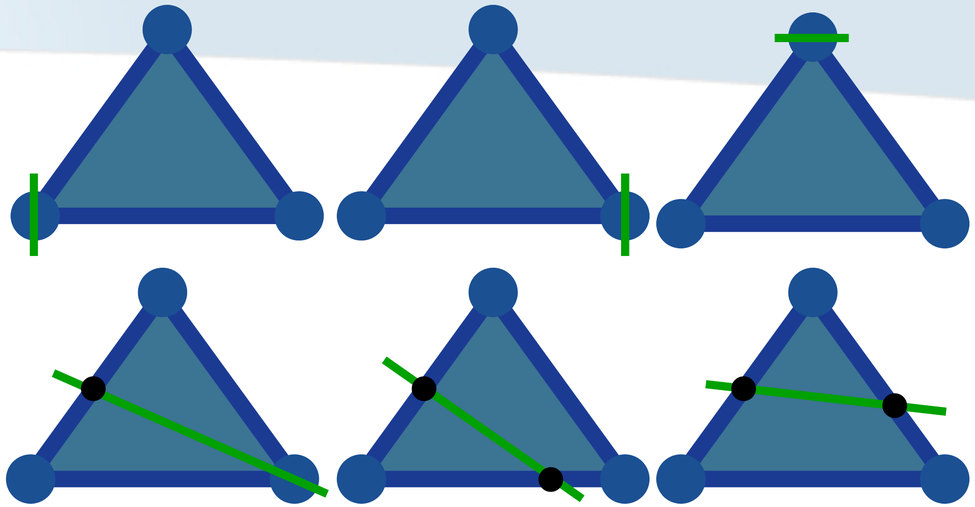

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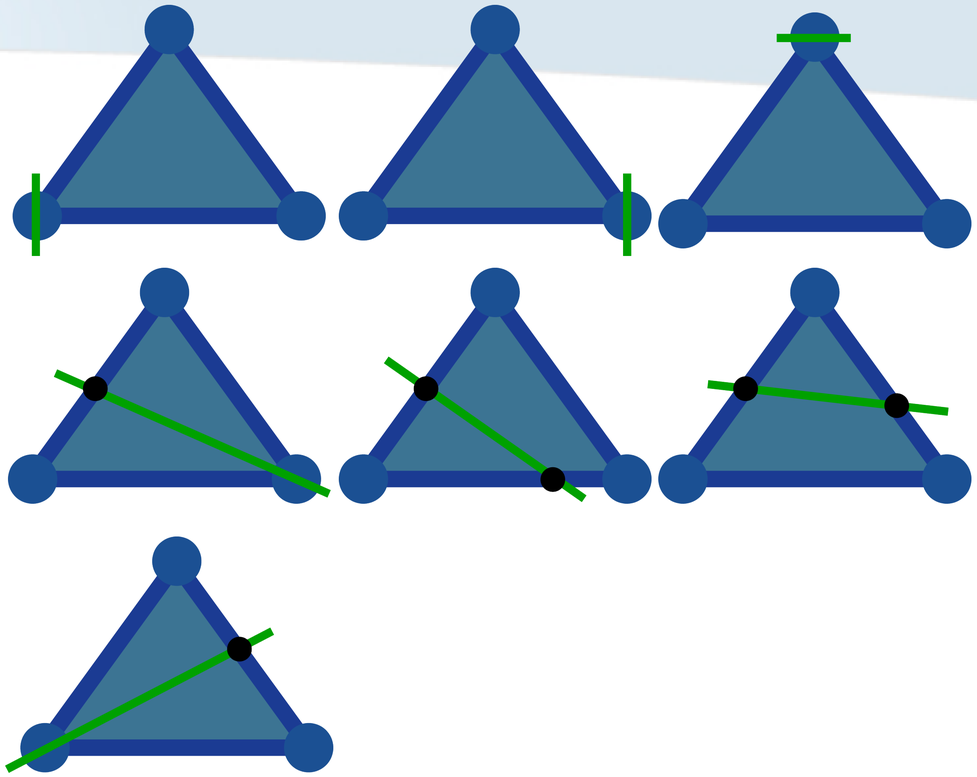

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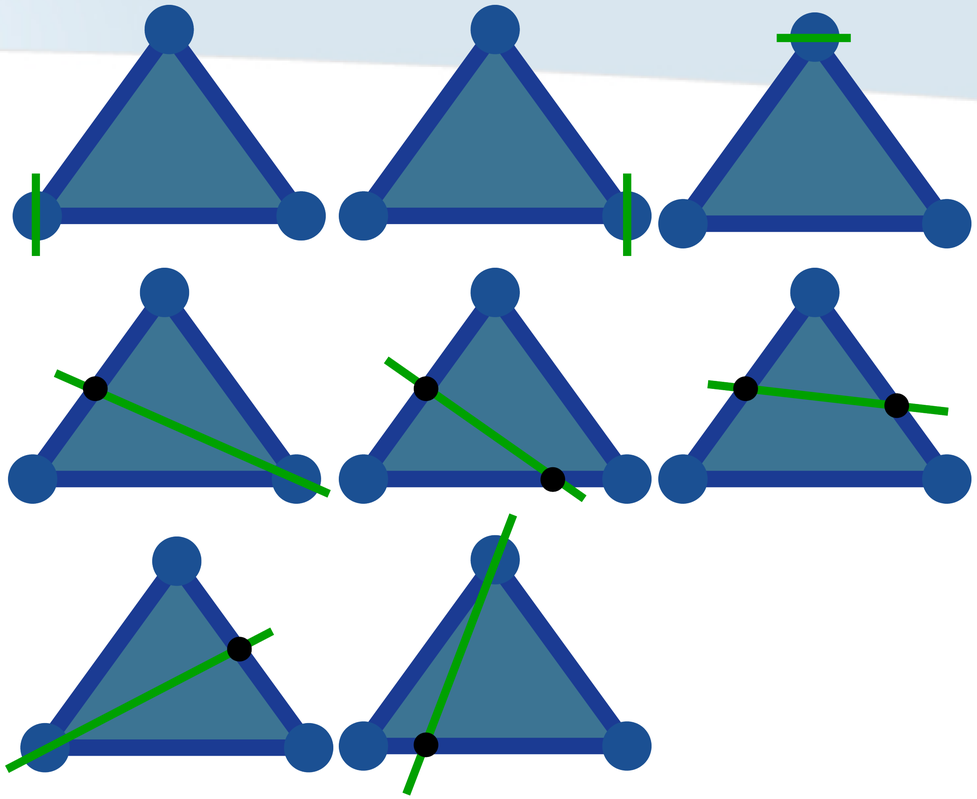

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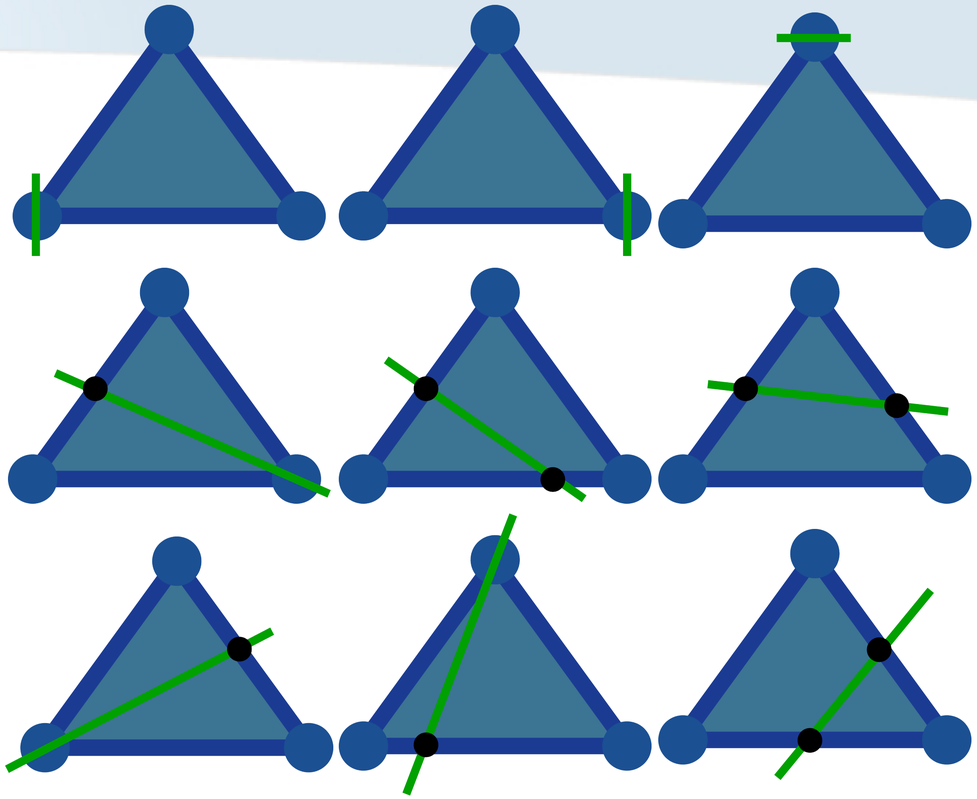

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Level set in a 3-simplex

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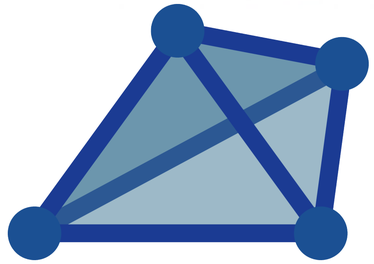

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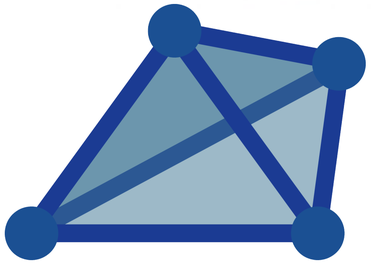

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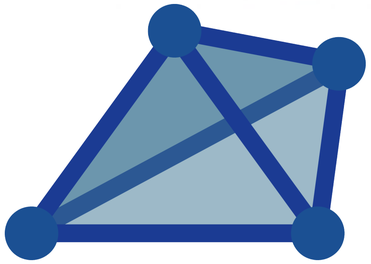
Level set in a 3-simplex



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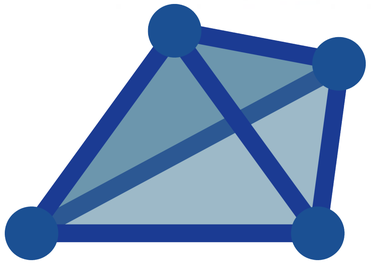

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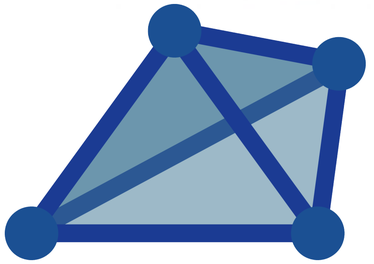

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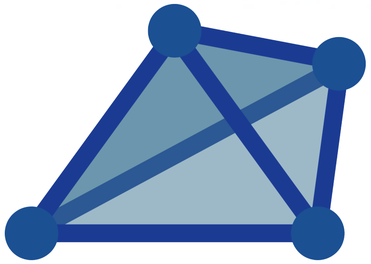

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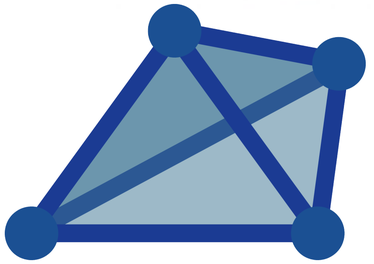

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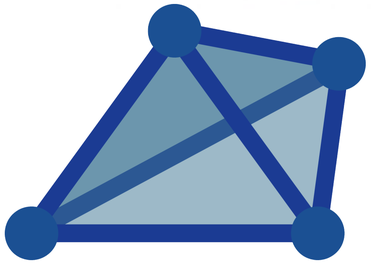
  - Many cases

Level set in a 3-simplex



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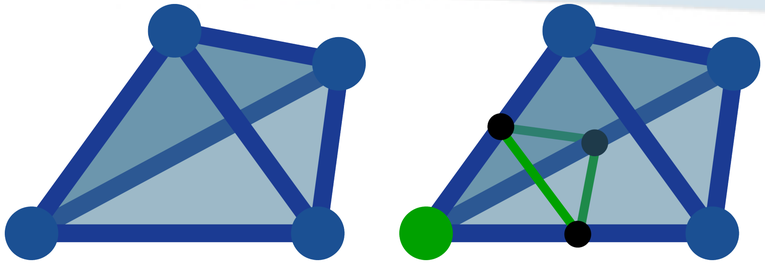

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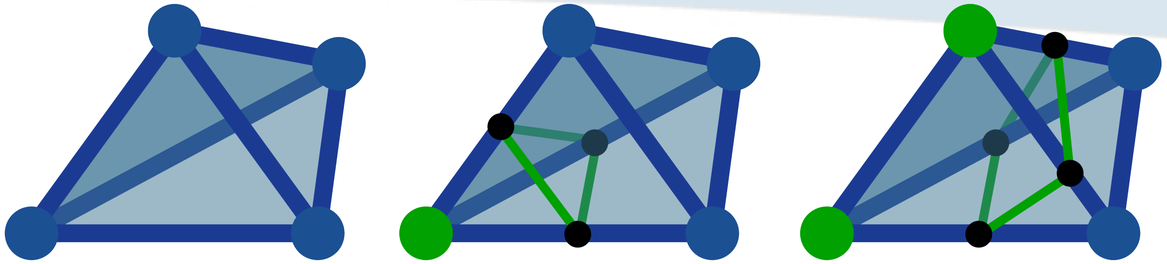

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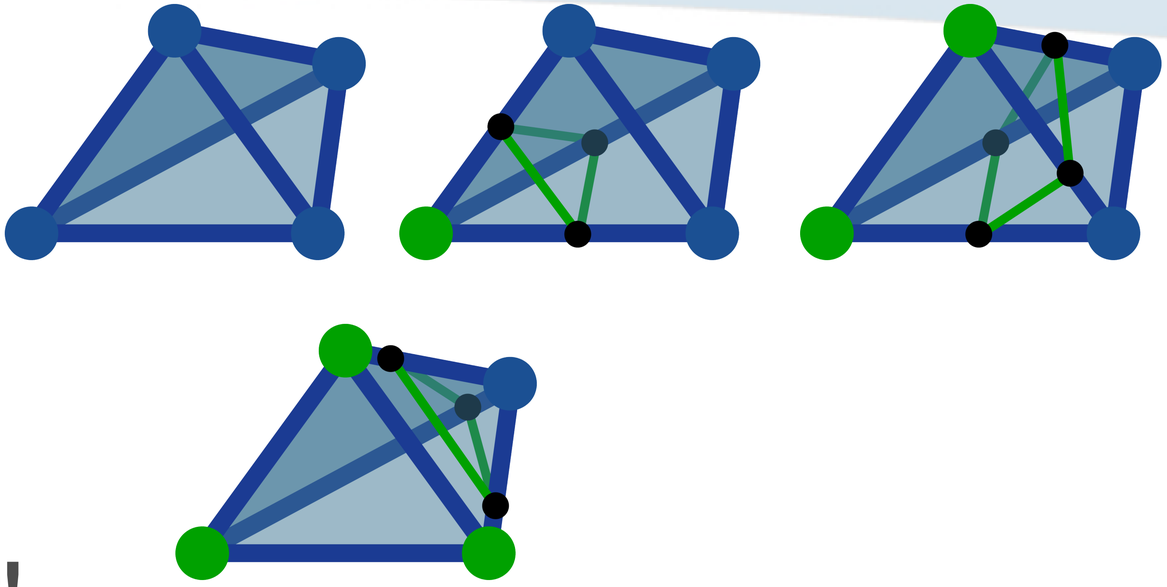

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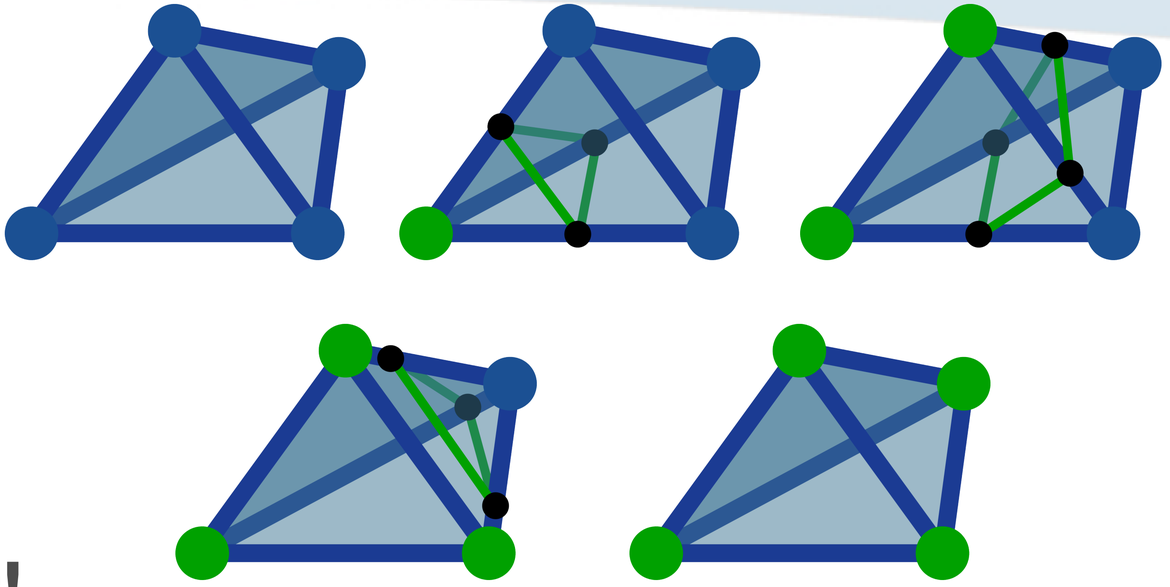

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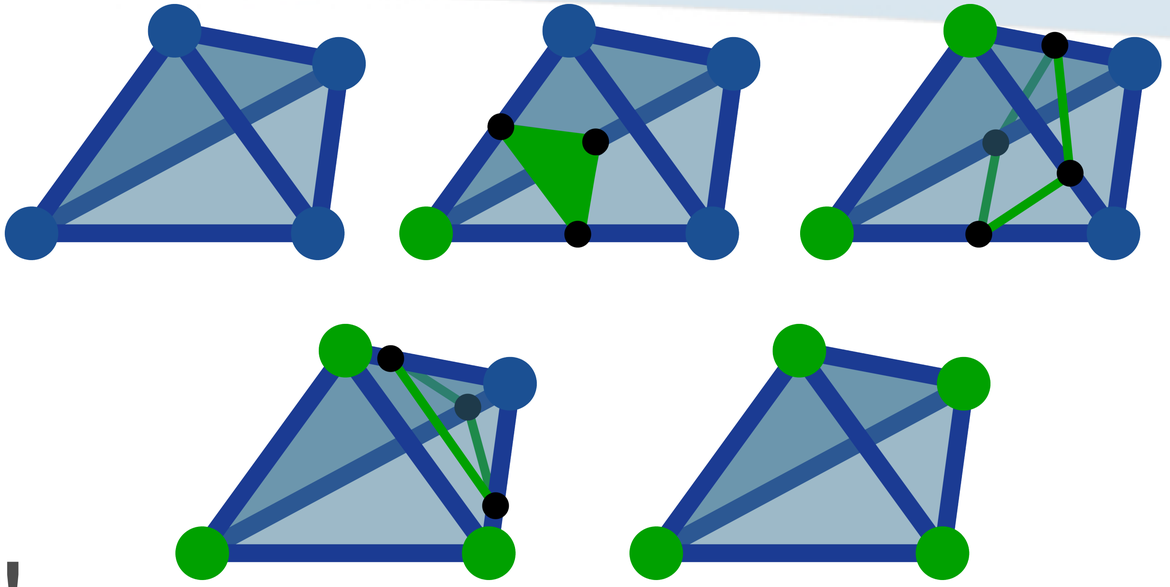

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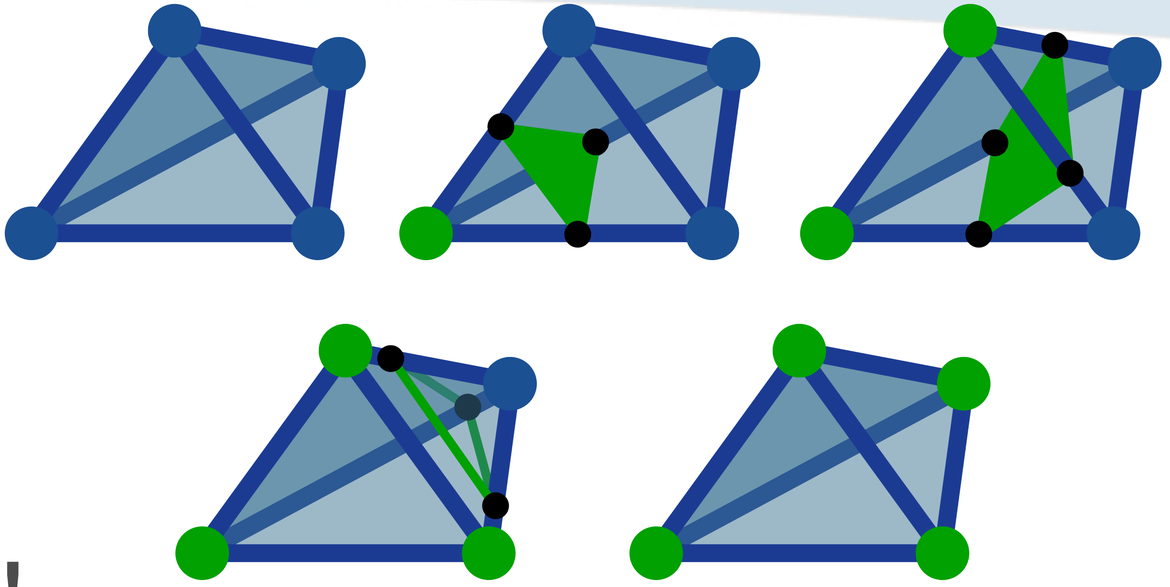

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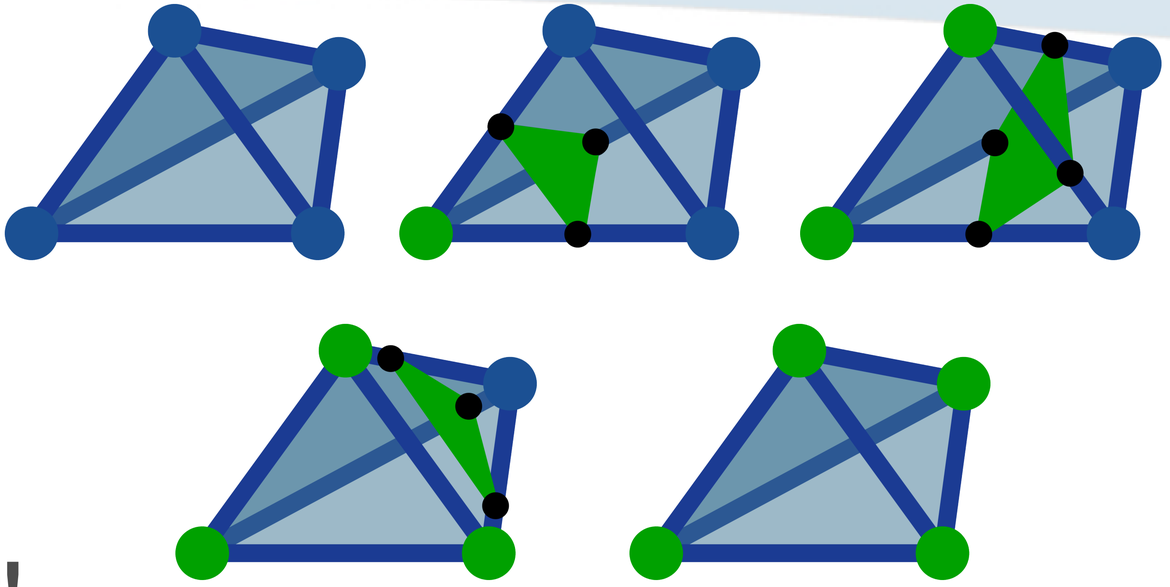

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Level set in a d-simplex

- Let $\mathcal{D}$ be a single d-simplex
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Level set in a d-simplex

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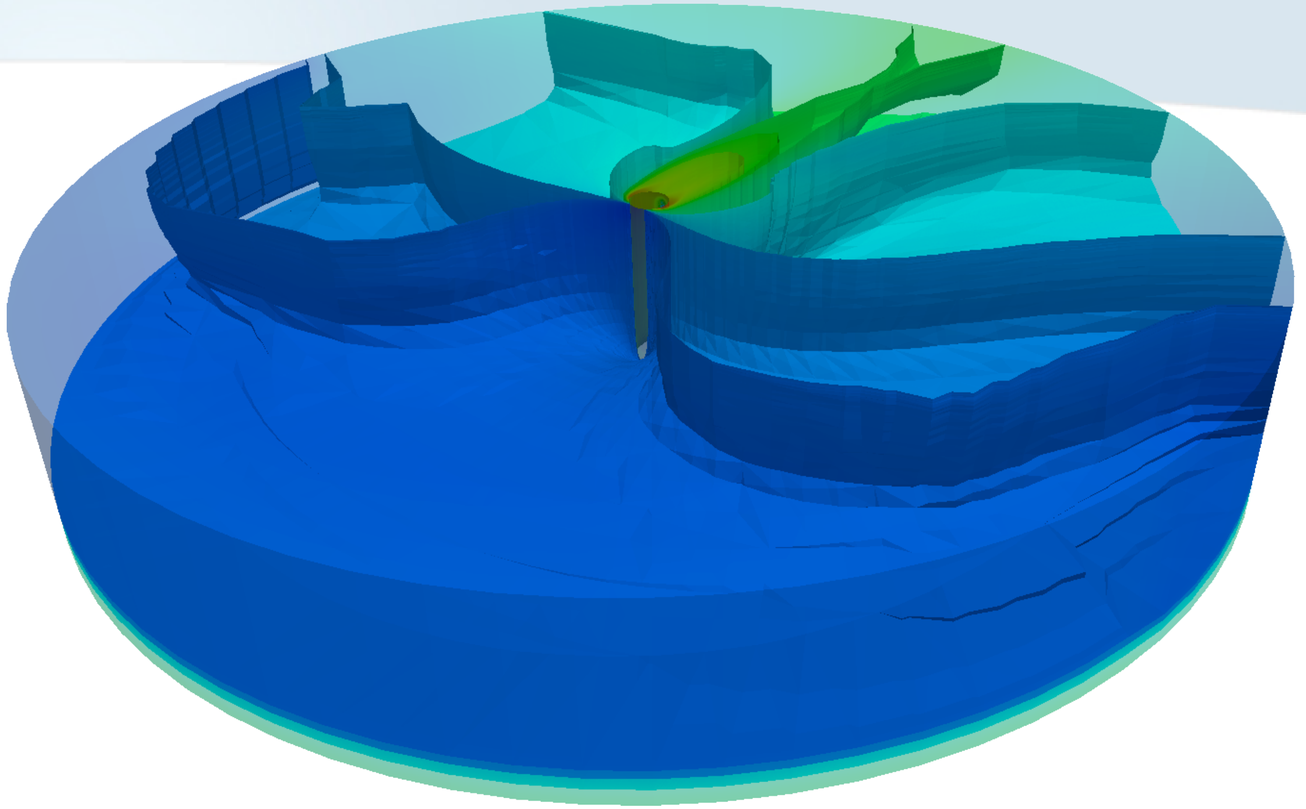
      - Level sets on (d-1)-faces: boundary of the level set
      - Recursive process

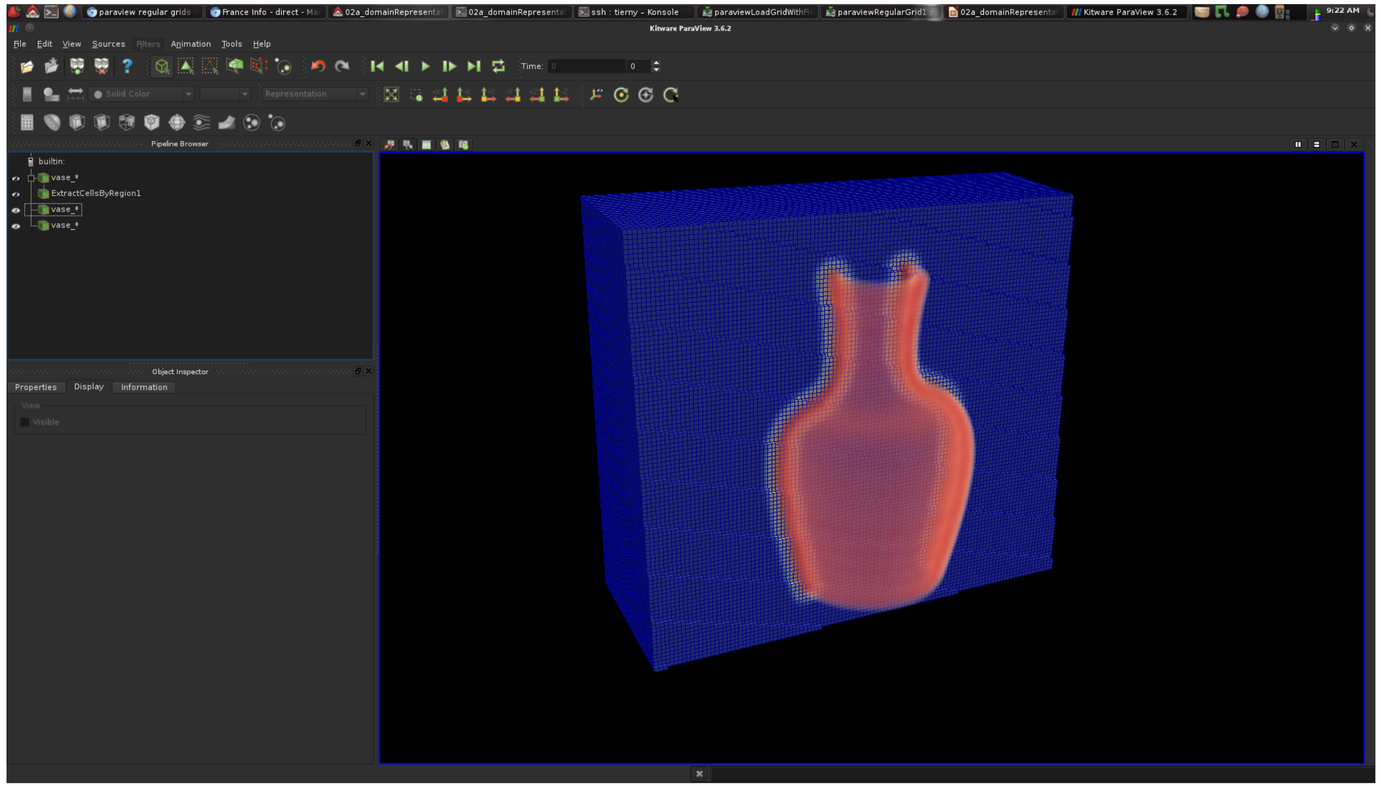
Level set extraction



- Now we know
  - How to compute a level set
  - In arbitrary dimension
  - But in only one d-simplex :(
- General algorithm
  - Flip through the list of d-simplex
  - Appy the algorithm on a per d-simplex basis
  - *“Marching Tetrahedra”*

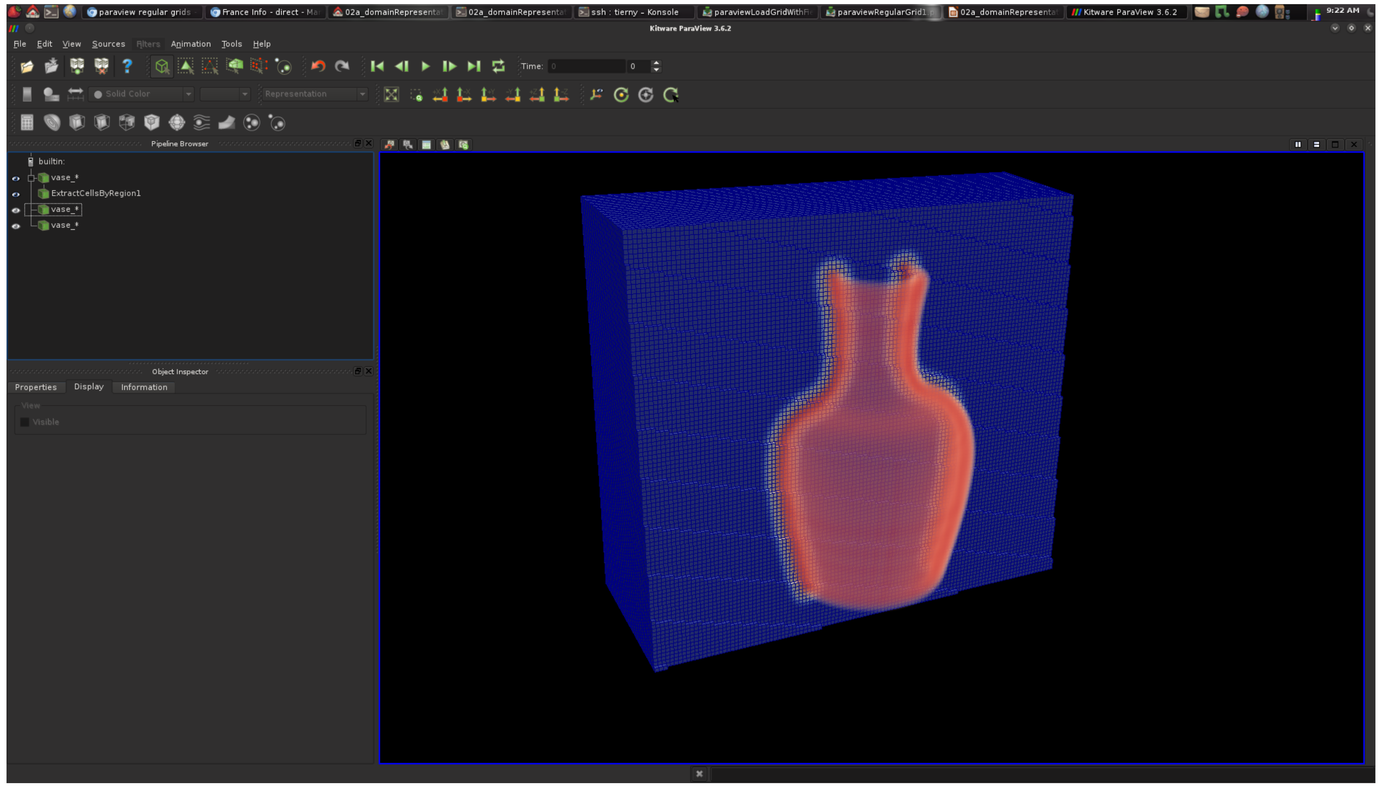
Level sets on regular grids

- What about regular grids?



Level sets on regular grids

- What about regular grids?
  - In principle
    - Same rationale



Level sets on regular grids

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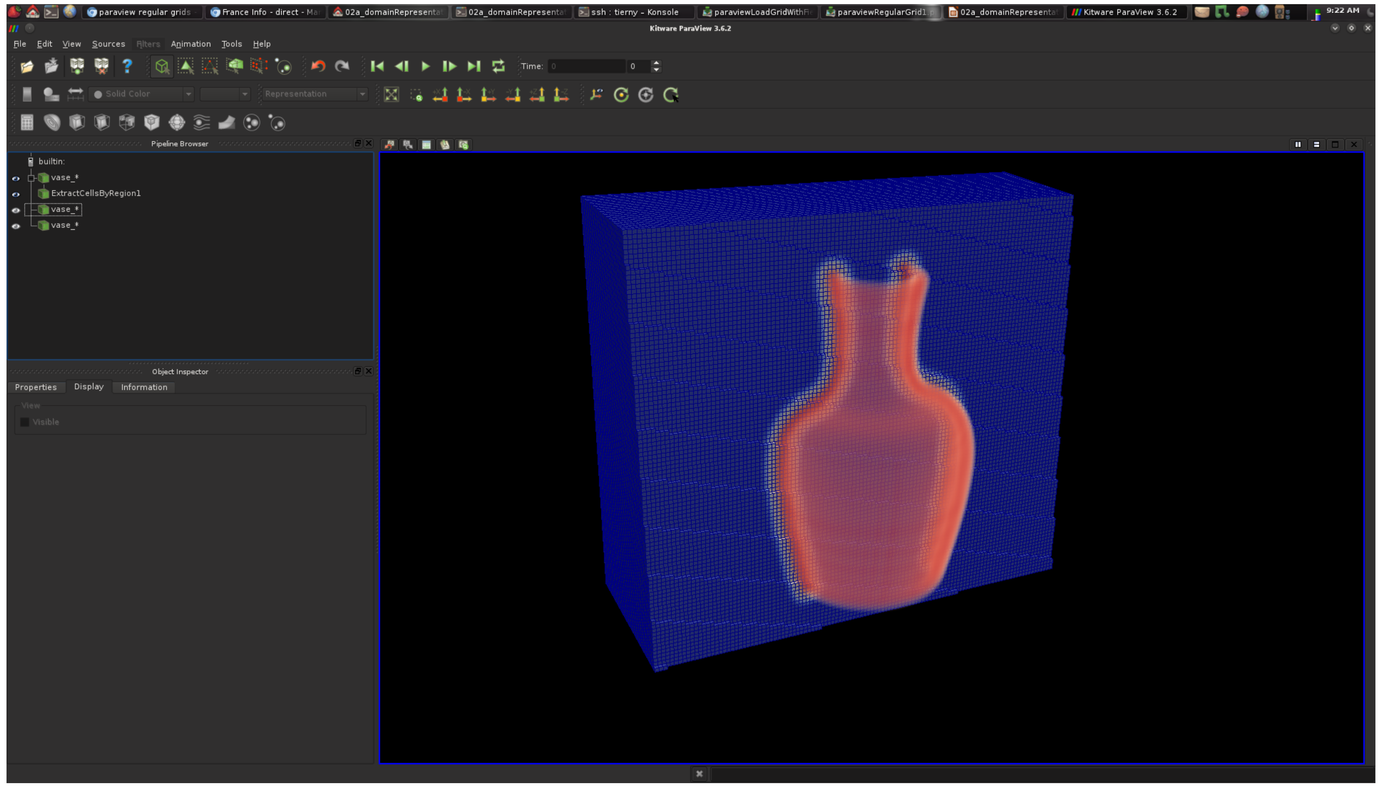
- 2D regular grids:
  - Marching squares



Level sets on regular grids

- What about regular grids?
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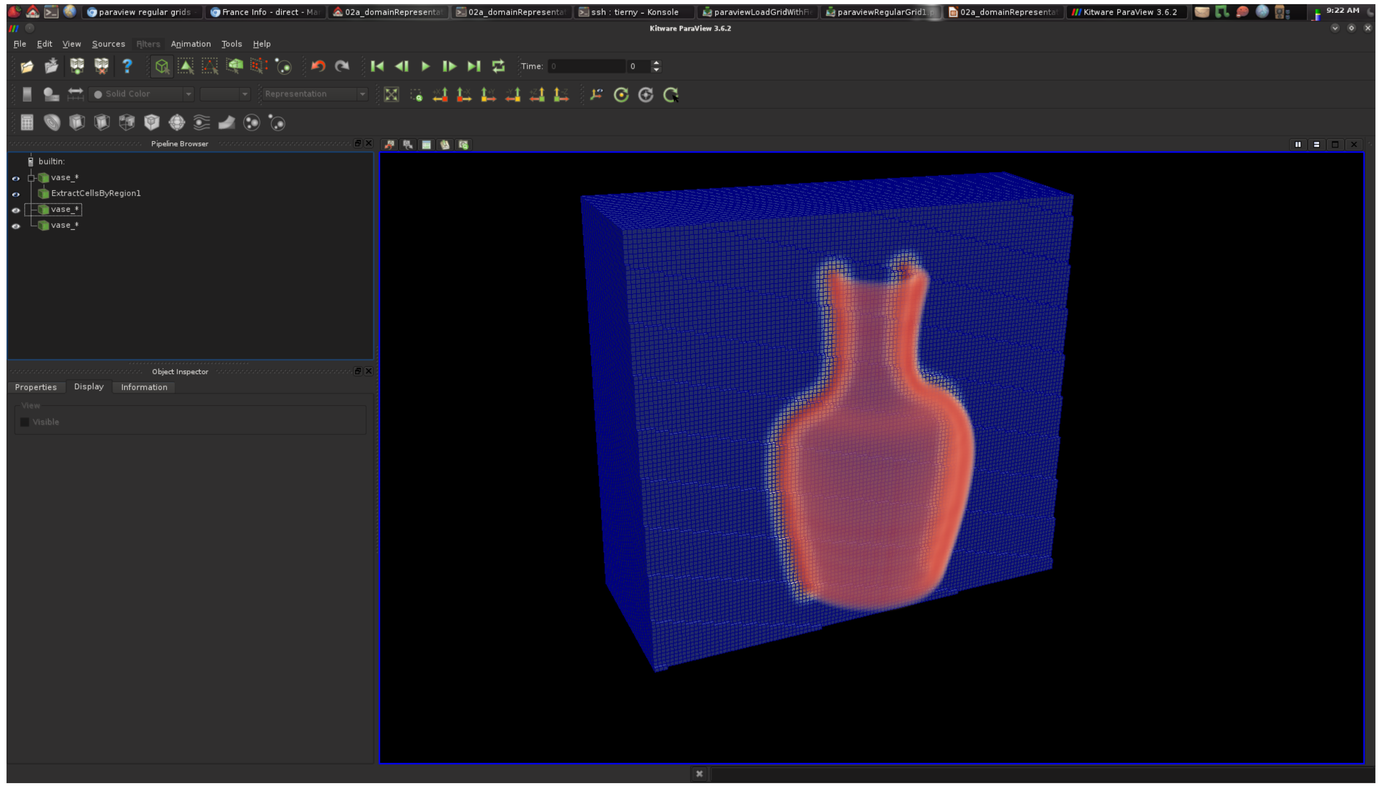
    - Same rationale

- 2D regular grids:
  - Marching squares
- 3D regular grids:
  - Marching cubes



Marching squares

- Let $\mathcal{D}$ be a 2-regular grid

[Wikipedia]

Marching squares

- Let $\mathcal{D}$ be a 2-regular grid
  - With bilinear interpolant

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Marching squares

- Let $\mathcal{D}$ be a 2-regular grid
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- Level set extraction
  - Loop over the unit cells

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  - Cases on a 2D unit cell

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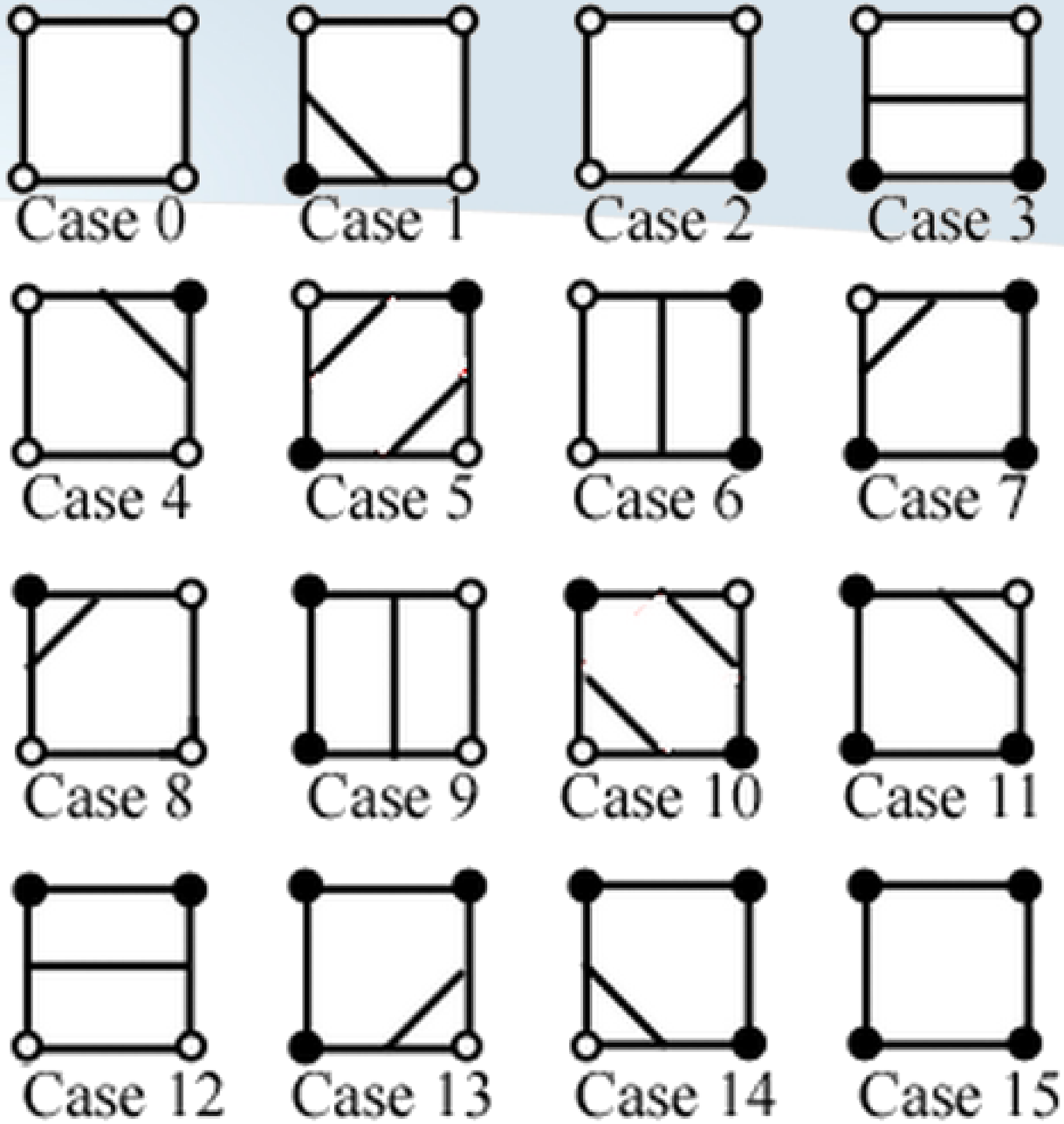
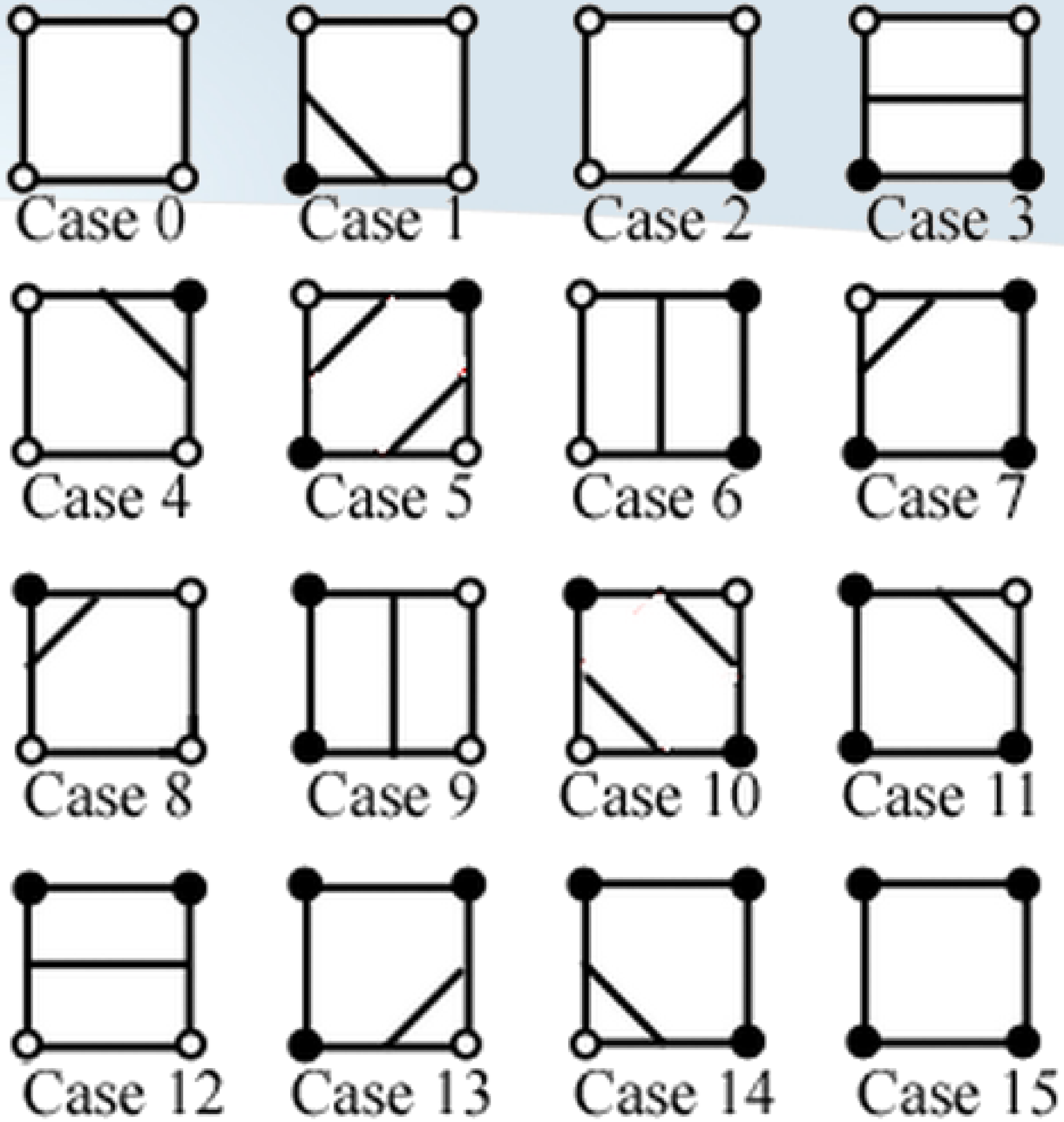


[Wikipedia]

Marching squares

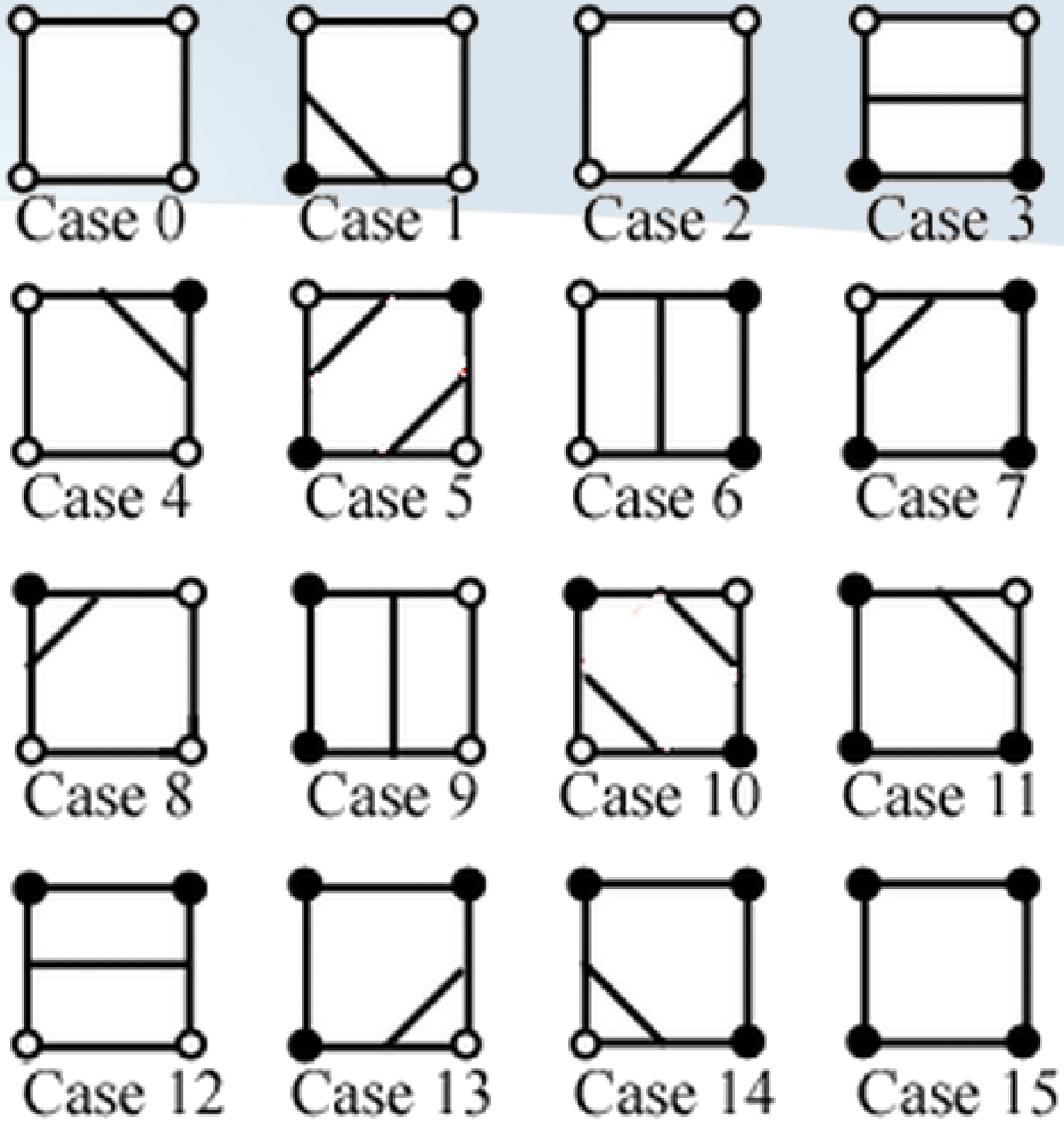
- Let $\mathcal{D}$ be a 2-regular grid
  - With bilinear interpolant
- Level set extraction
  - Loop over the unit cells
  - Cases on a 2D unit cell
    - Much more than in a triangle



[Wikipedia]

Marching squares

- Let $\mathcal{D}$ be a 2-regular grid
  - With bilinear interpolant
- Level set extraction
  - Loop over the unit cells
  - Cases on a 2D unit cell
    - Much more than in a triangle
    - More than in a tet!



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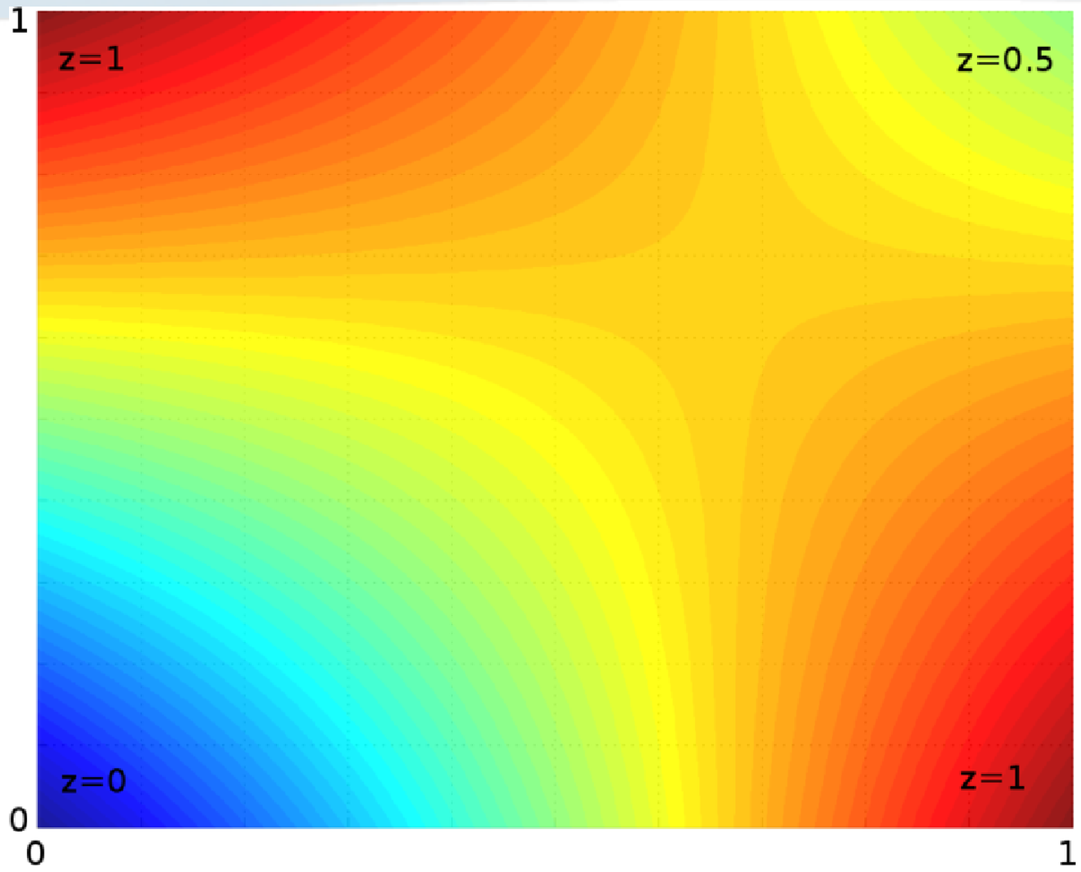
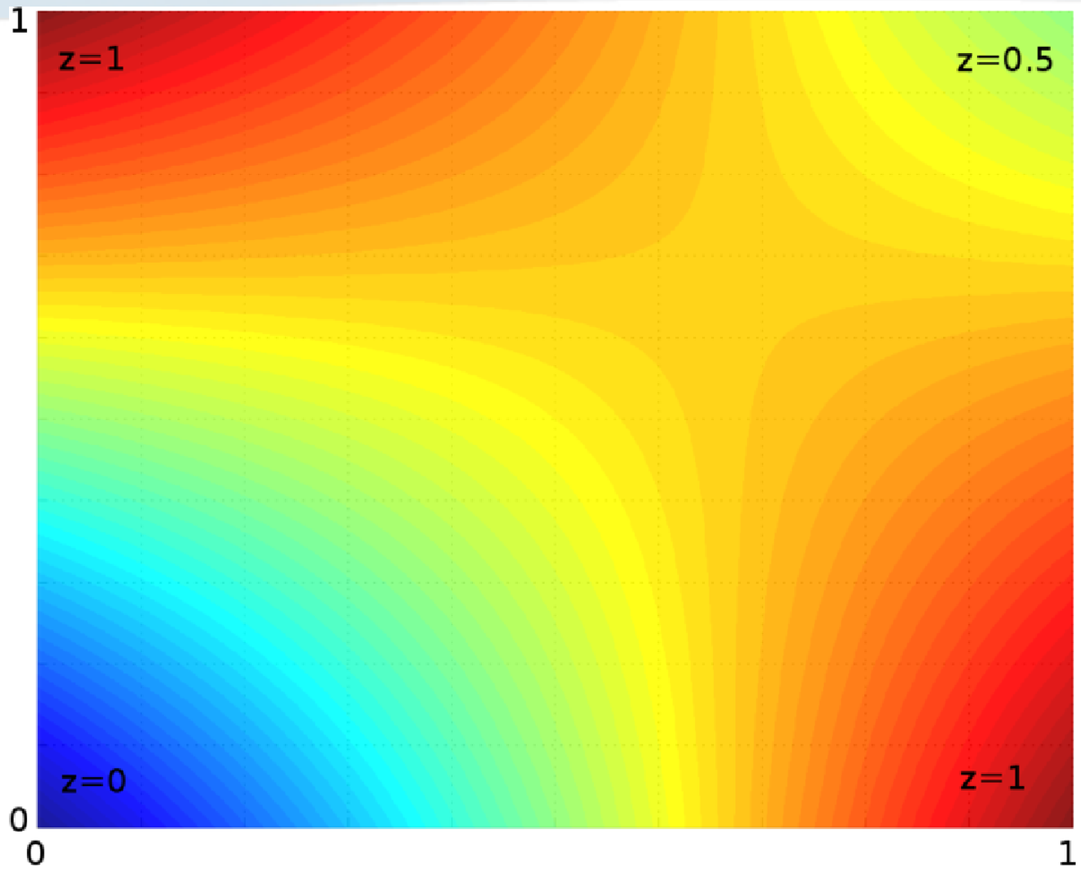
Issues of marching squares



Issues of marching squares

$f^{-1}(0.95)$



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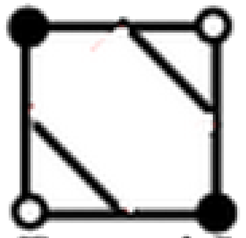
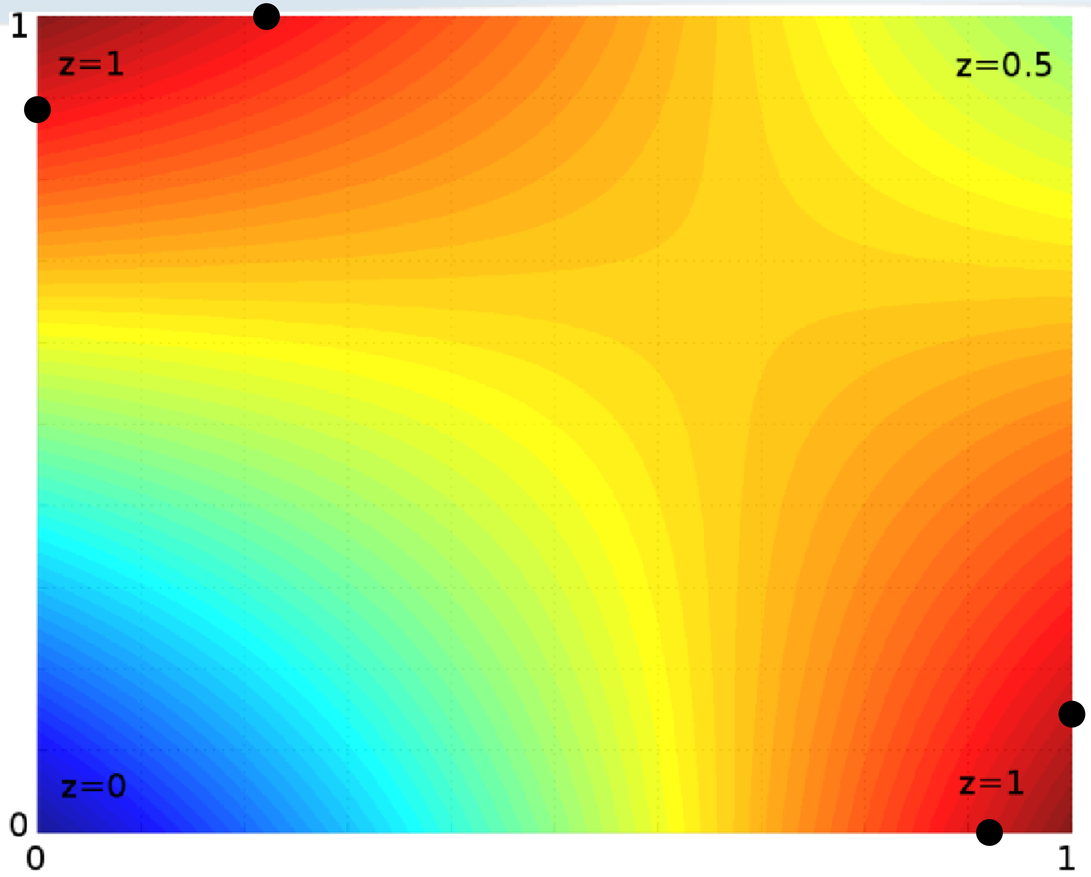


Case 10



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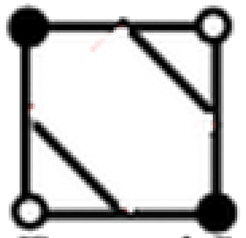

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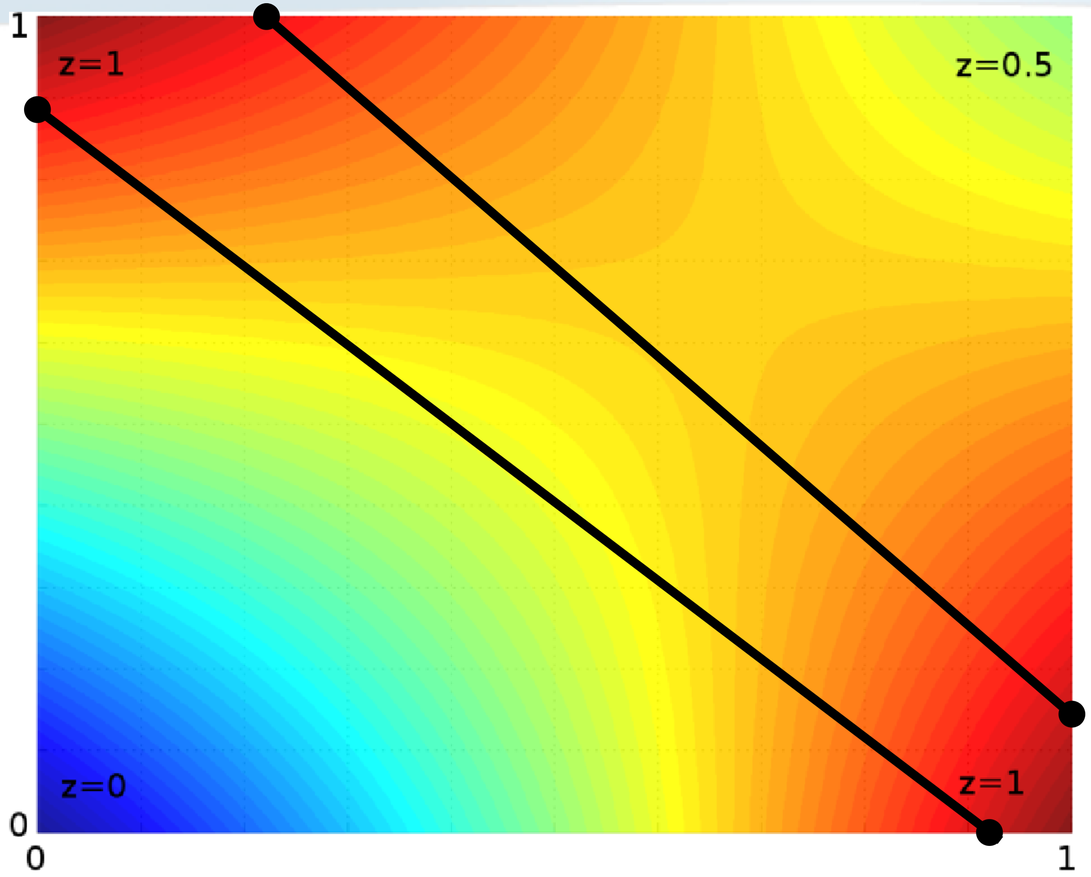

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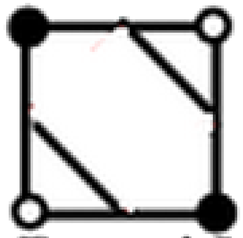

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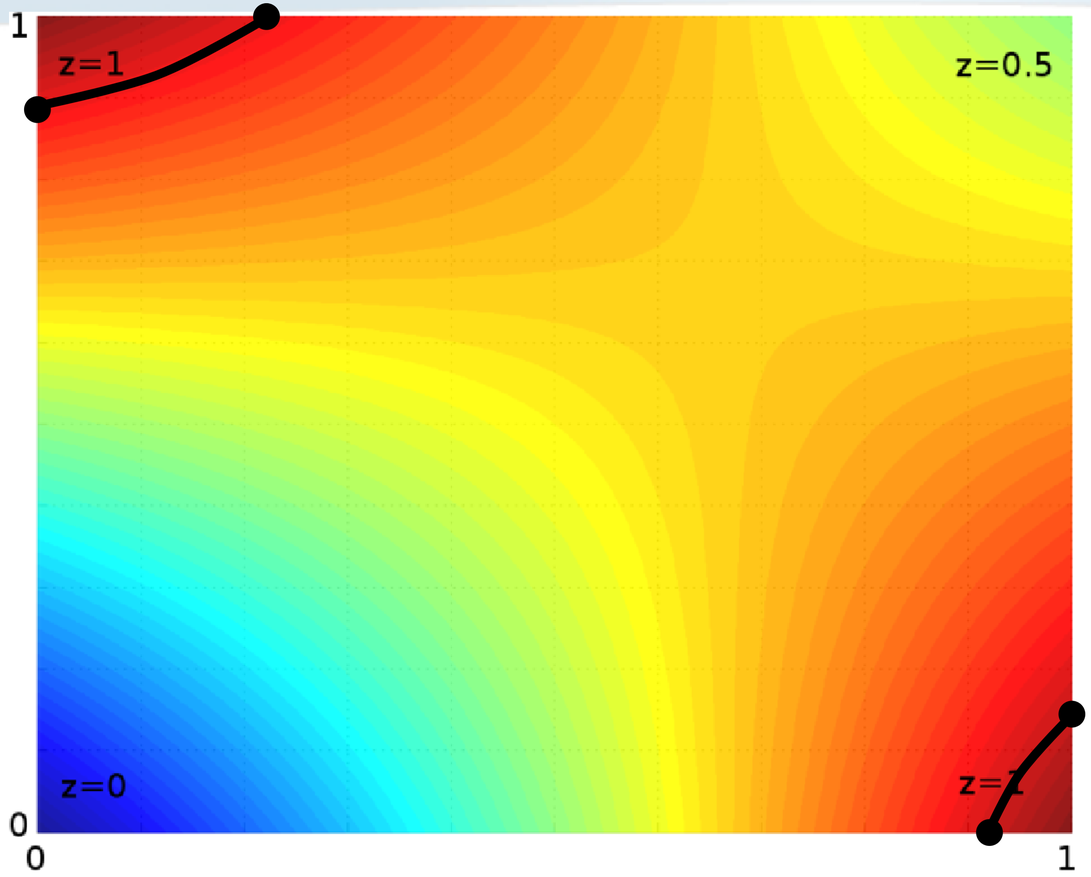

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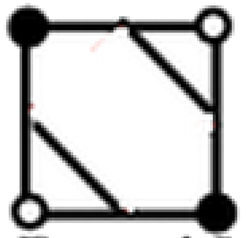

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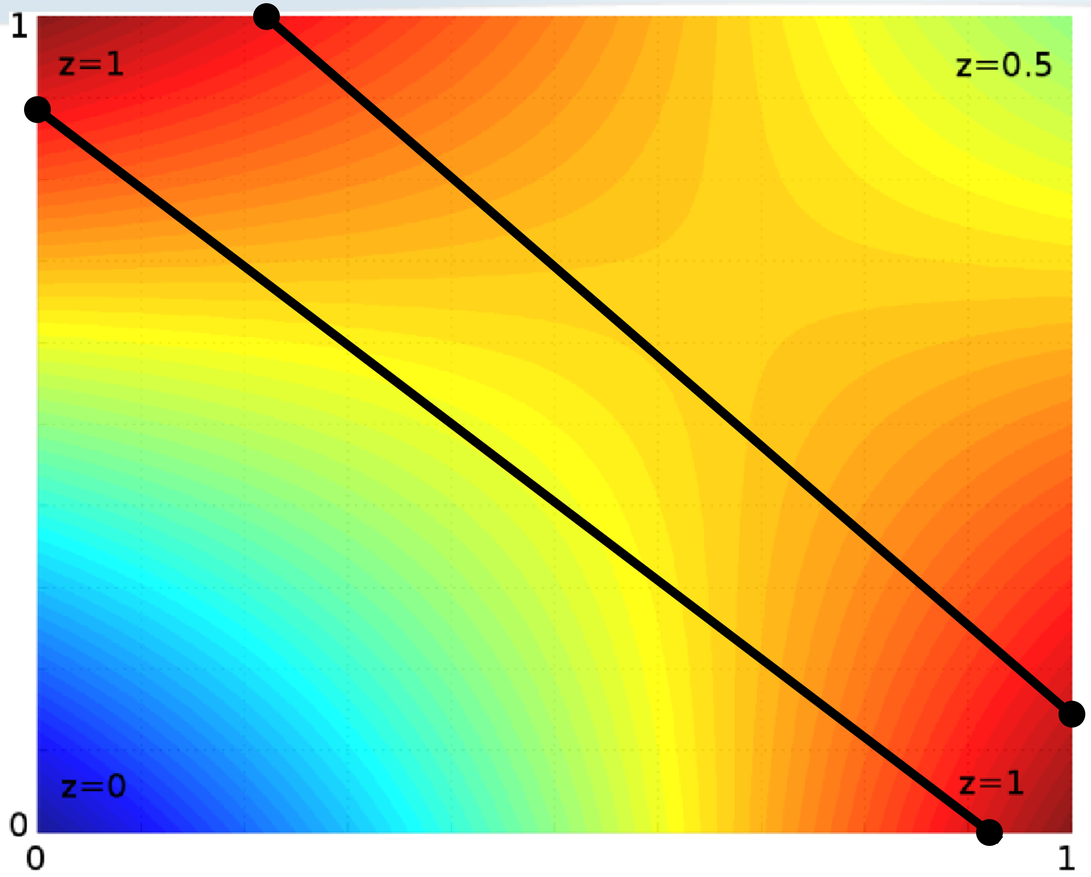

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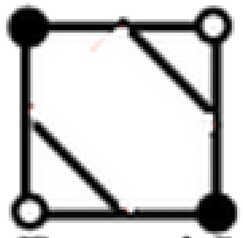

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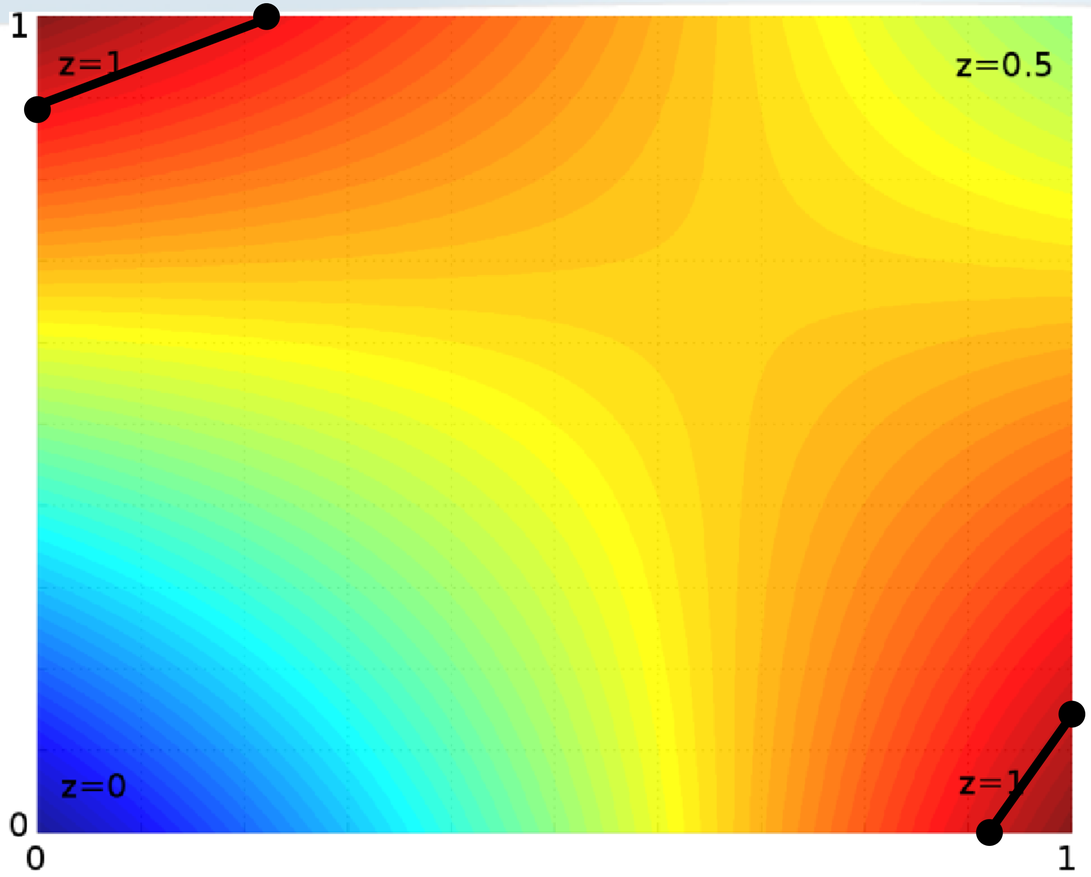

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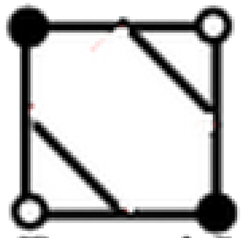

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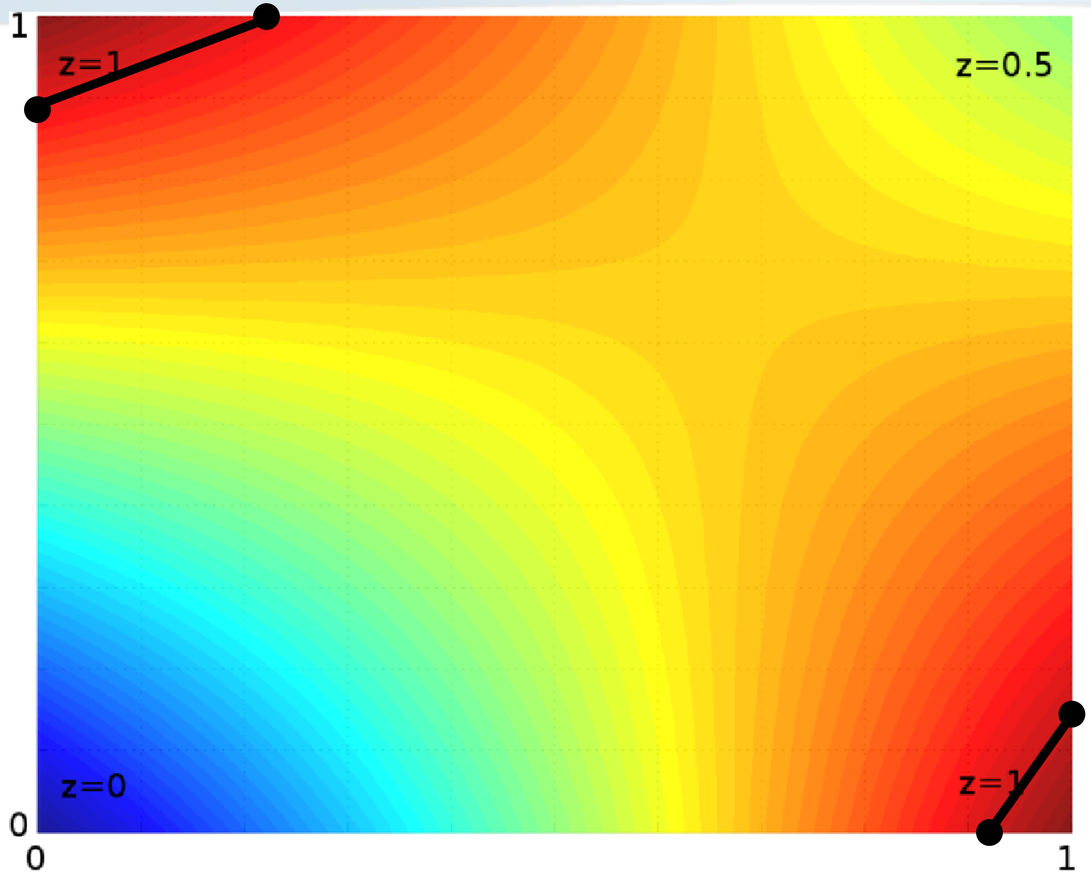

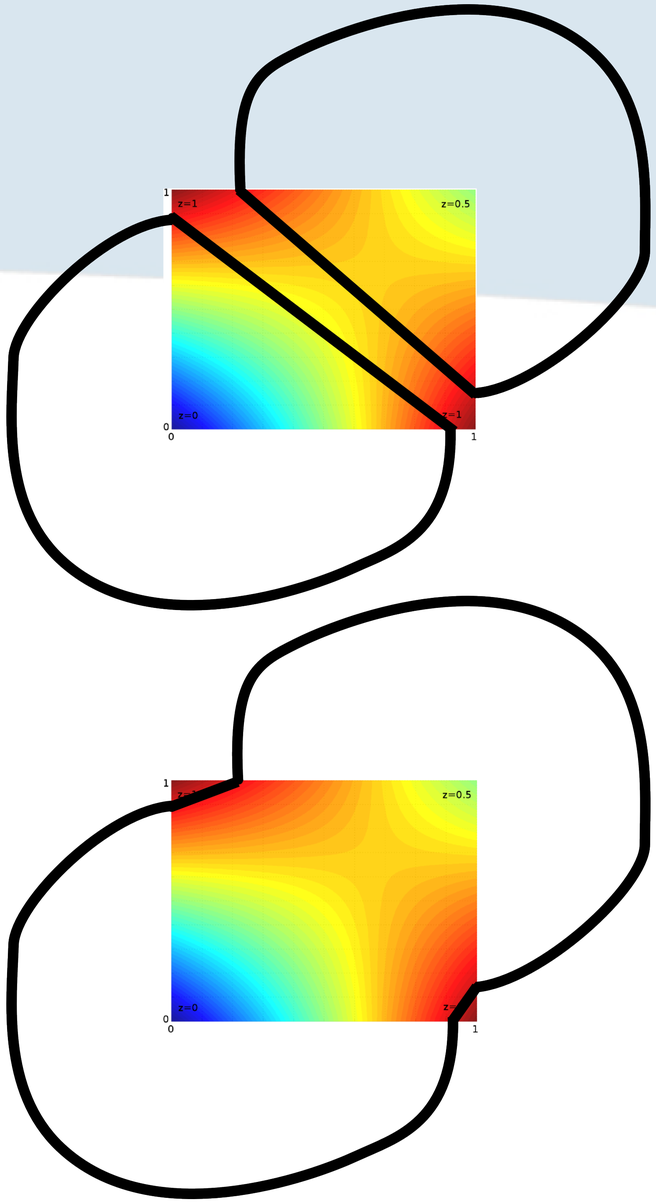

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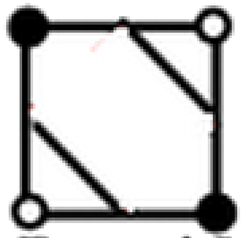
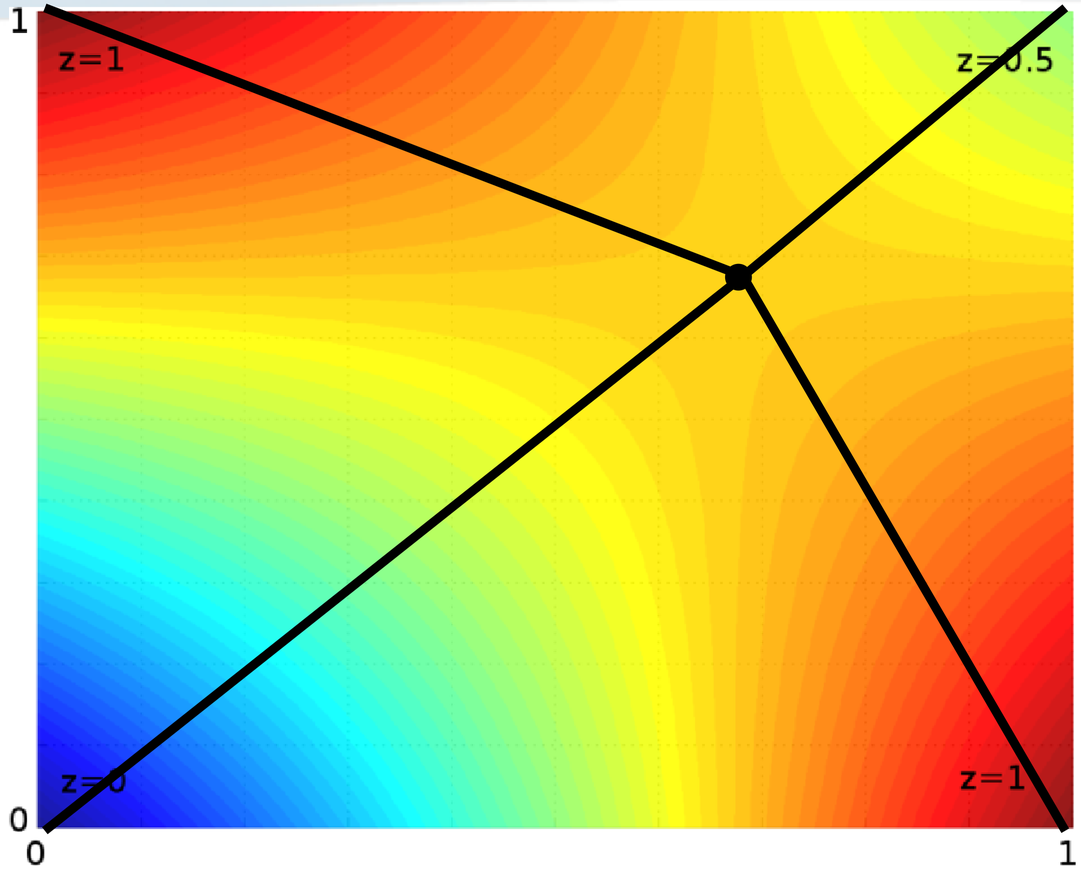
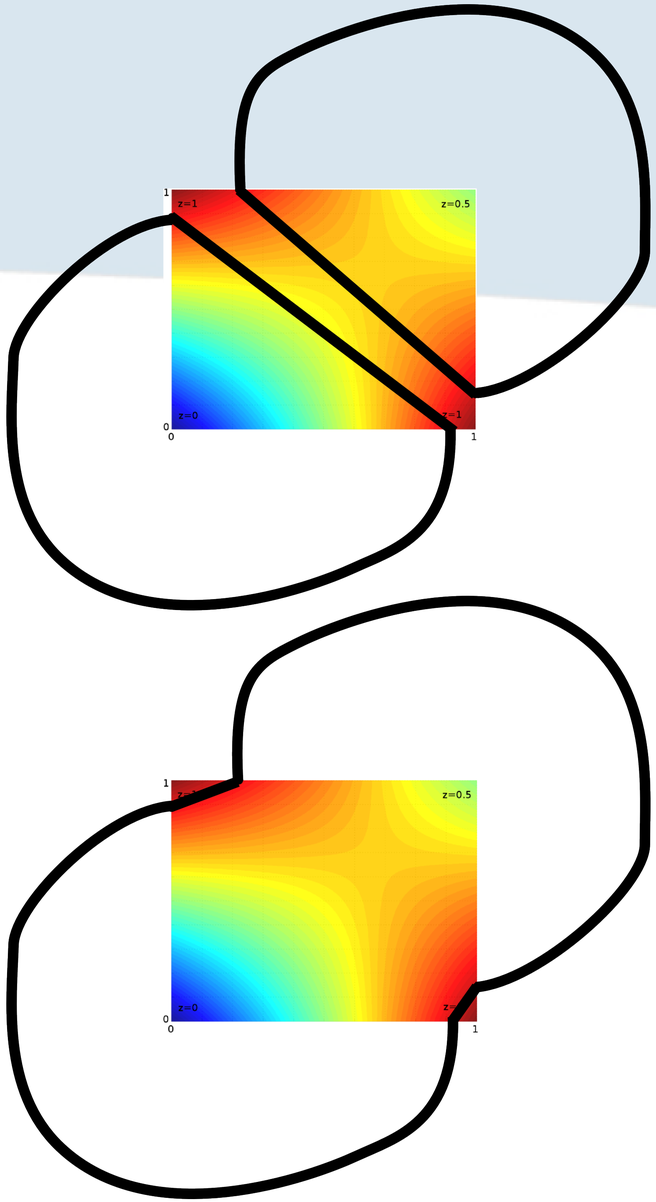


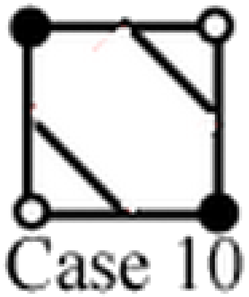
Case 10

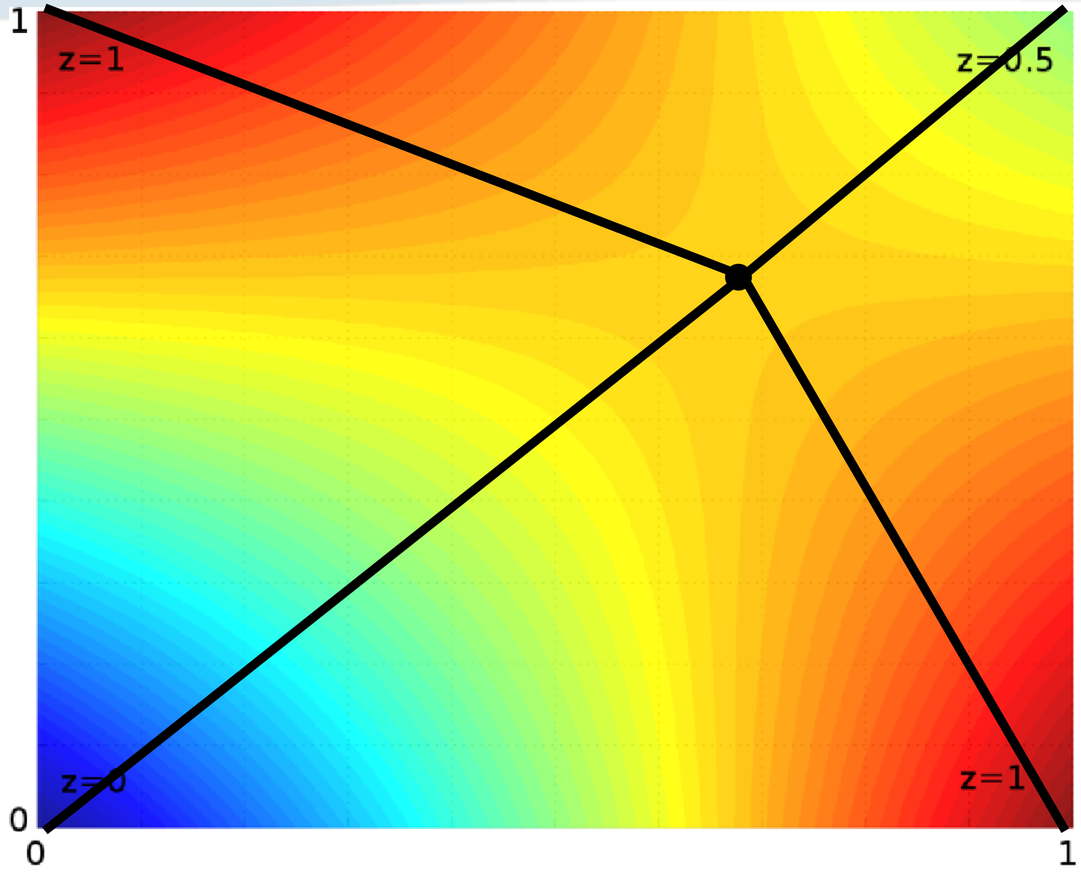


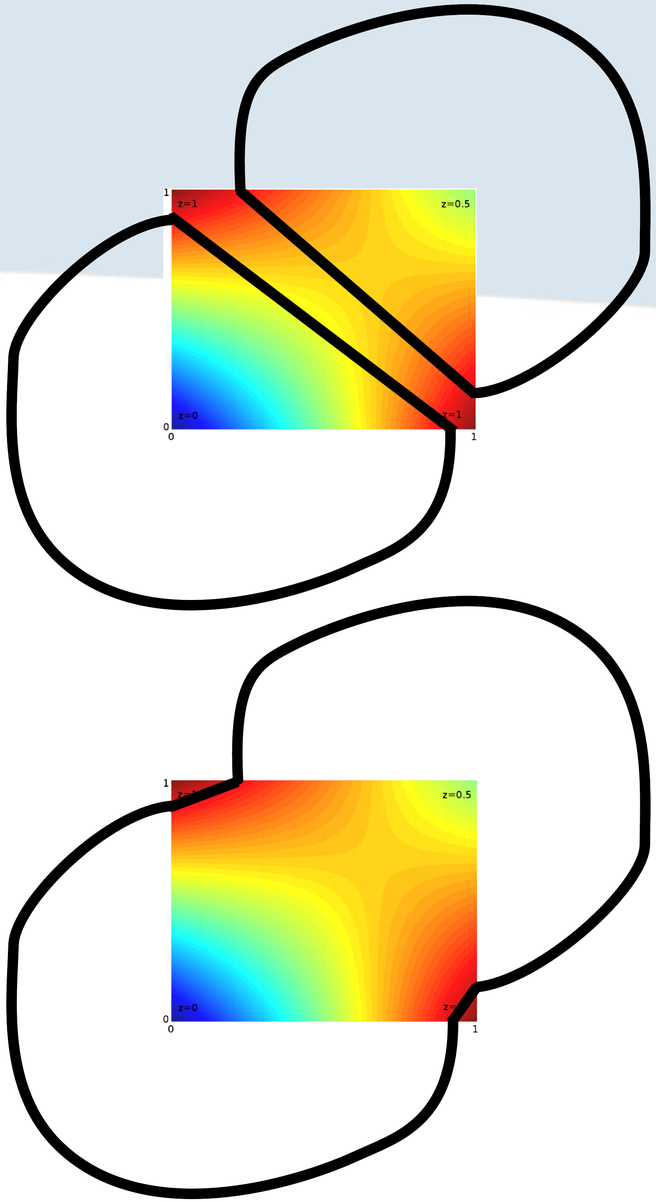


Issues of marching squares

$f^{-1}(0.95)$







- Geometrically inaccurate (lines)

Issues of marching squares

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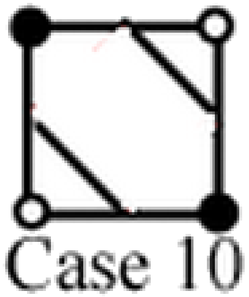
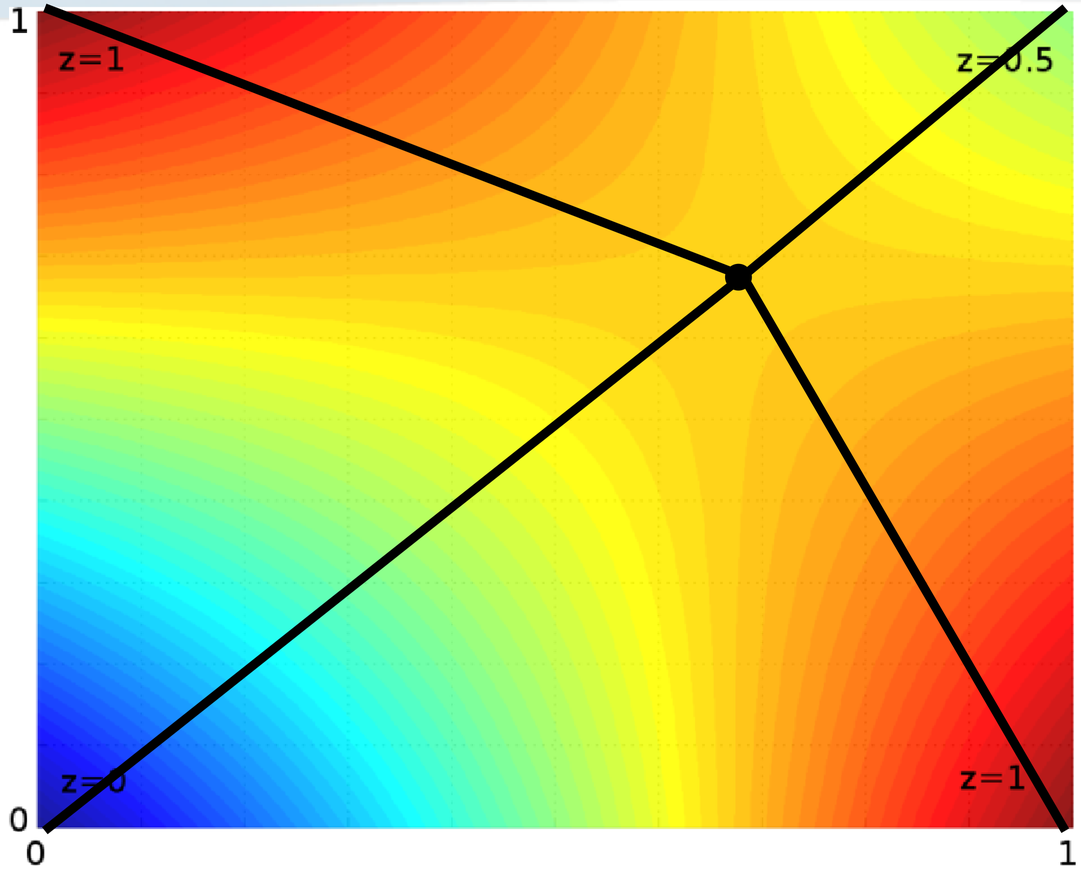
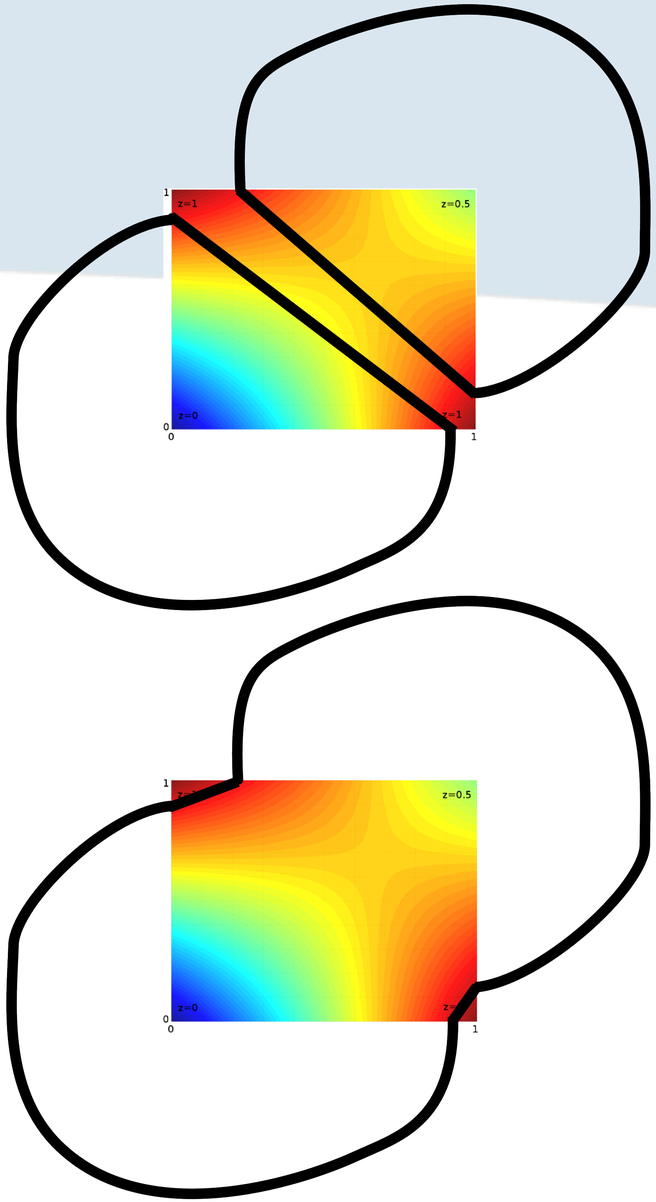


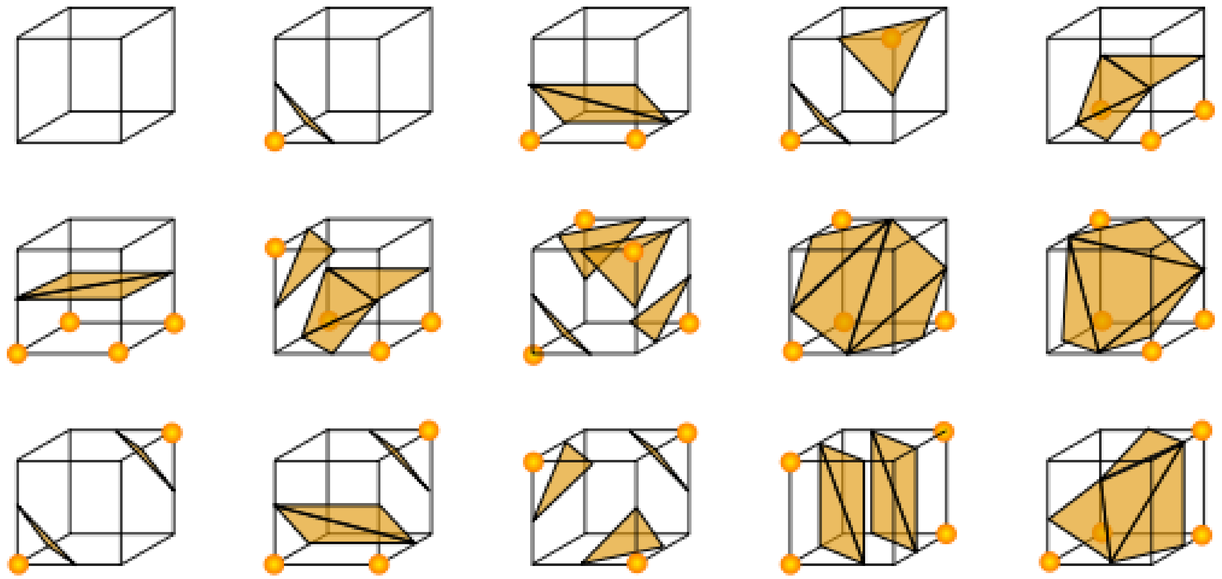




- Geometrically inaccurate (lines)
- Topologically inconsistent (**numerical** estimation of the saddle)

Marching cubes

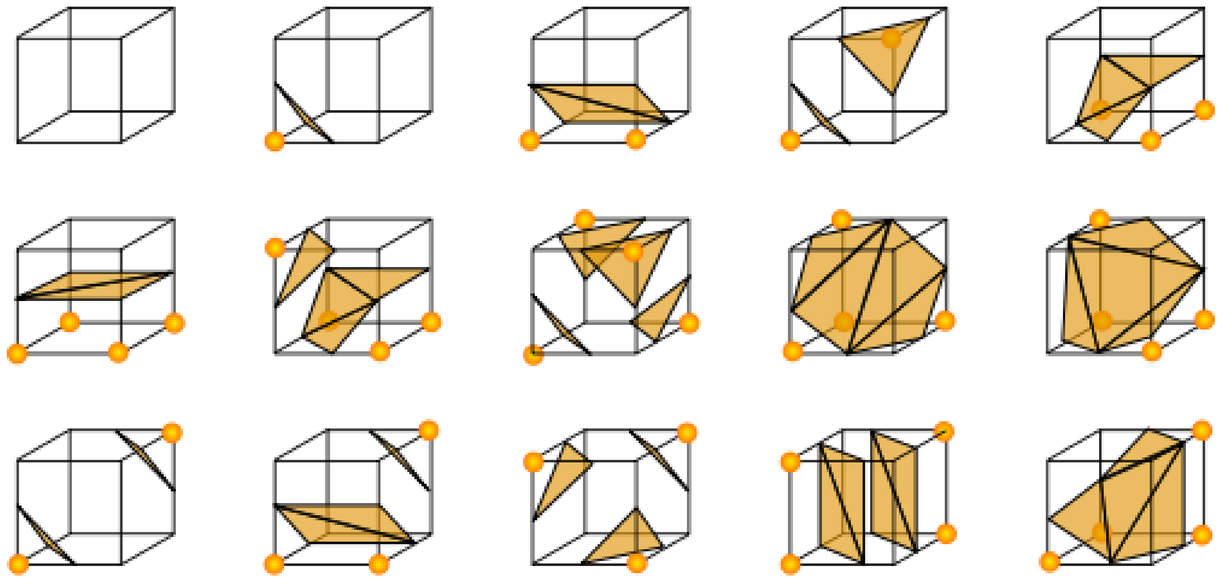
- Let $\mathcal{D}$ be a 3-regular grid
  - With trilinear interpolant



[Wikipedia]

Marching cubes

- Let $\mathcal{D}$ be a 3-regular grid
  - With trilinear interpolant
- Level set extraction
  - Loop over the unit cells



[Wikipedia]

Marching cubes

- Let $\mathcal{D}$ be a 3-regular grid
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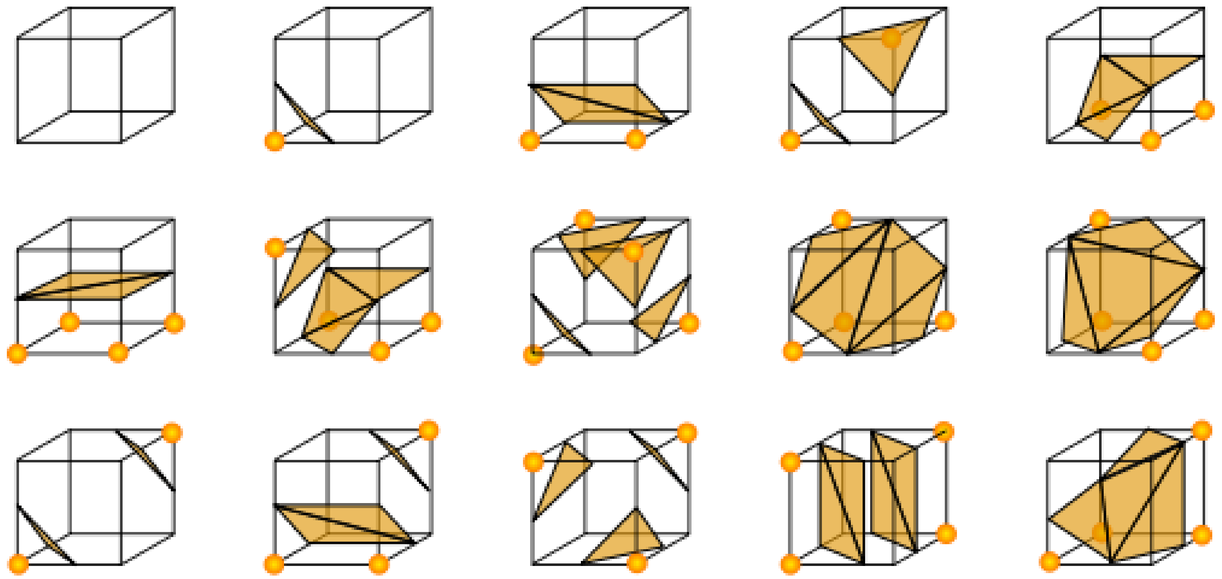
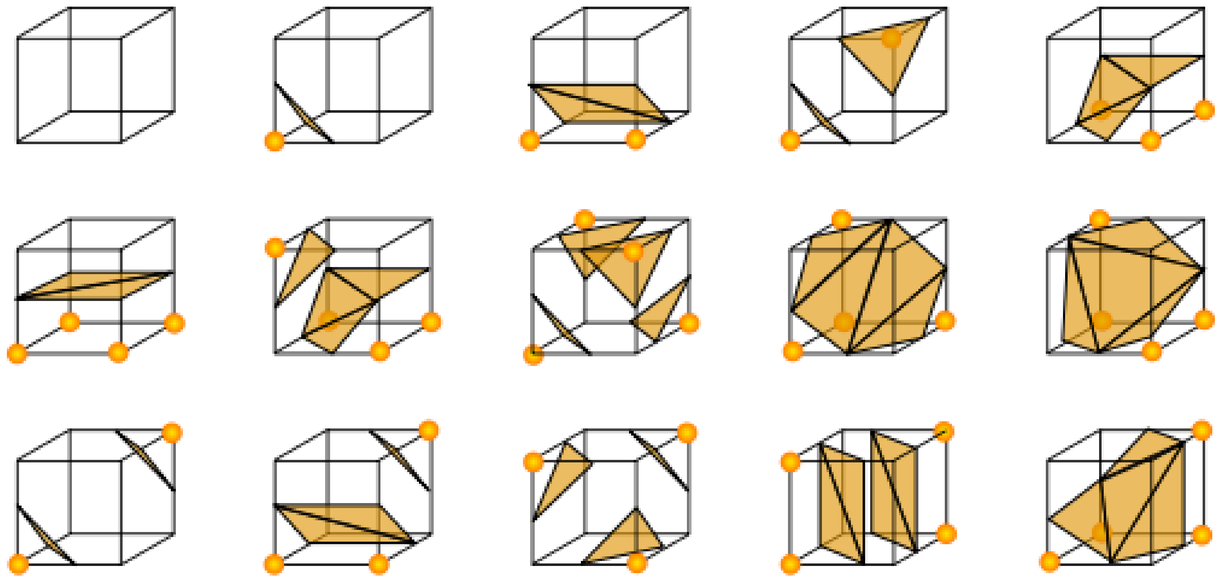
  - Cases on a 3D unit cell



[Wikipedia]

Marching cubes

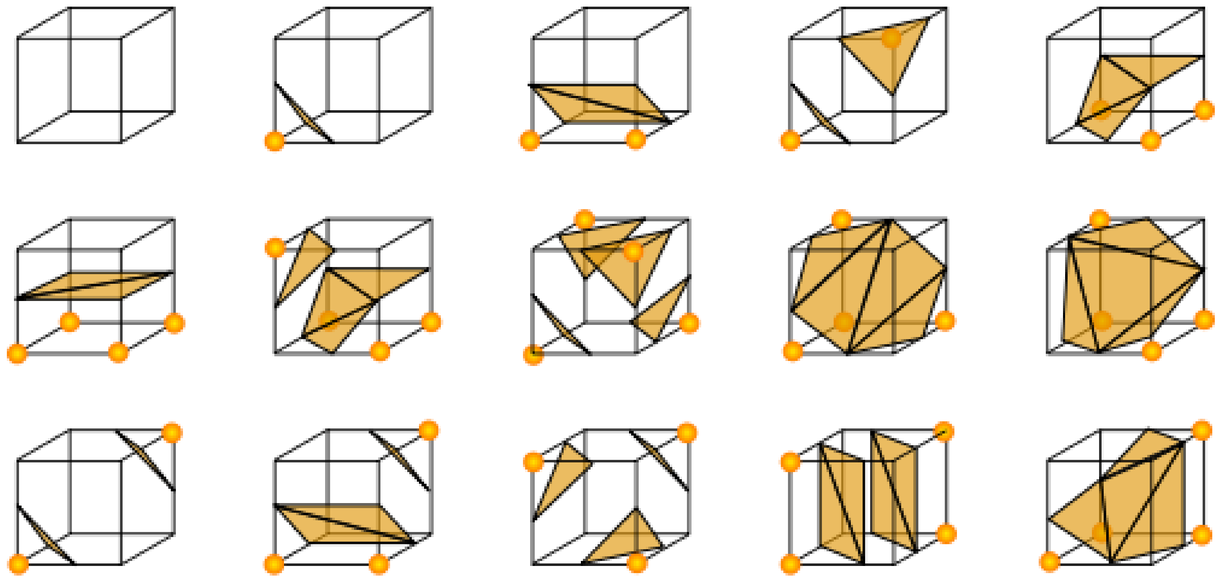
- Let $\mathcal{D}$ be a 3-regular grid
  - With trilinear interpolant
- Level set extraction
  - Loop over the unit cells
  - Cases on a 3D unit cell
    - 256 cases



[Wikipedia]

Issues of marching cubes

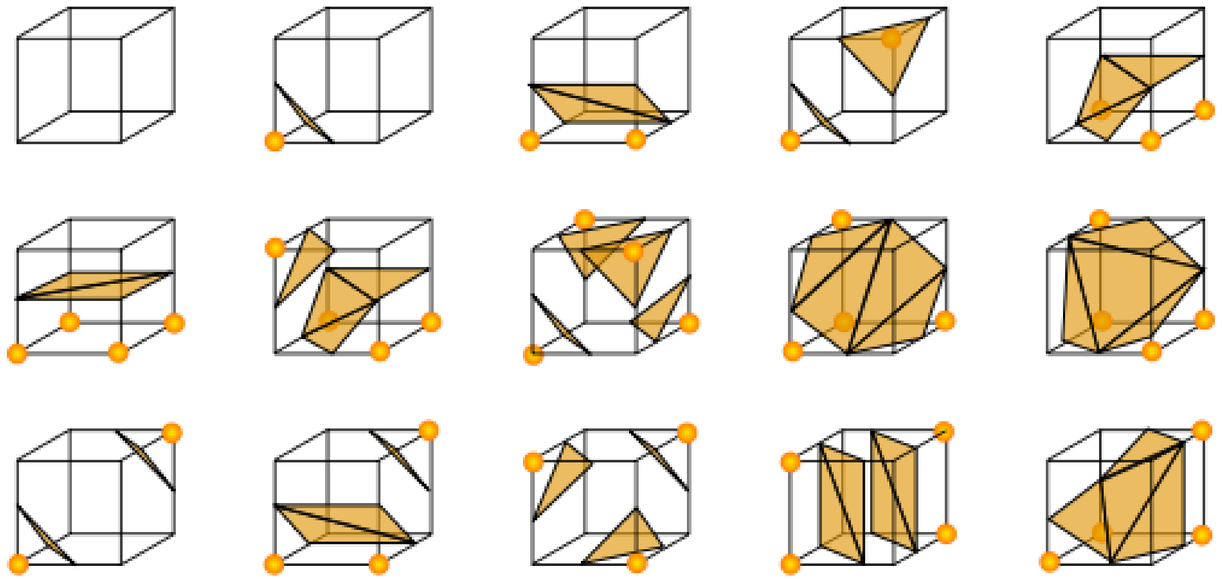
- Same as before



[Wikipedia]

Issues of marching cubes

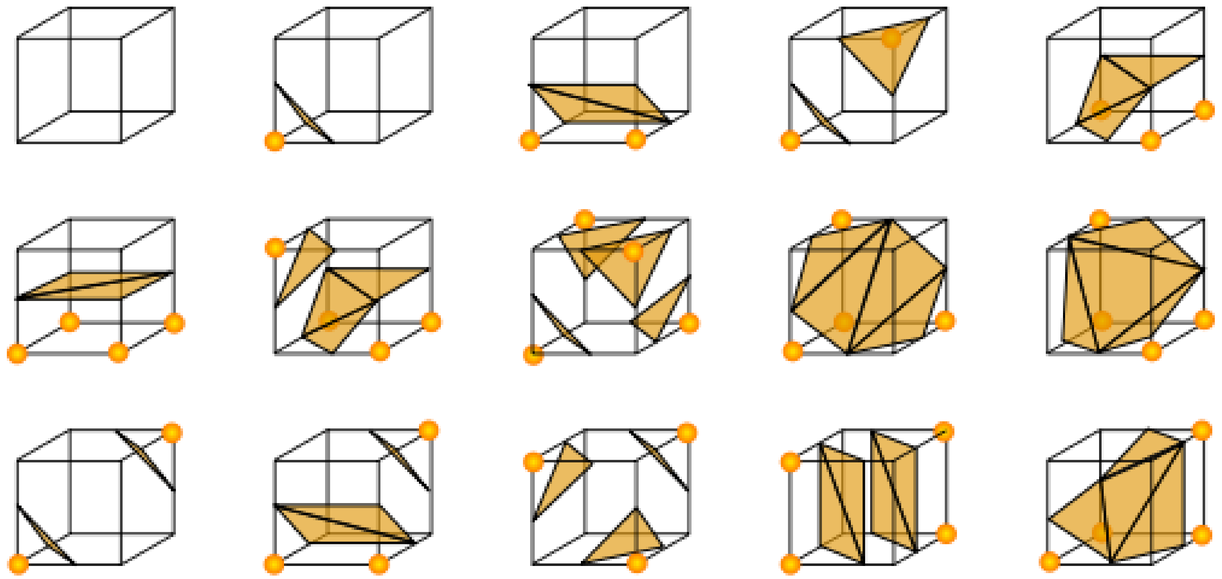
- Same as before
  - But worse!



[Wikipedia]

Issues of marching cubes

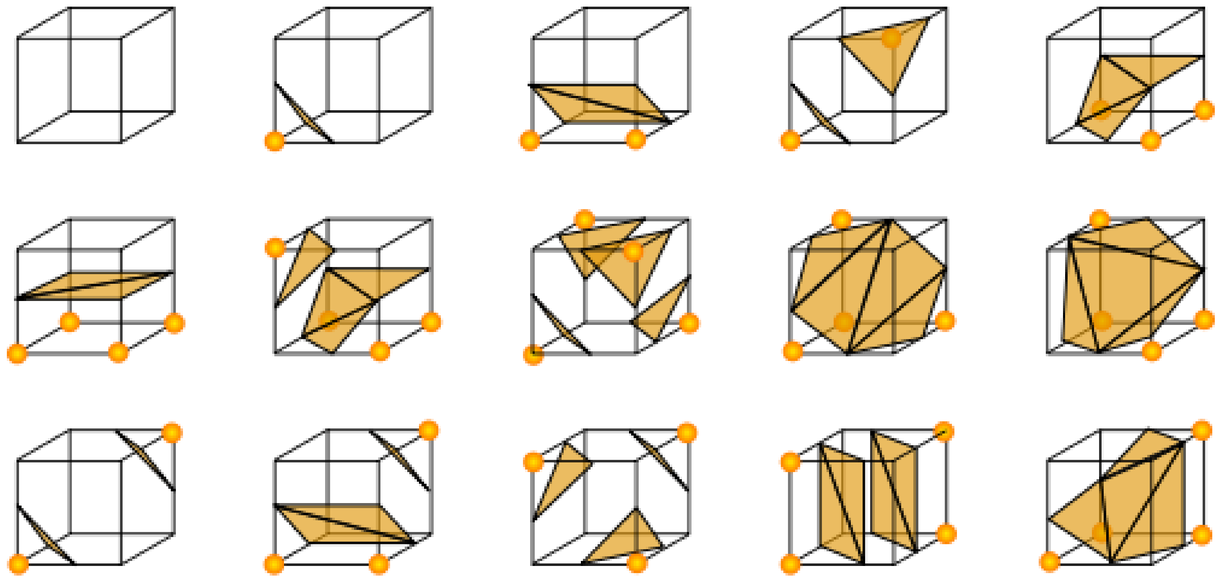
- Same as before
  - But worse!
- Lorensen and Cline 1987



[Wikipedia]

Issues of marching cubes

- Same as before
  - But worse!
- Lorensen and Cline 1987
  - 16 cases (symmetries)



[Wikipedia]

Issues of marching cubes

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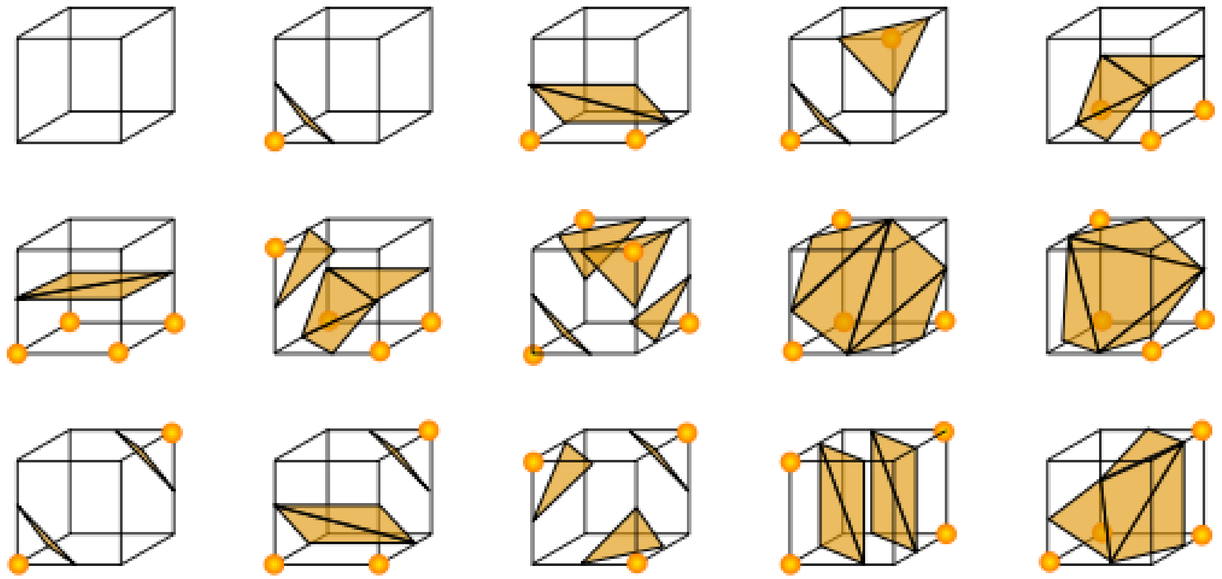
  - Possible cracks



[Wikipedia]

Issues of marching cubes

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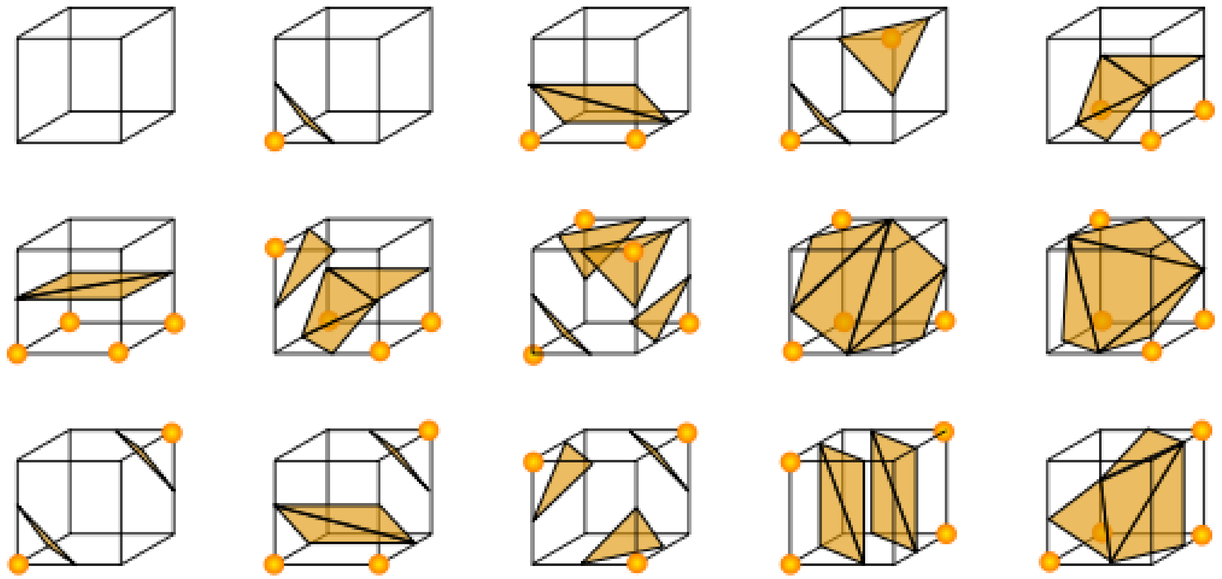
  - Improvement 28 cases



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Issues of marching cubes

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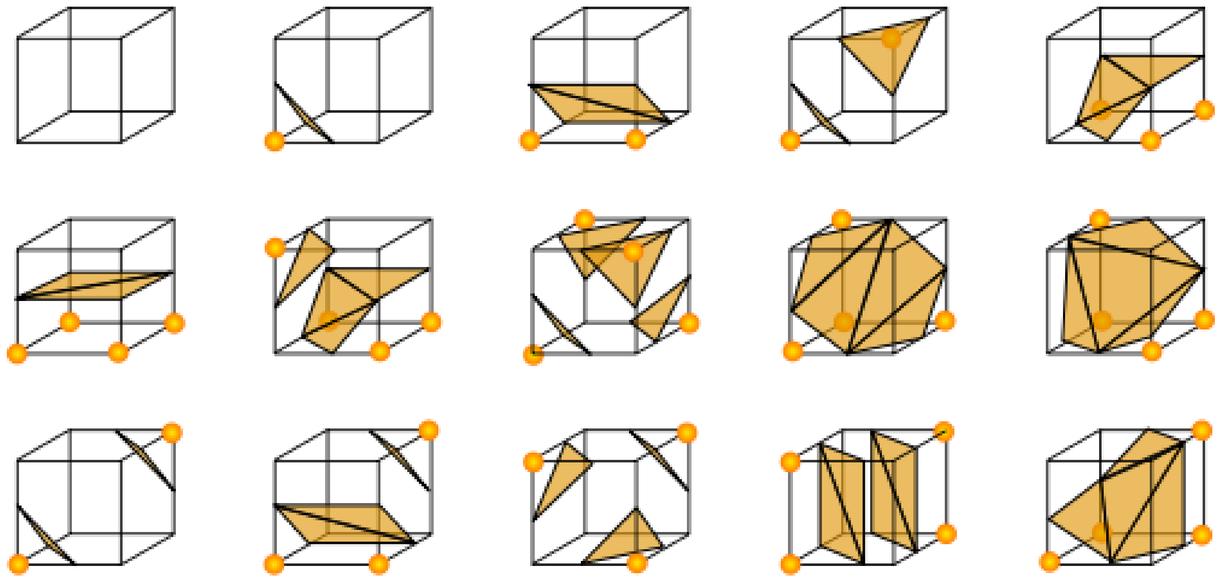
  - Improvement 28 cases
- Saddle curse (components and genus):



[Wikipedia]

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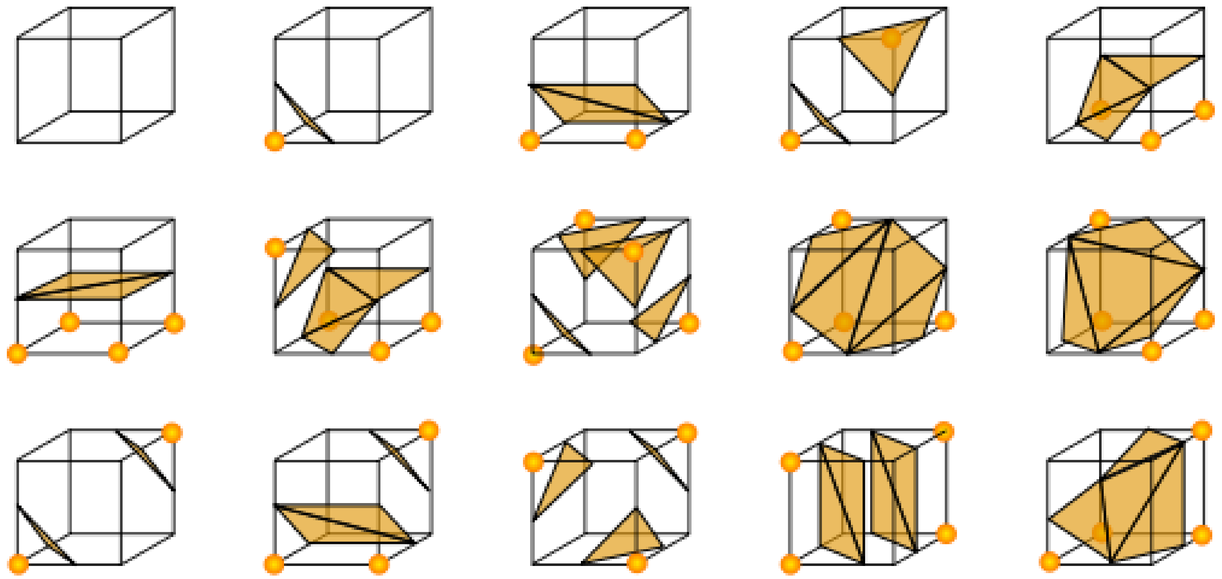

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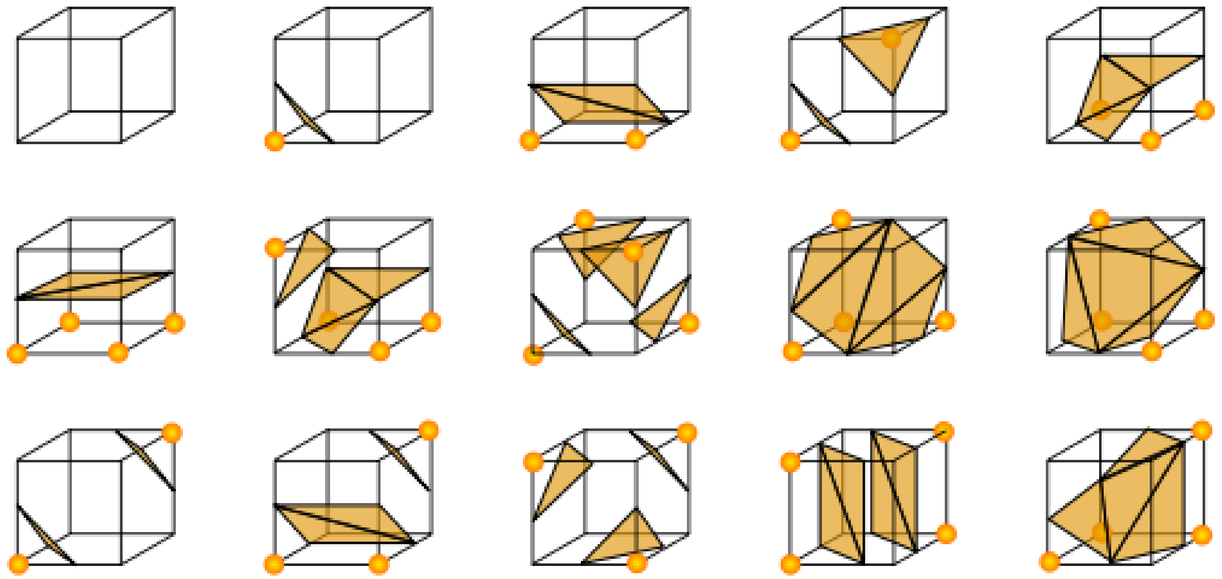

[Wikipedia]

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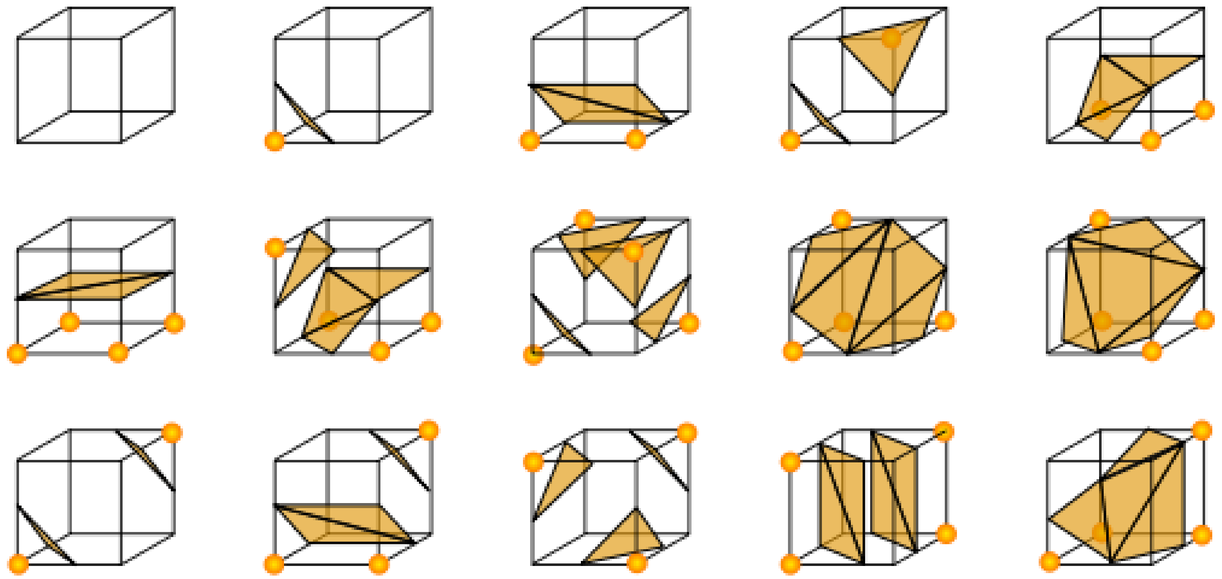

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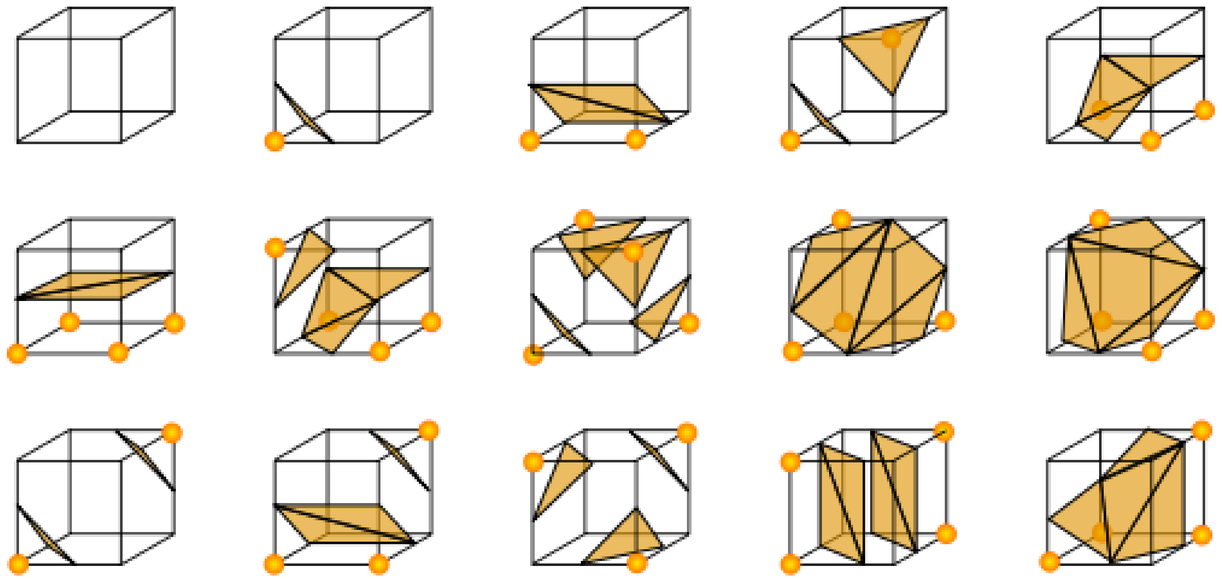

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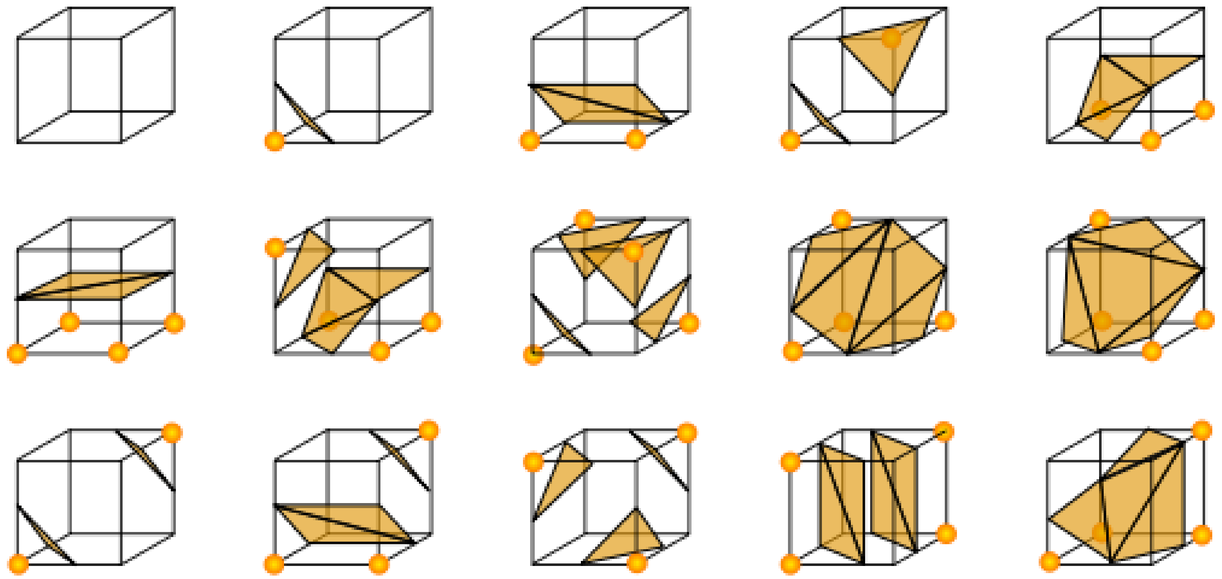

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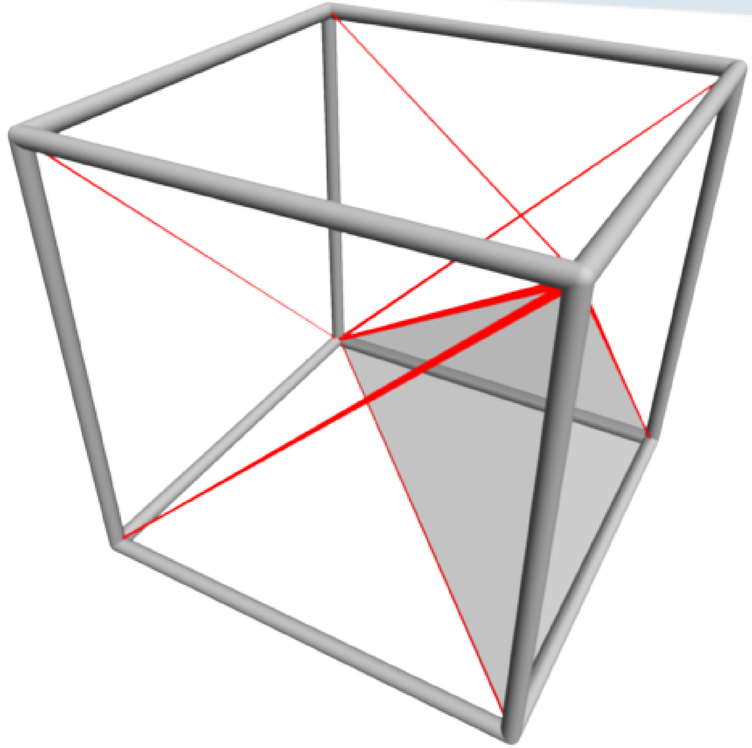
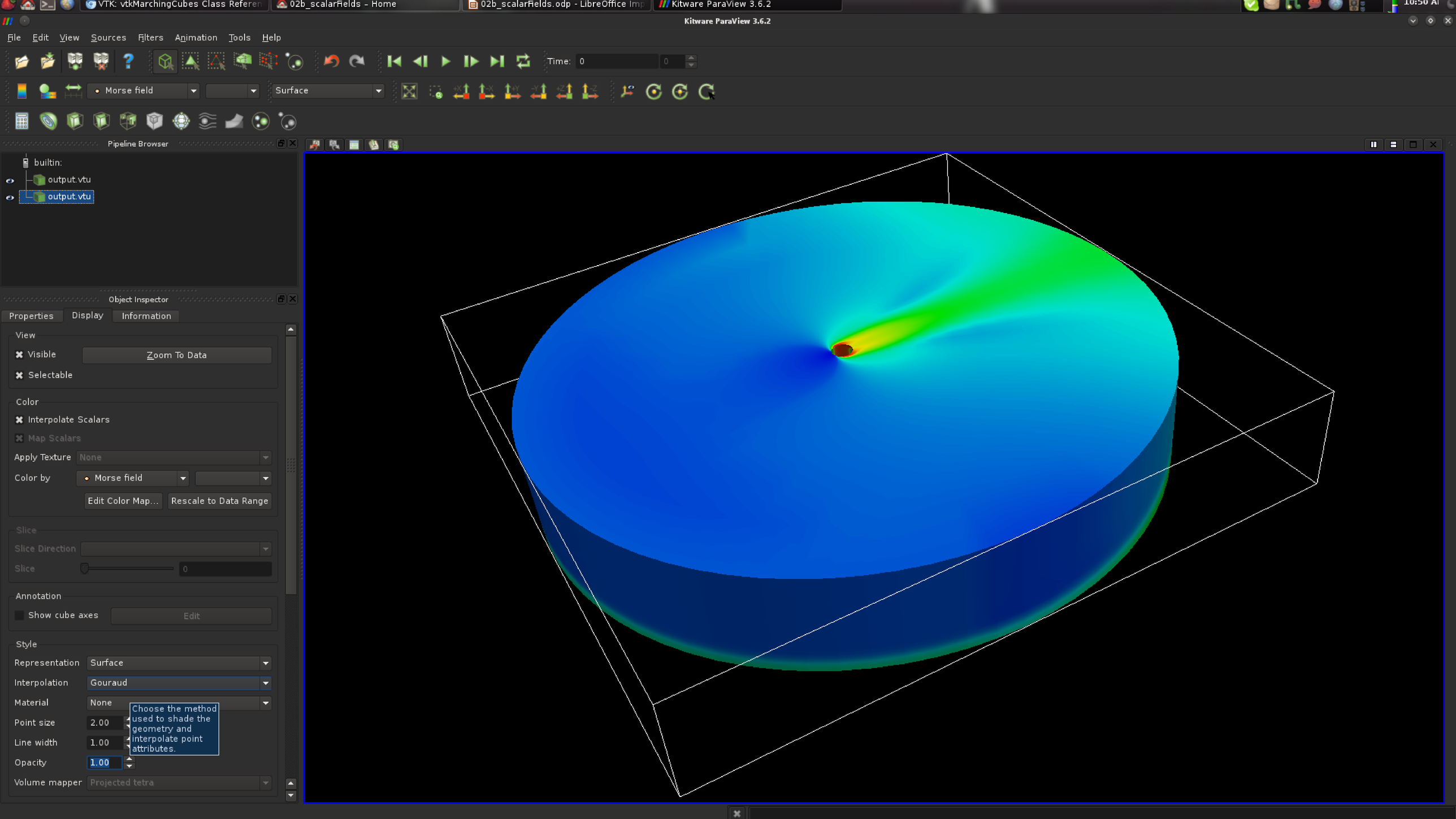


[Wikipedia]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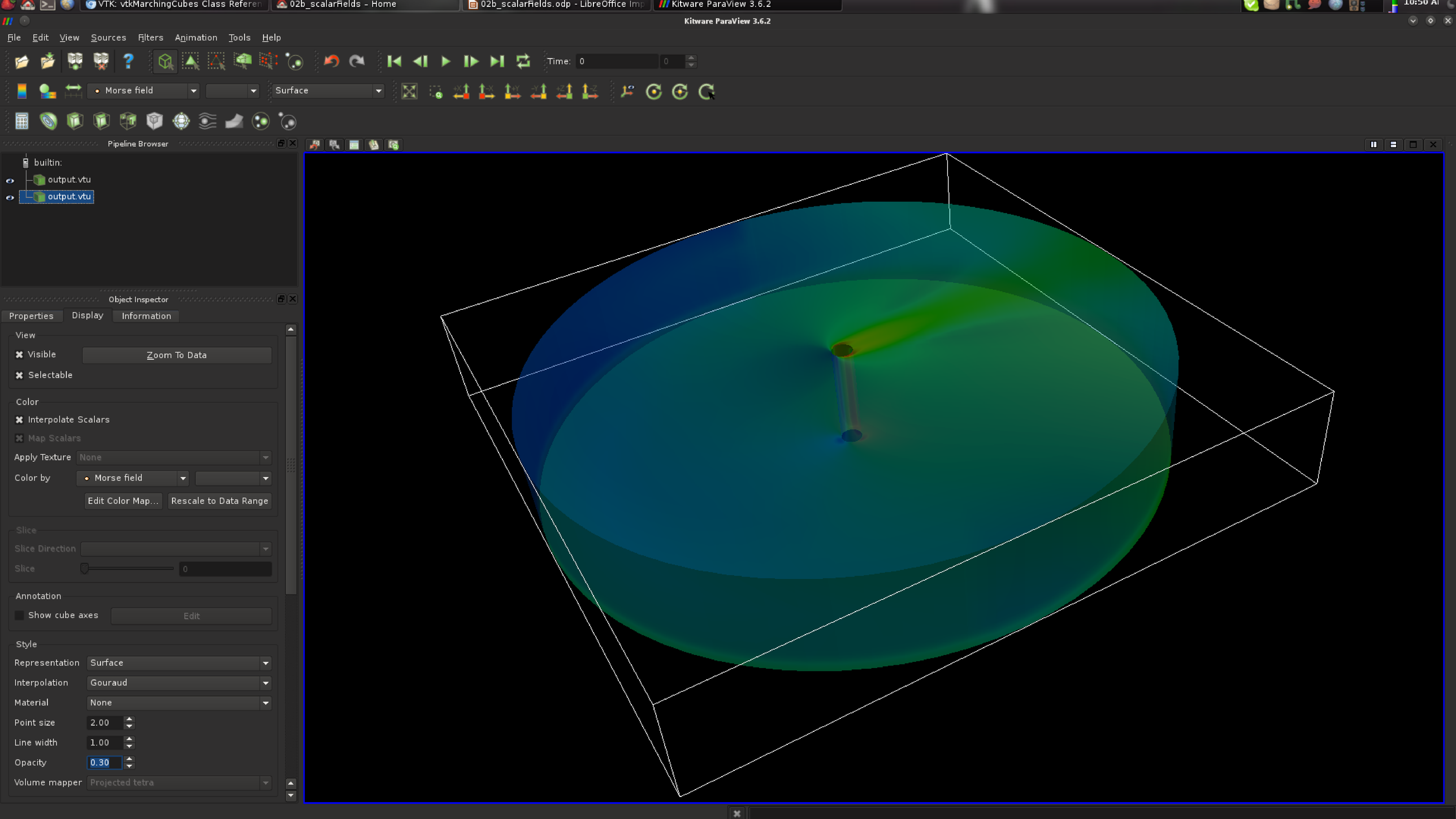
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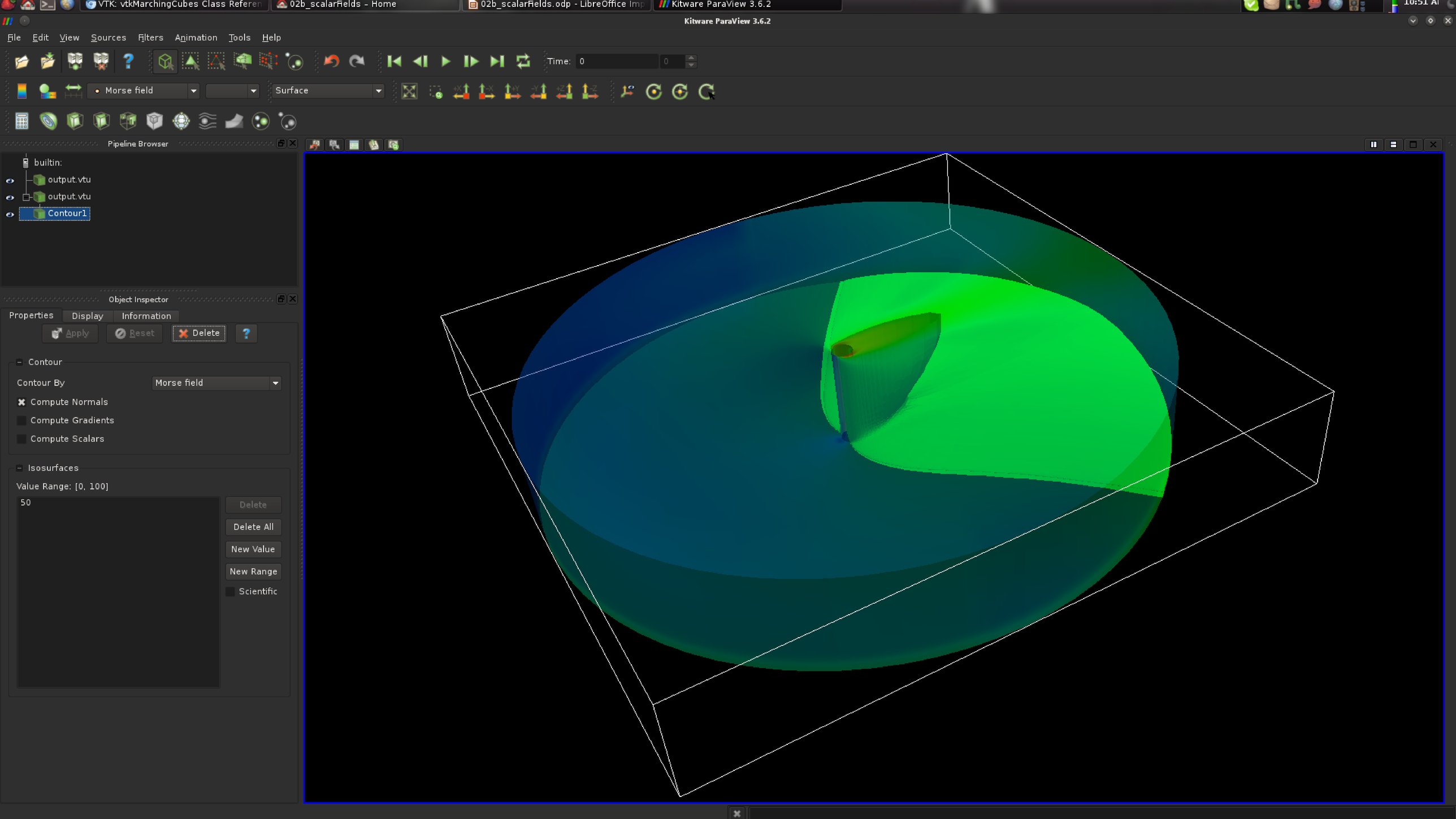
Volume mapper Projected tetra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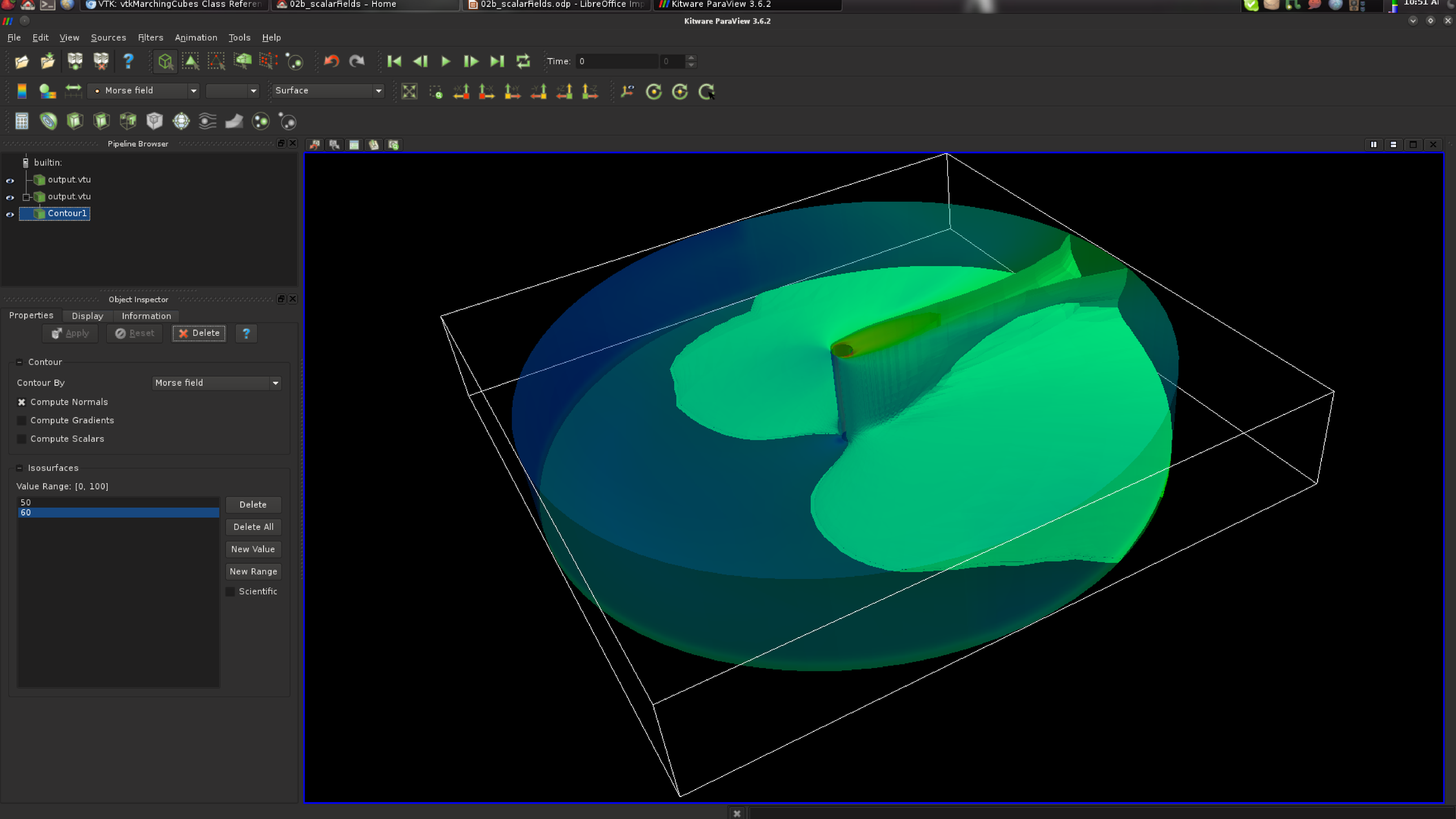
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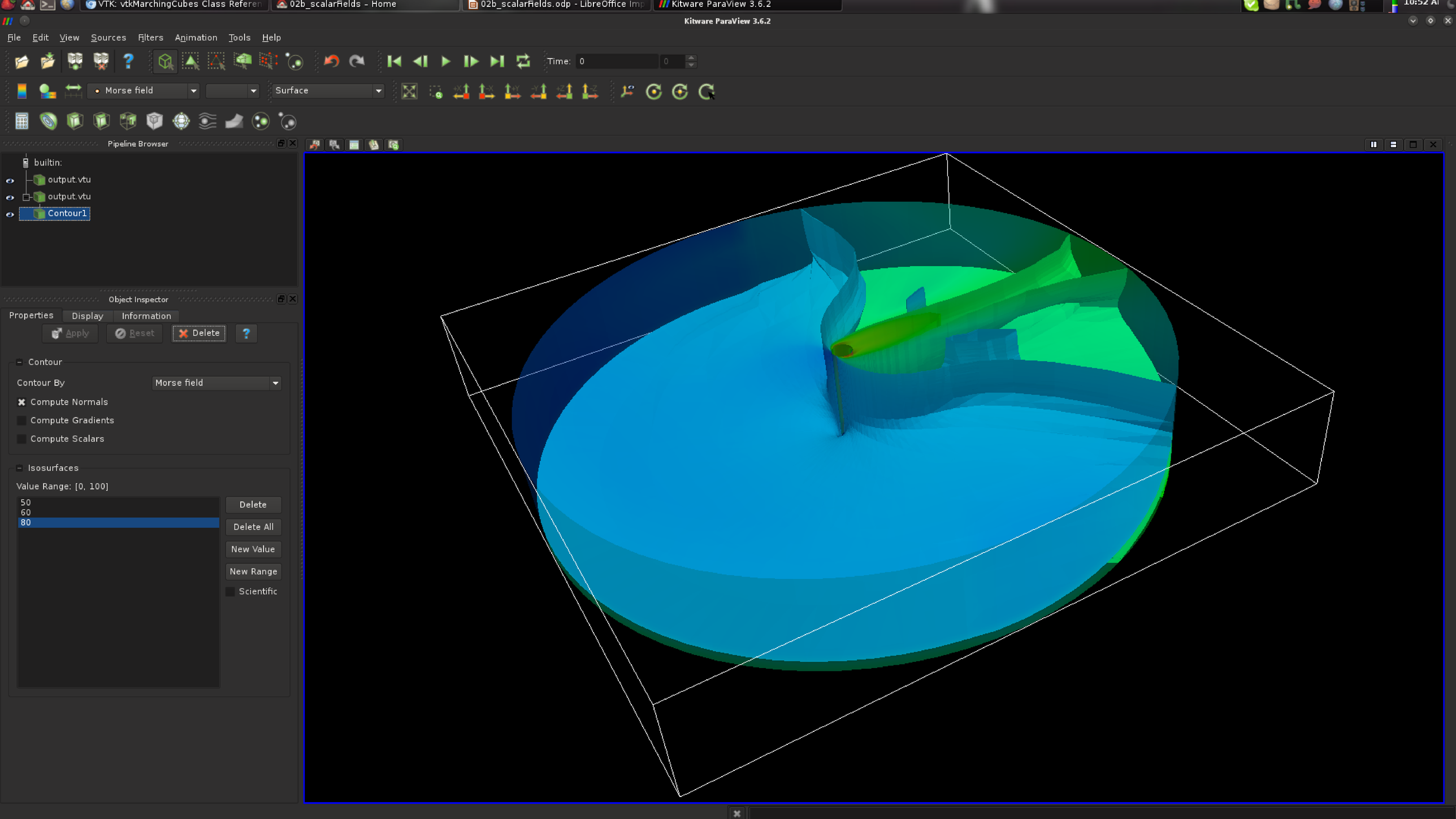
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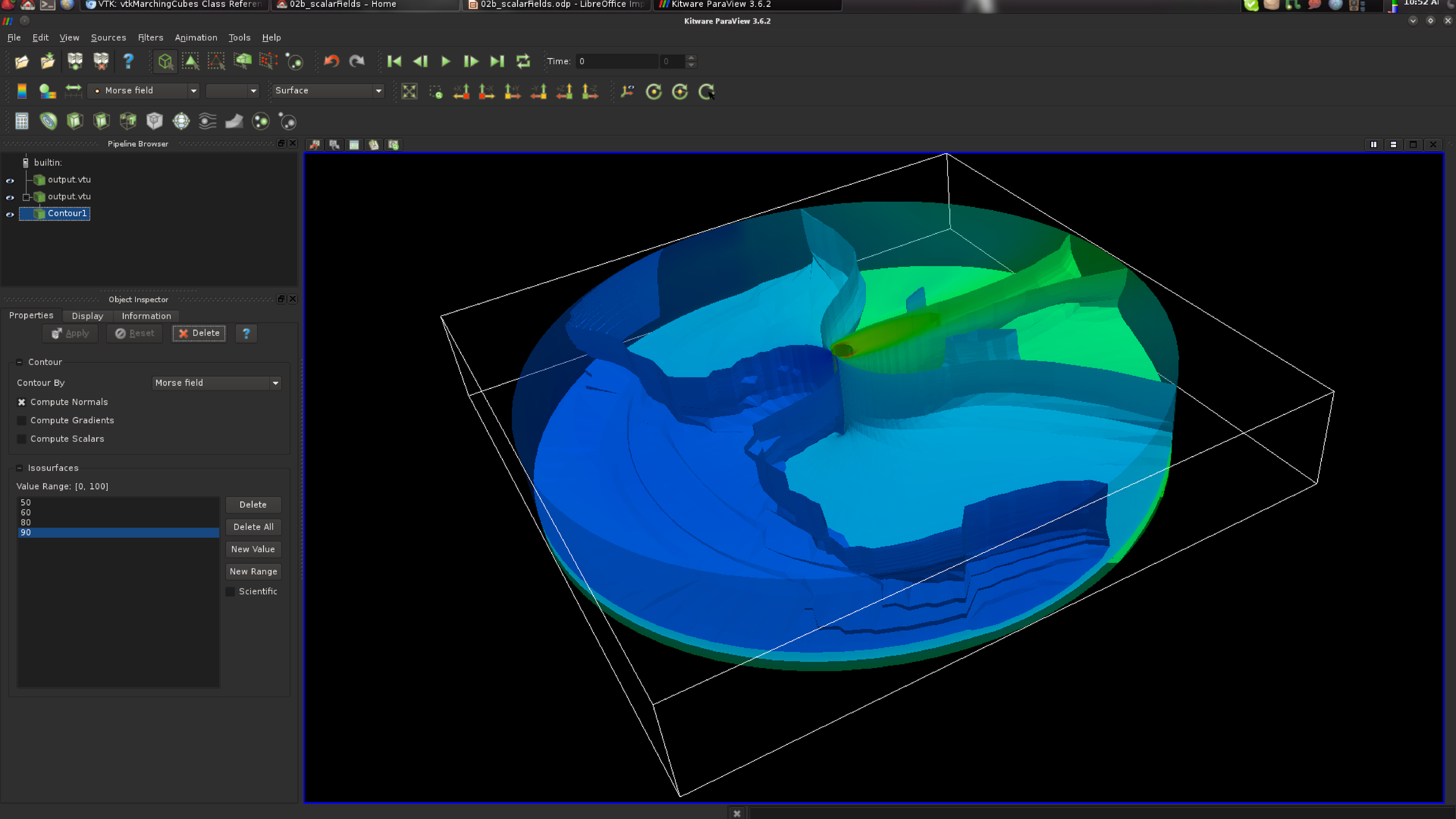
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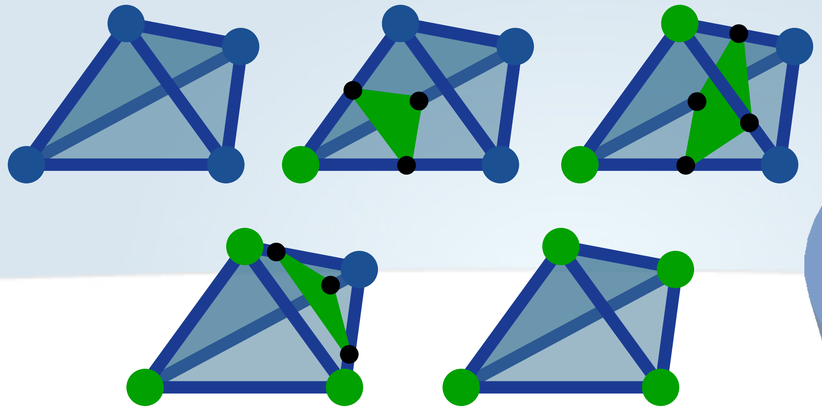
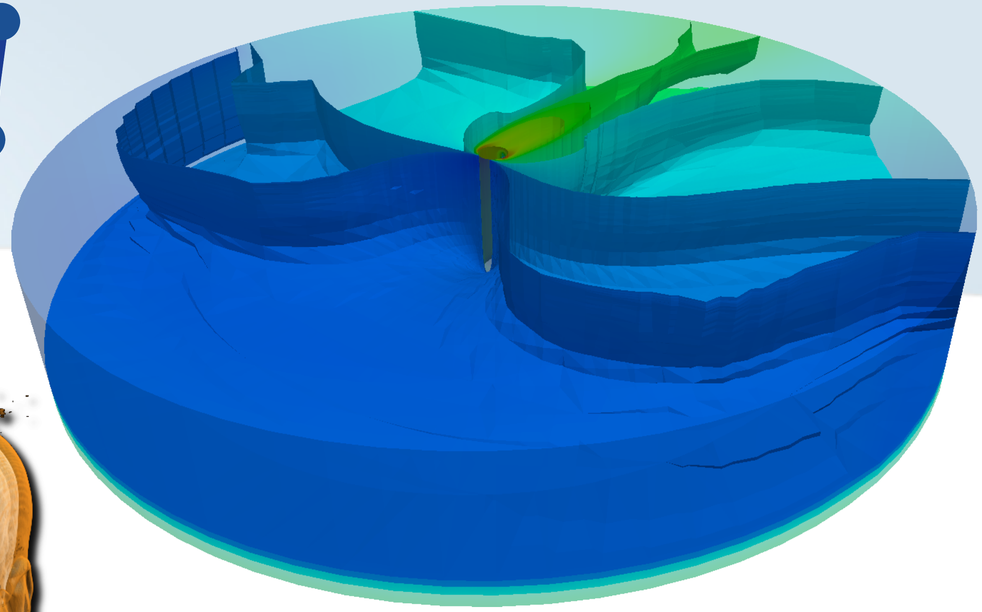
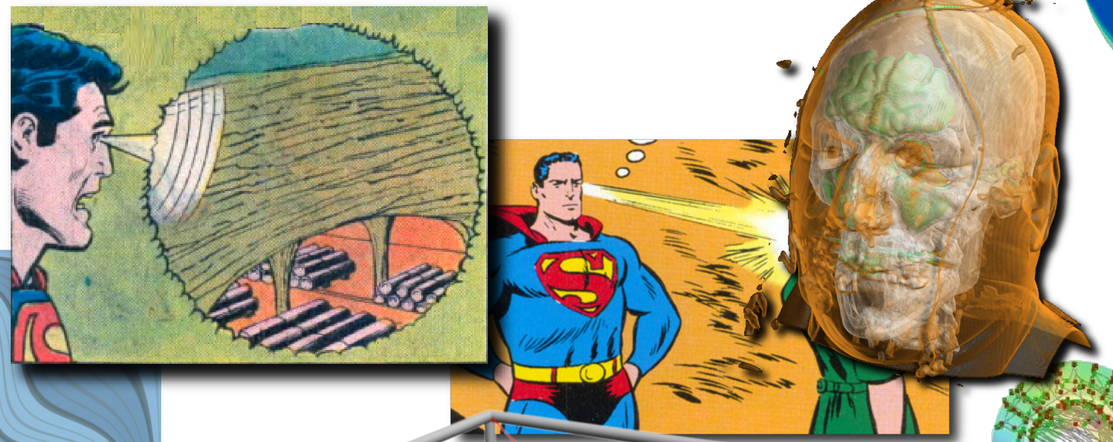
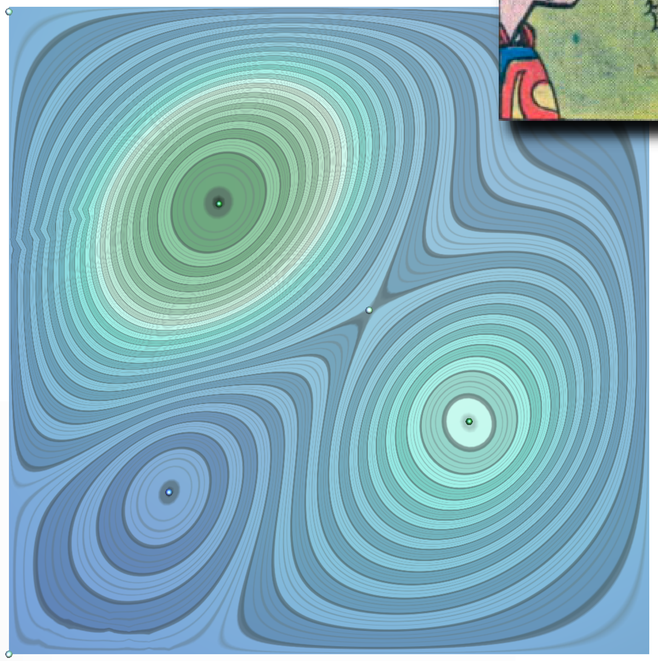
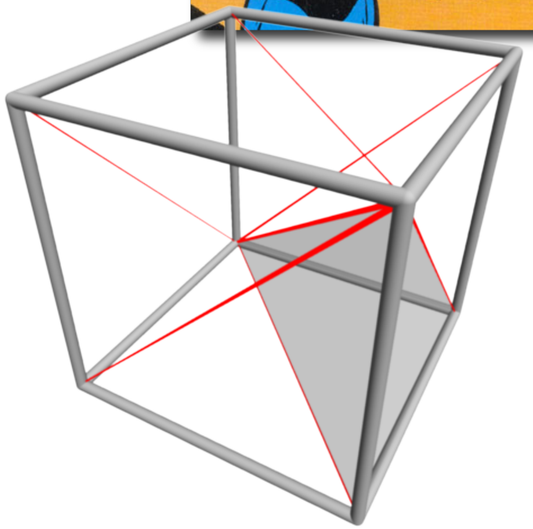
Scalars

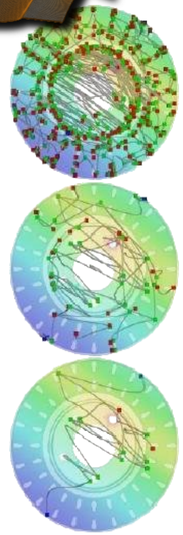


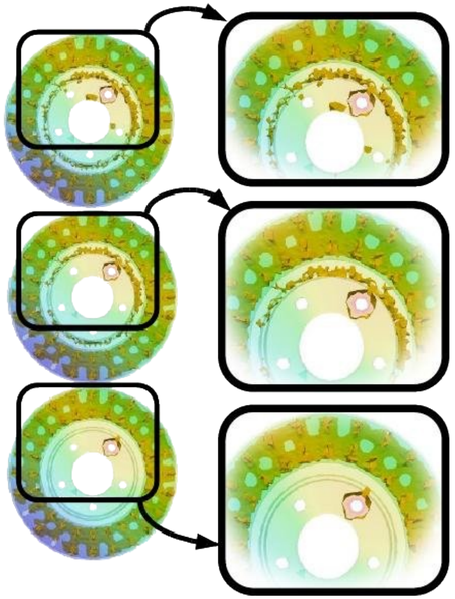


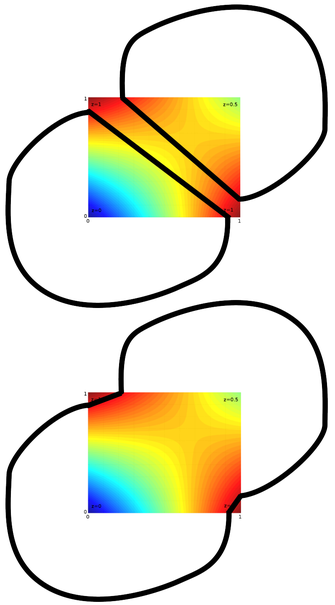


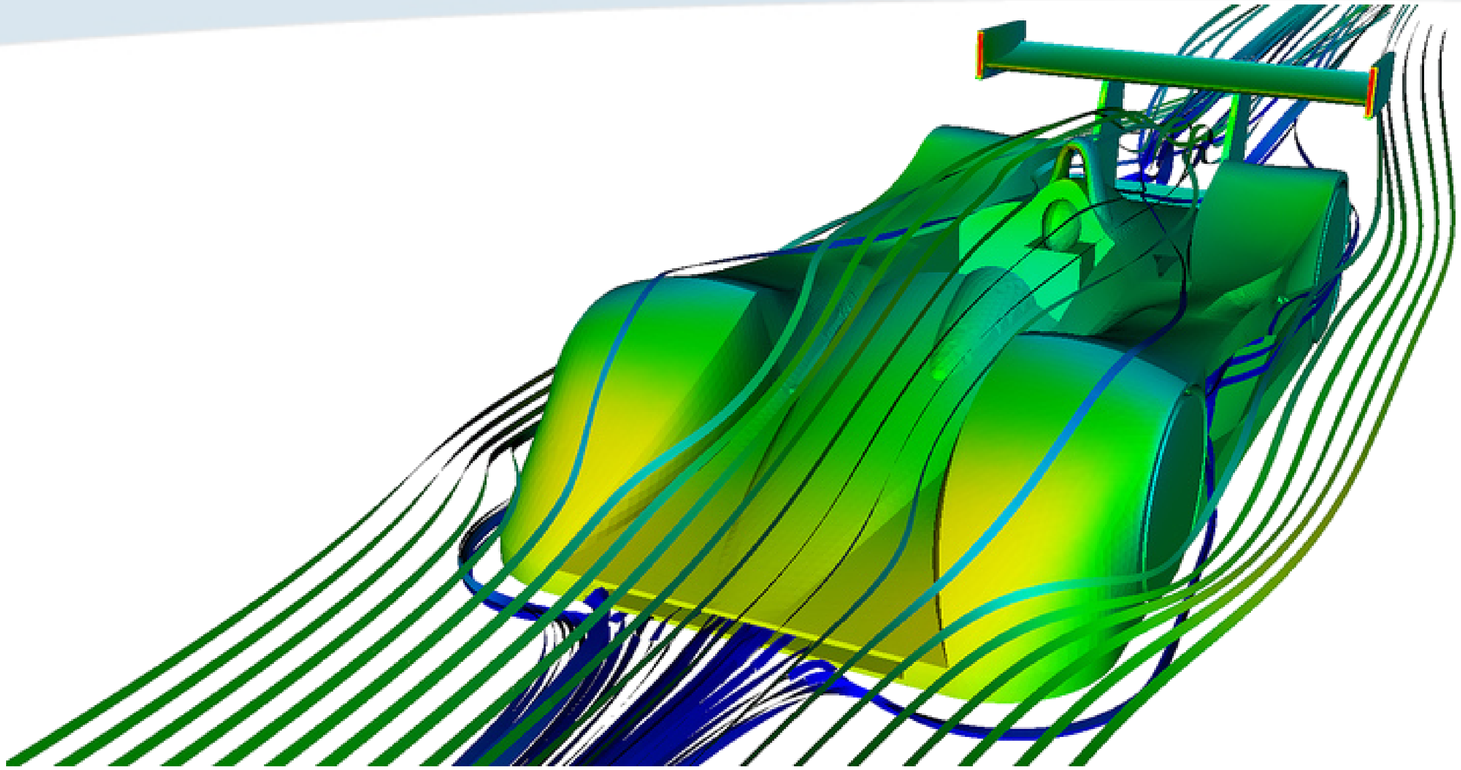












[Kitware]

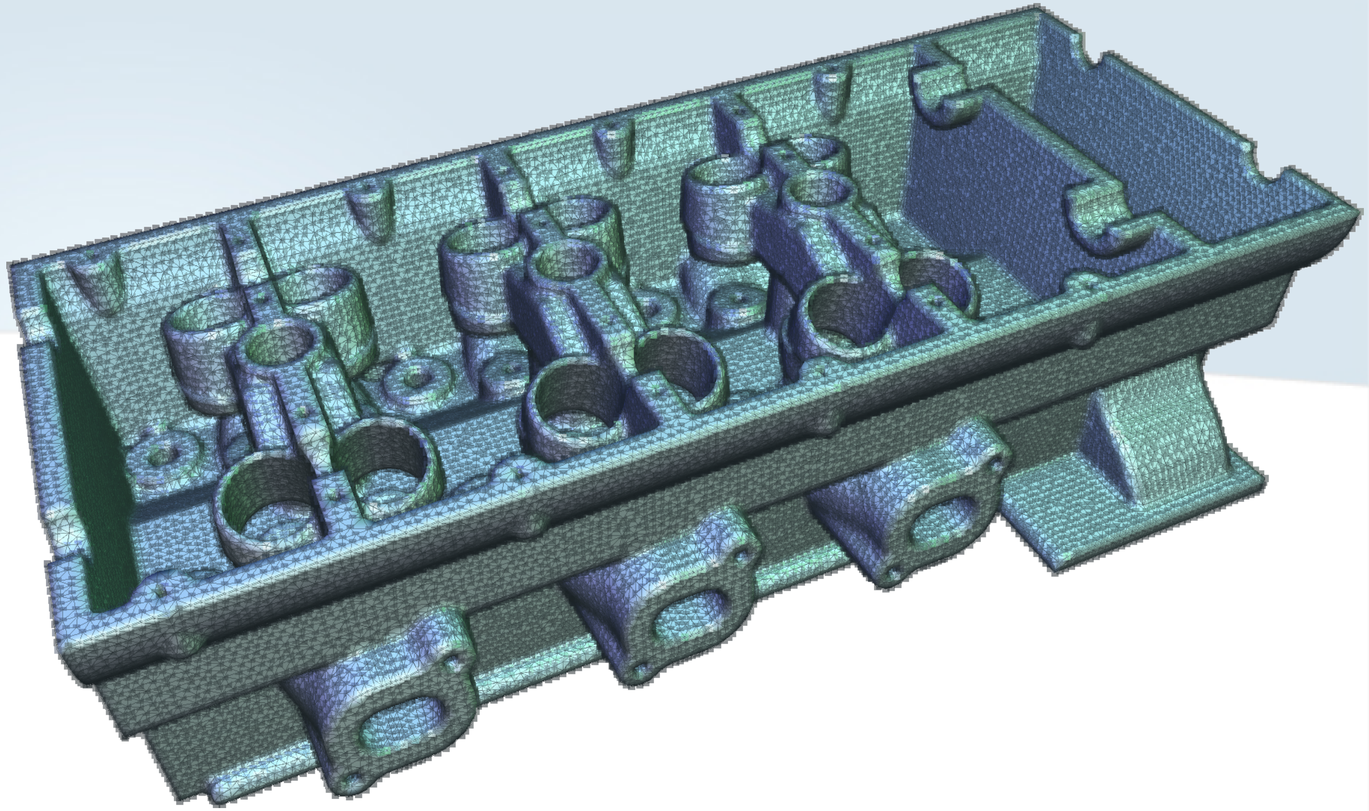
Vector Field Visualization

In practice

- Given a domain $\mathcal{D}$

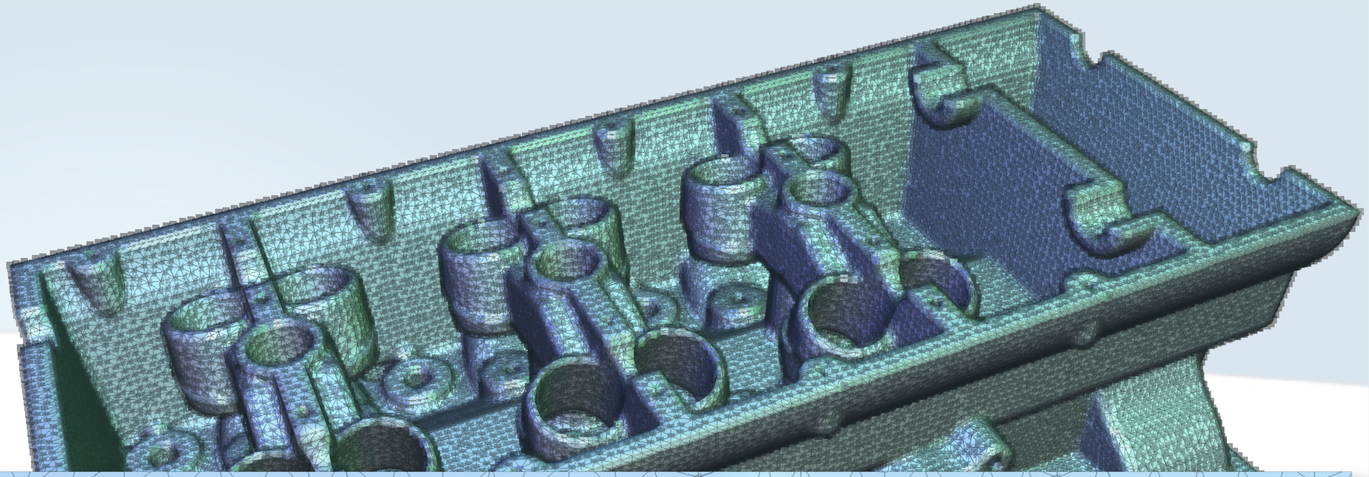
In practice

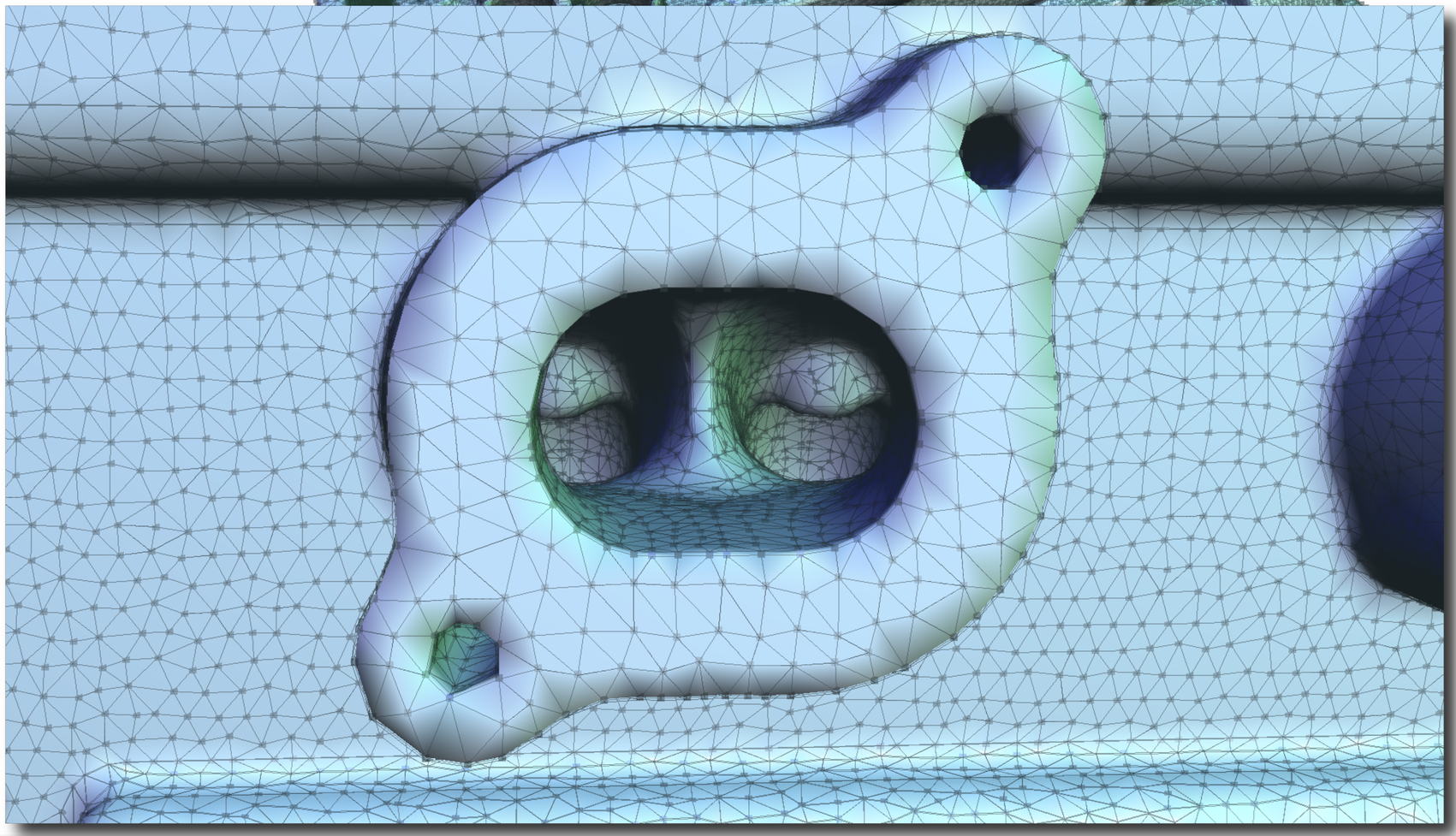
- Given a domain $\mathcal{D}$



In practice

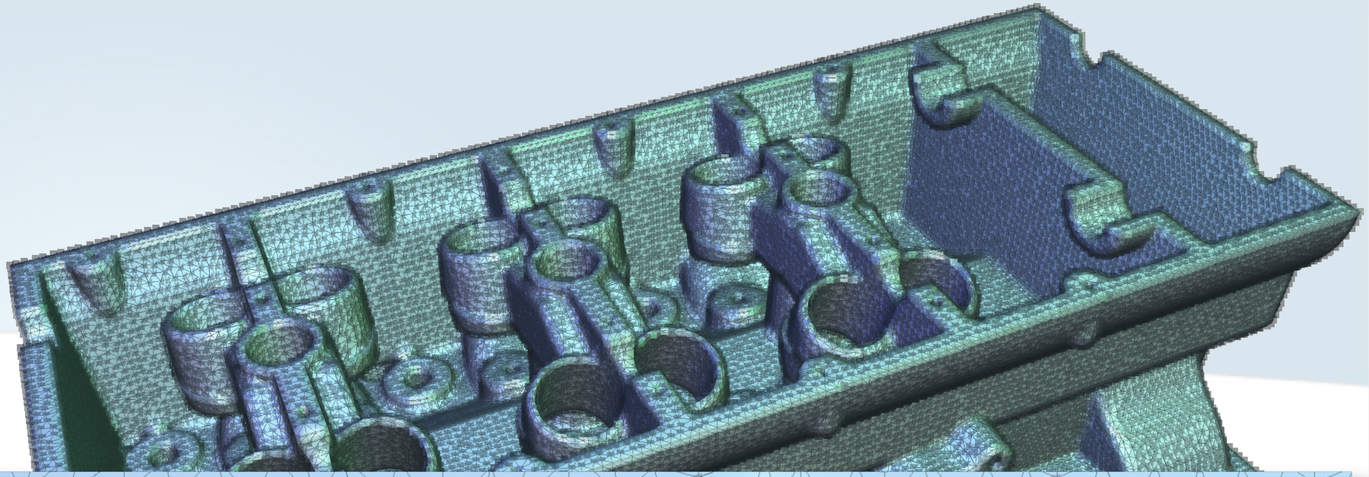
- Given a domain $\mathcal{D}$

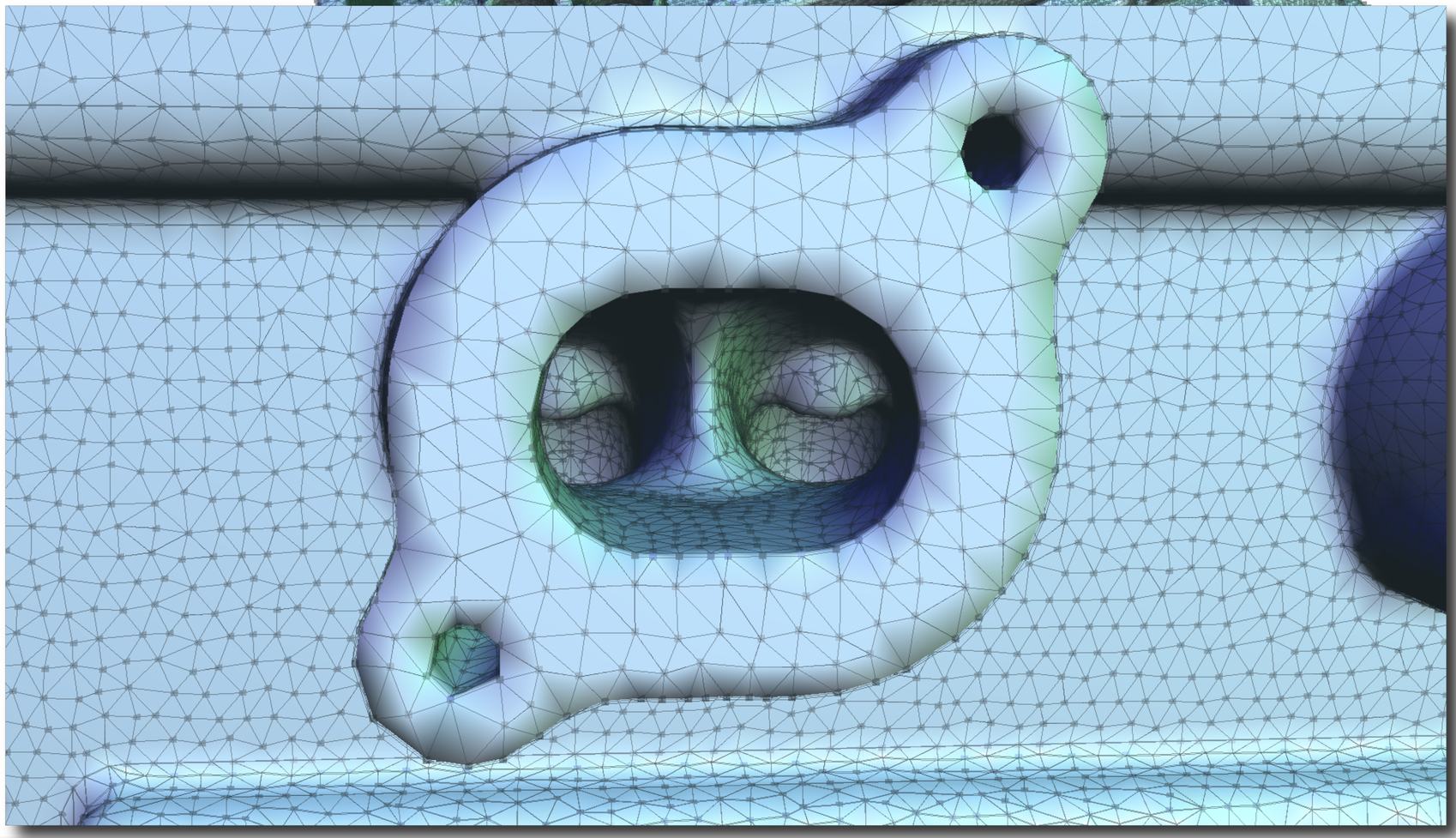




In practice

- Given a domain $\mathcal{D}$
- For each vertex v





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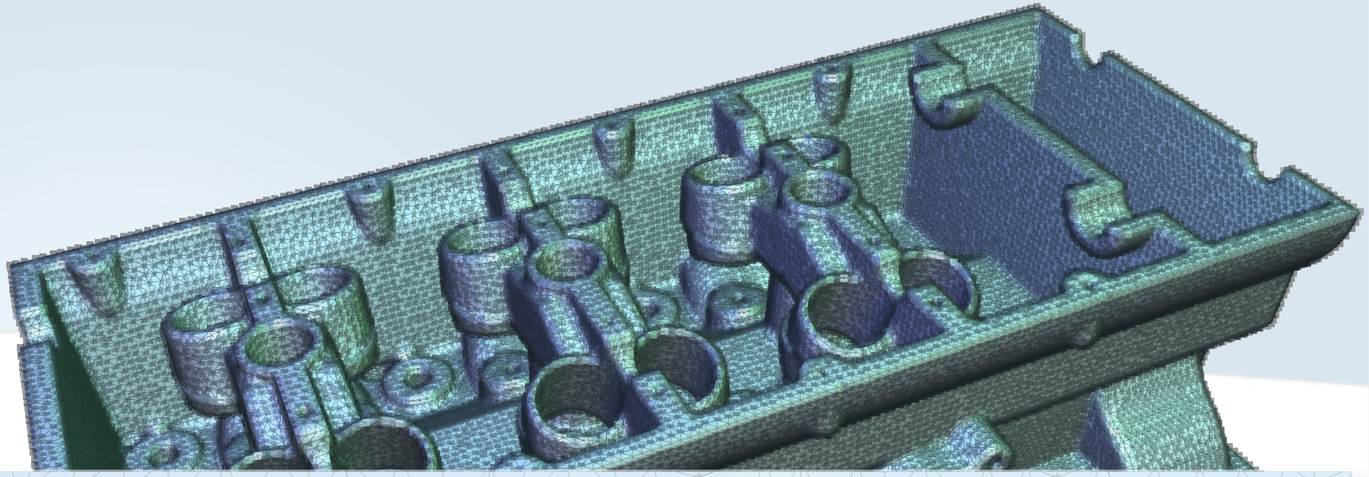
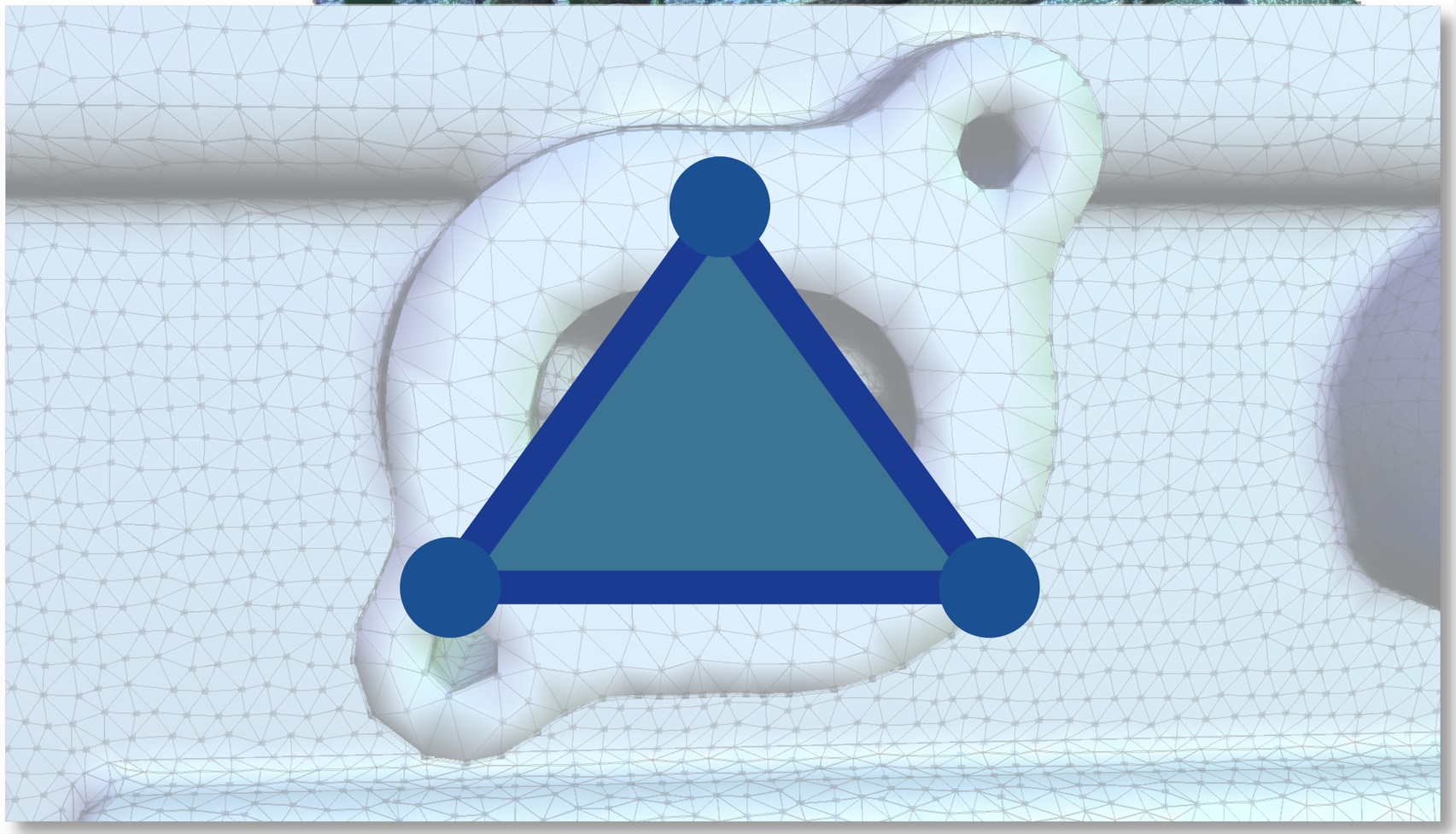
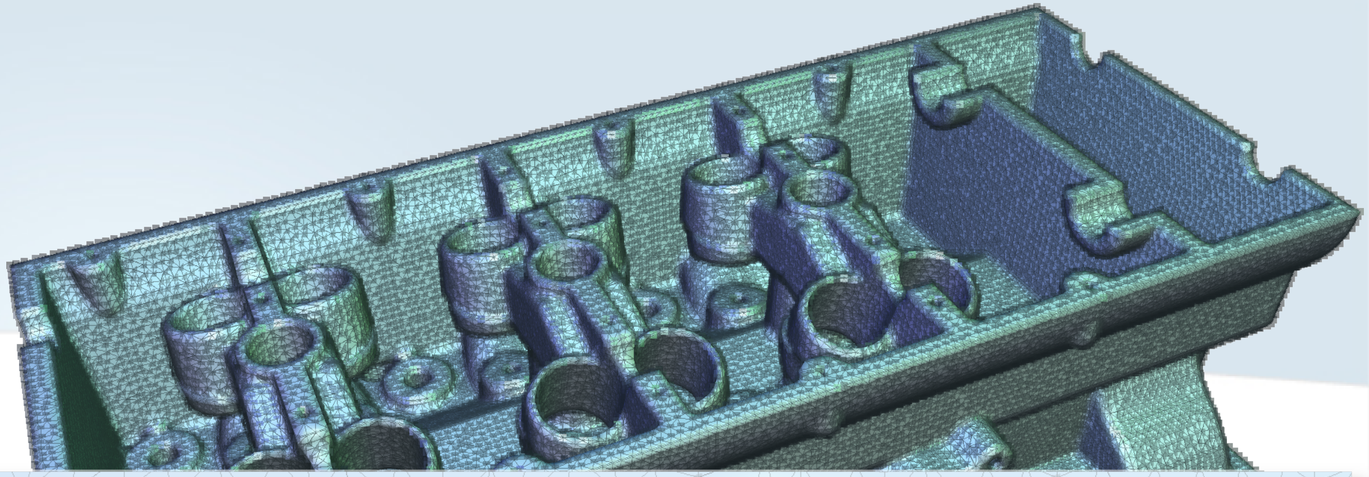
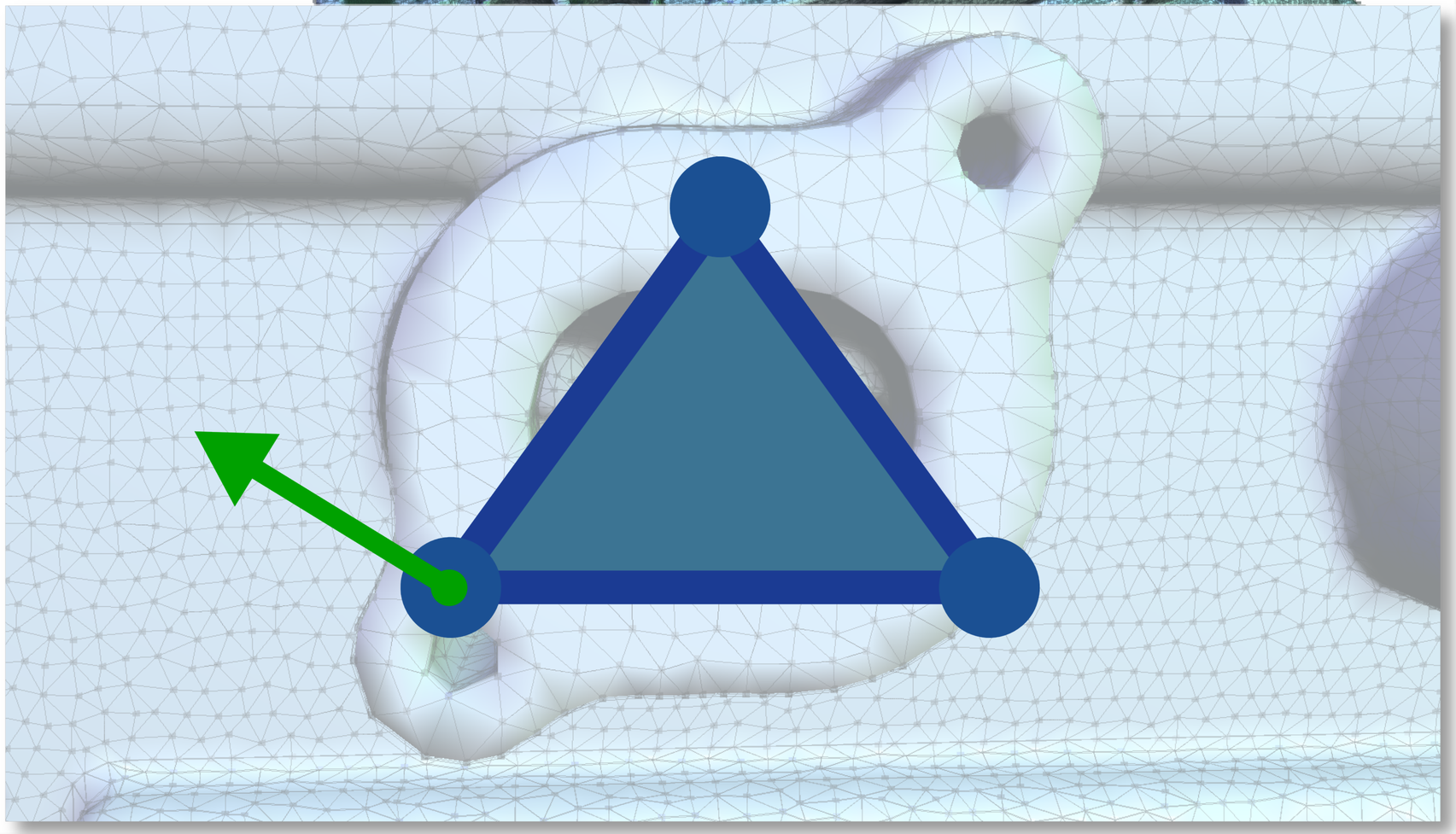




In practice

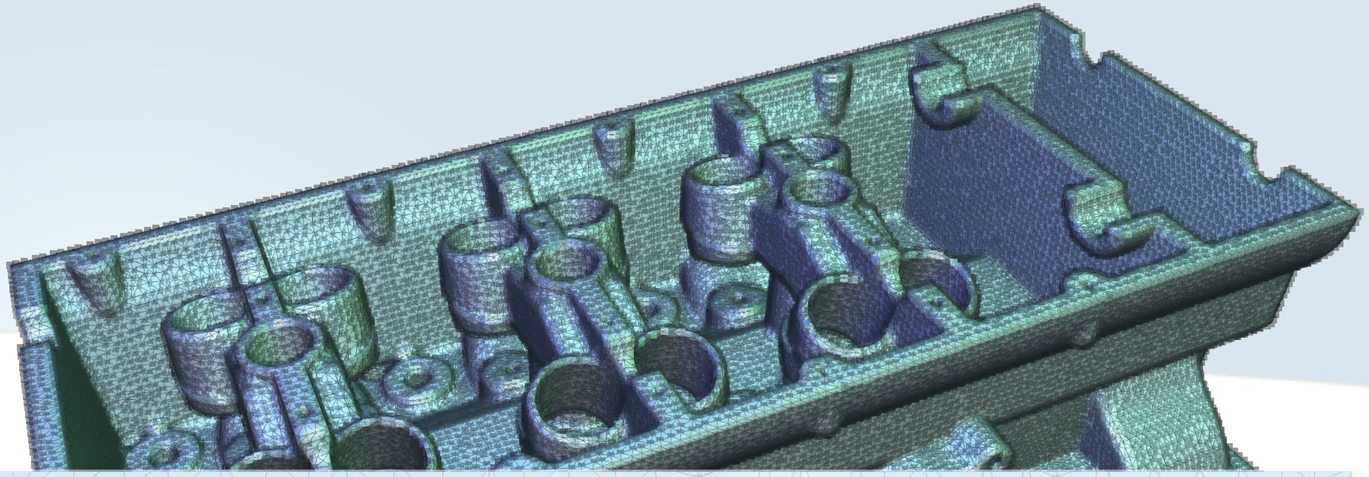
- Given a domain $\mathcal{D}$
- For each vertex v
- One vector $\vec{f}(v)$

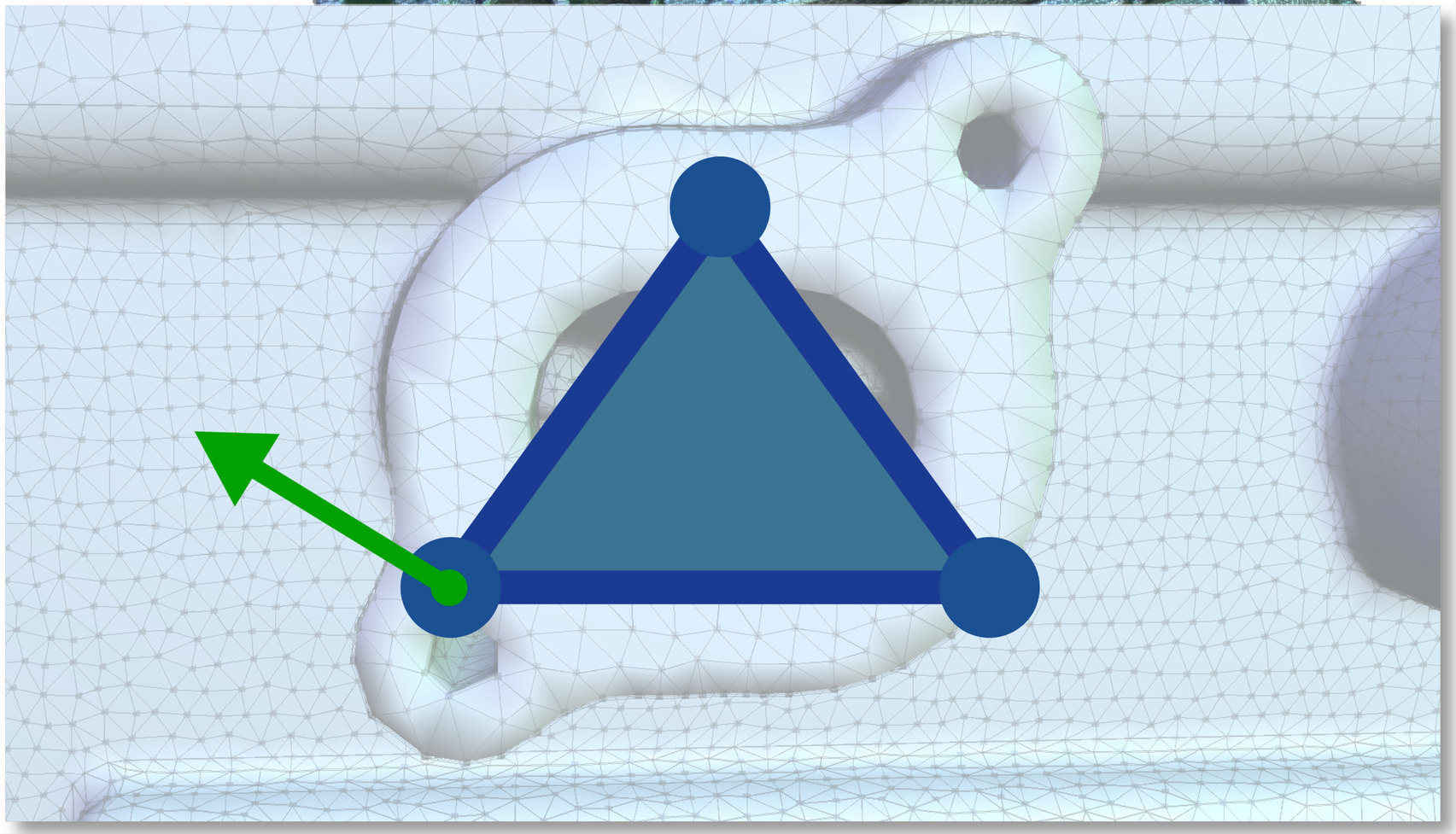




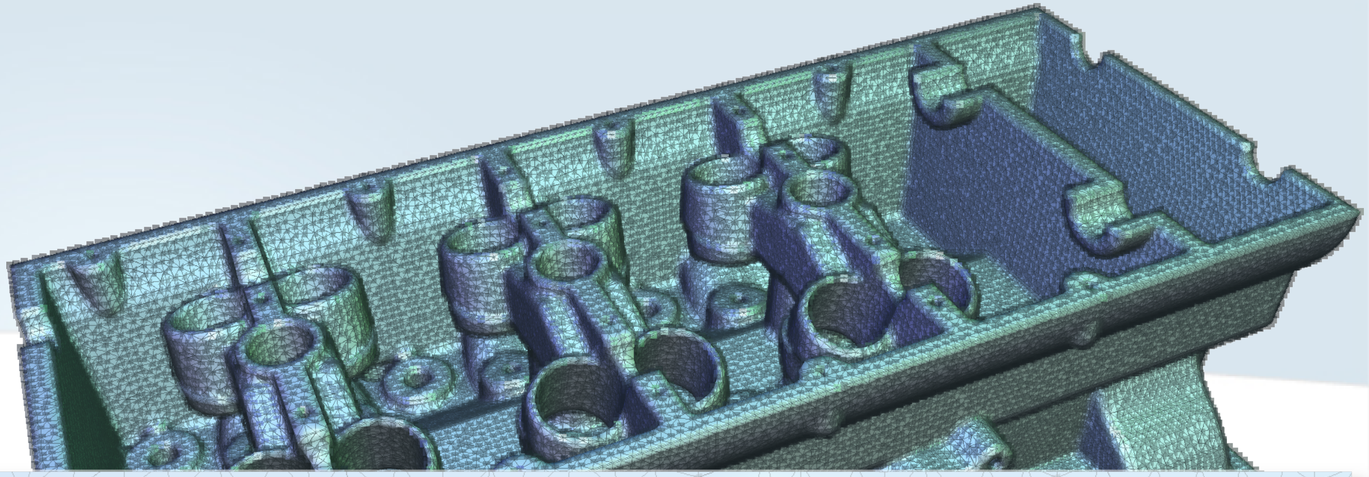
In practice

- Given a domain $\mathcal{D}$
- For each vertex v
- One vector $\vec{f}(v)$
  - *Coordinates in* $\mathbb{E}$

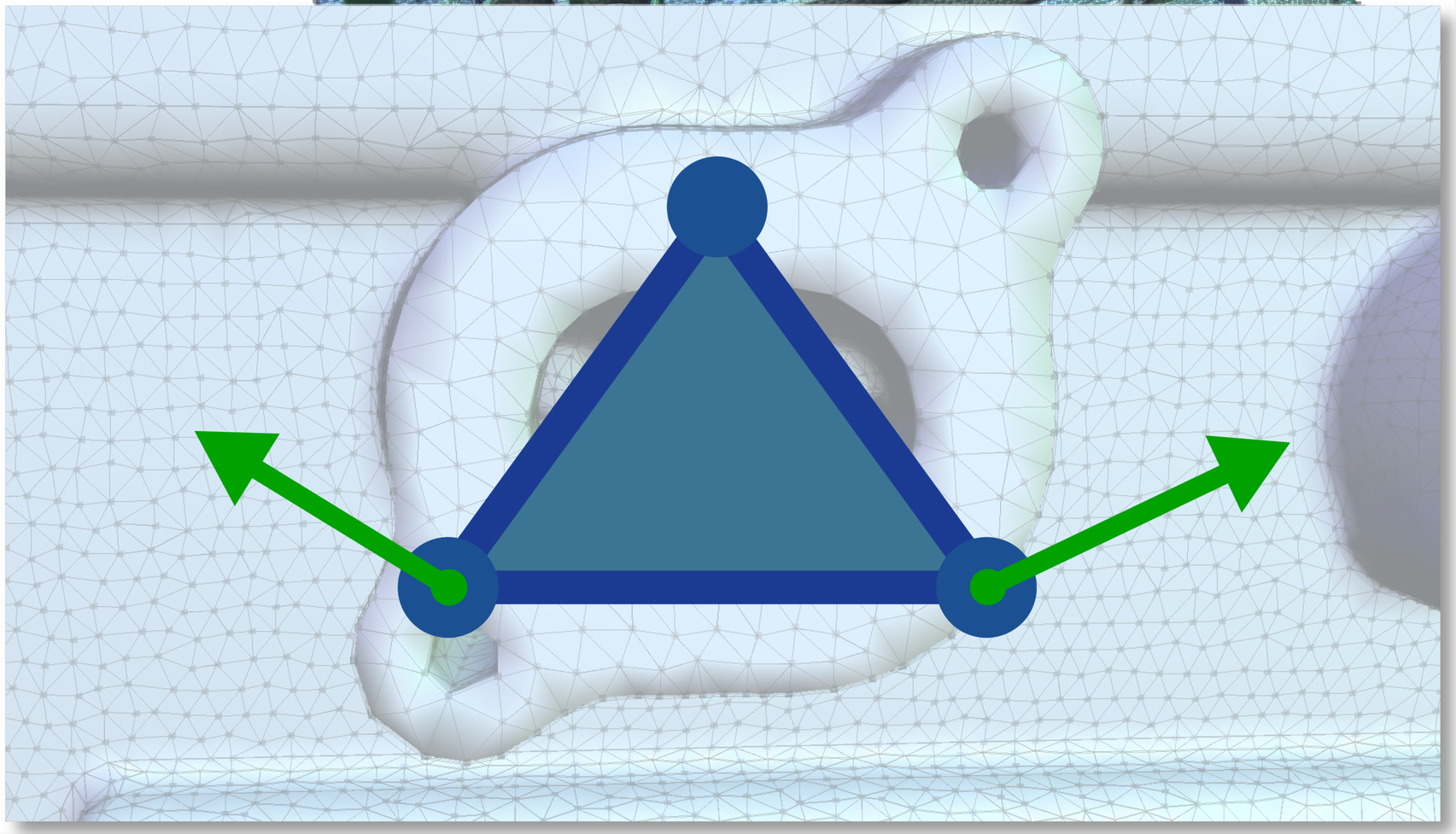




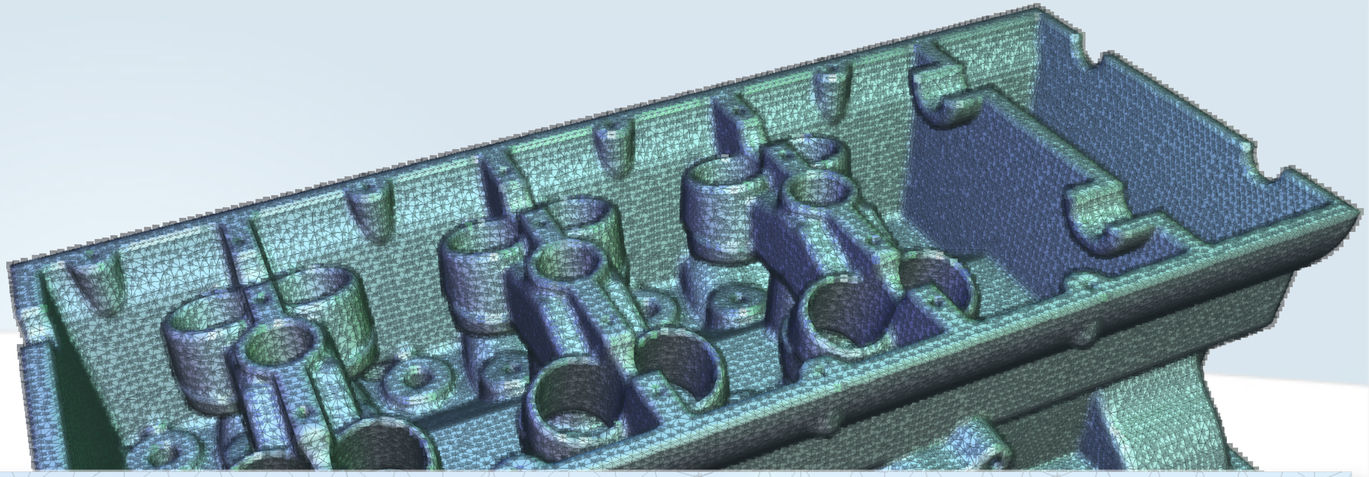
In practice



- Given a domain $\mathcal{D}$
- For each vertex v
- One vector $\vec{f}(v)$
  - Coordinates in $\mathbb{E}$
- Interpolation on the other simplices:



In practice



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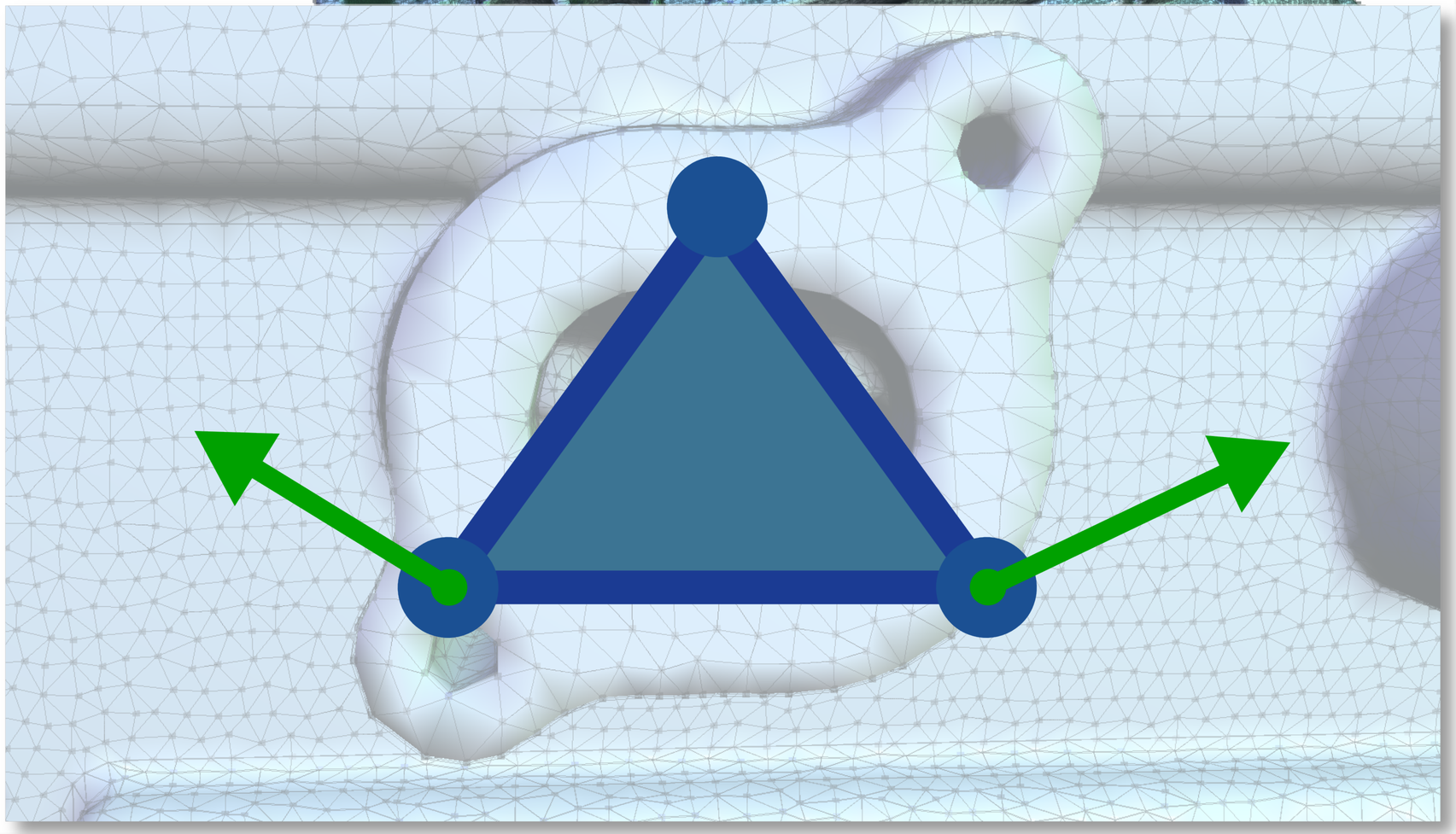
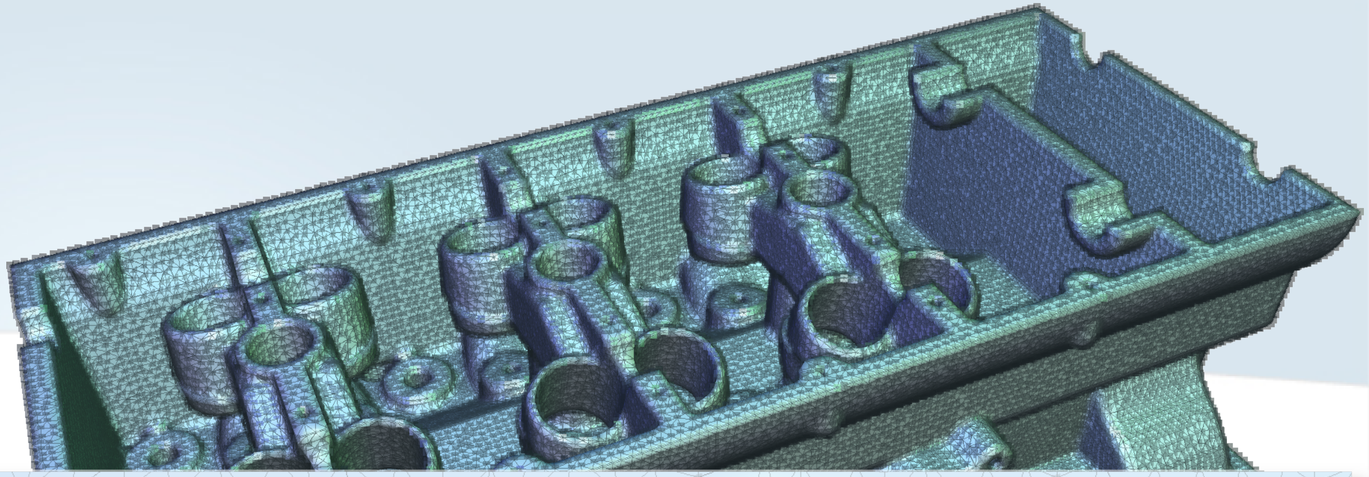
- For each vertex v
- One vector $\vec{f}(v)$
  - Coordinates in $\mathbb{E}$
- Interpolation on the other simplices:
  - Coordinates



In practice



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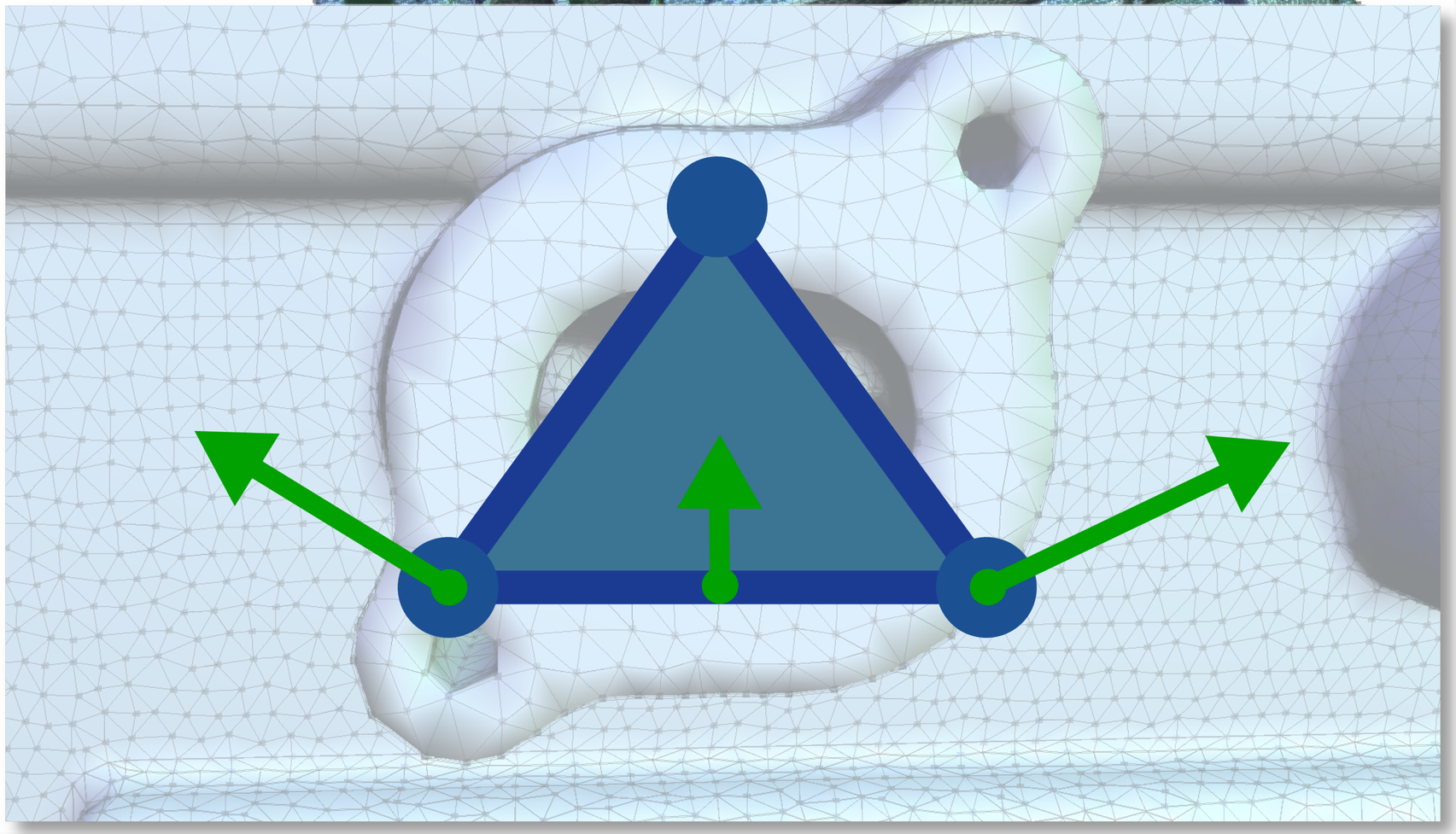
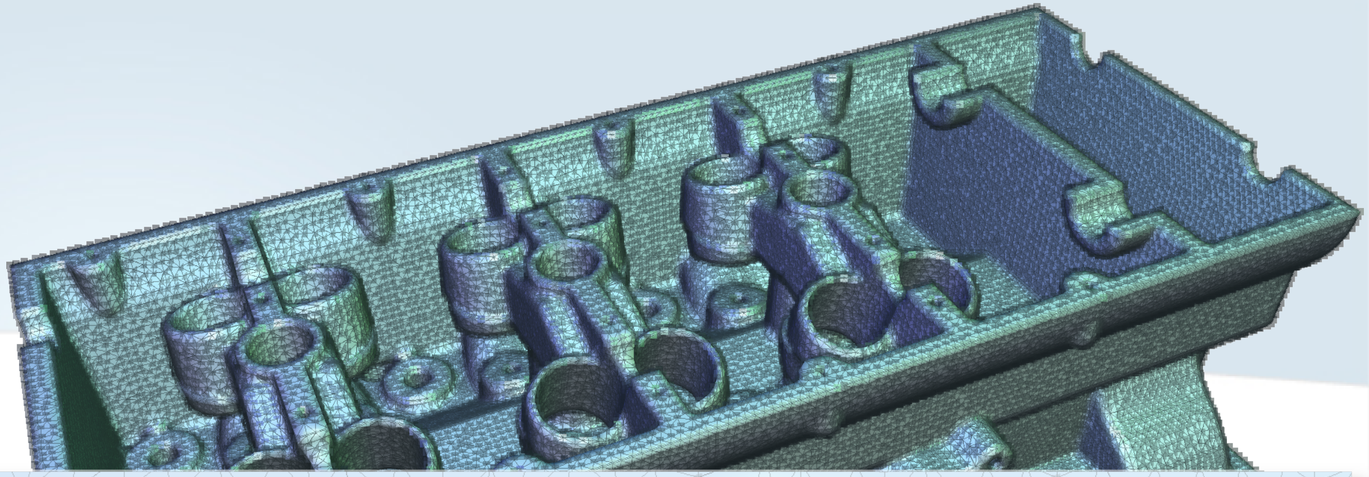
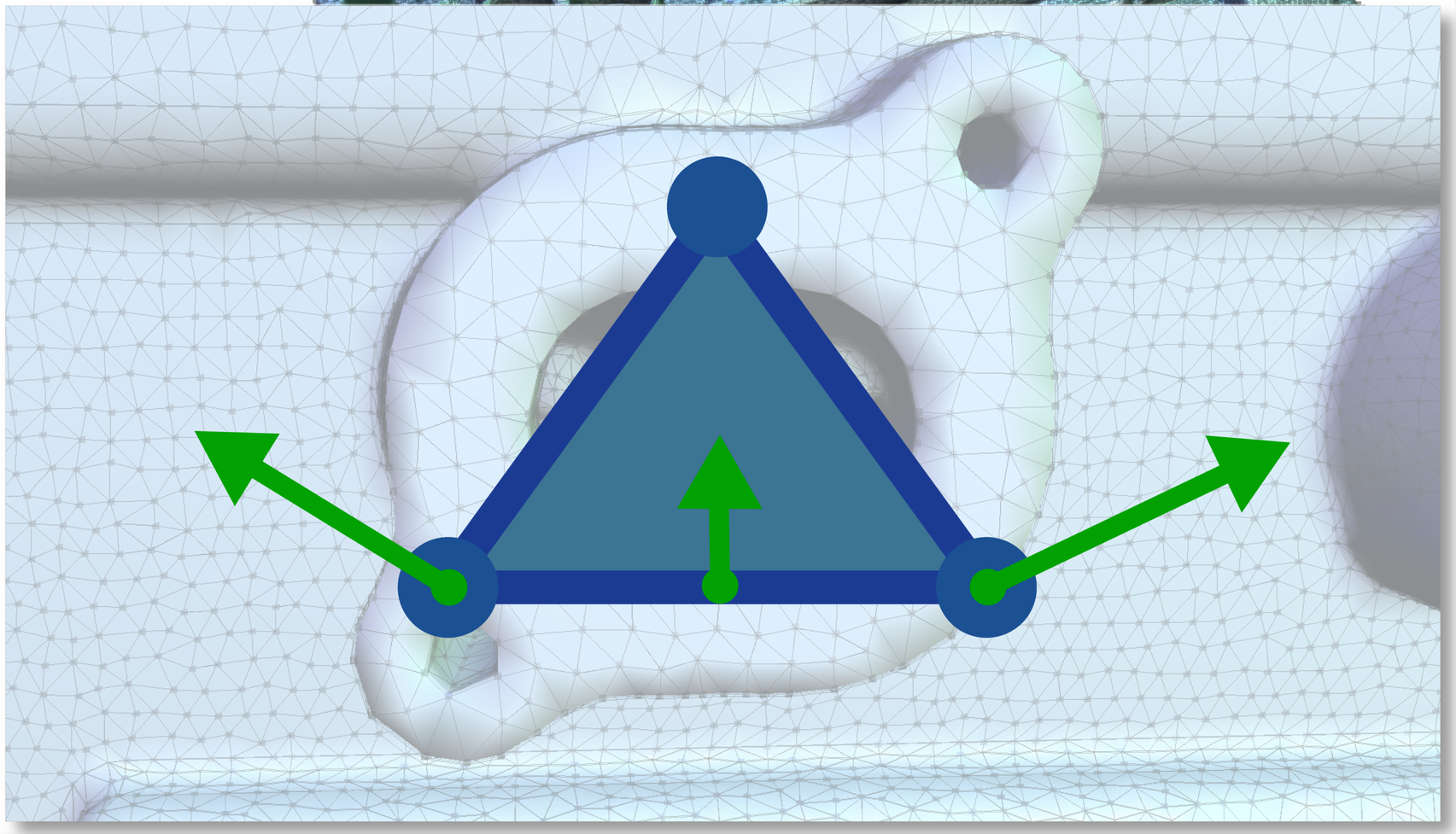


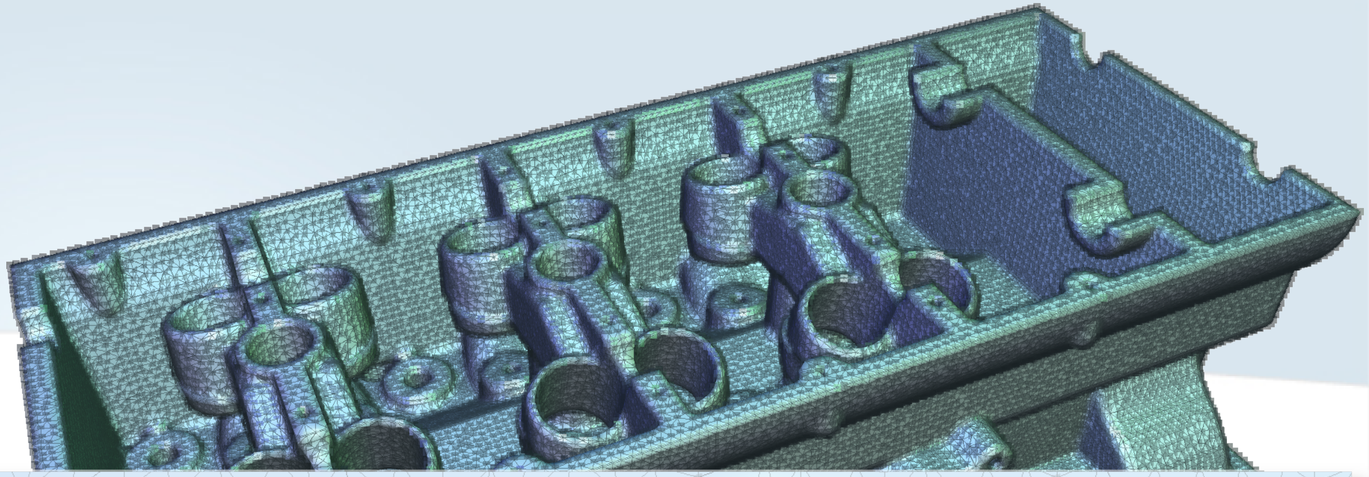
In practice



- Given a domain $\mathcal{D}$
- For each vertex v
- One vector $\vec{f}(v)$
  - Coordinates in $\mathbb{E}$
- Interpolation on the other simplices:
  - Coordinates
  - Magnitude/angle



In practice



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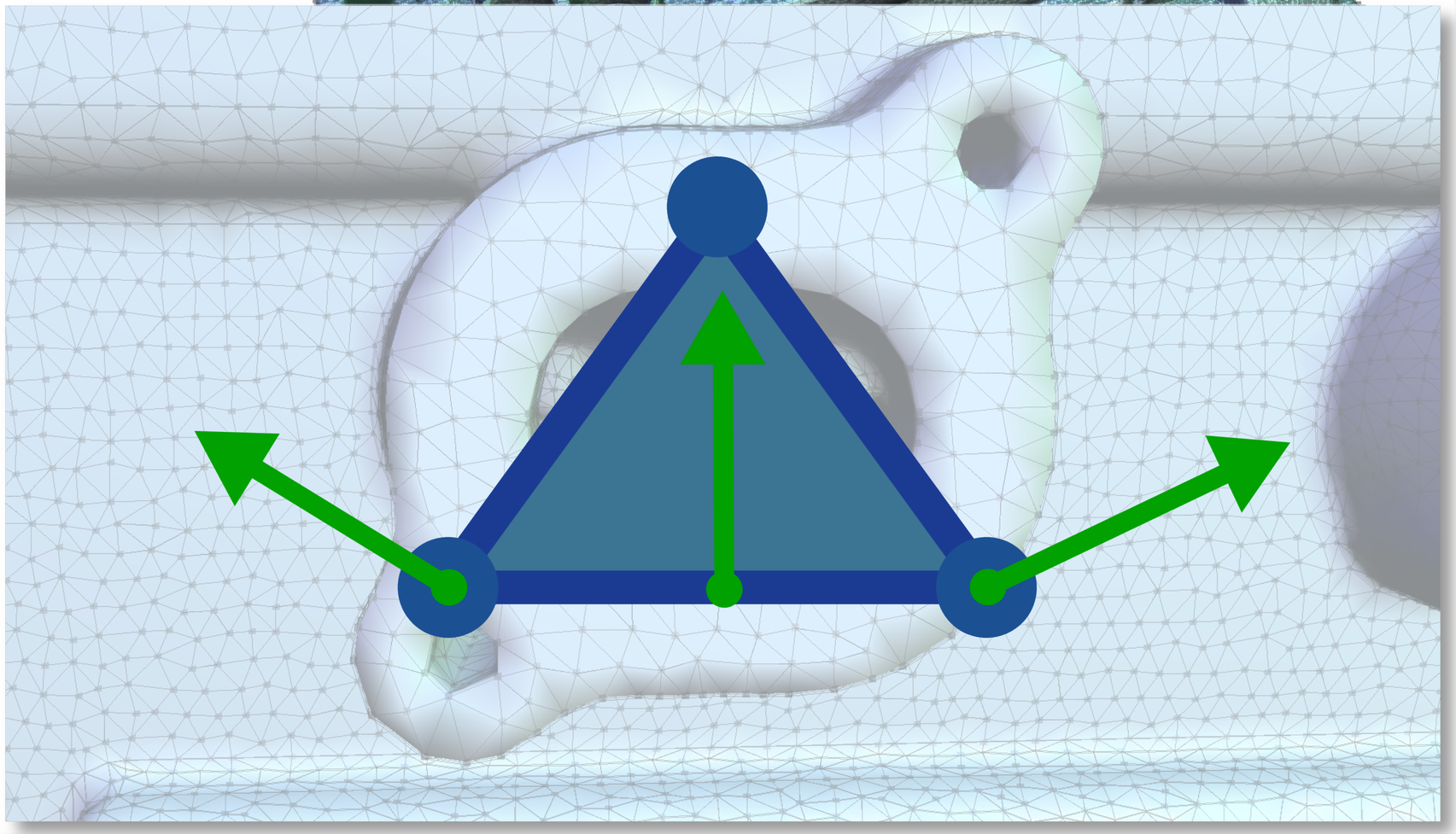
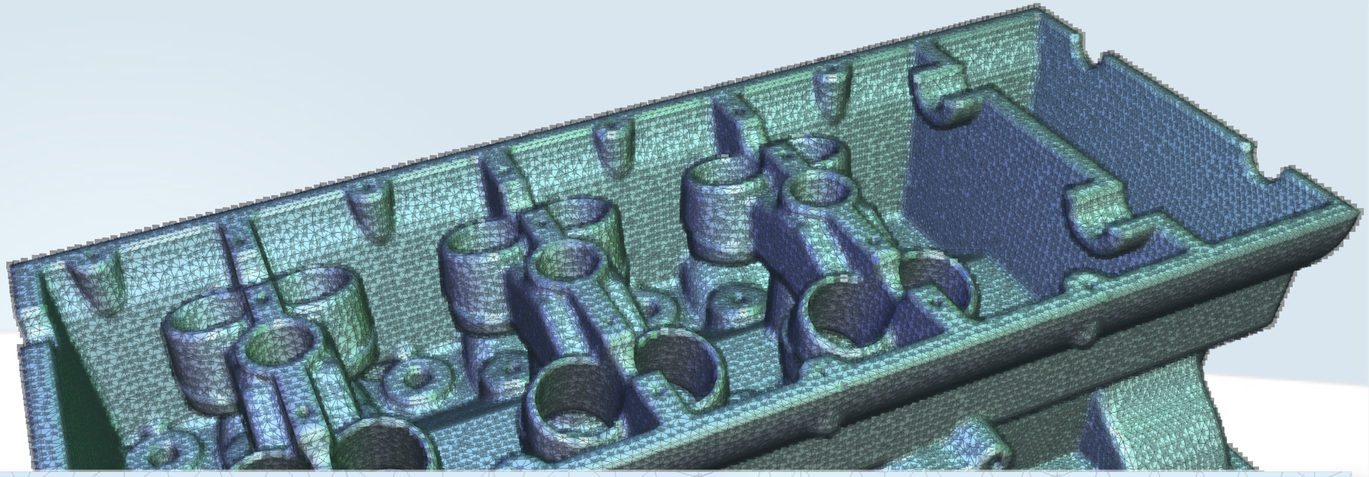


In practice



- Given a domain $\mathcal{D}$
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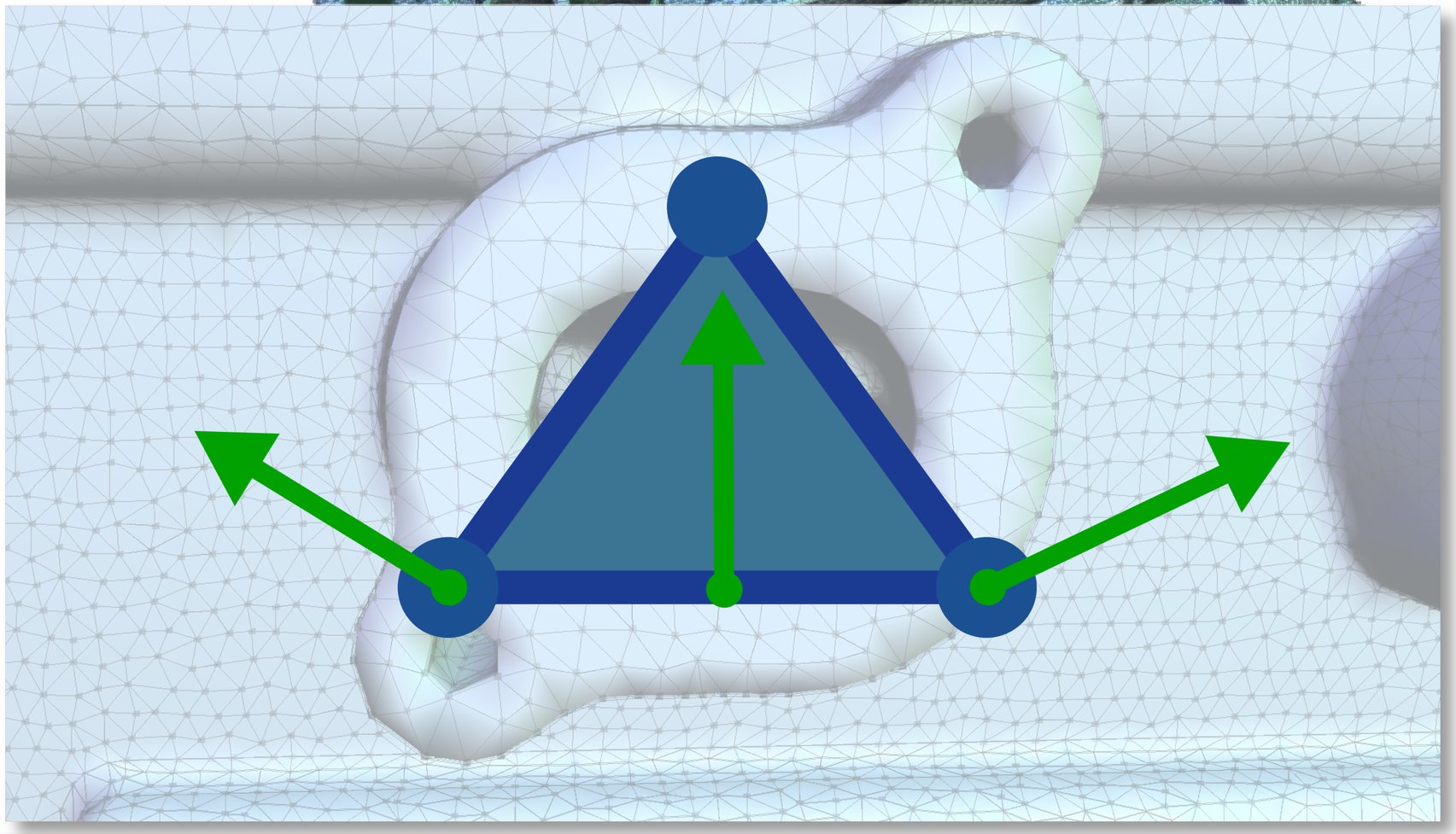
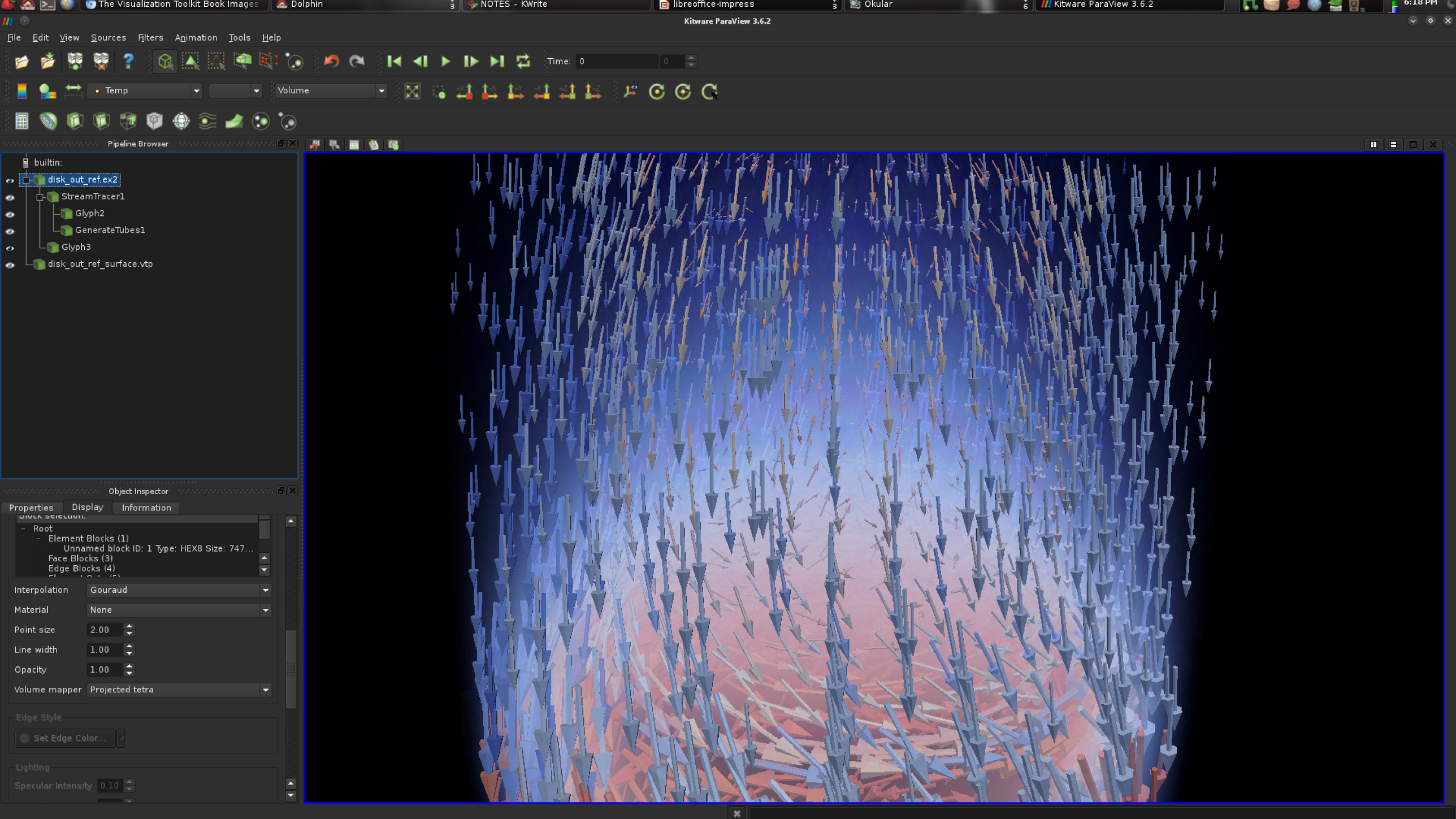
- One vector $\vec{f}(v)$
  - Coordinates in $\mathbb{E}$
- Interpolation on the other simplices:
  - Coordinates
  - **Magnitude/angle**




The Visualization Toolkit Book Images
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NOTES - KWrite
libreoffice-impress
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Kitware ParaView 3.6.2
Kitware ParaView 3.6.2
File Edit View Sources Filters Animation Tools Help
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Edge Style
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Lighting
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What is there to visualize?

- Getting inspiration from engineering sciences

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[www.speedhunter.com]

What is there to visualize?

- Getting inspiration from engineering sciences
  - Localized visualization

[www.speedhunter.com]

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  - Localized visualization
  - Explicit representations

[www.speedhunter.com]

What is there to visualize?

- Getting inspiration from engineering sciences
  - Localized visualization
  - Explicit representations
  - *Stream lines and surfaces*

[www.speedhunter.com]

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- Analogy to scalar fields:
  - Isocontours
  - Isosurfaces

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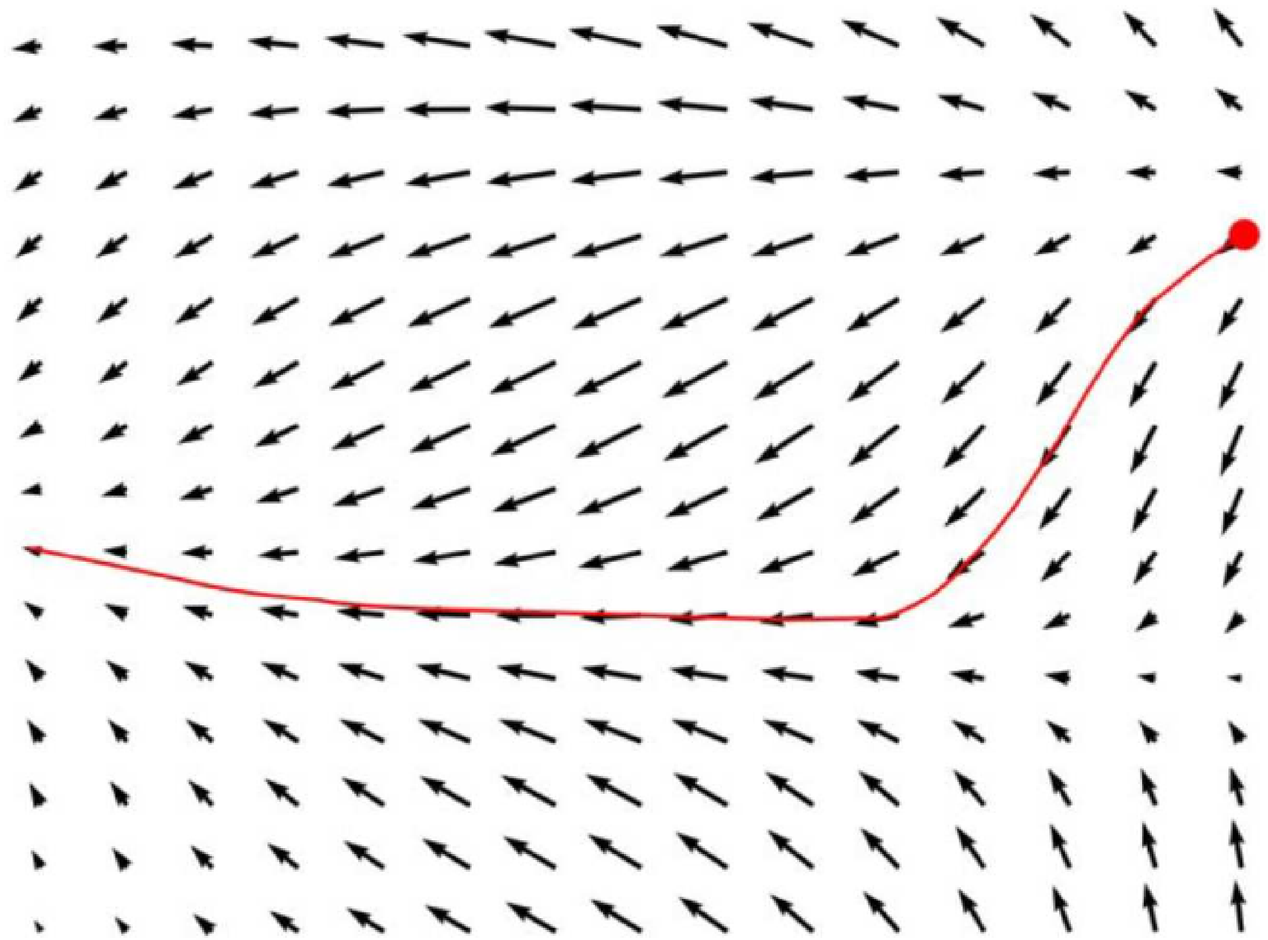
Streamlines

- What is there to visualize?
  - Integral curves
  - “Streamlines”



[Chen]

Streamlines

- What is there to visualize?
  - Integral curves
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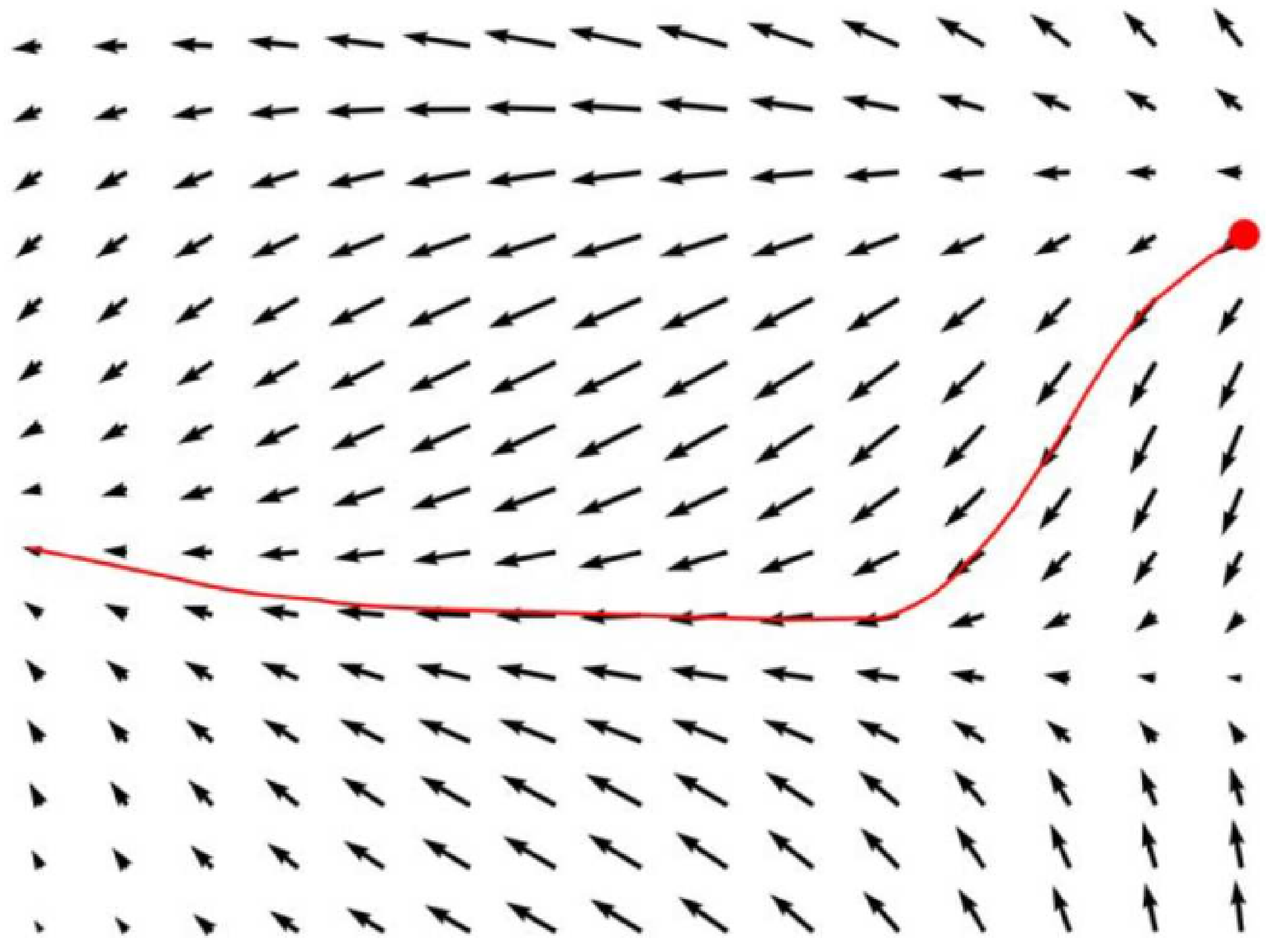
- Curve $\mathcal{C} \in \mathcal{D}$ such that:



[Chen]

Streamlines

- What is there to visualize?
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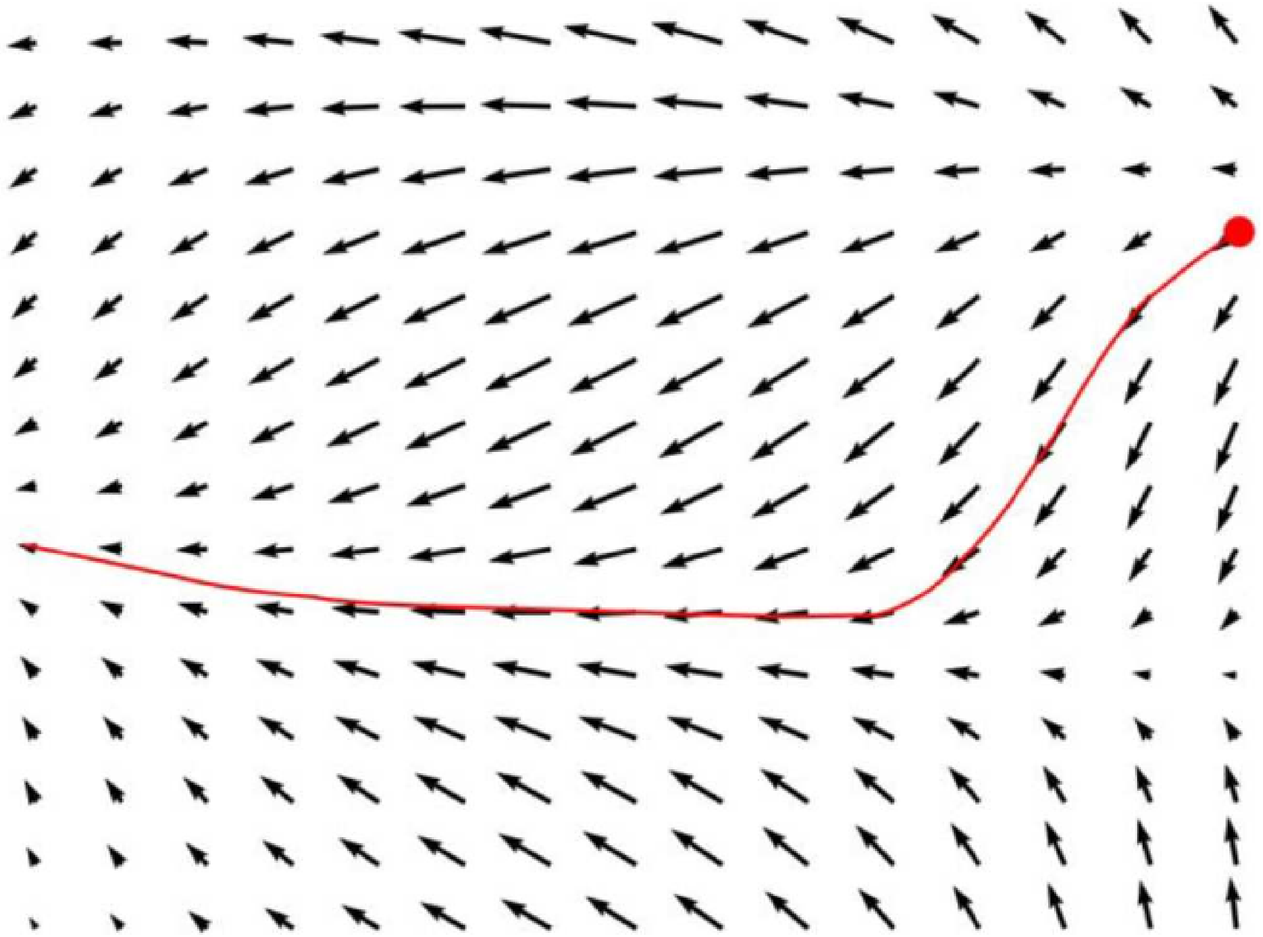
- Curve $\mathcal{C} \in \mathcal{D}$ such that:
  - Given a bijection
    - $c : \mathcal{C} \rightarrow [0, 1]$



[Chen]

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- Curve $\mathcal{C} \in \mathcal{D}$ such that:
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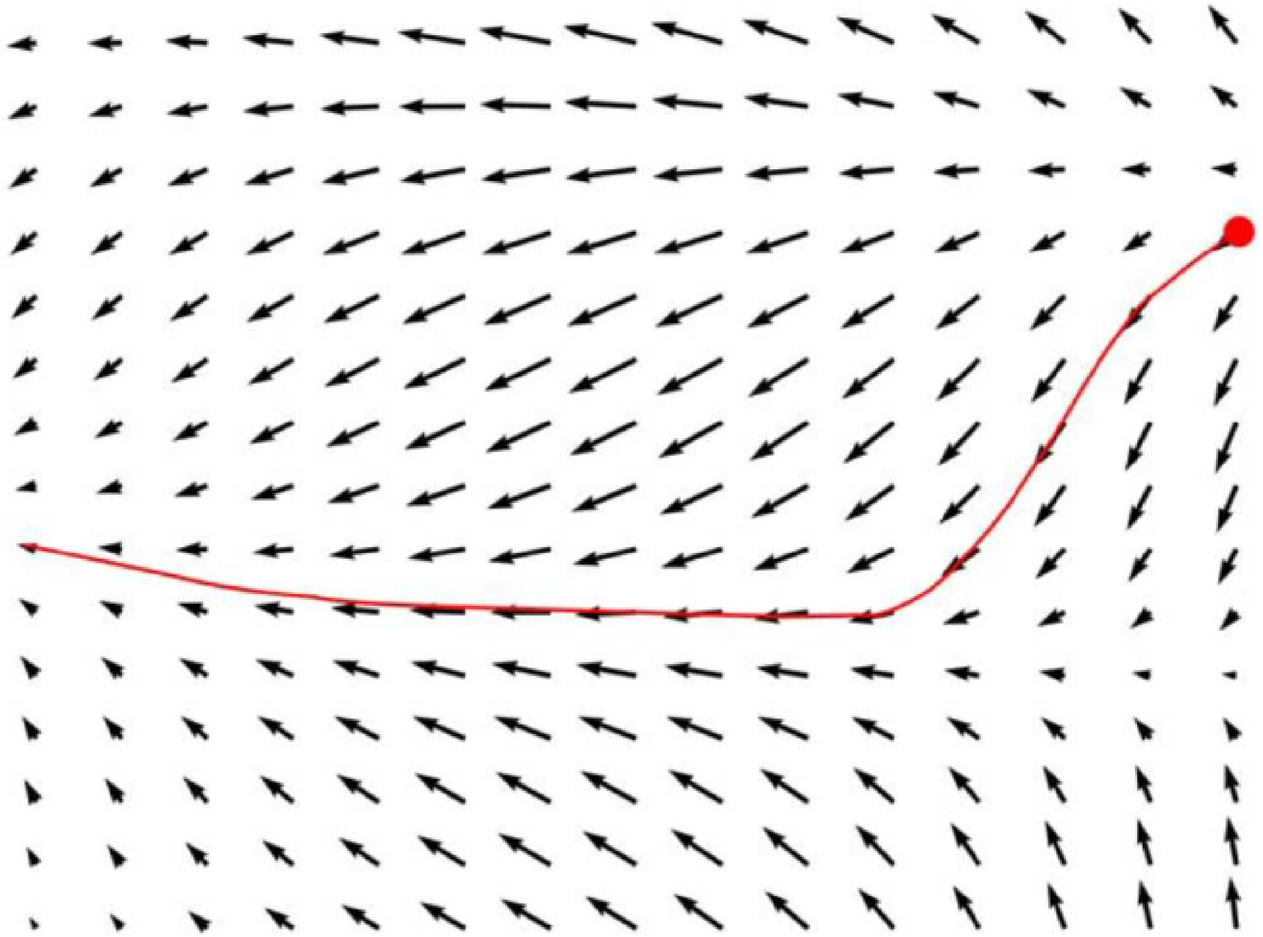

[Chen]

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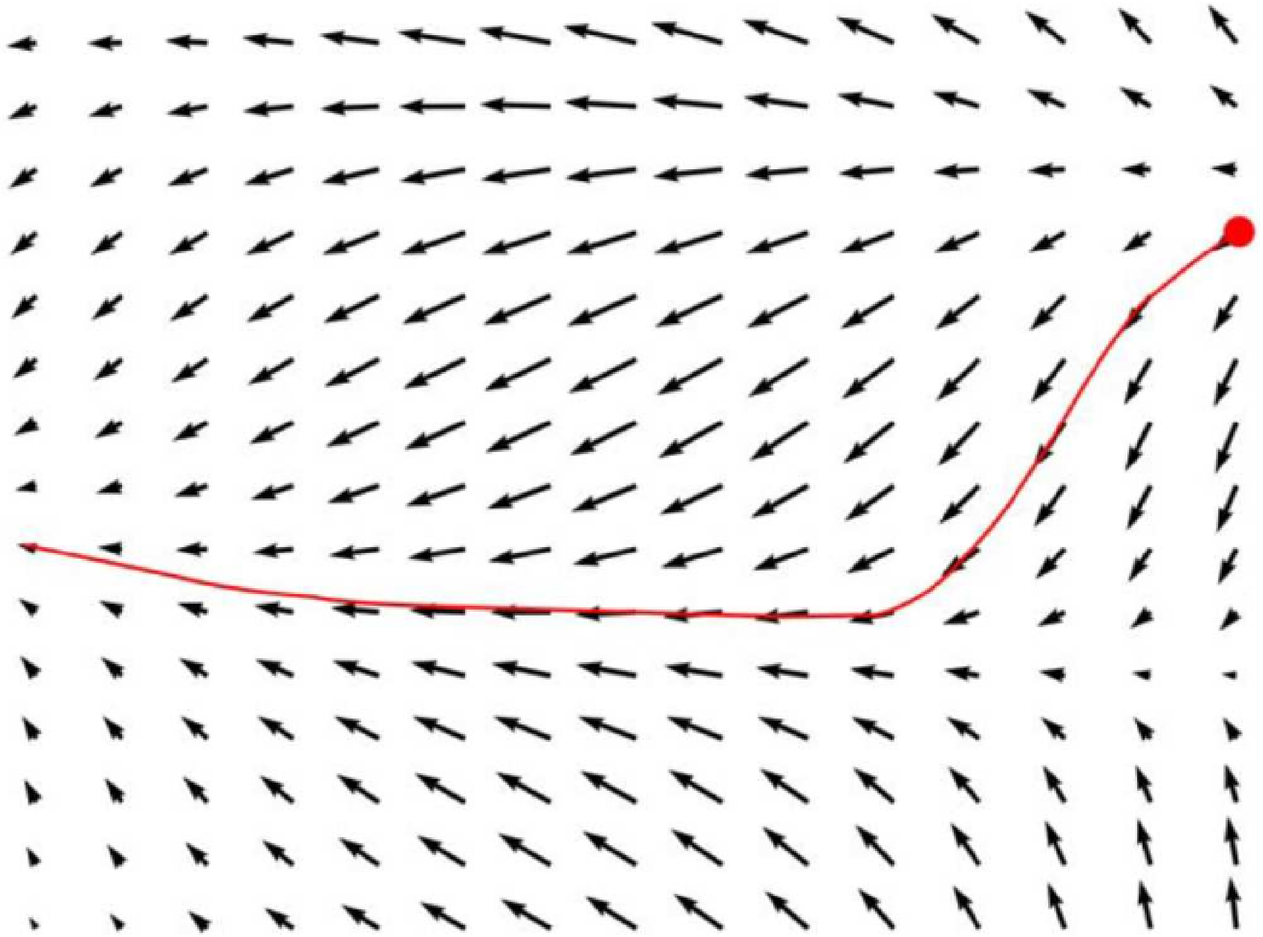
    - $c : \mathcal{C} \rightarrow [0, 1]$
  - $\frac{\partial p}{\partial c} \times \vec{f}(p) = \vec{0}, \quad \forall p \in \mathcal{C}$
- Everywhere tangential to the flow



[Chen]

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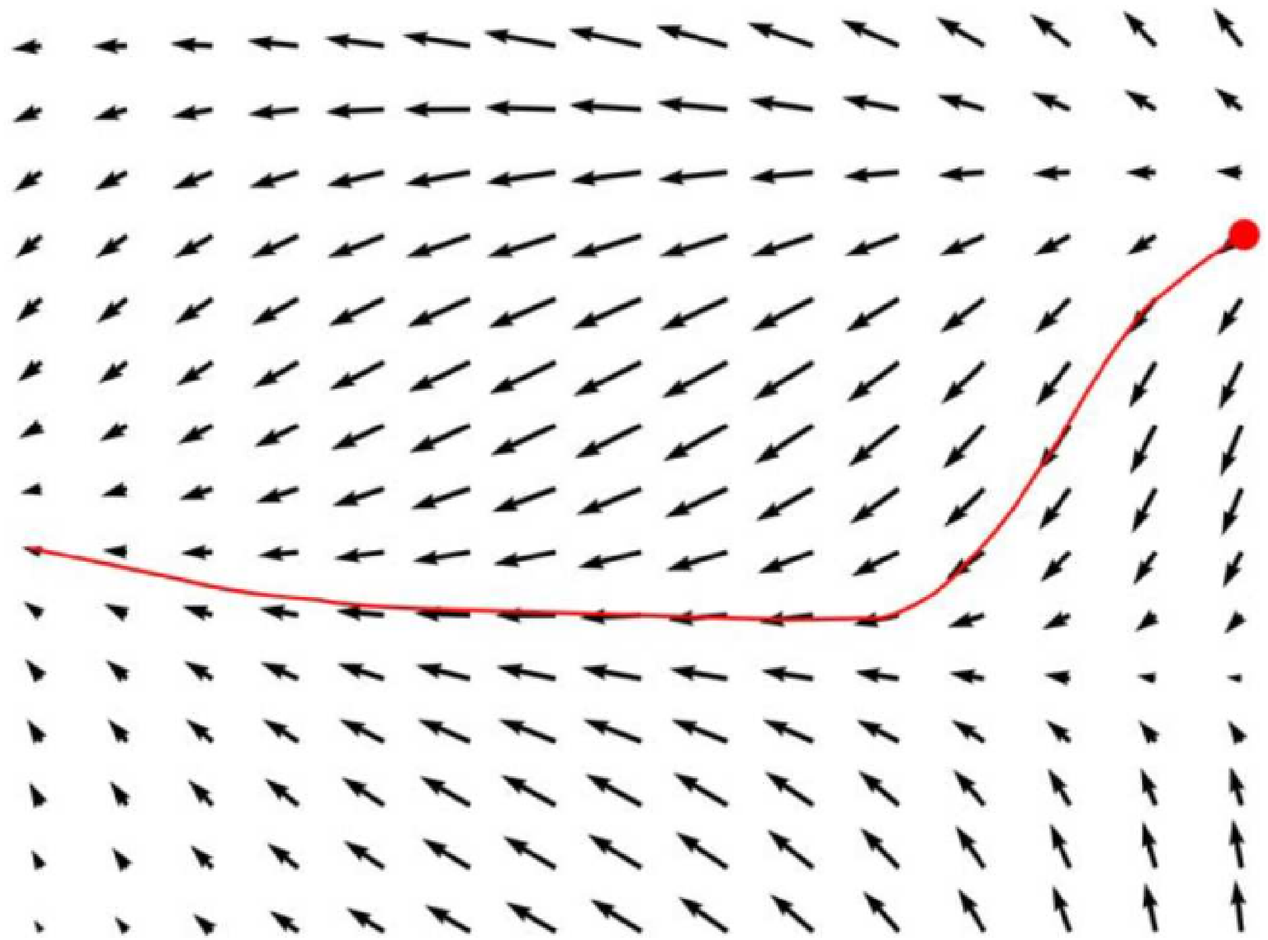

[Chen]

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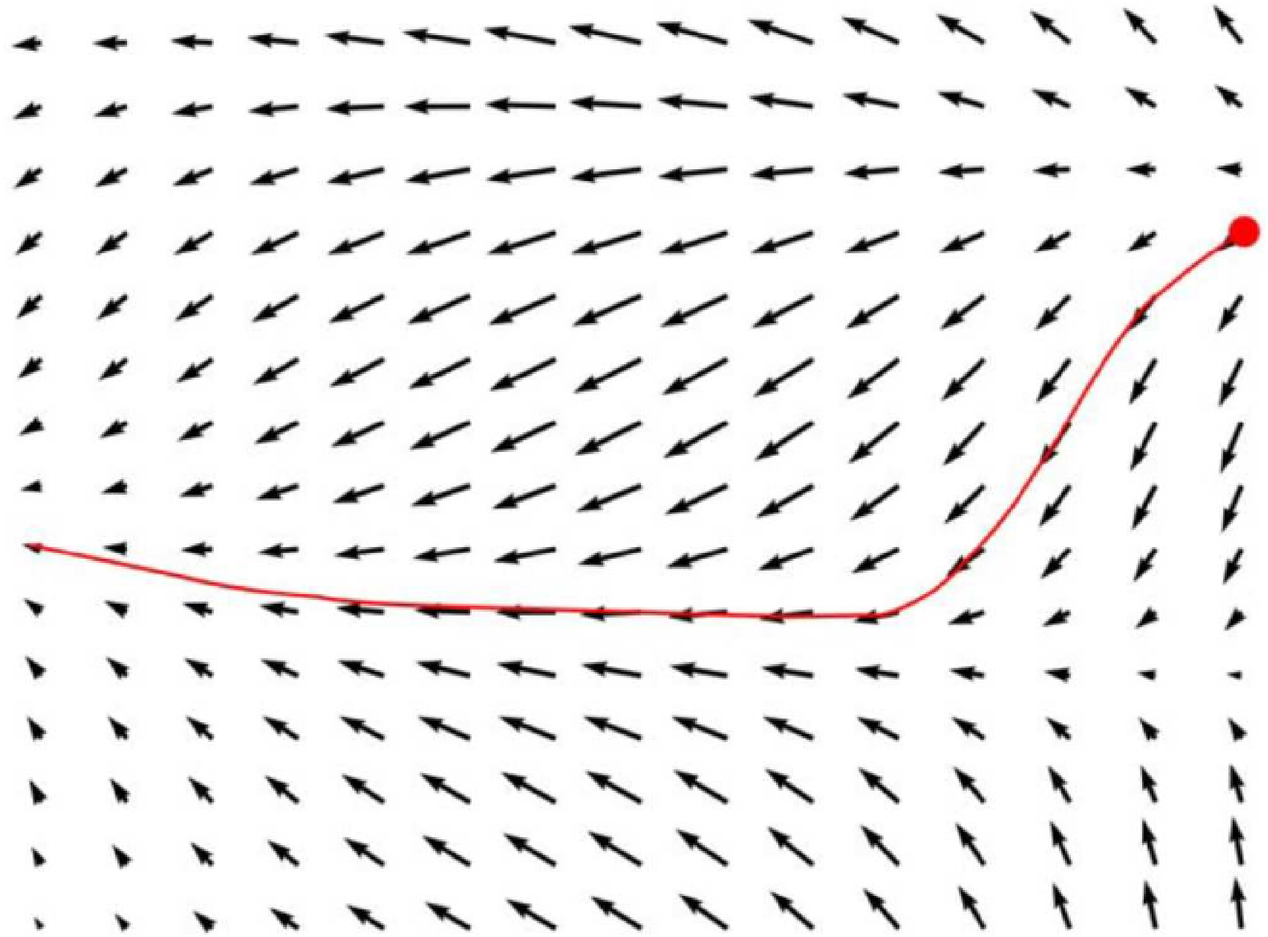

[Chen]

Streamlines

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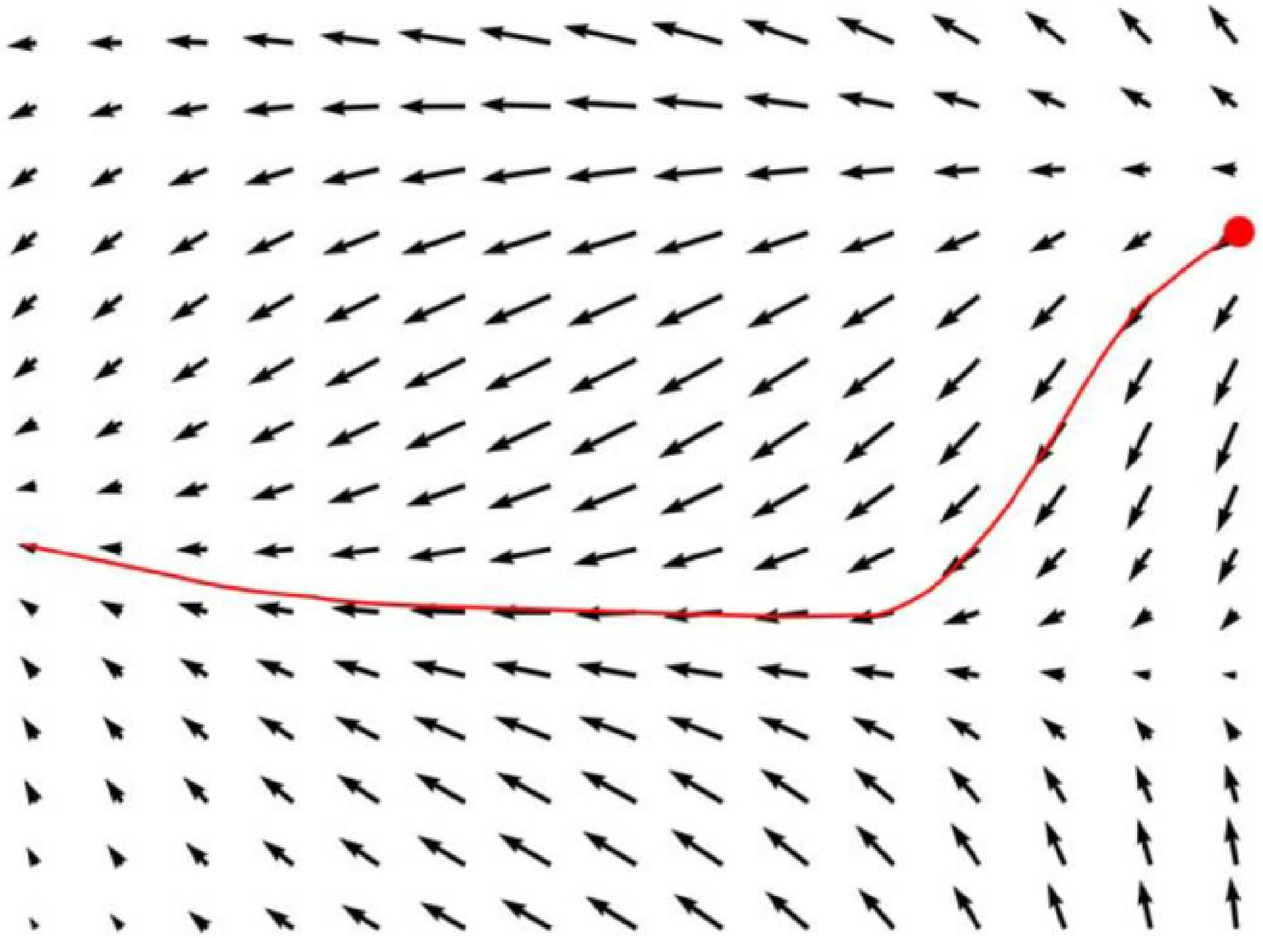
- Solution to an ODE
  - $c : \mathcal{C} \rightarrow [0, 1]$
  - $p = c^{-1}(0) + \int_0^{c(p)} \vec{f}(c^{-1}(u))du$



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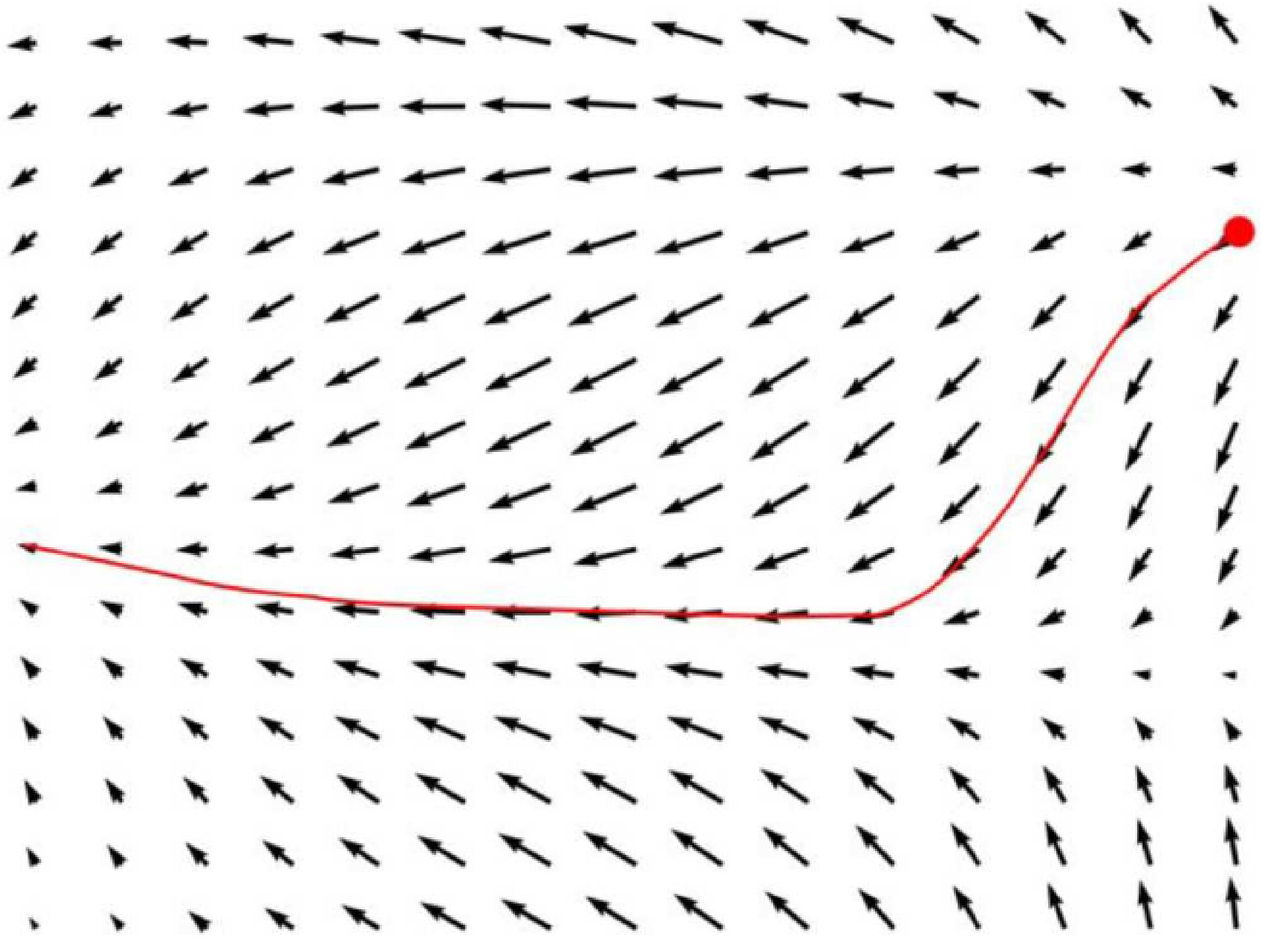

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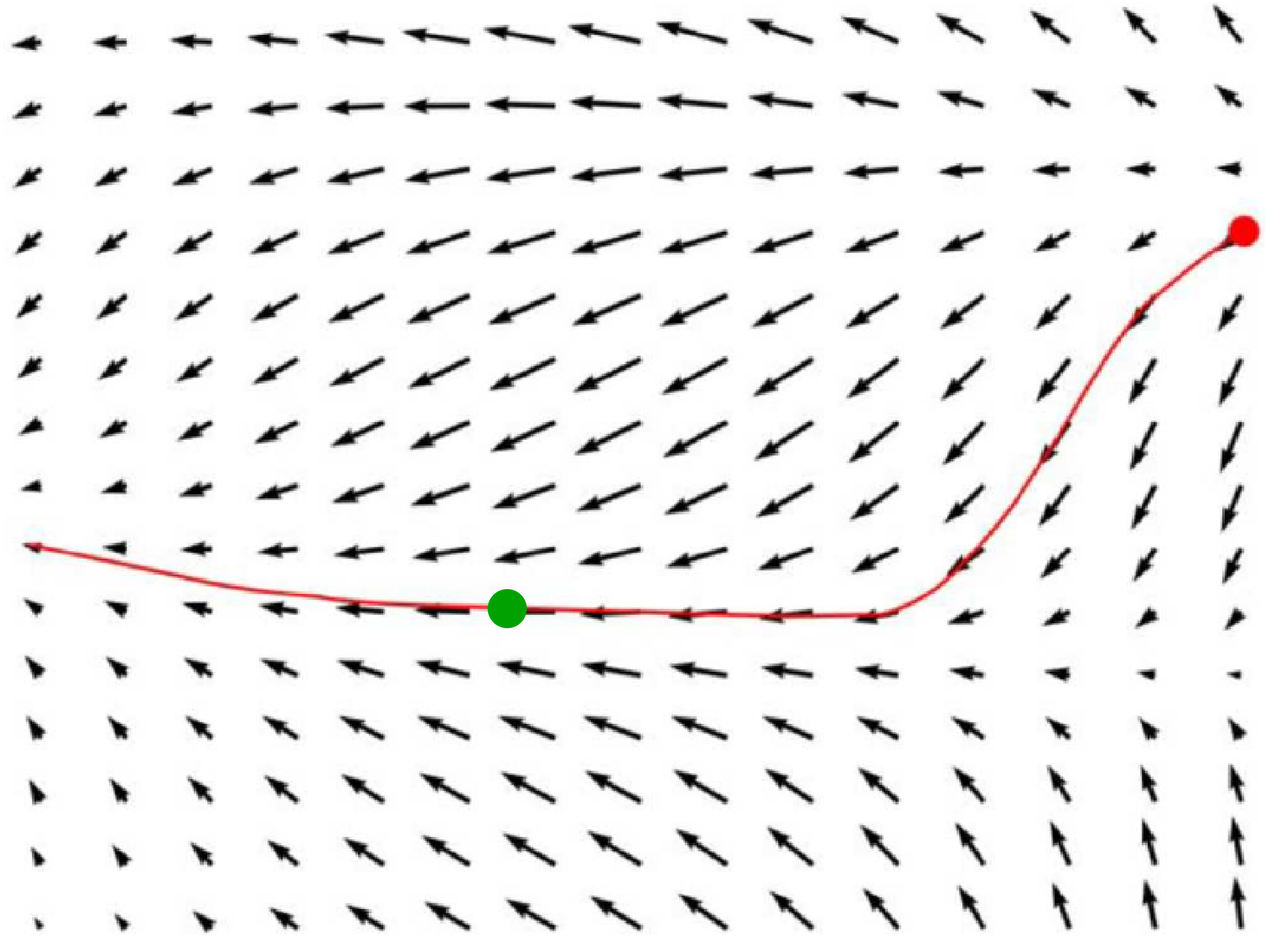

[Chen]

Streamlines

- What is there to visualize?
  - Integral curves
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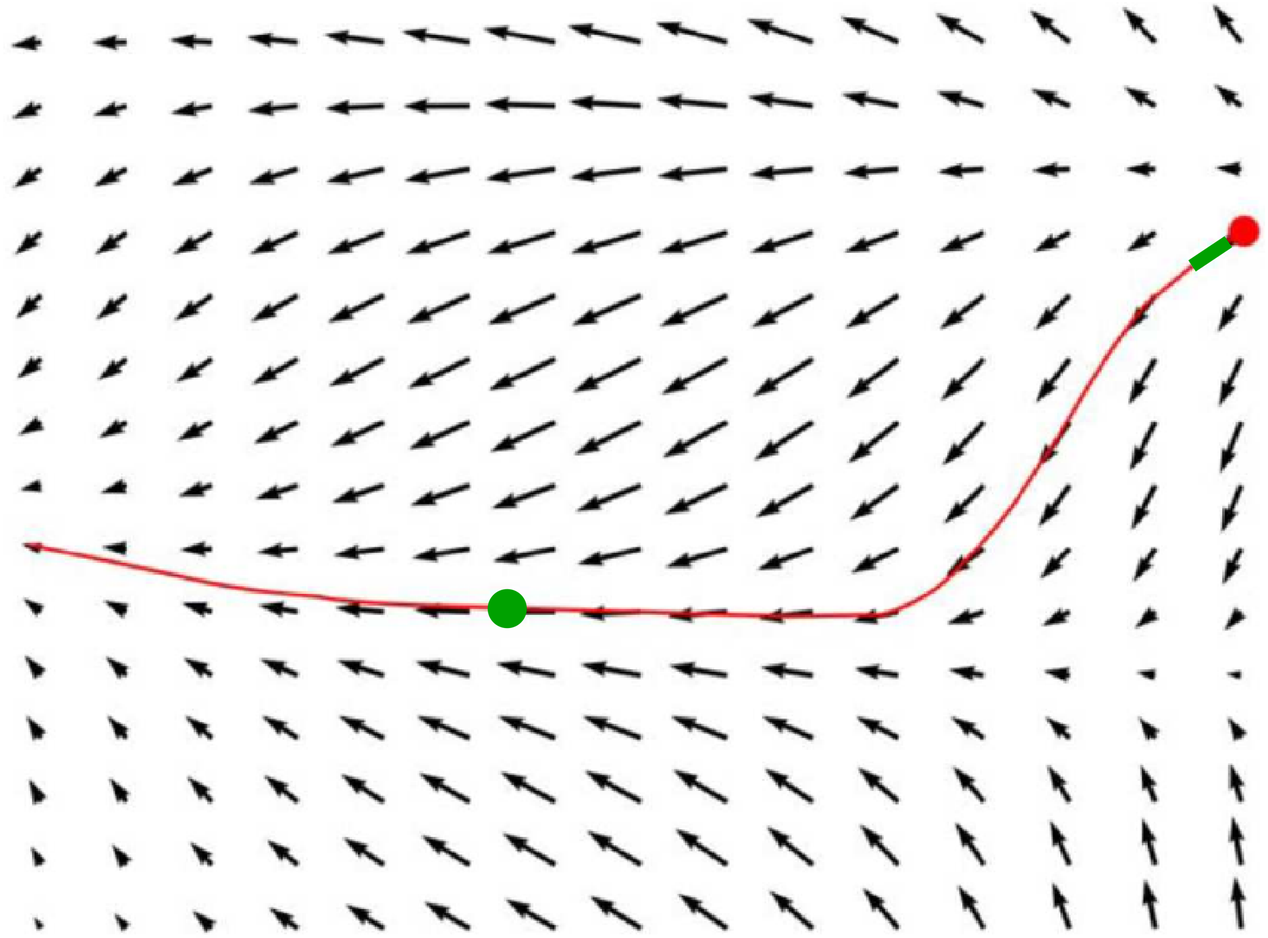

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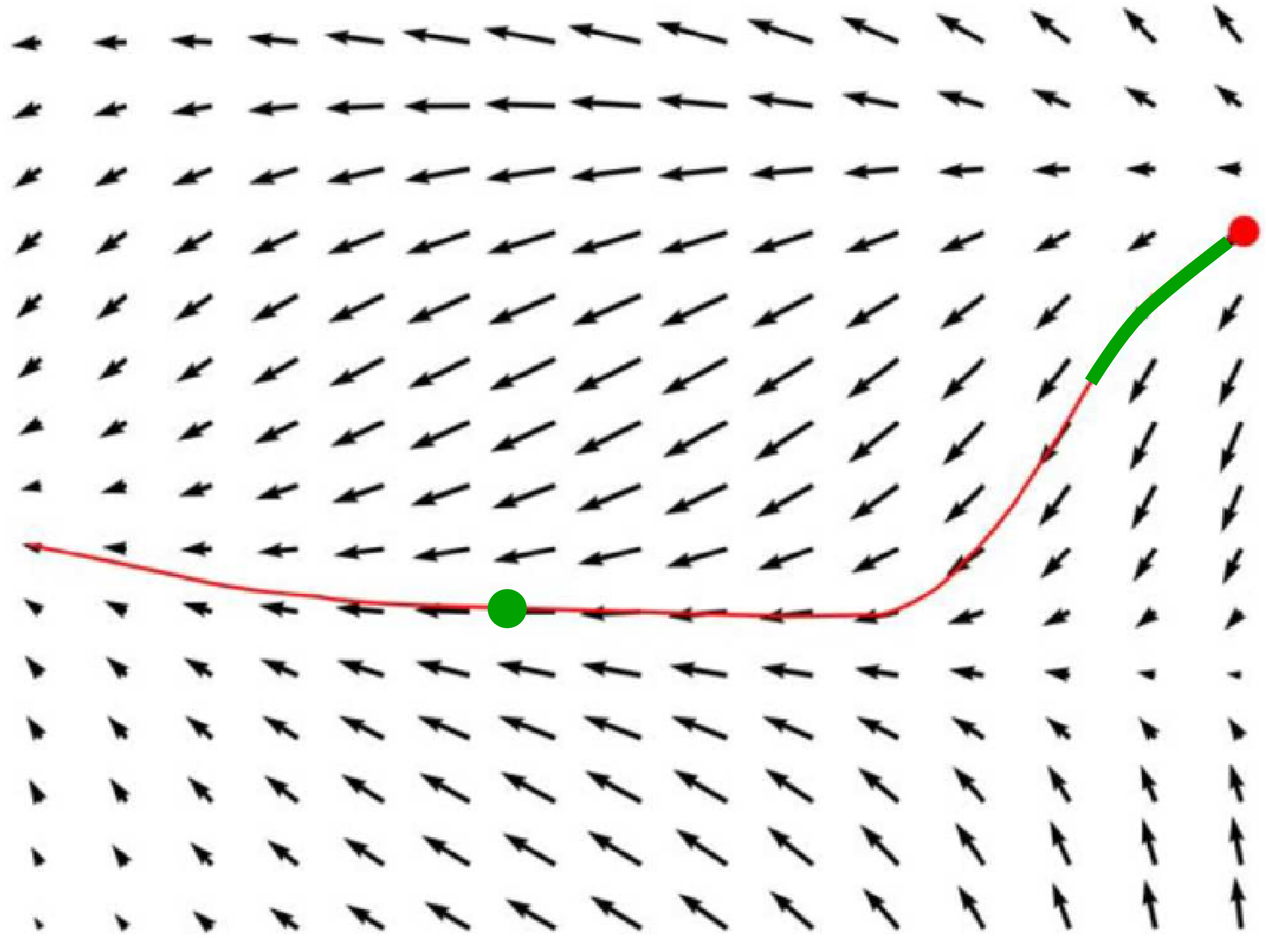

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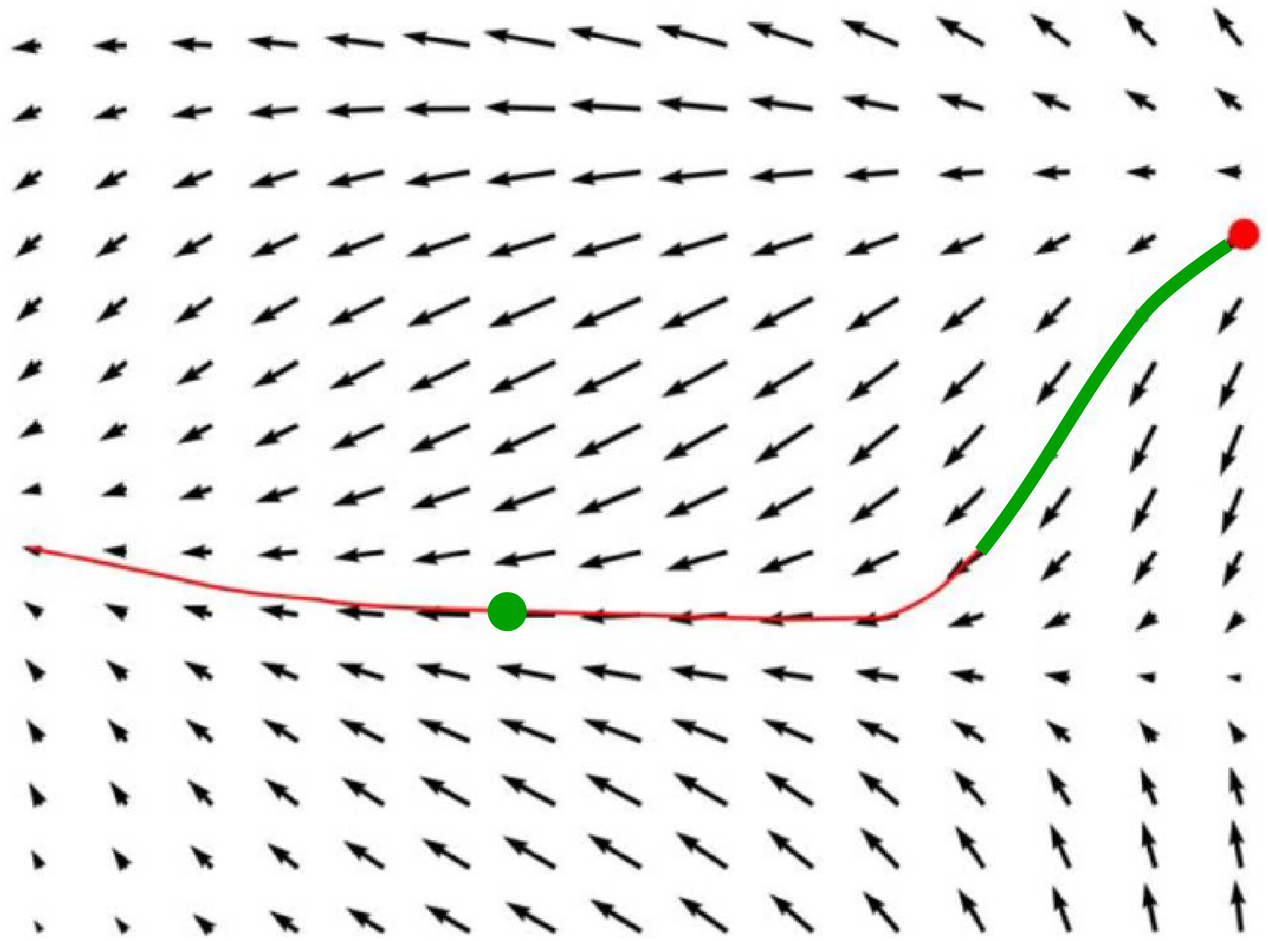

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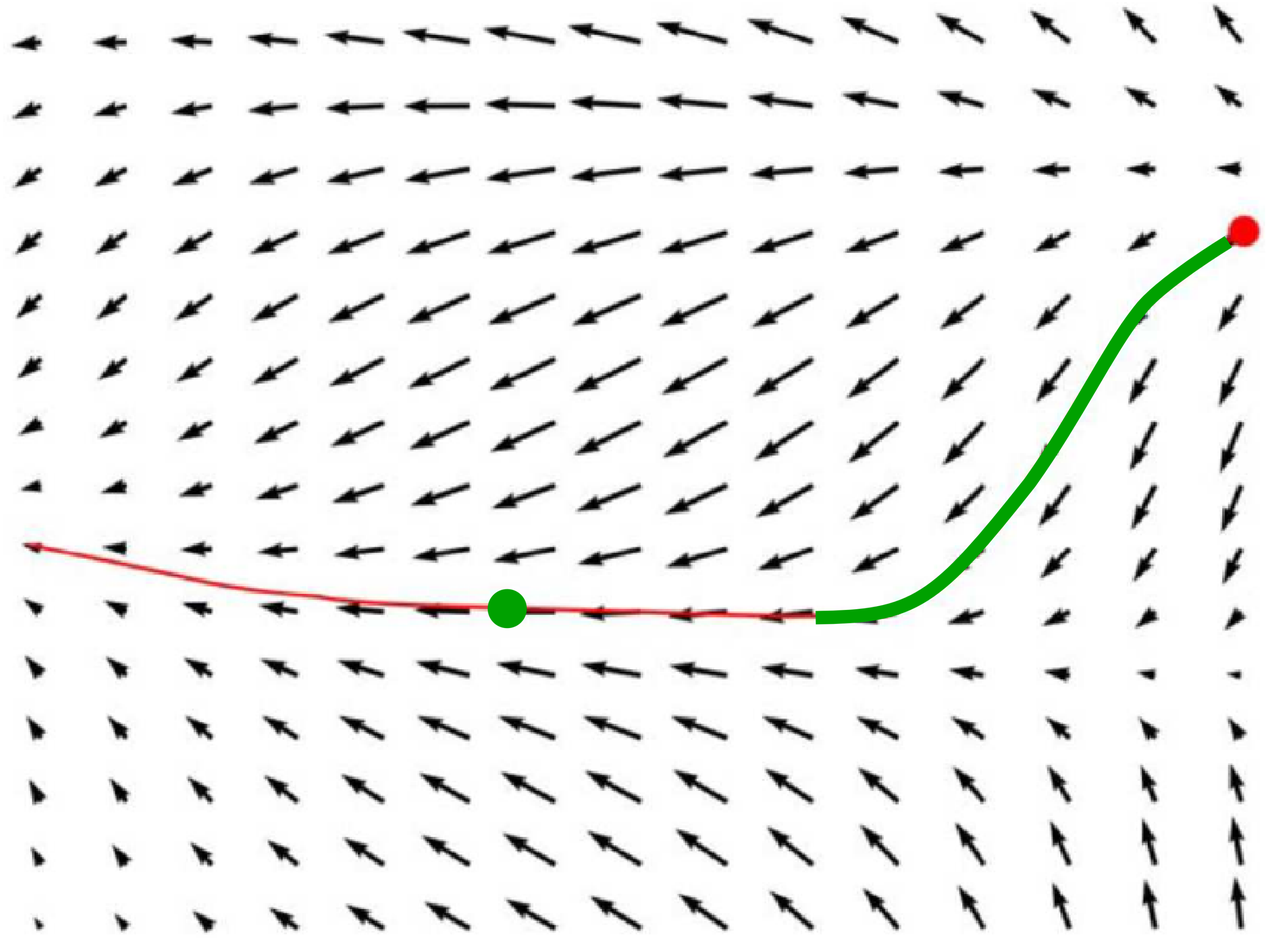

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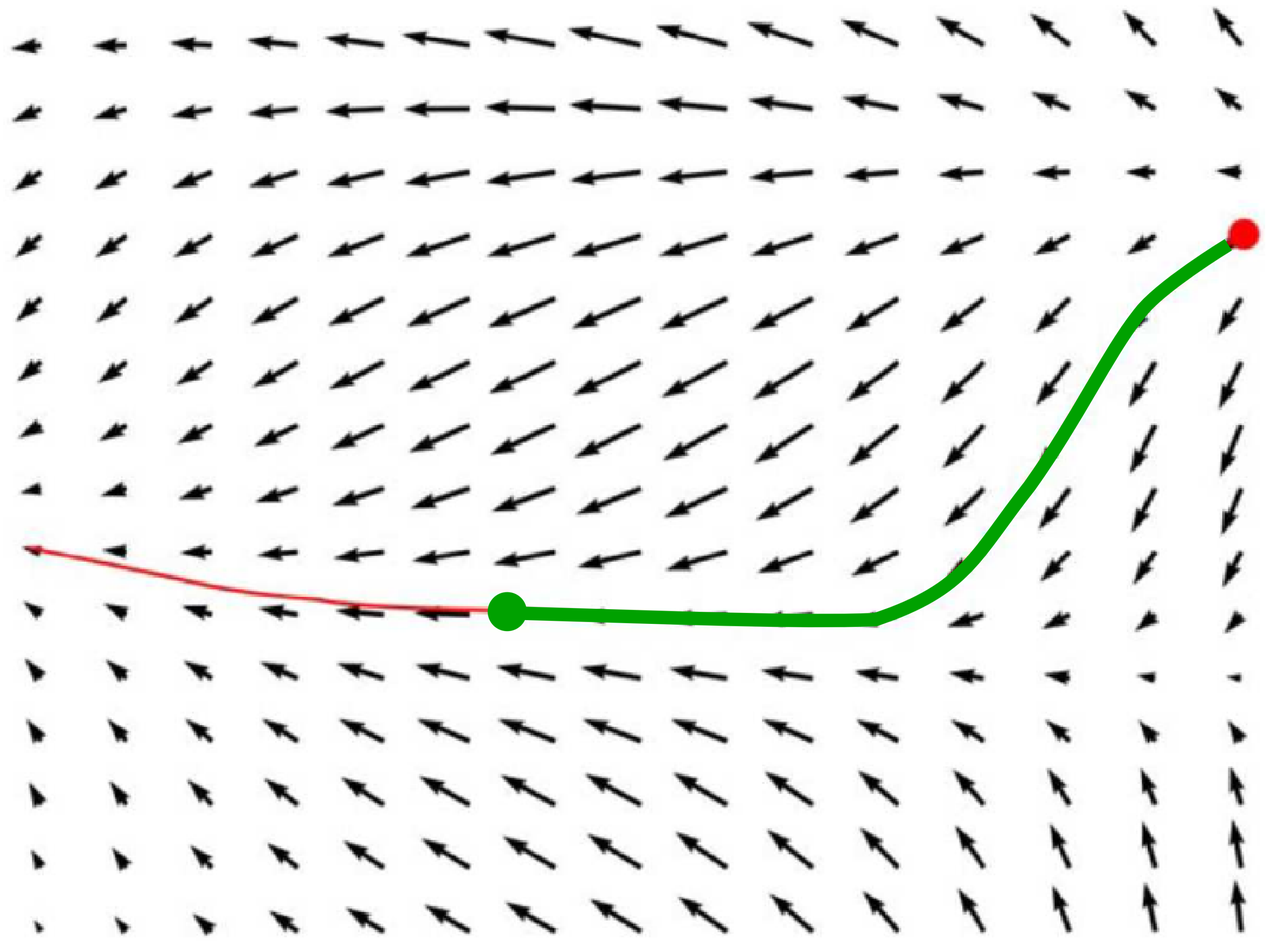

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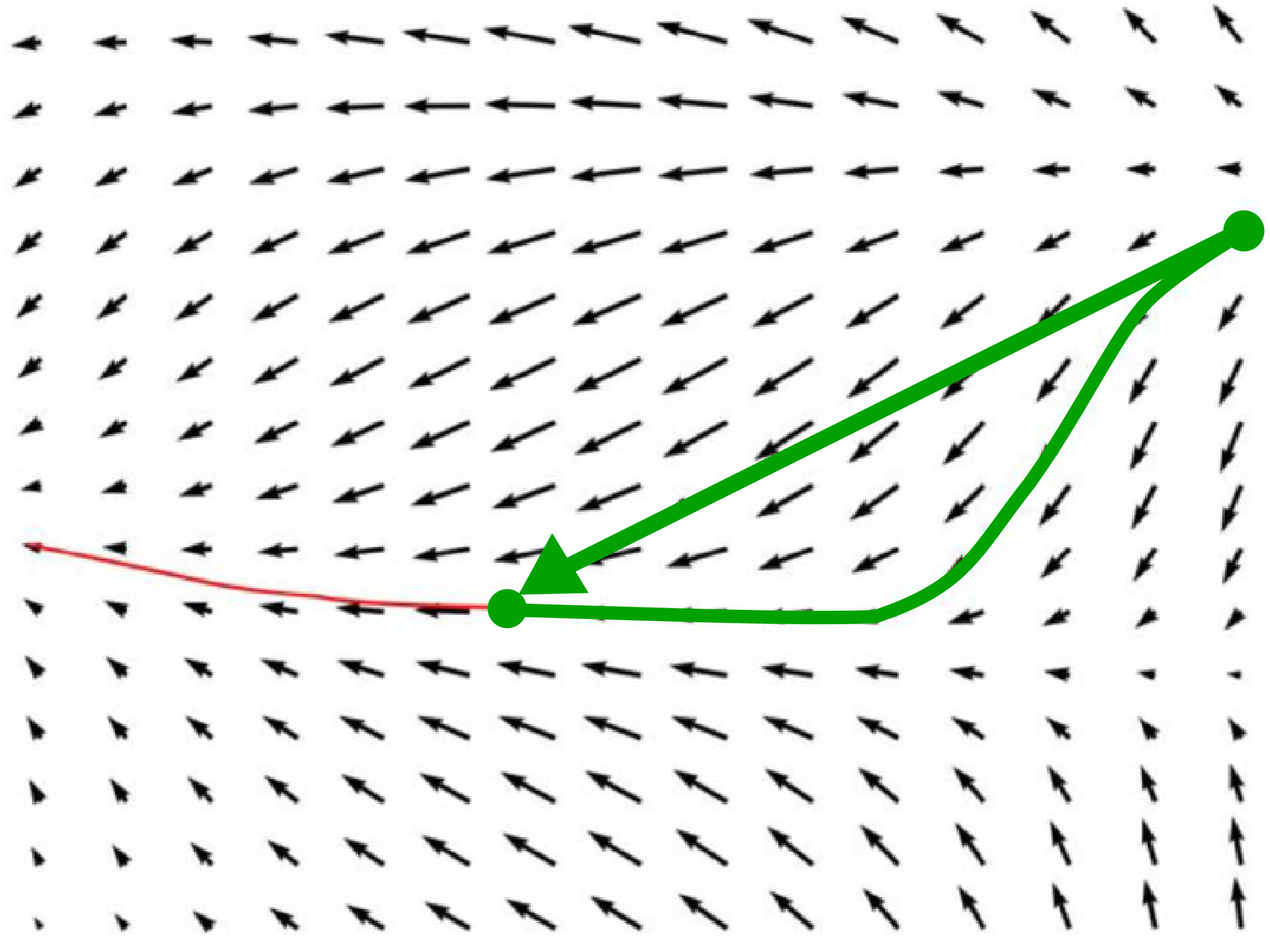

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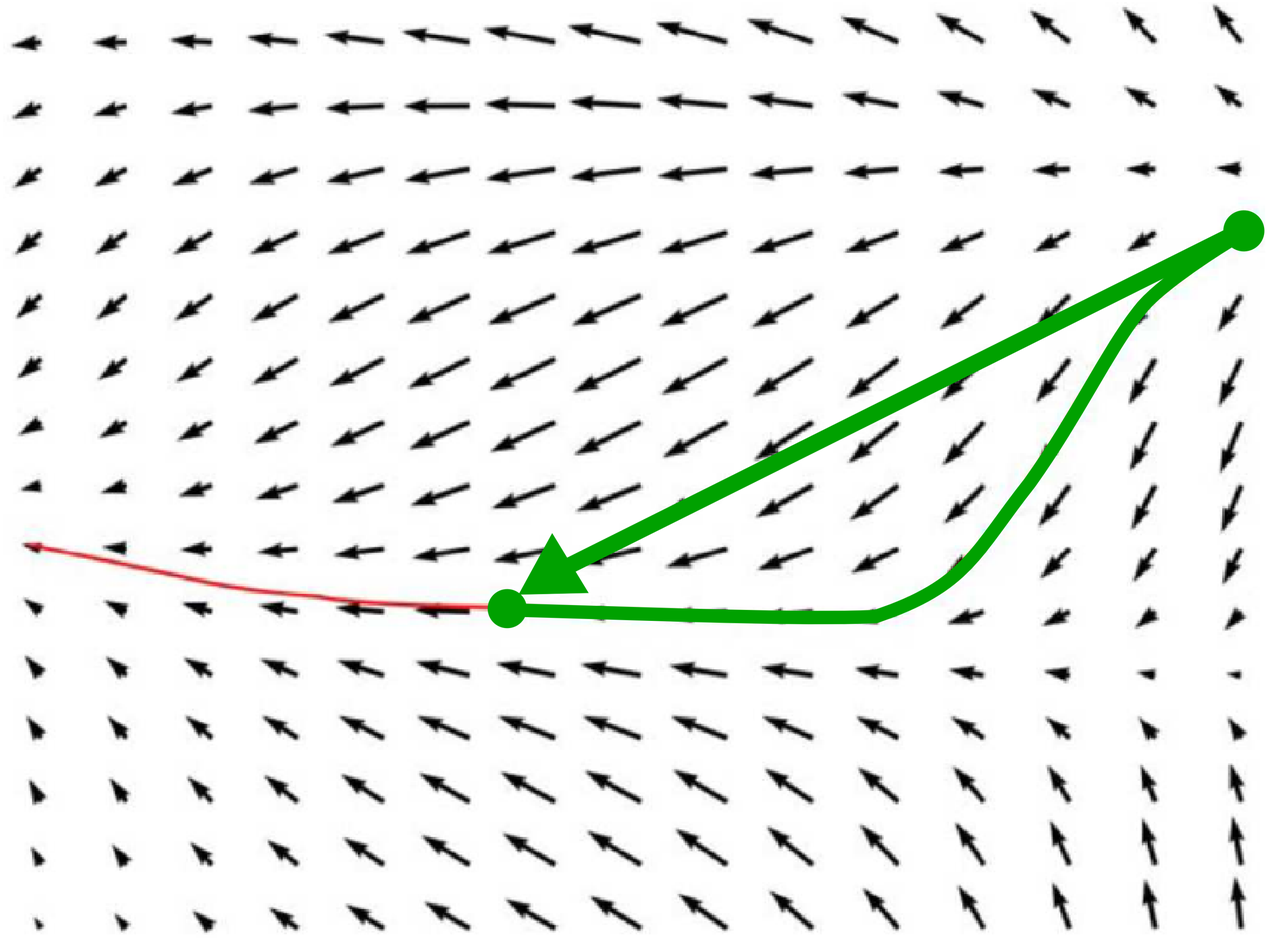
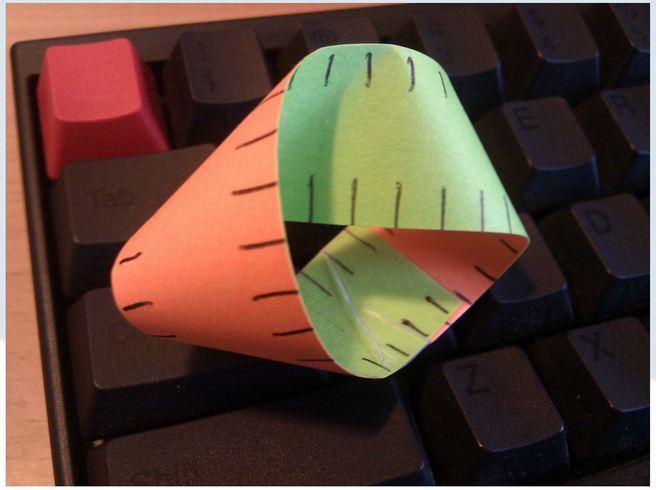
  - Unique solution



[Chen]

Streamlines on a computer



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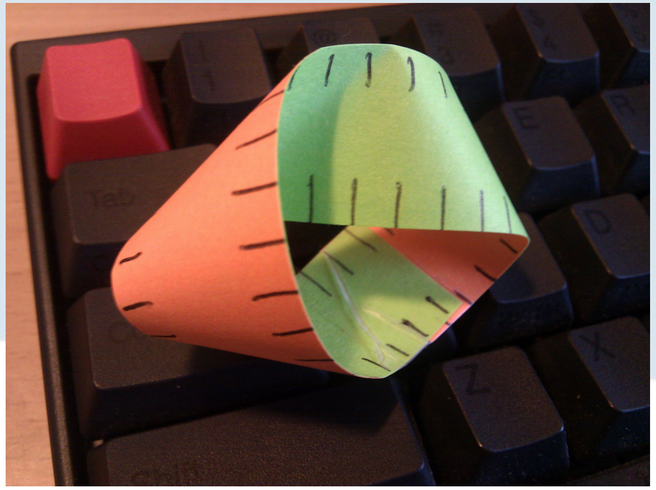
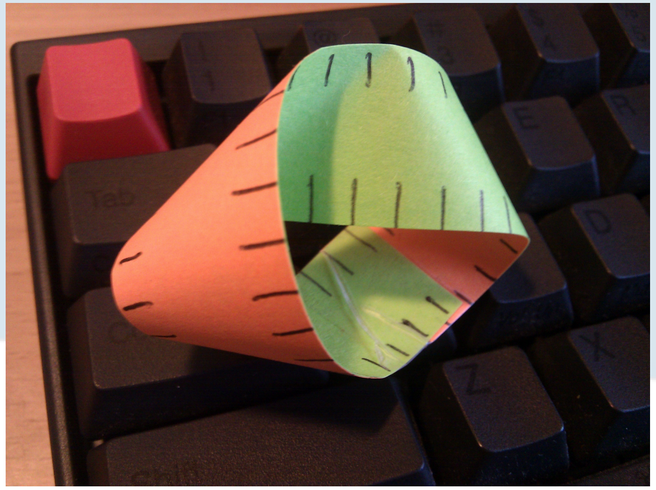


- Numerical integration of the ODE

Streamlines on a computer



- Numerical integration of the ODE
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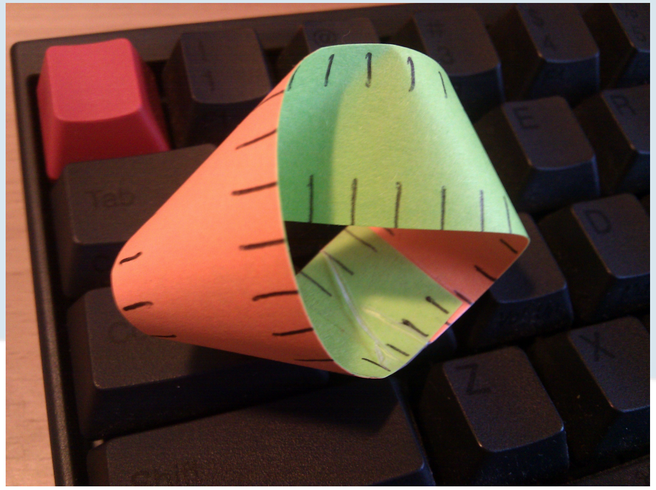
Streamlines on a computer



- Numerical integration of the ODE
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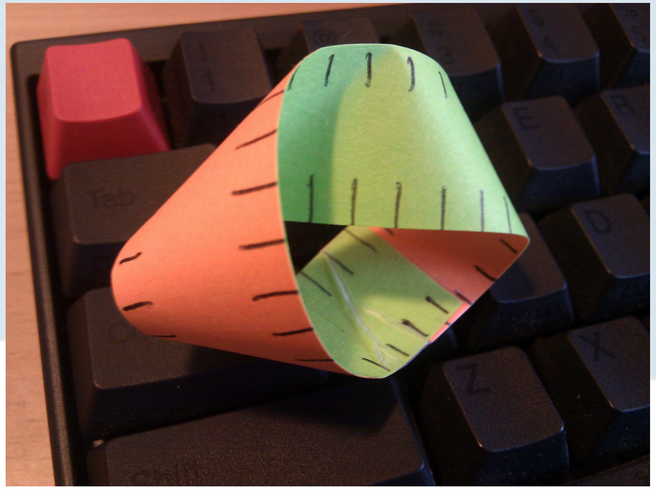
Streamlines on a computer



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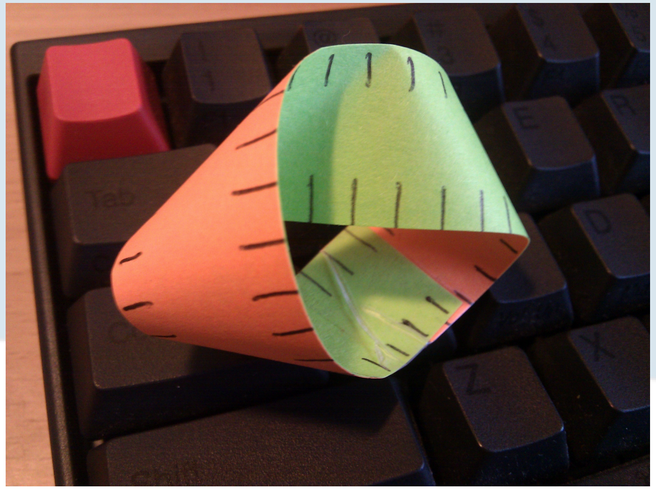
Streamlines on a computer



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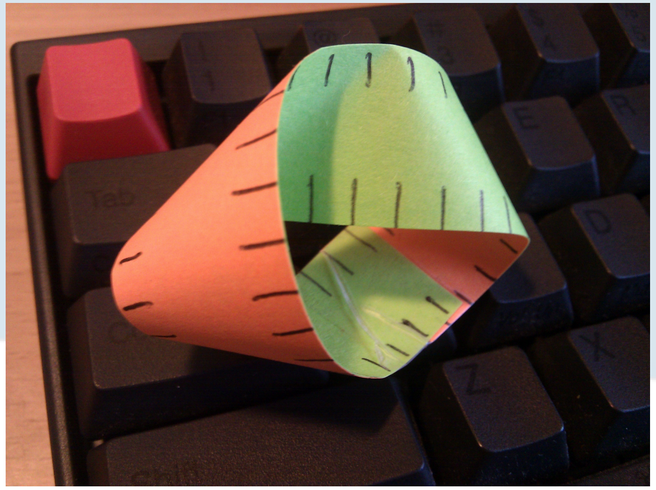

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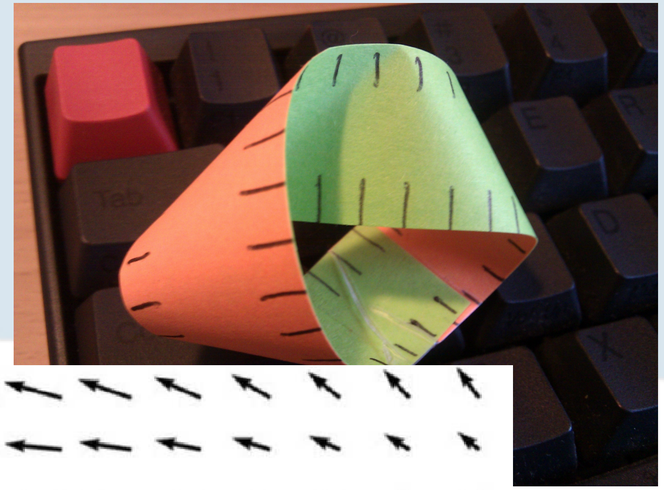

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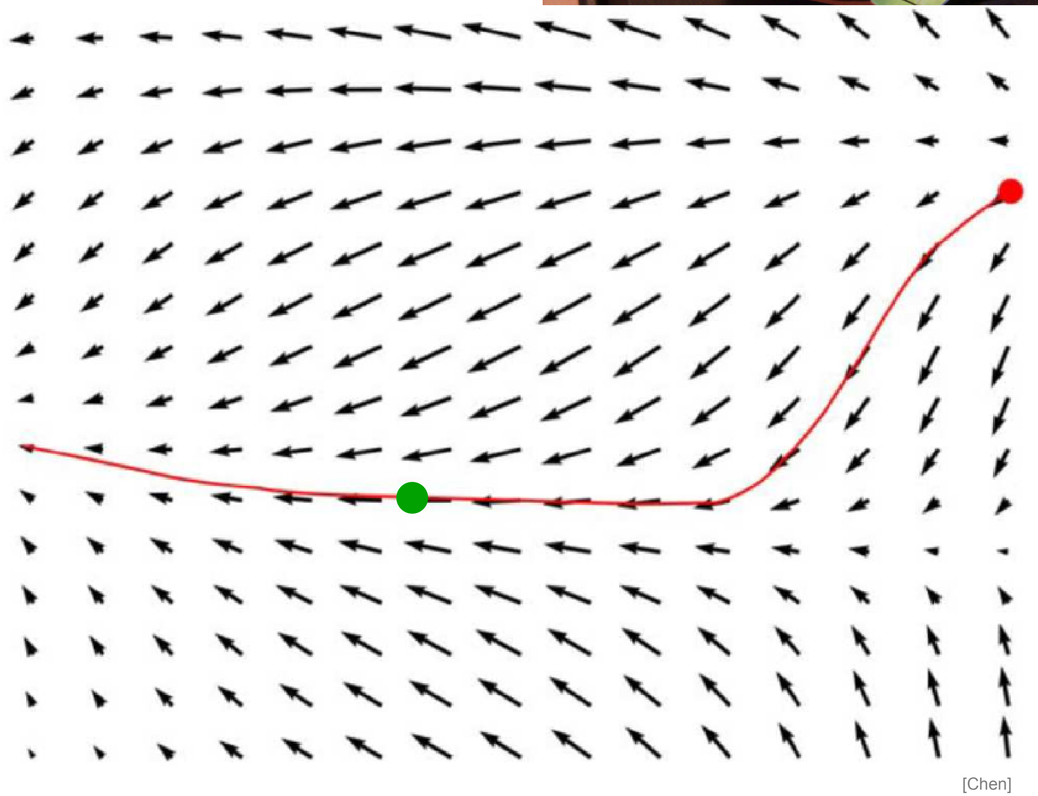

[Chen]

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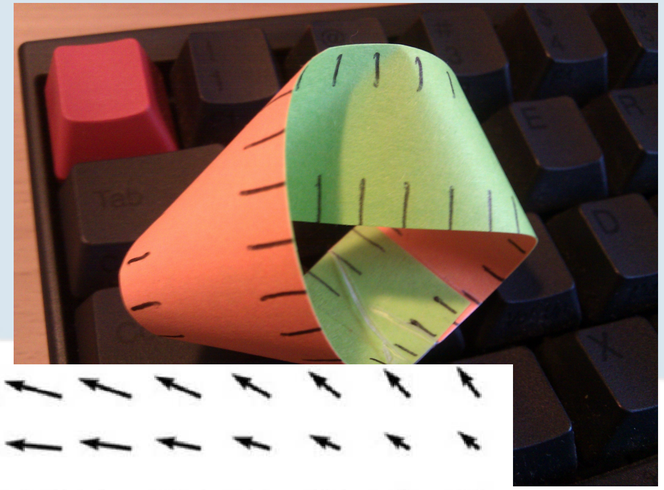

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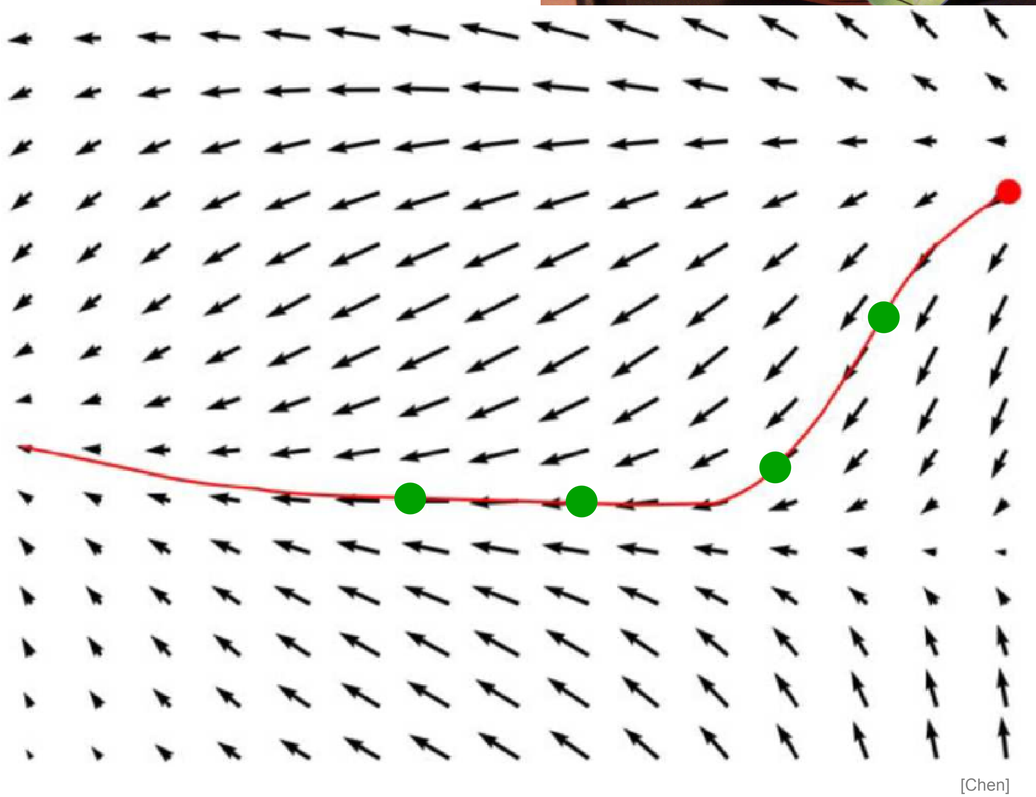

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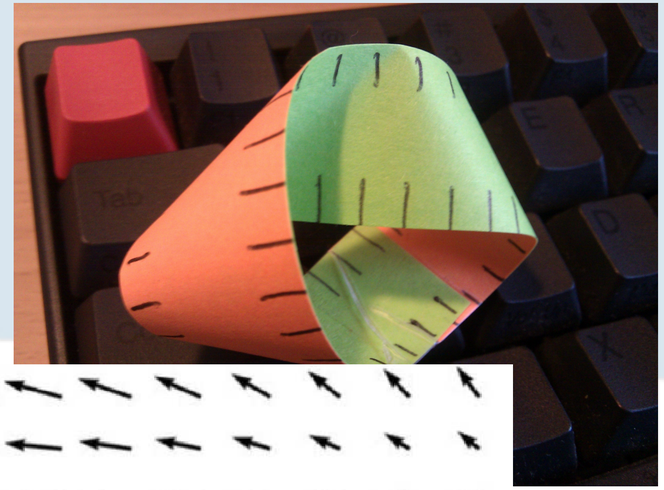

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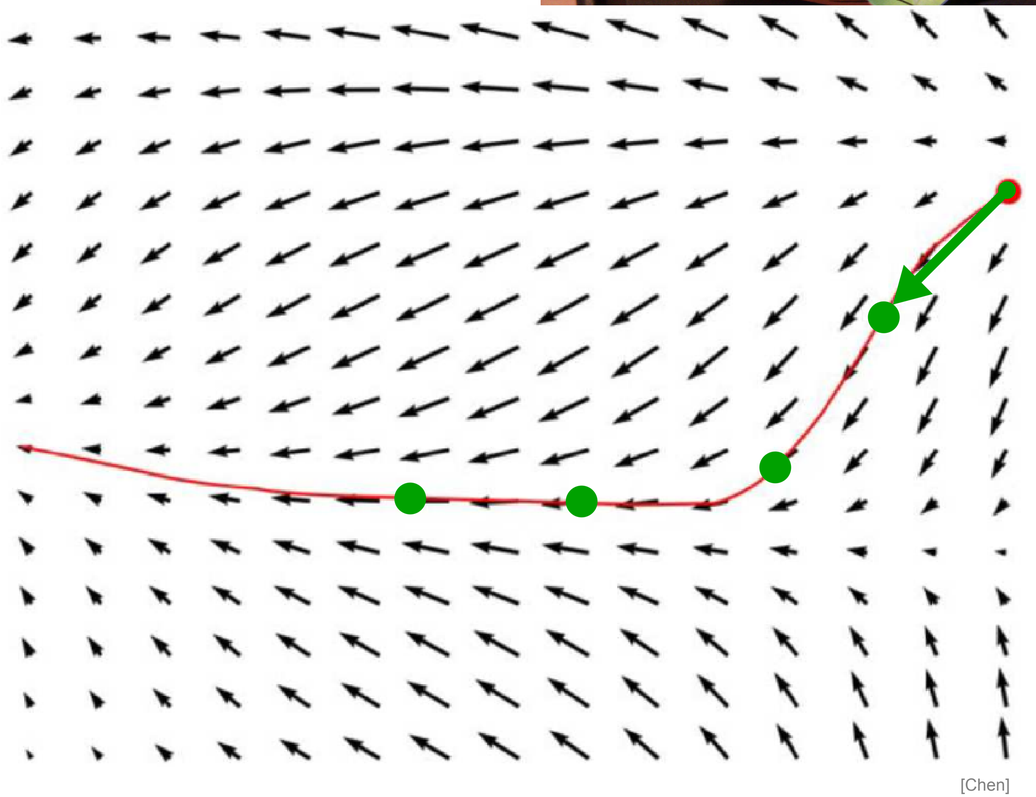

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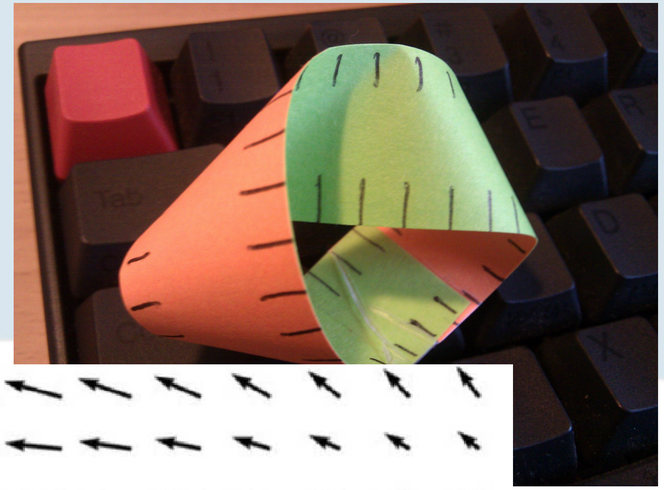

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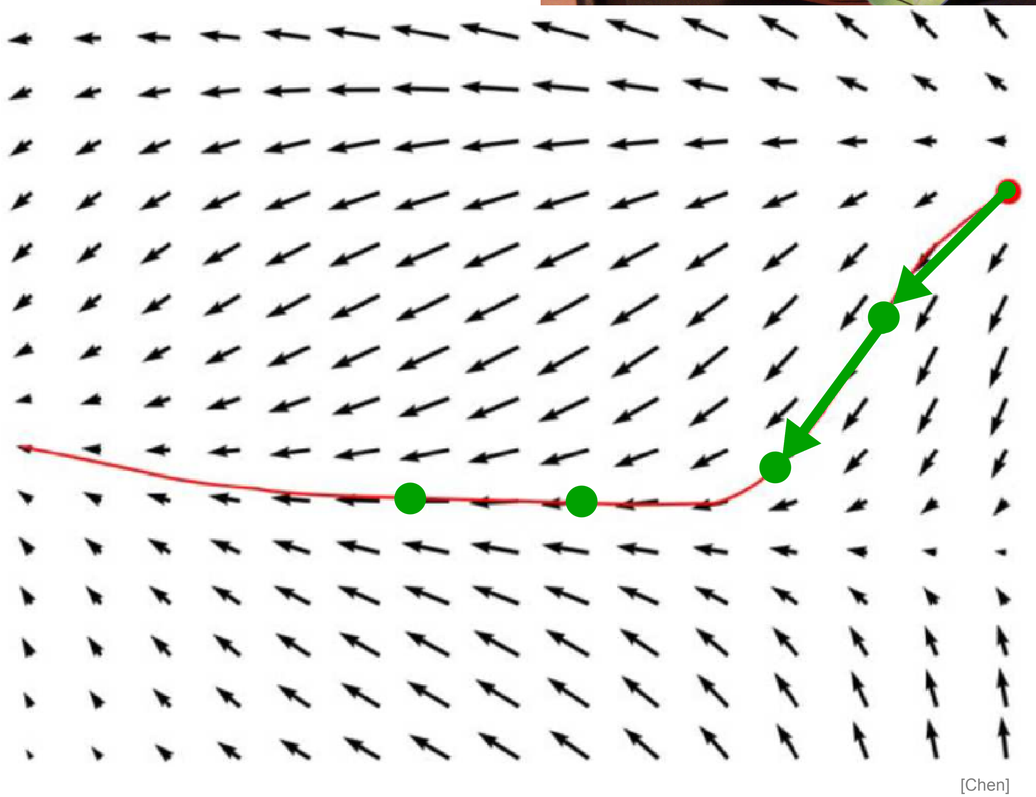

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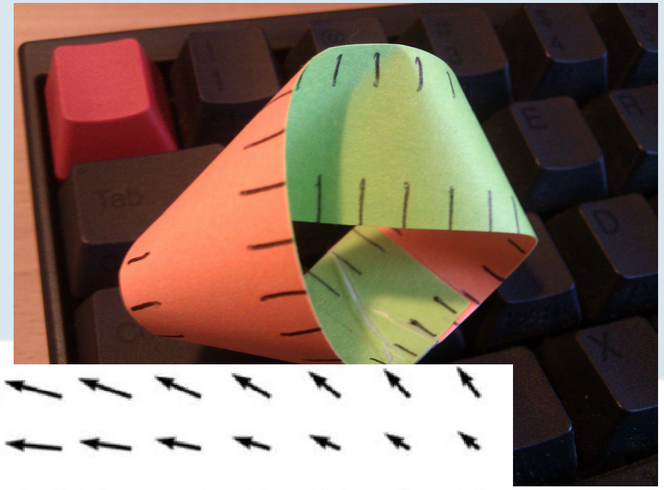

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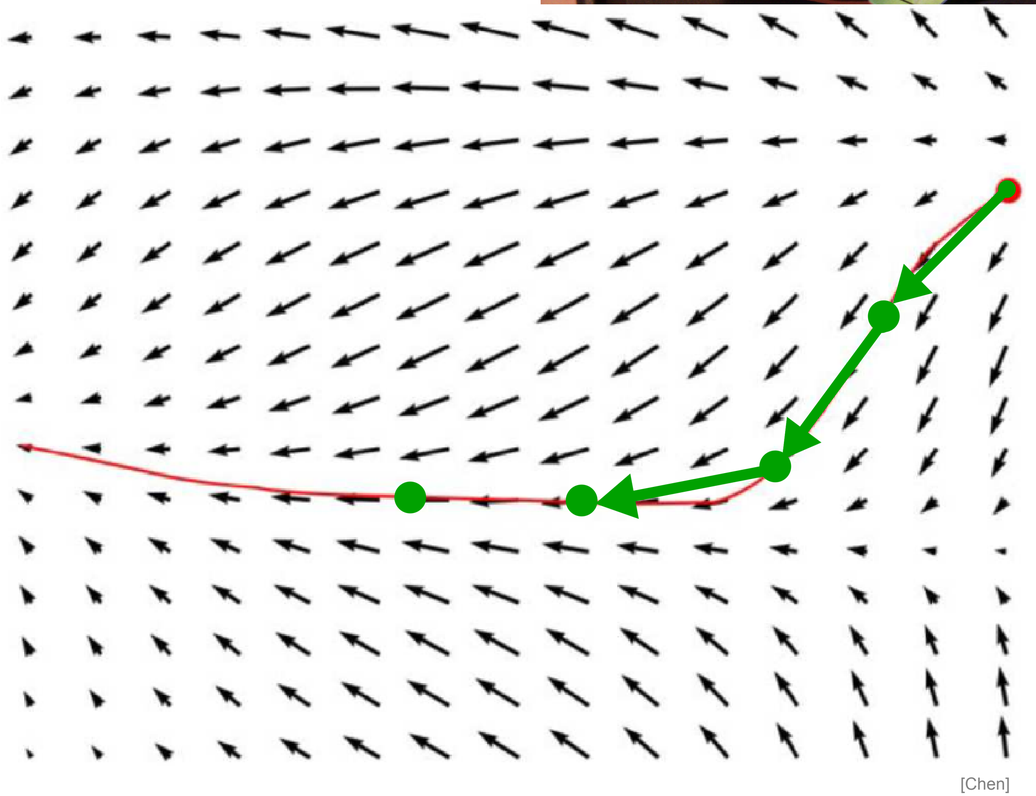

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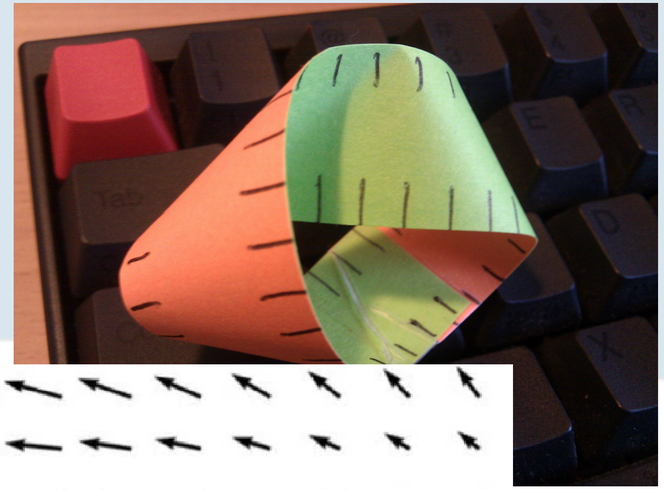

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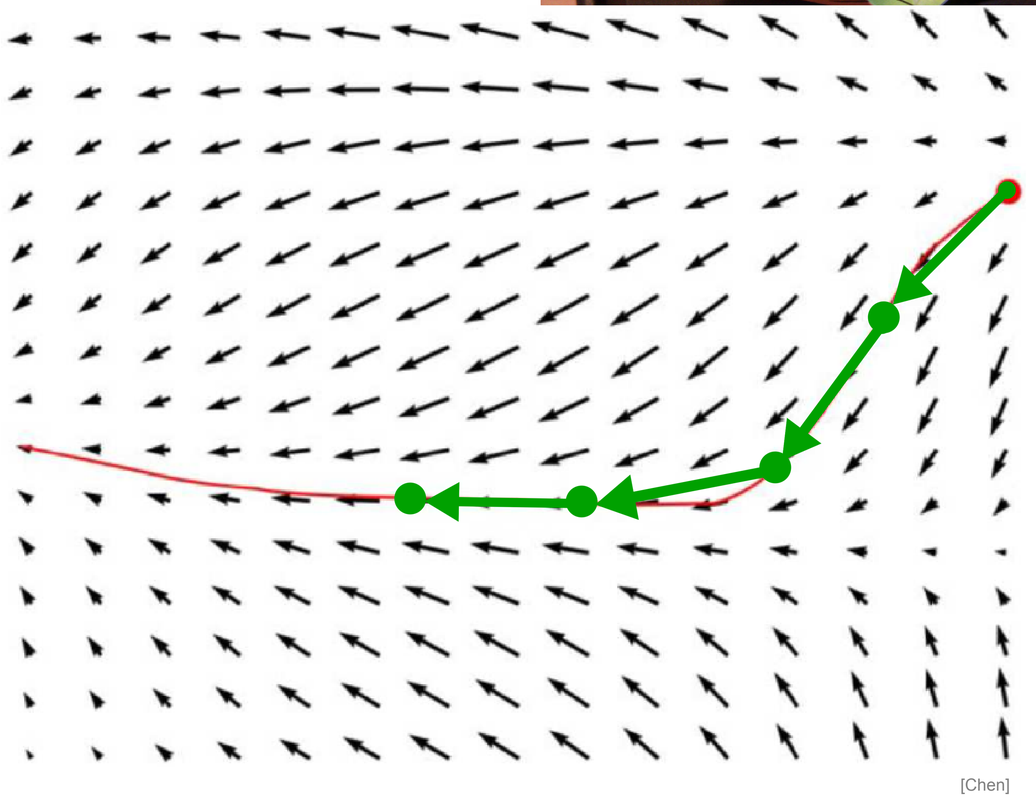


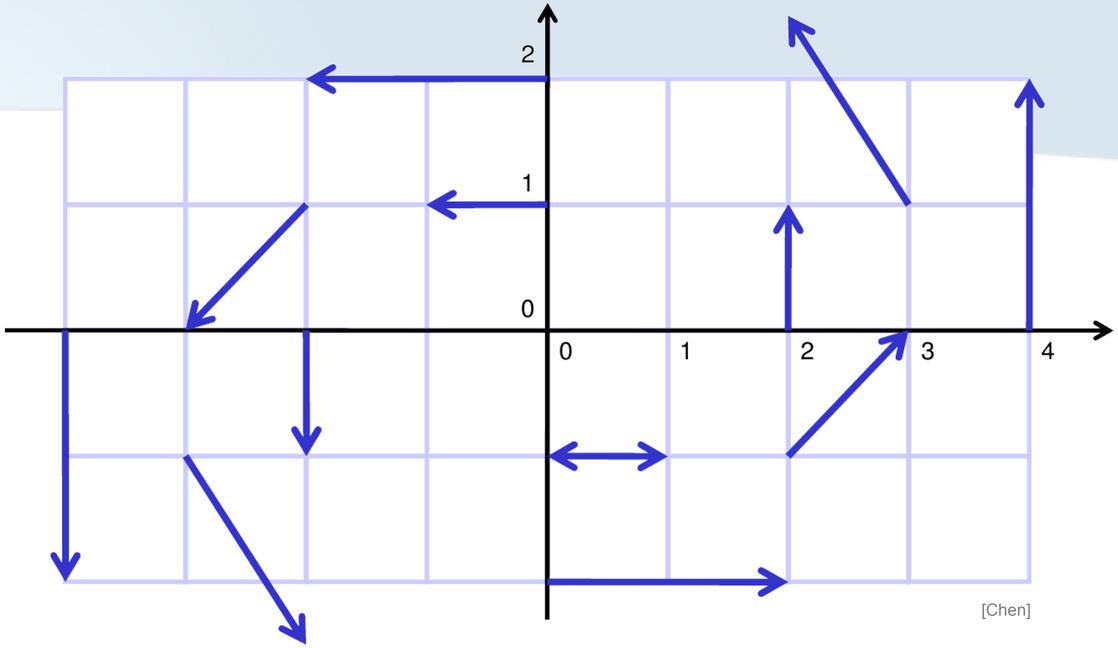
[Chen]

Streamlines on a computer

- Euler integration algorithm

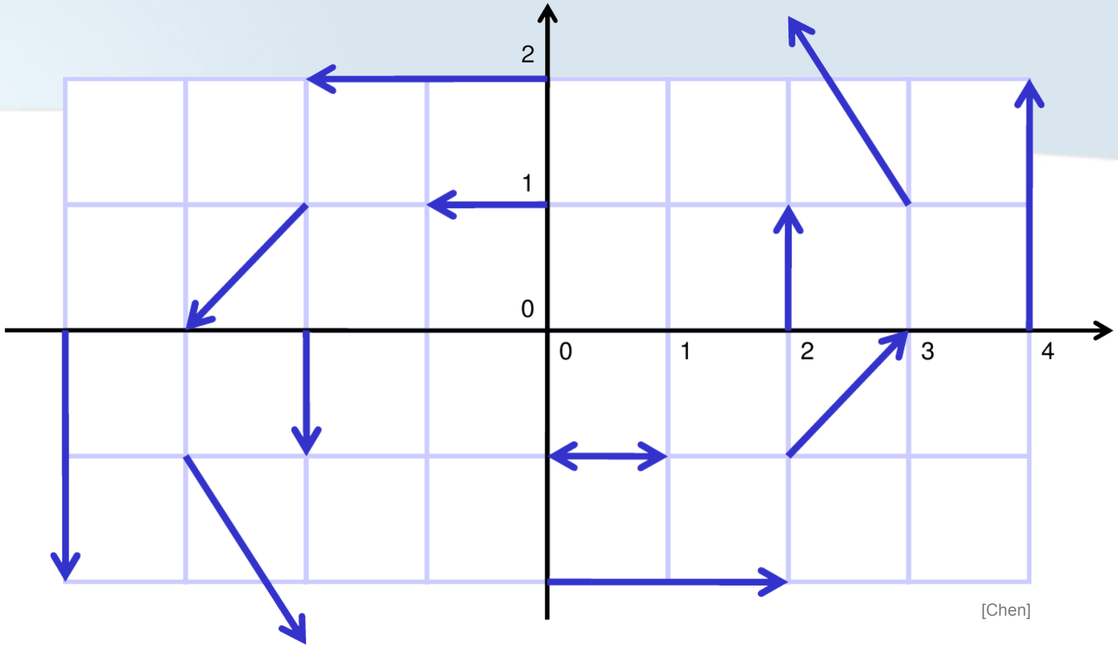
Streamlines on a computer

- Euler integration algorithm
  - Given a seed point $c^{-1}(0)$



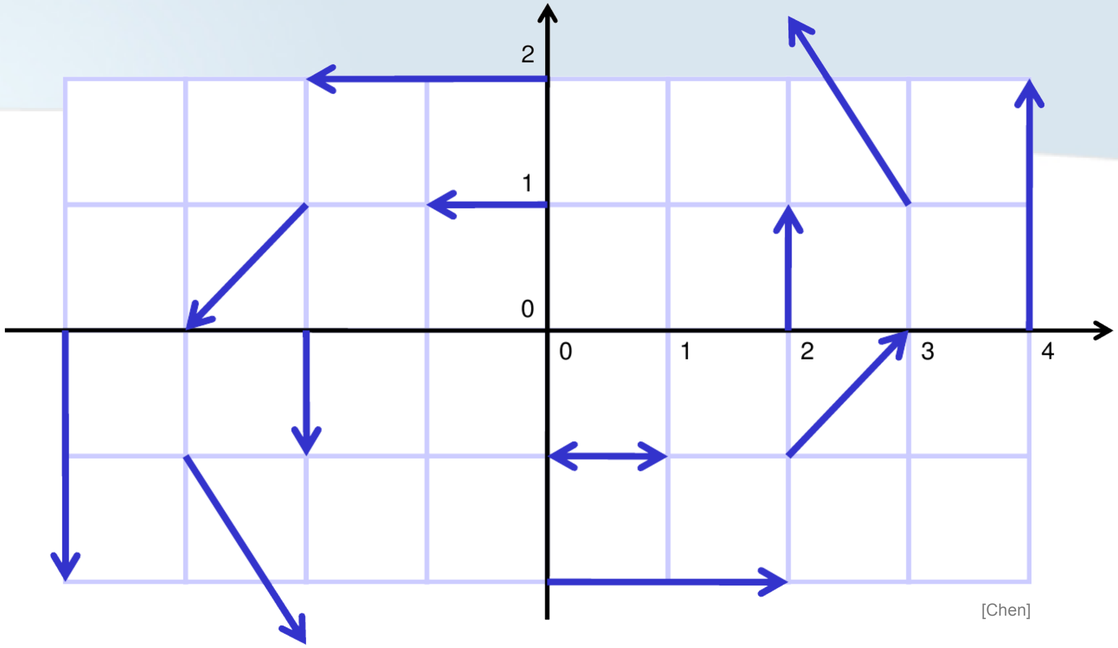
Streamlines on a computer

- Euler integration algorithm
  - Given a seed point $c^{-1}(0)$
  - $u \leftarrow 0$



Streamlines on a computer

- Euler integration algorithm
  - Given a seed point $c^{-1}(0)$
  - $u \leftarrow 0$
  - Repeat



Streamlines on a computer

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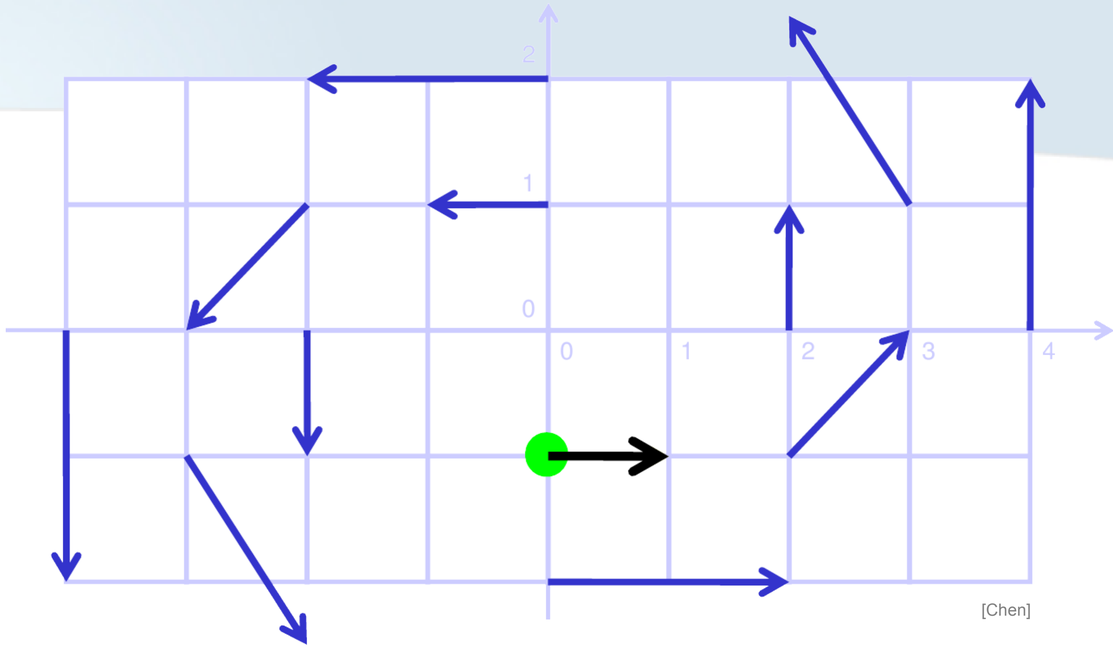
    - Evaluate $\vec{f}(c^{-1}(u.c(p)/q)$



Streamlines on a computer

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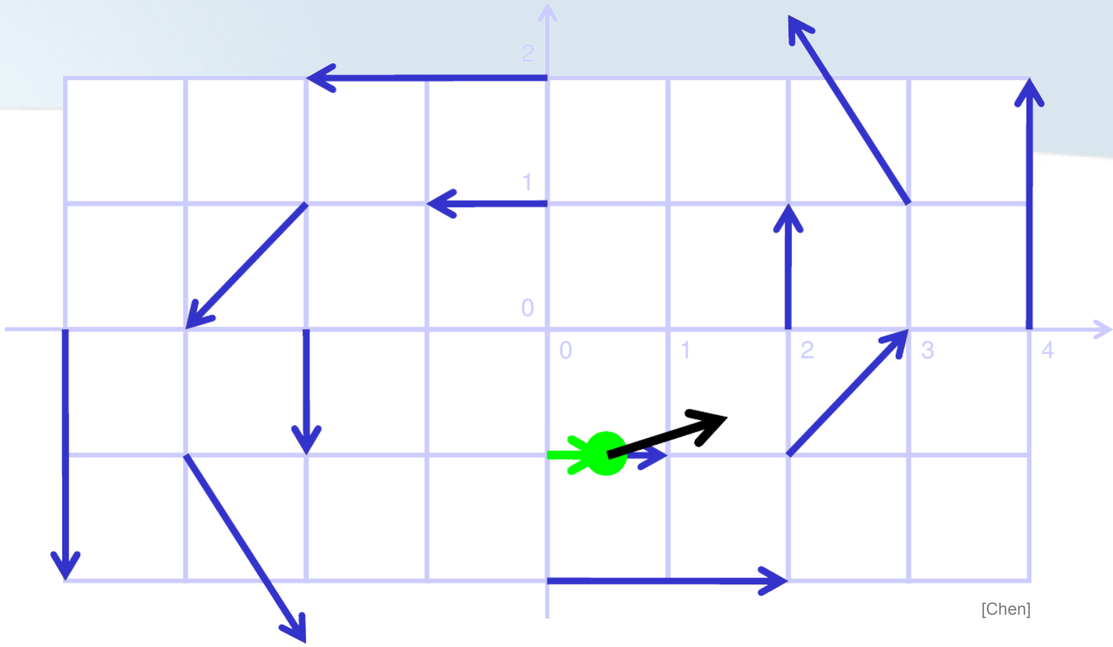

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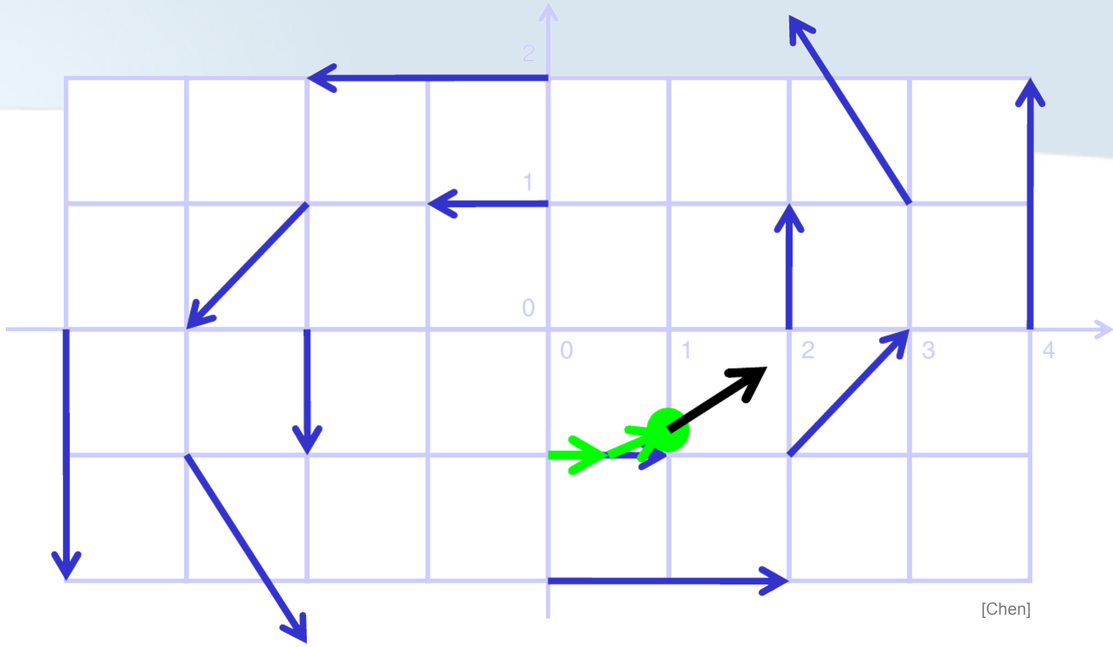

[Chen]

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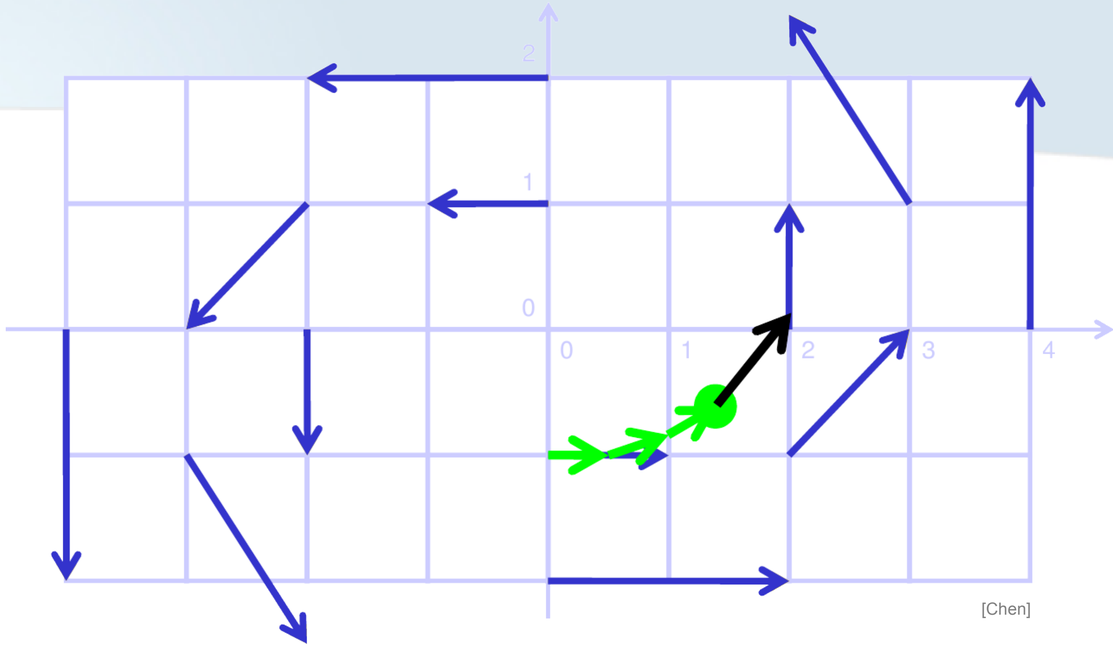

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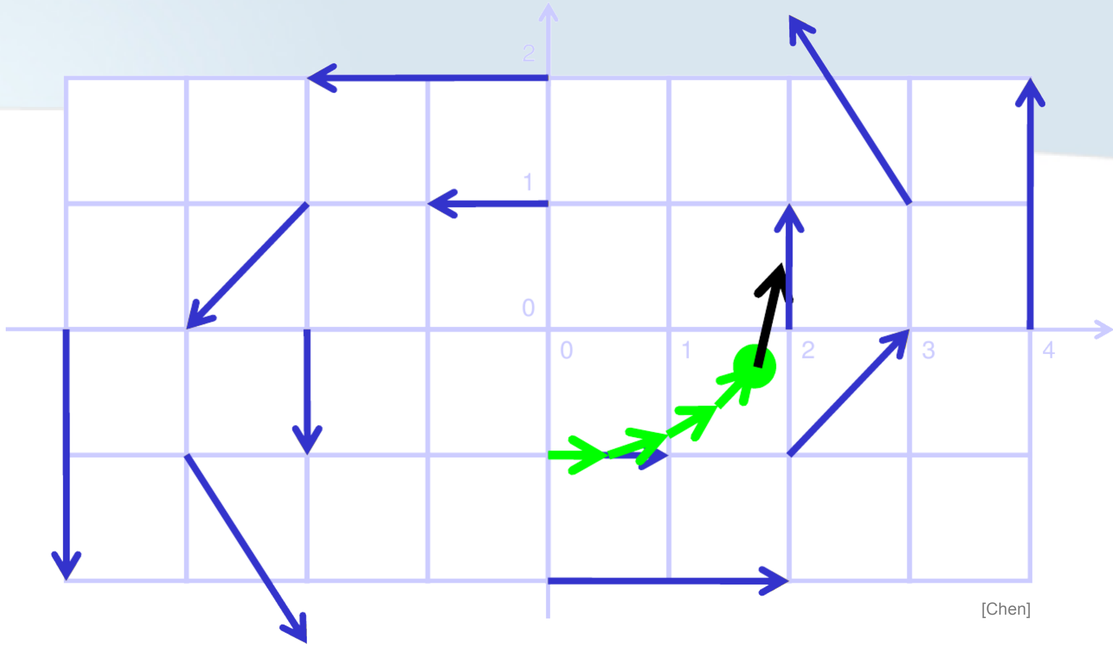

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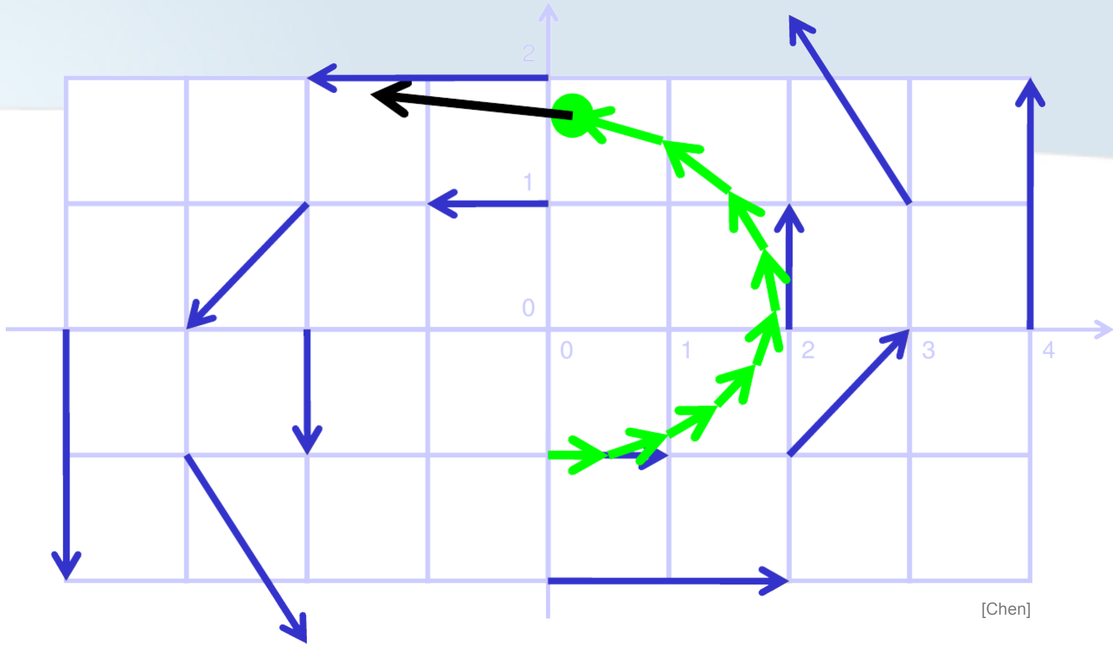

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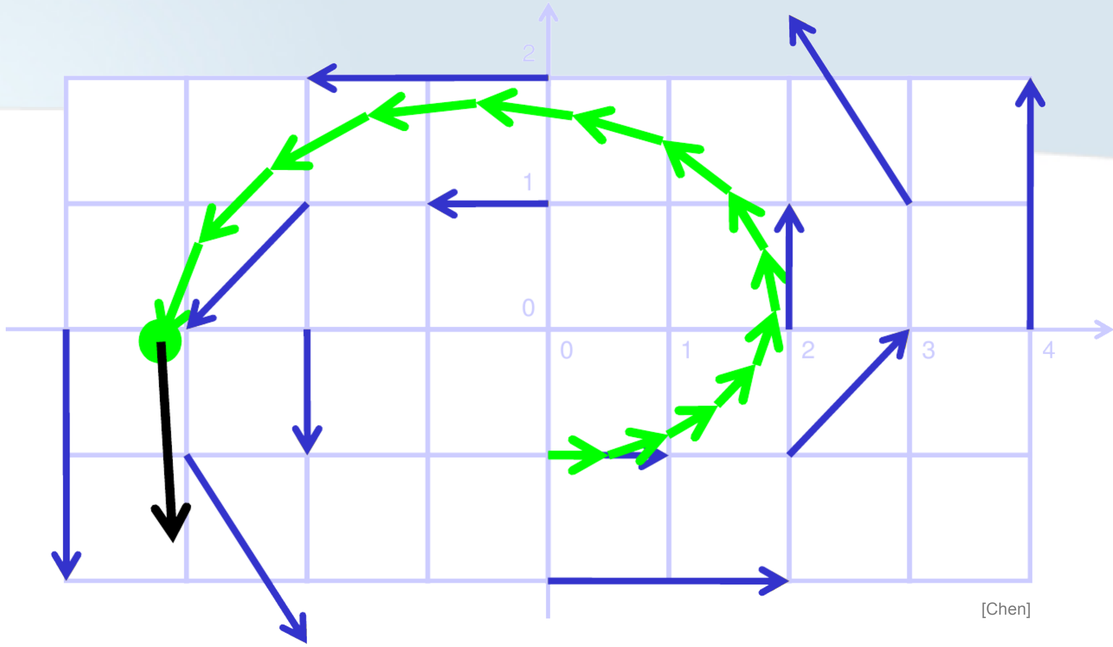
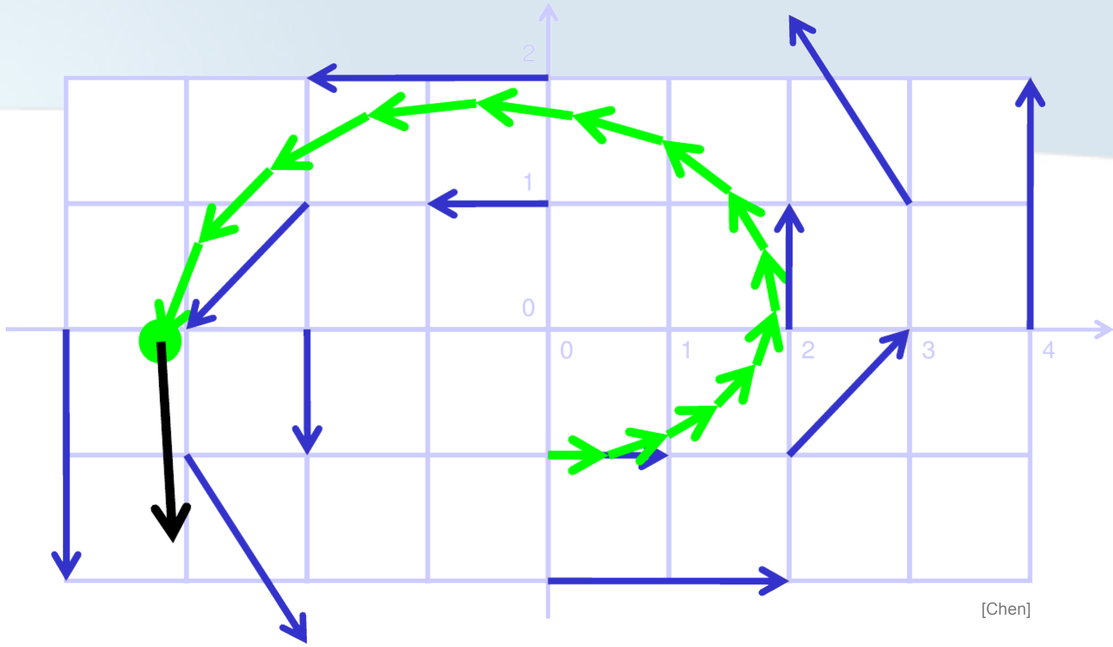


Streamlines on a computer



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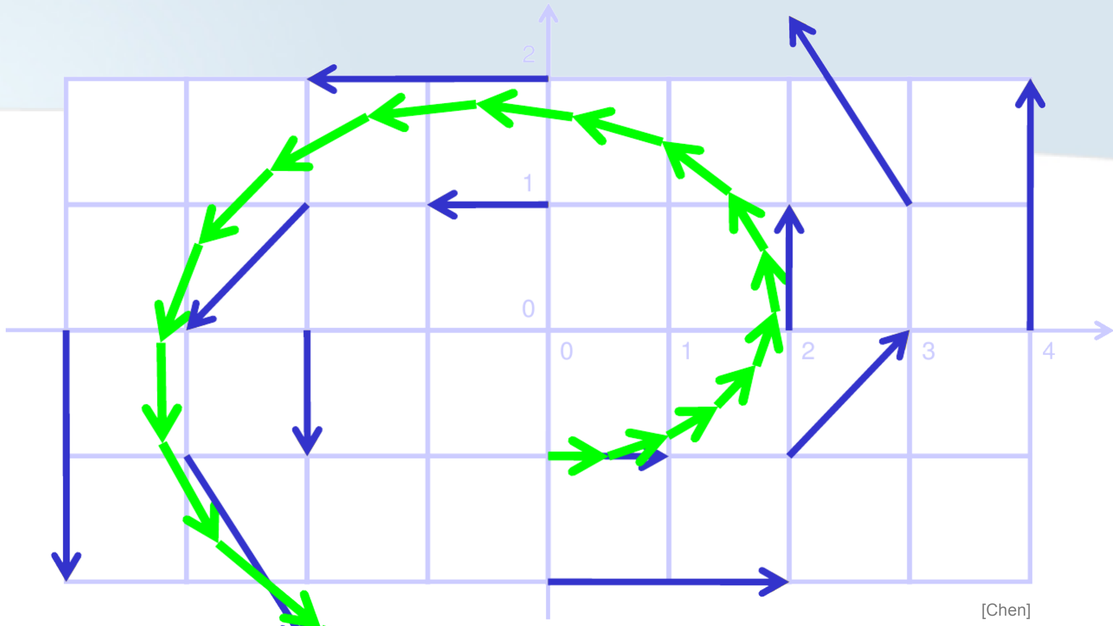
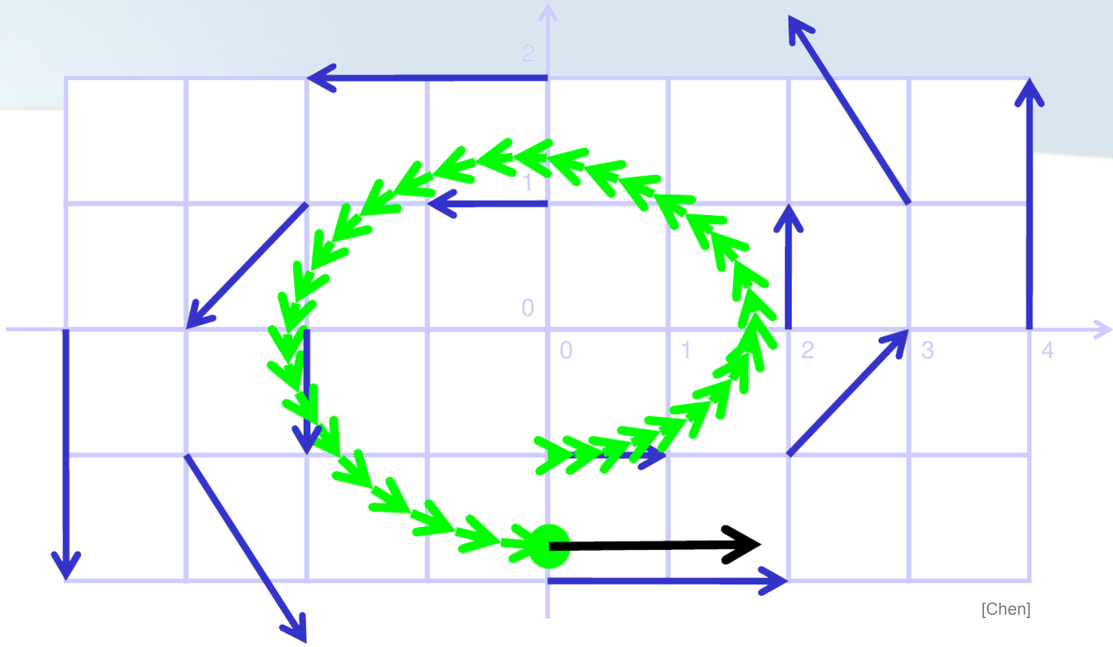


[Chen]

Streamlines on a computer



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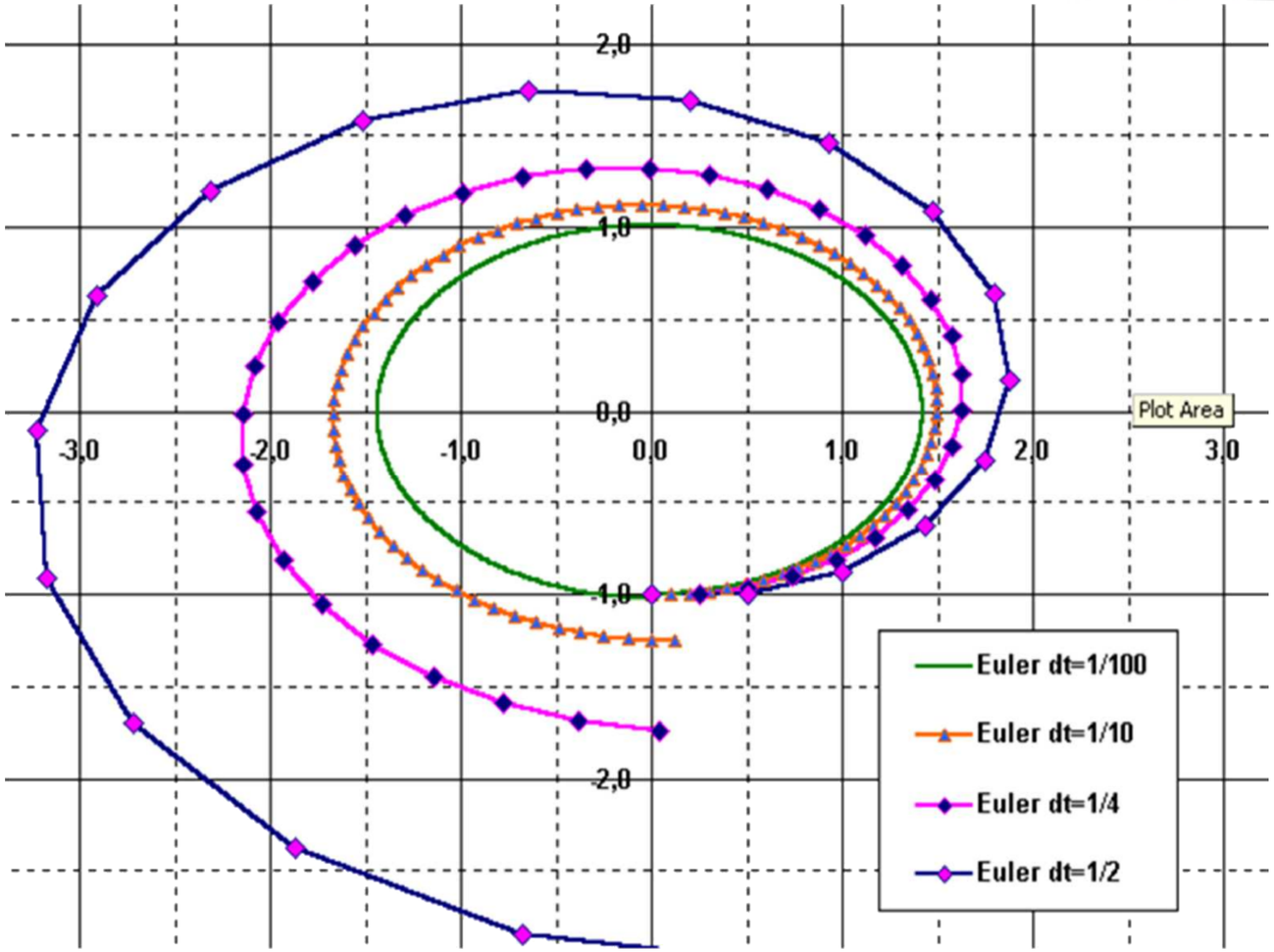
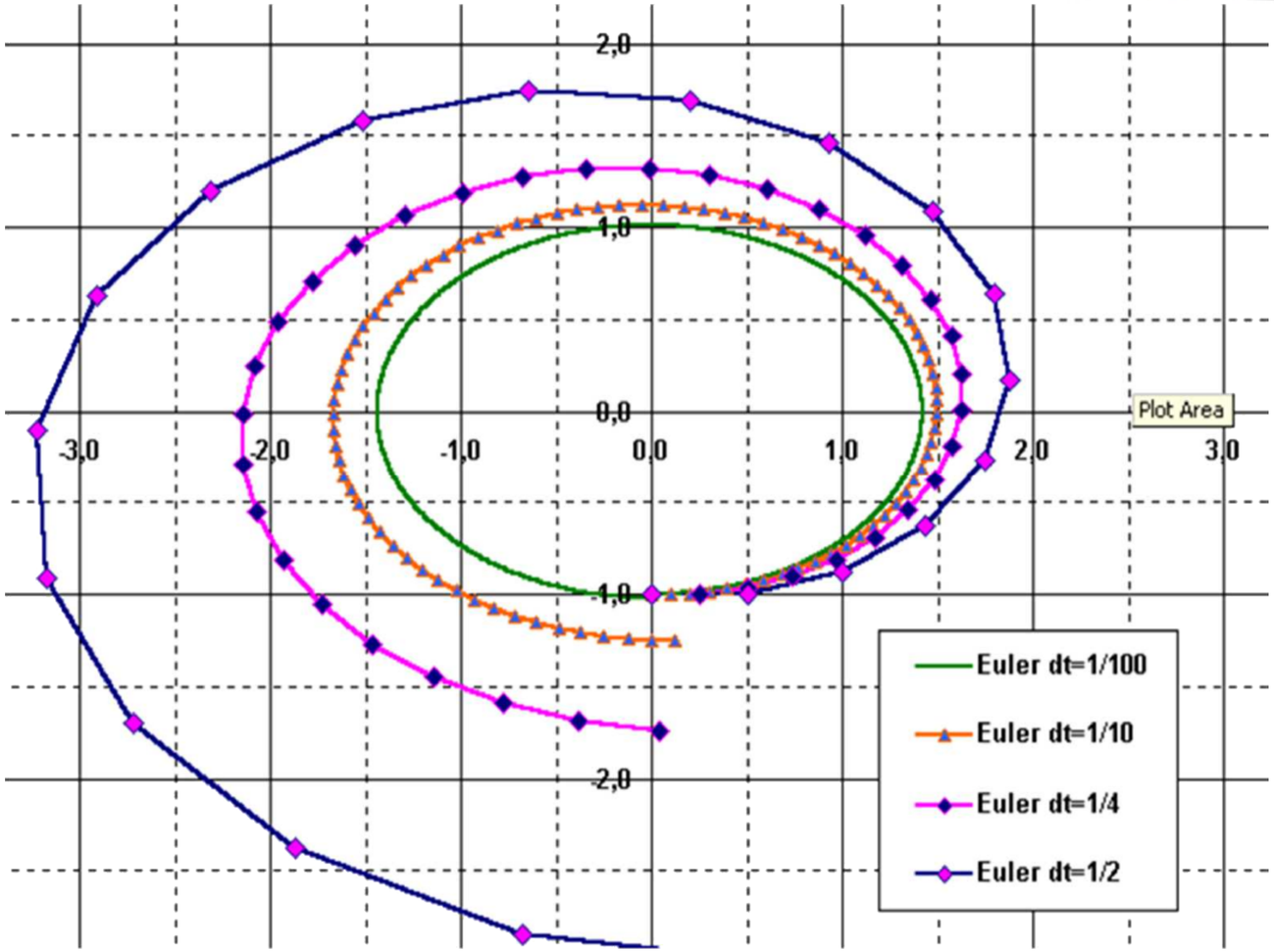


[Chen]

Streamlines on a computer

- Euler integration algorithm
  - Fixed step size
  - Ratio of the magnitude
- Trade-off
  - Sampling
  - Approximation quality



[Chen]

Special case

- Gradient fields on PL-manifolds

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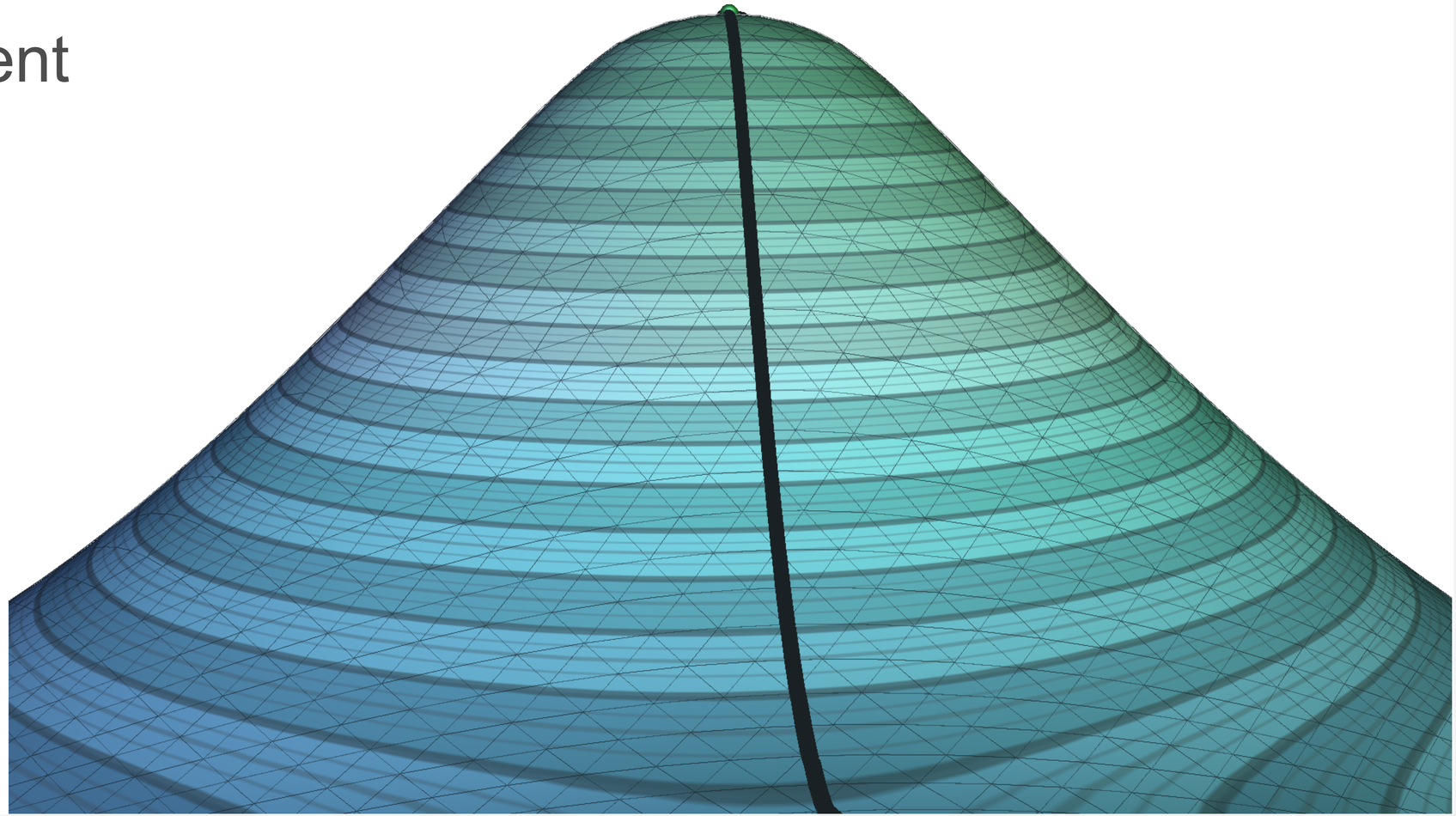
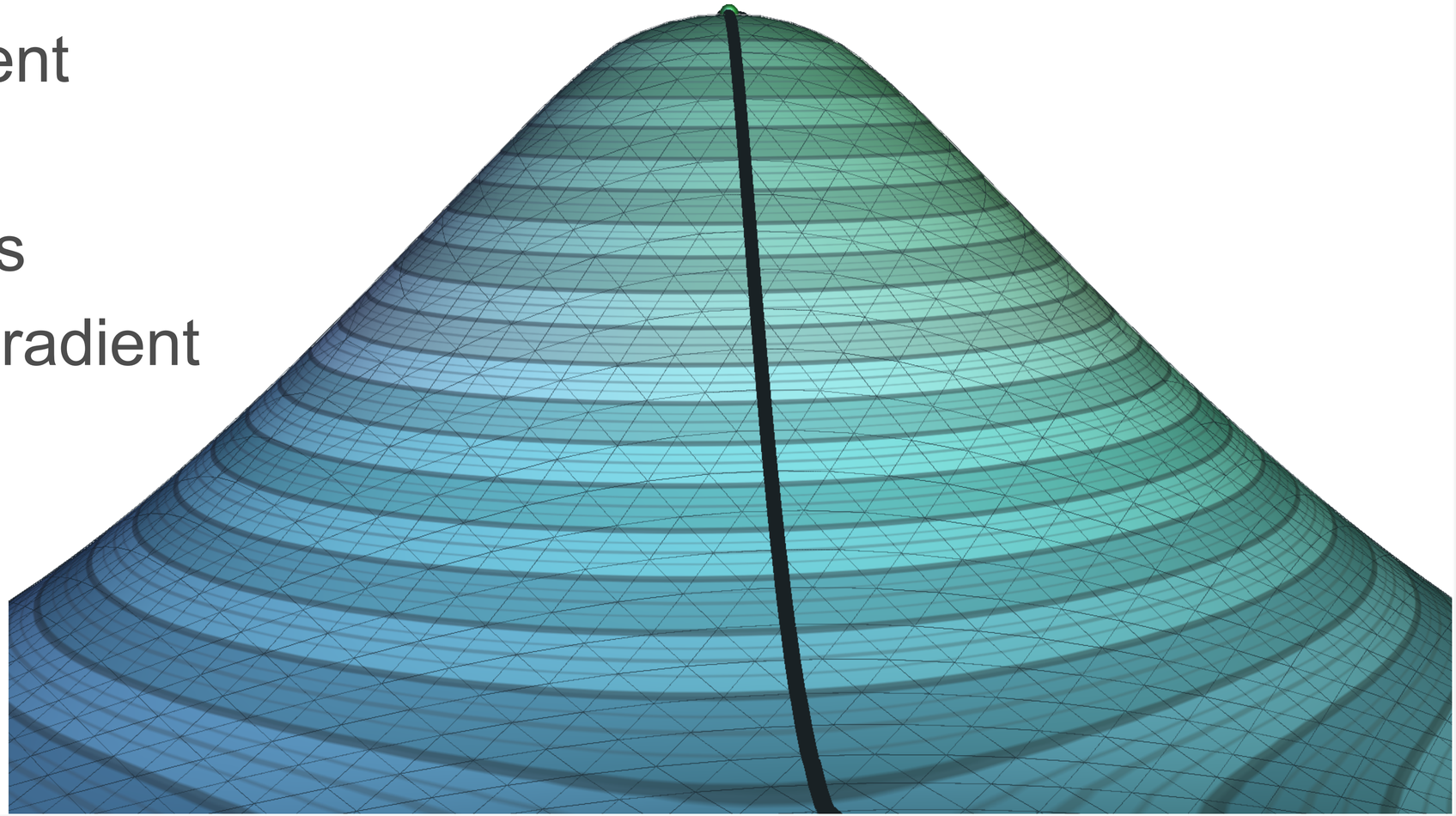


Special case

- Gradient fields on PL-manifolds
  - Path of steepest ascent
- Barycentric coordinates
  - Piecewise constant gradient



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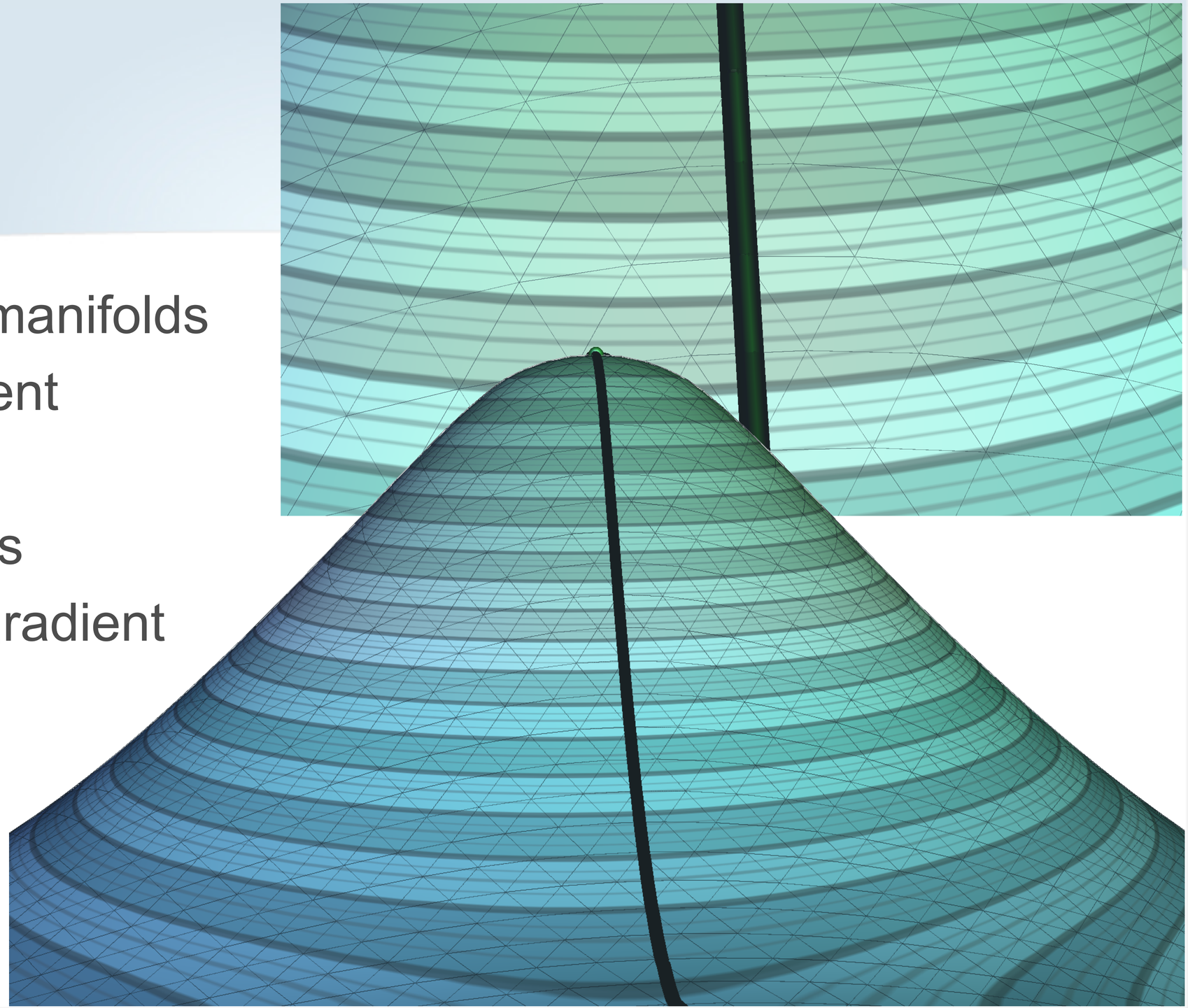

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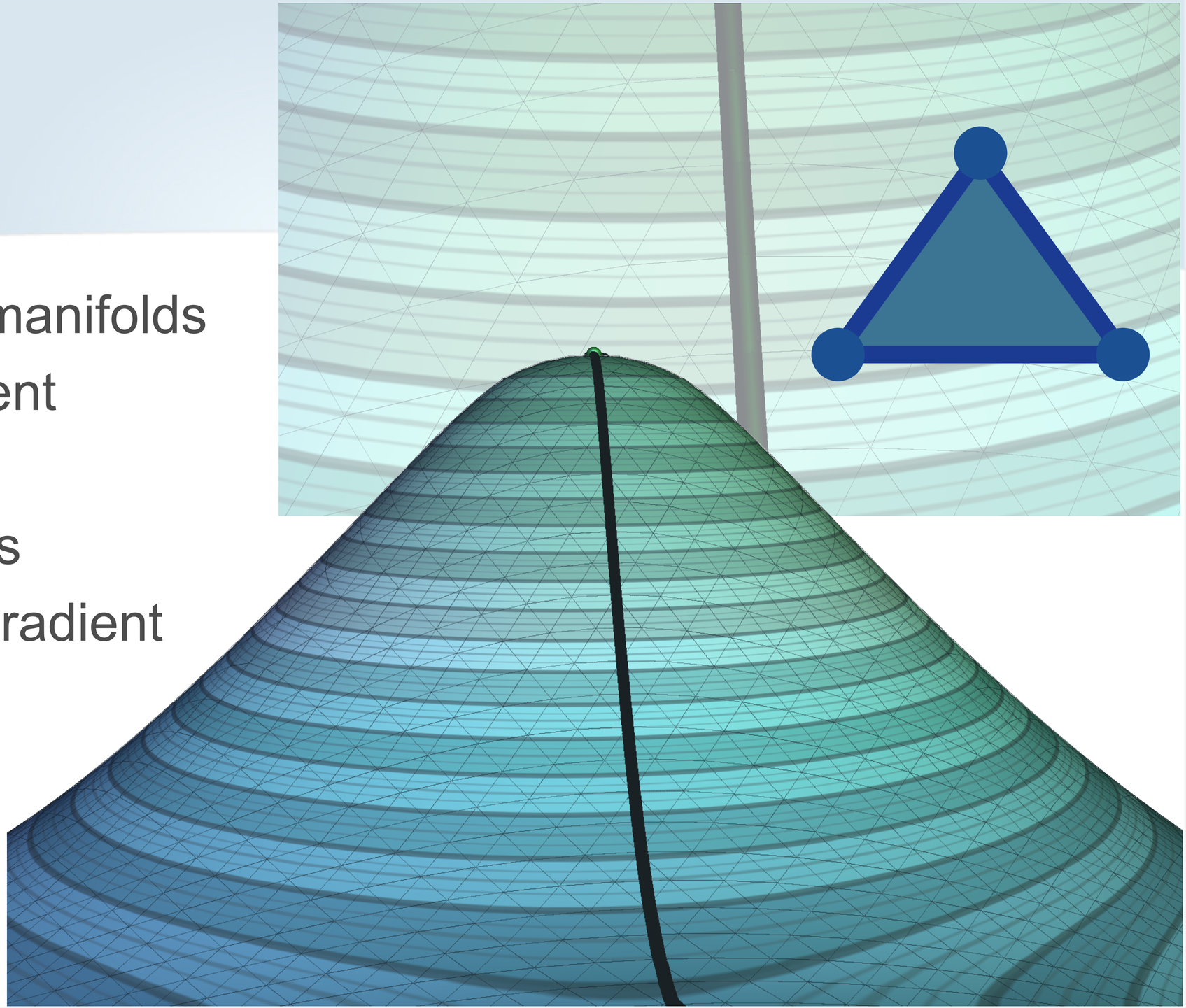

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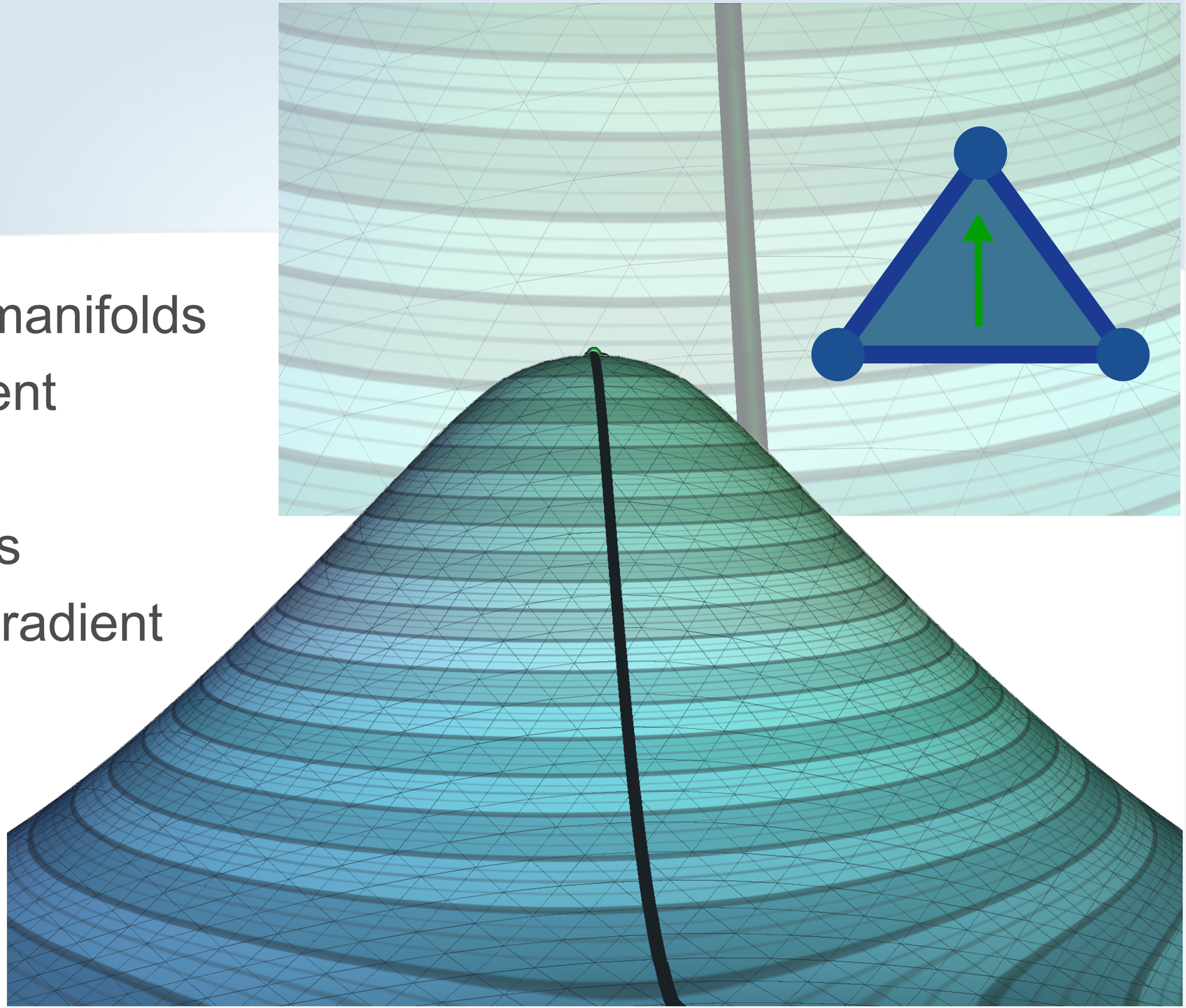

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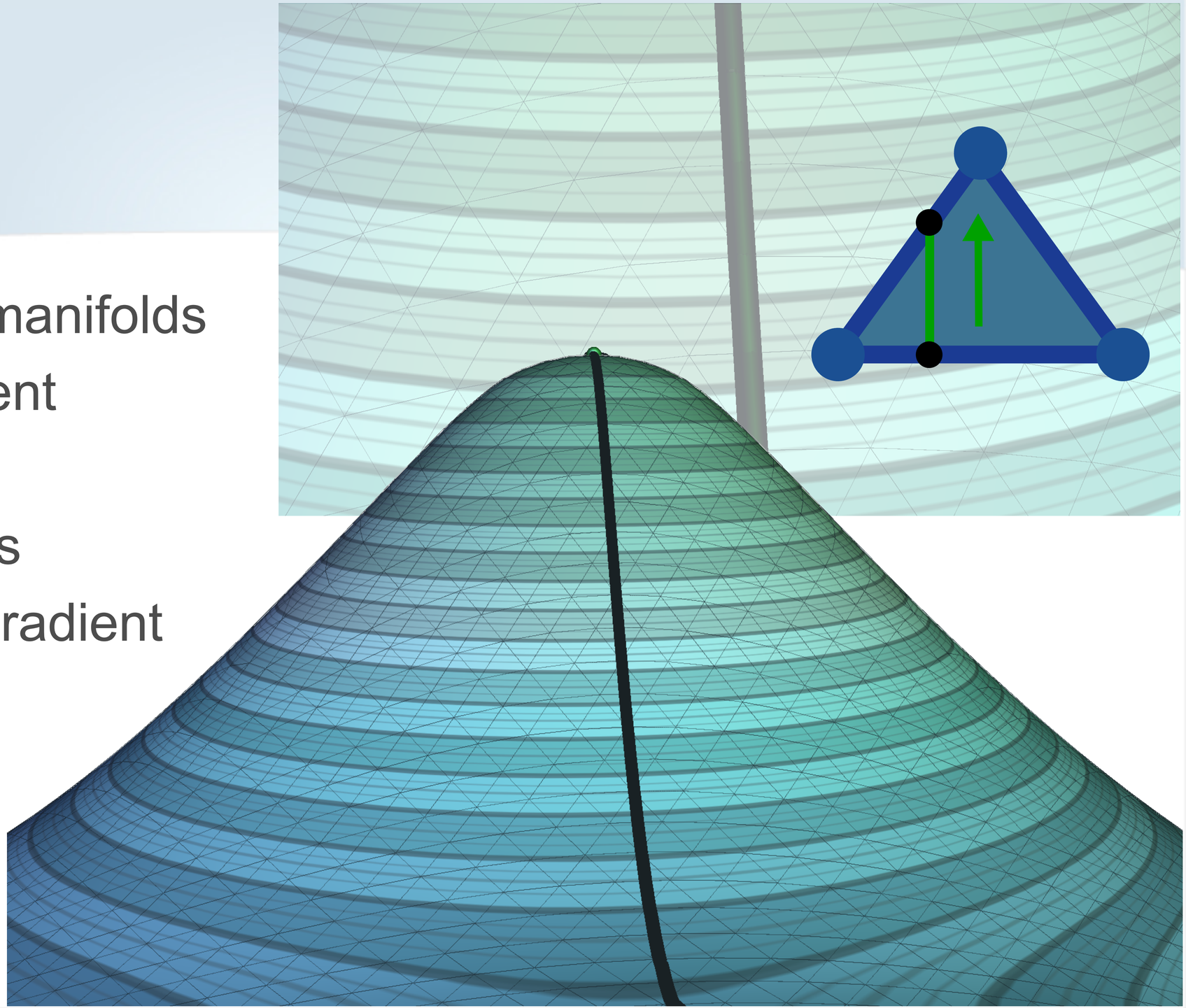

Special case

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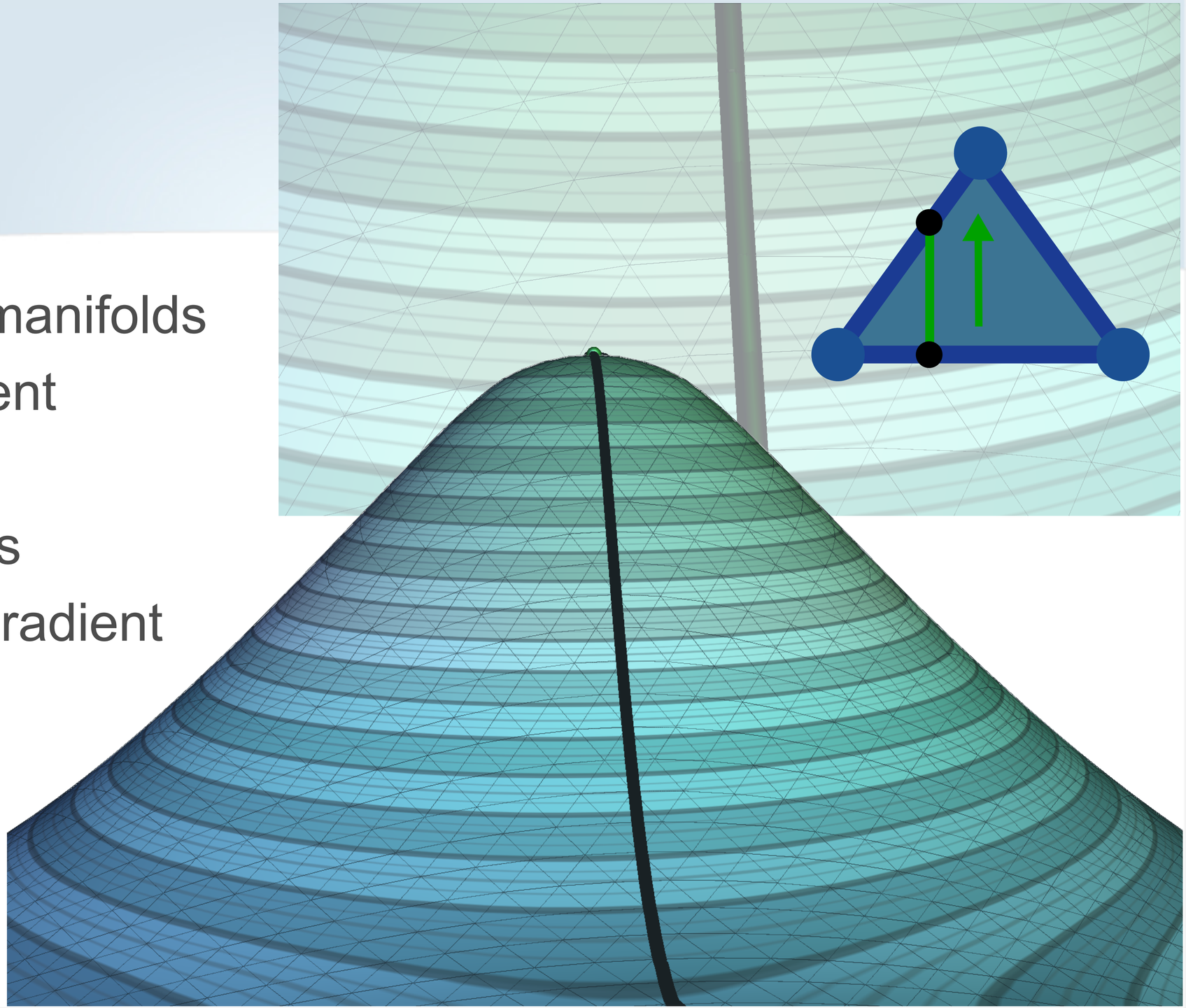
  - Piecewise constant gradient
  - No integration error
  - Nearly no ambiguity



Special case

- Gradient fields on PL-manifolds
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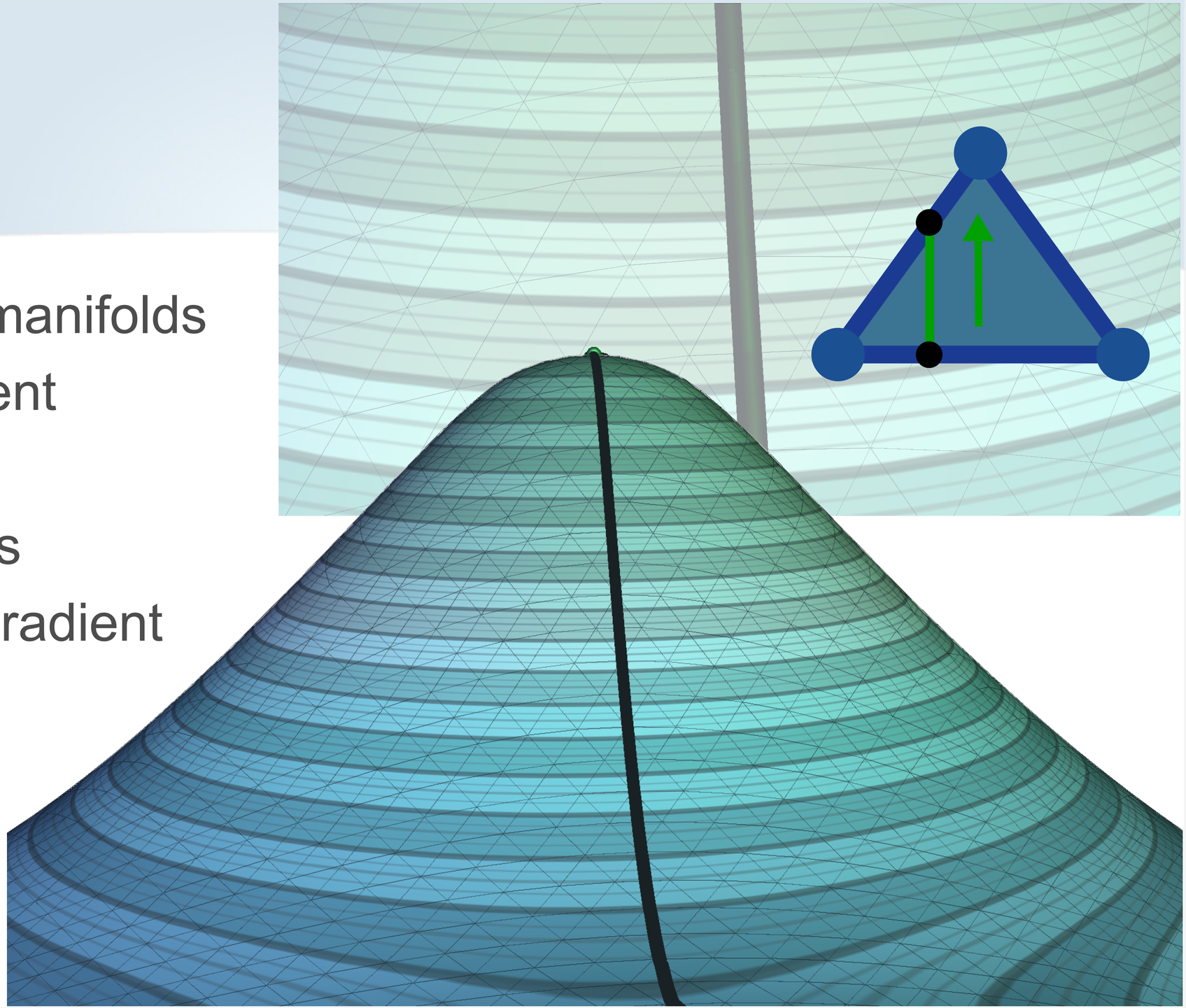
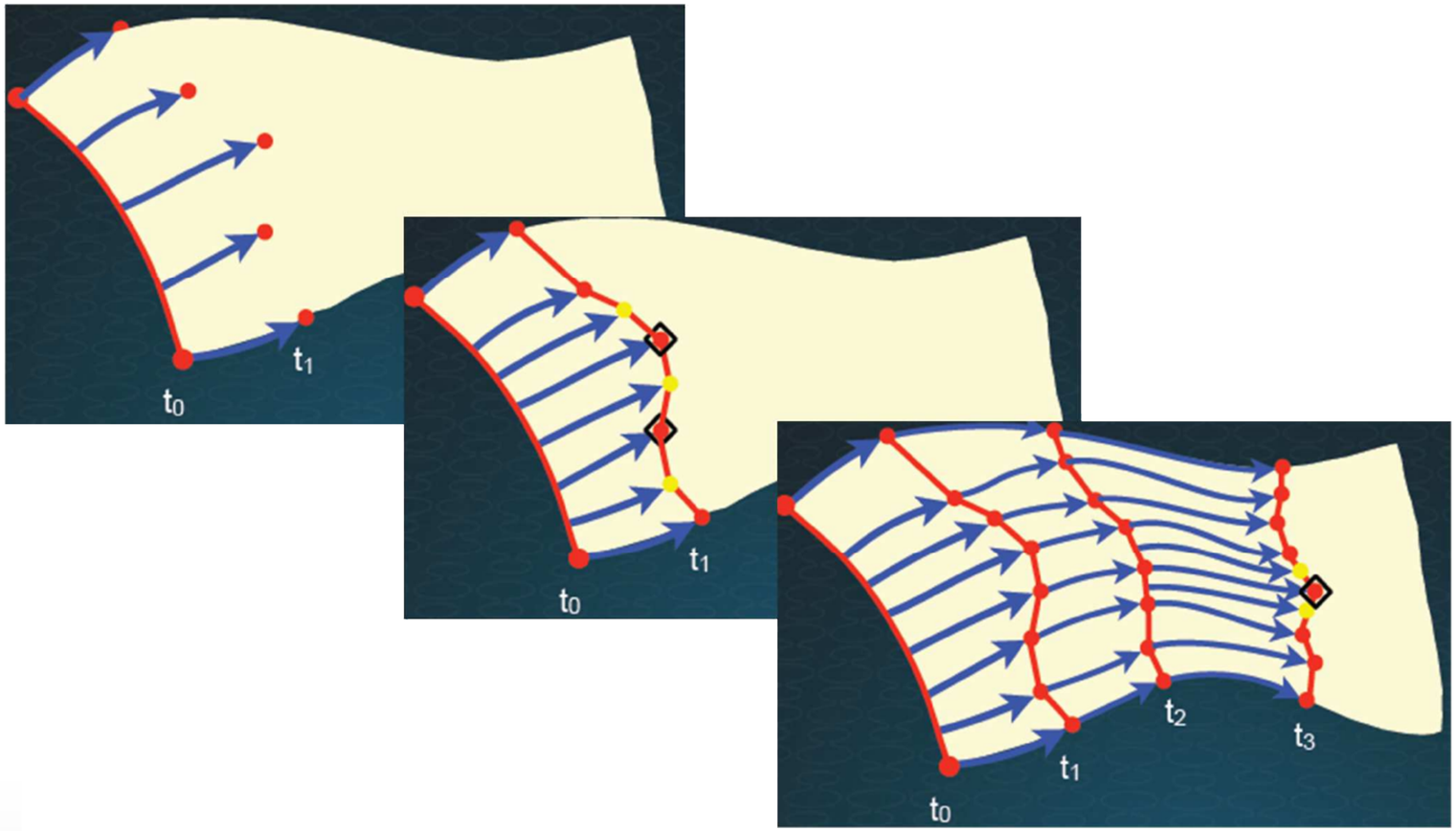
  - No integration error
  - Nearly no ambiguity
- Primal/dual meshes



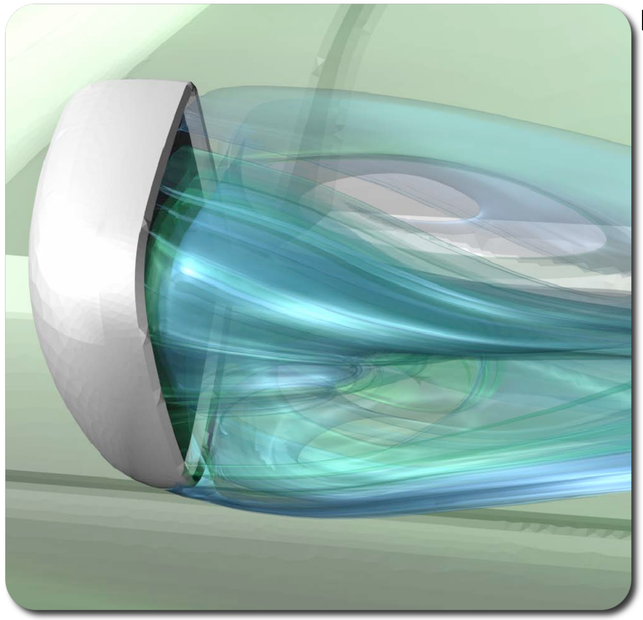
Streamsurfaces

- Adaptive sampling depending on the curvature of the seed curve

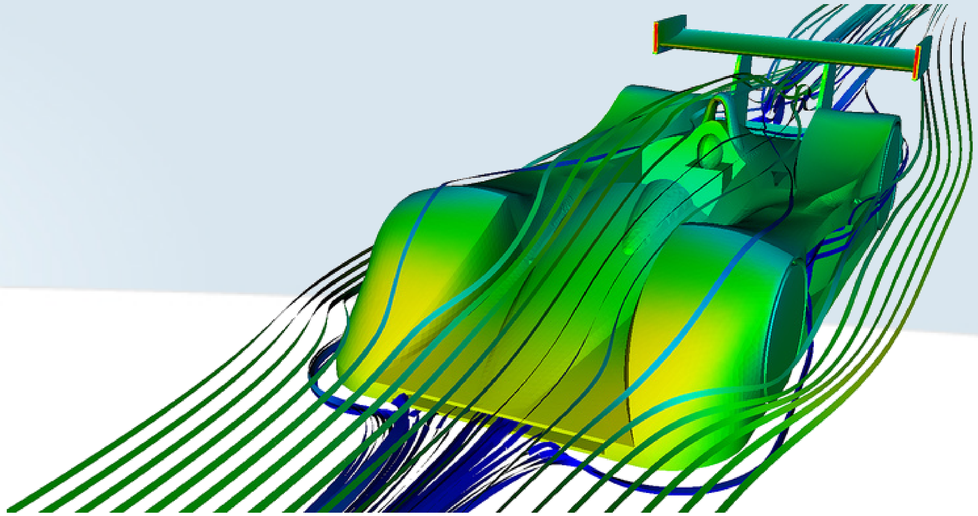


[Garth 09]

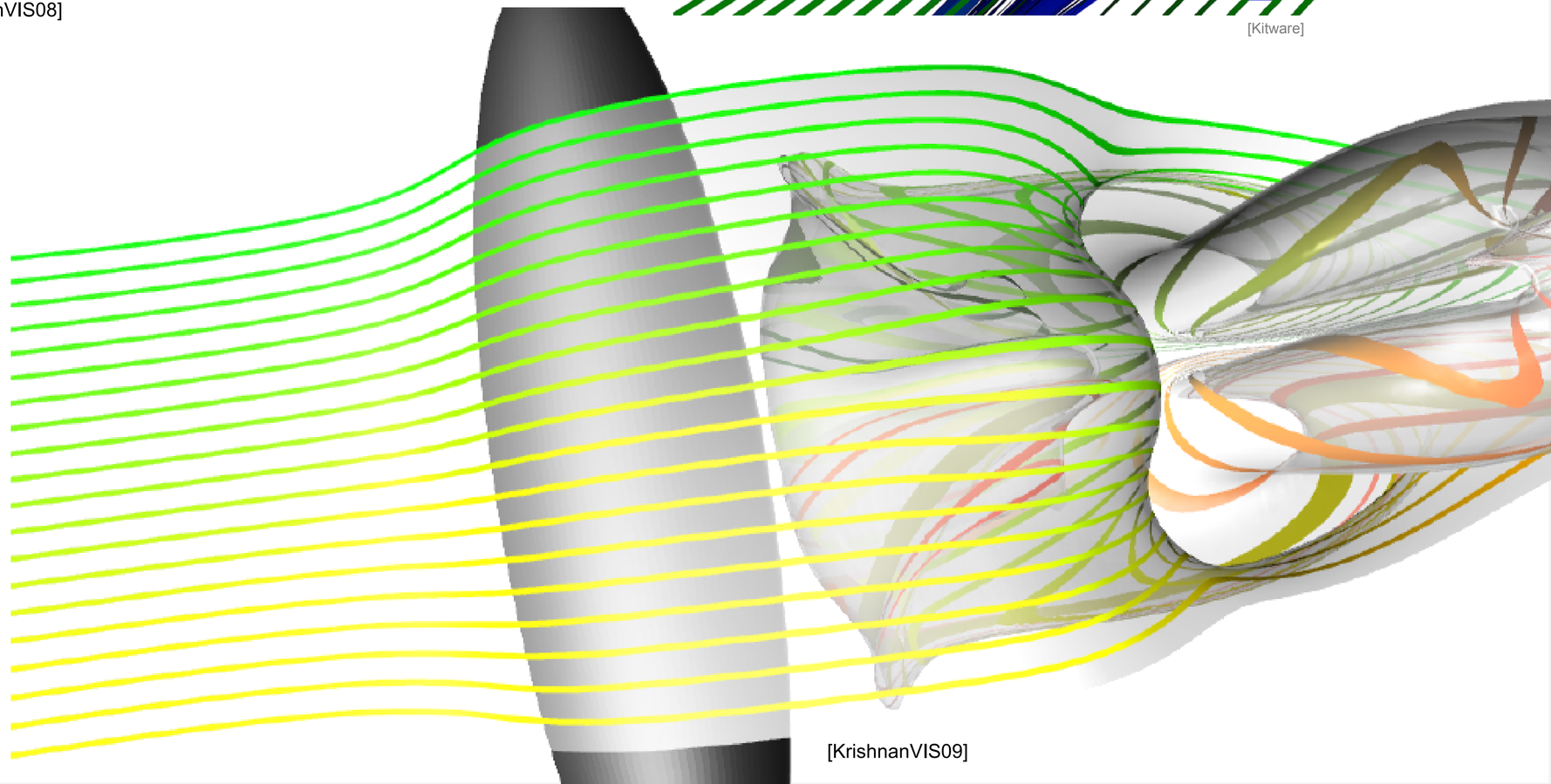
Examples



[GarthVIS08]



[Kitware]



[KrishnanVIS09]

Steady vector fields

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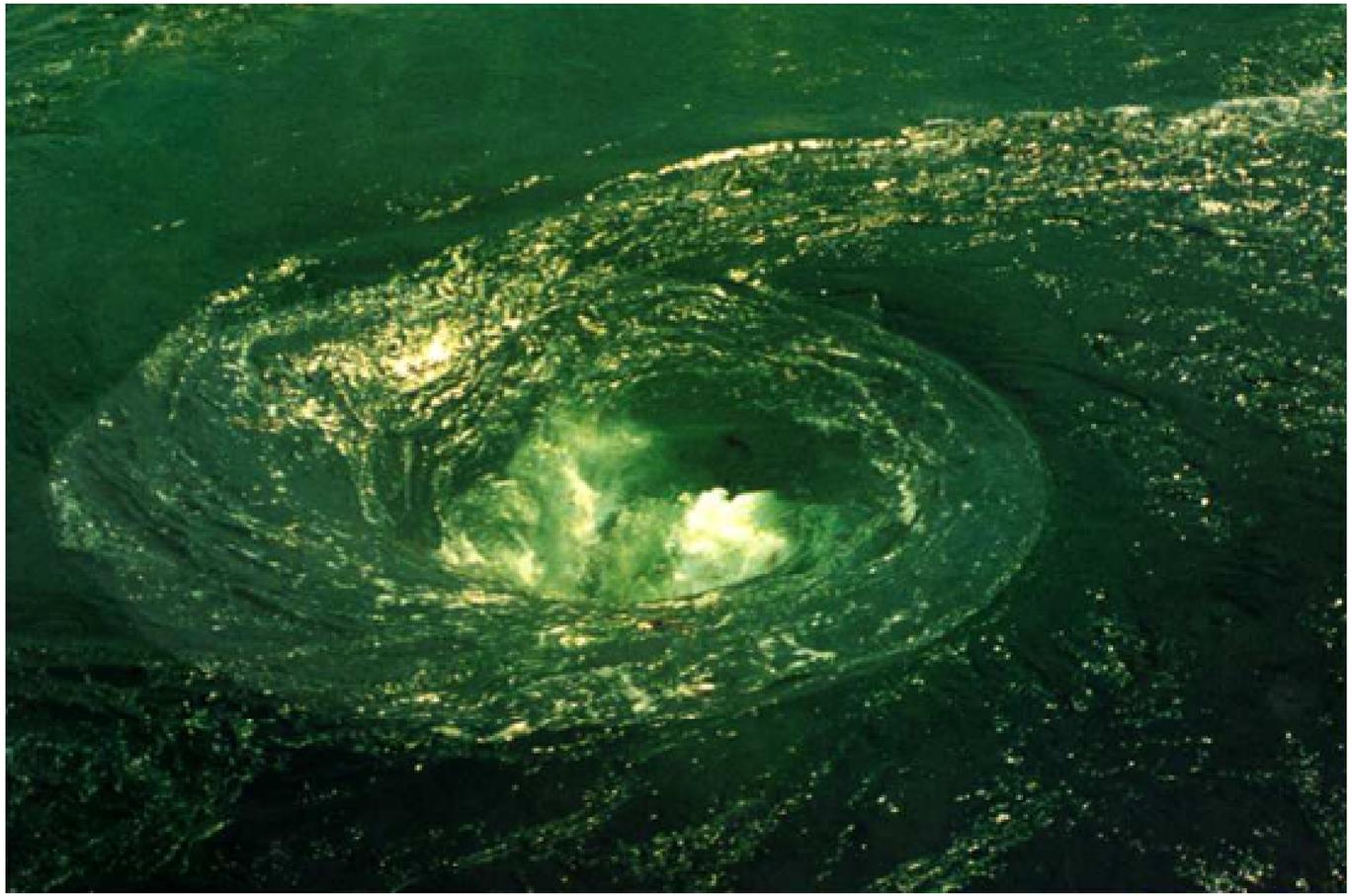

[Post et al. 2003]

Steady vector fields

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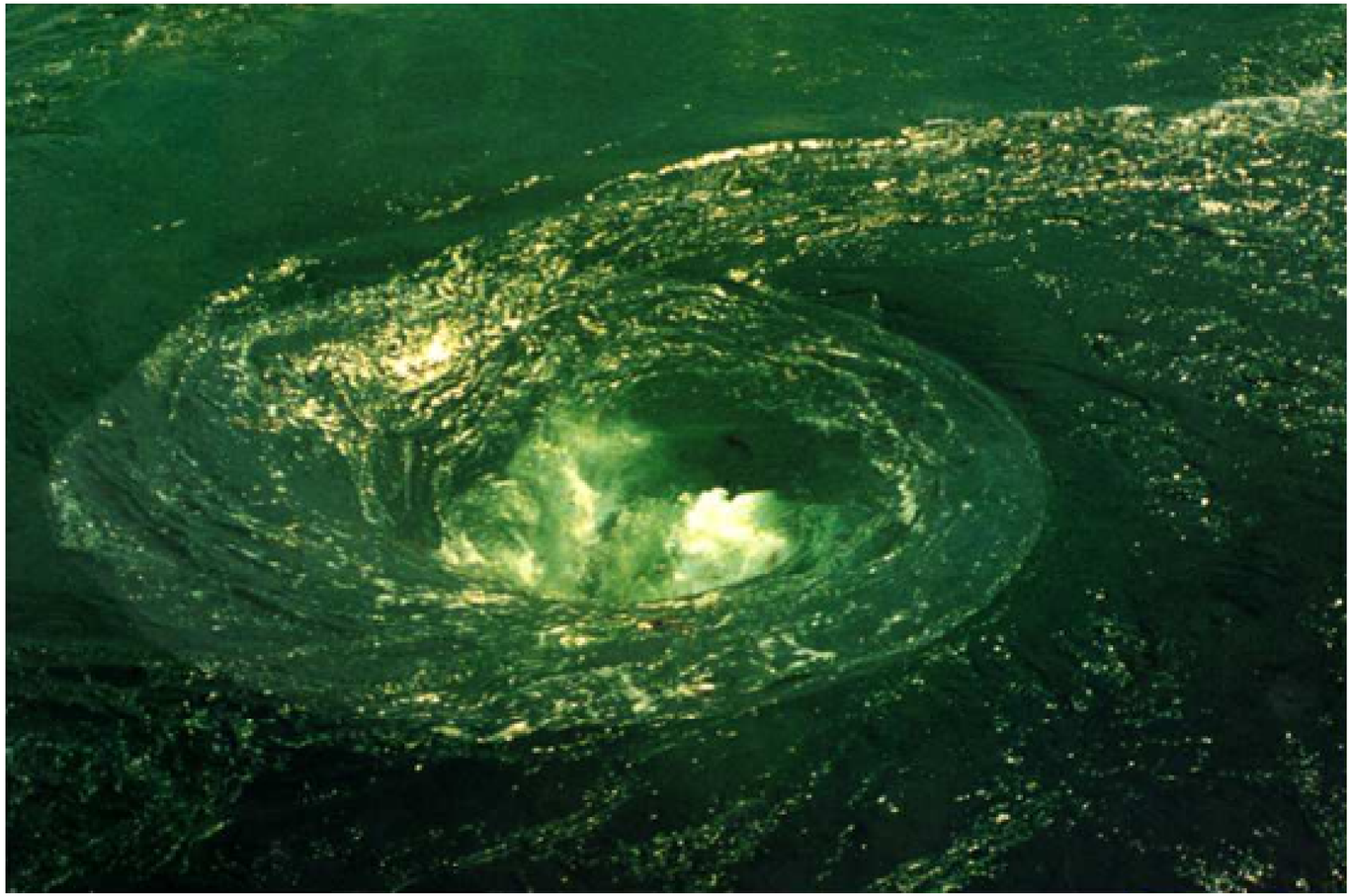
  - For each point of the domain



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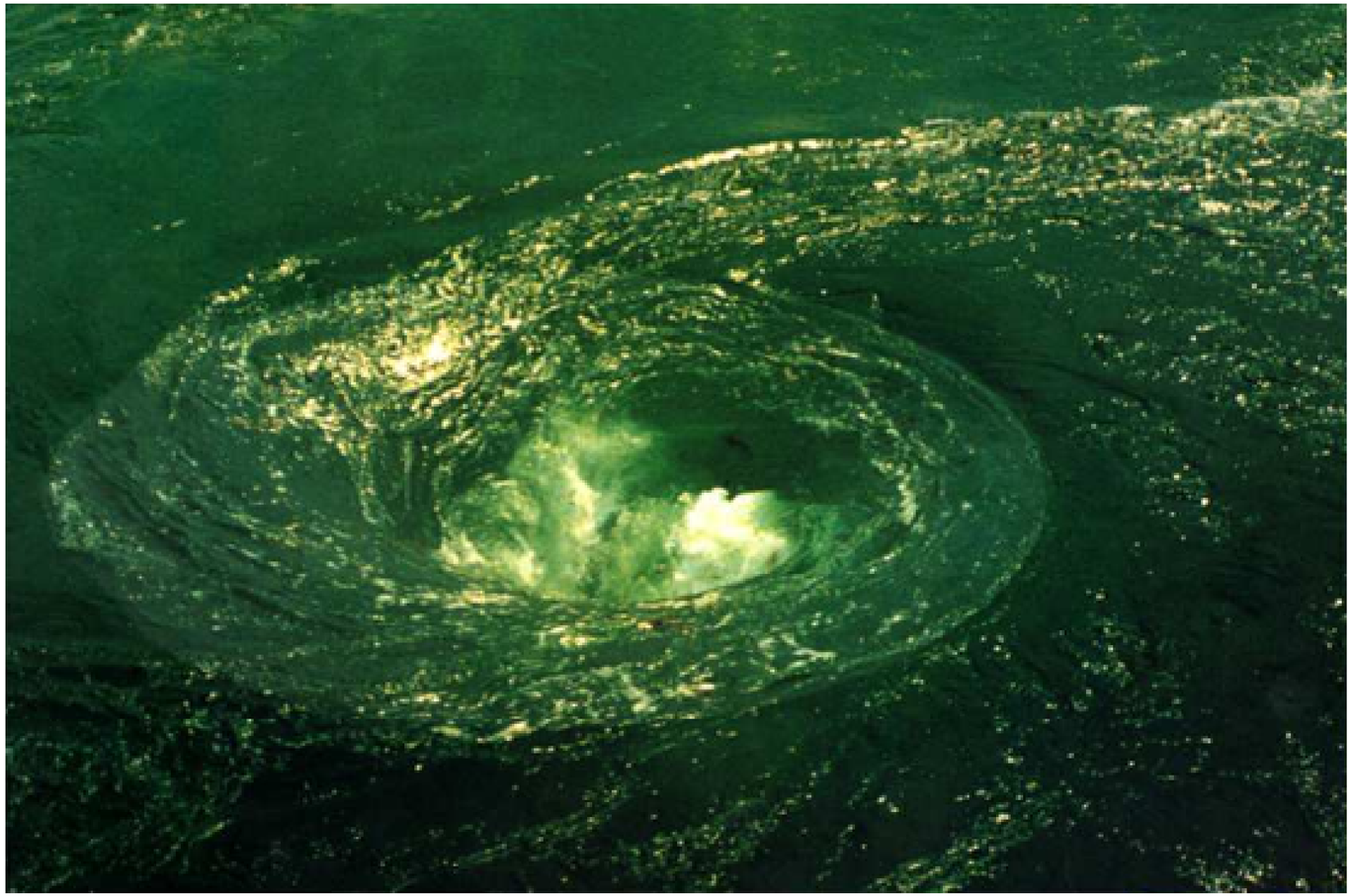

[Post et al. 2003]

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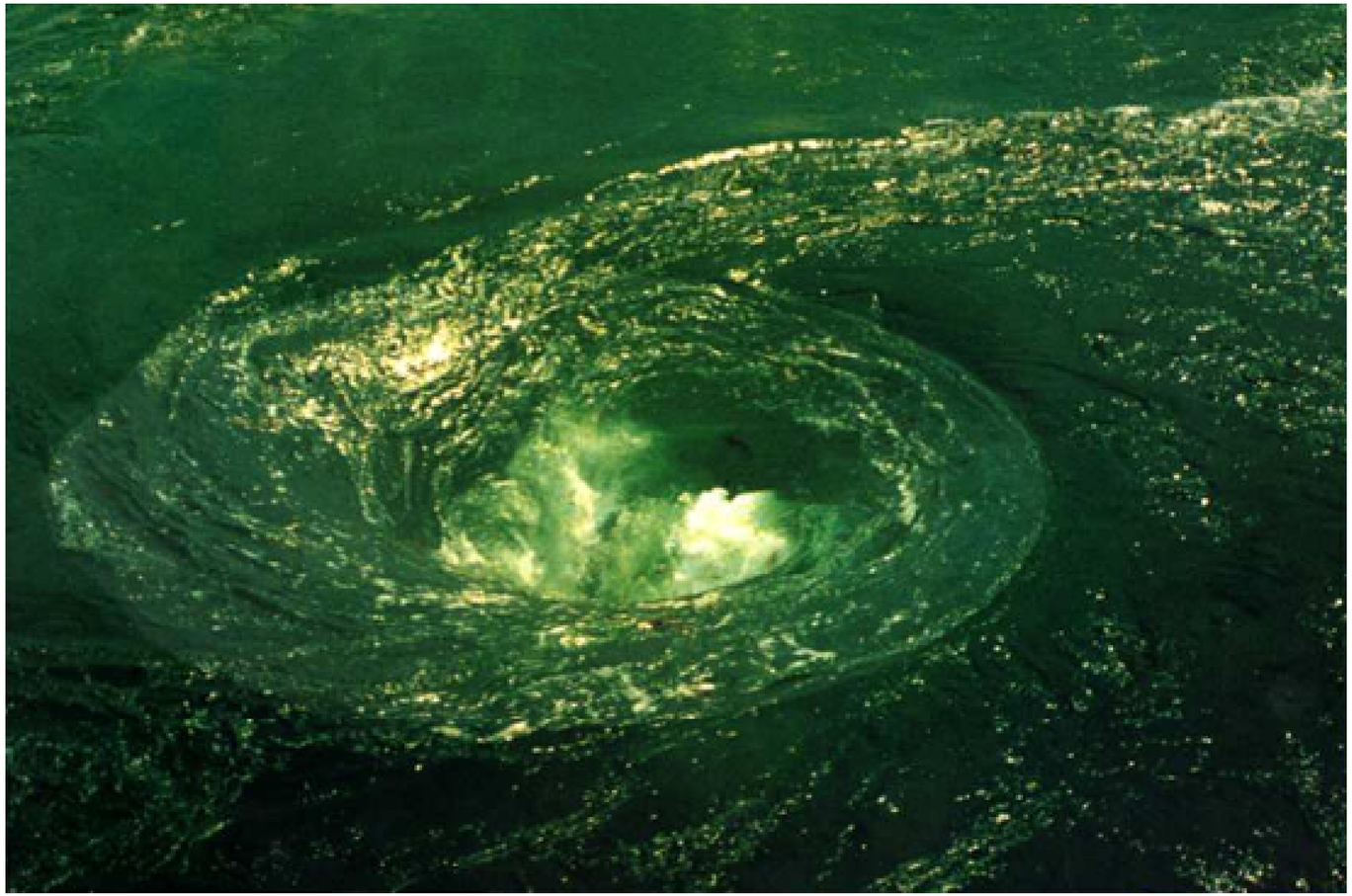
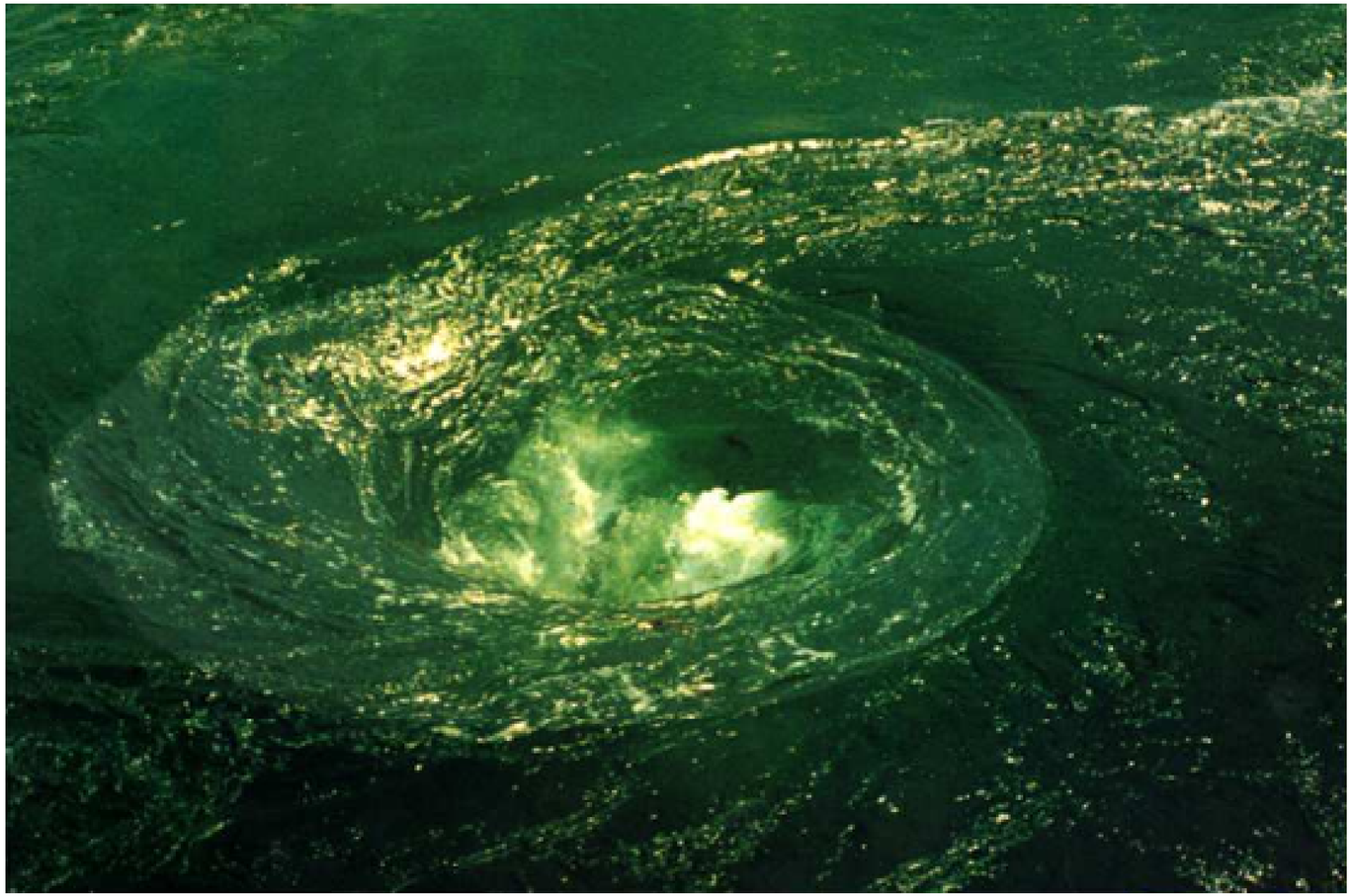
- Line Integral Convolution



[Post et al. 2003]

Line Integral Convolution

- Basic idea



[Post et al. 2003]

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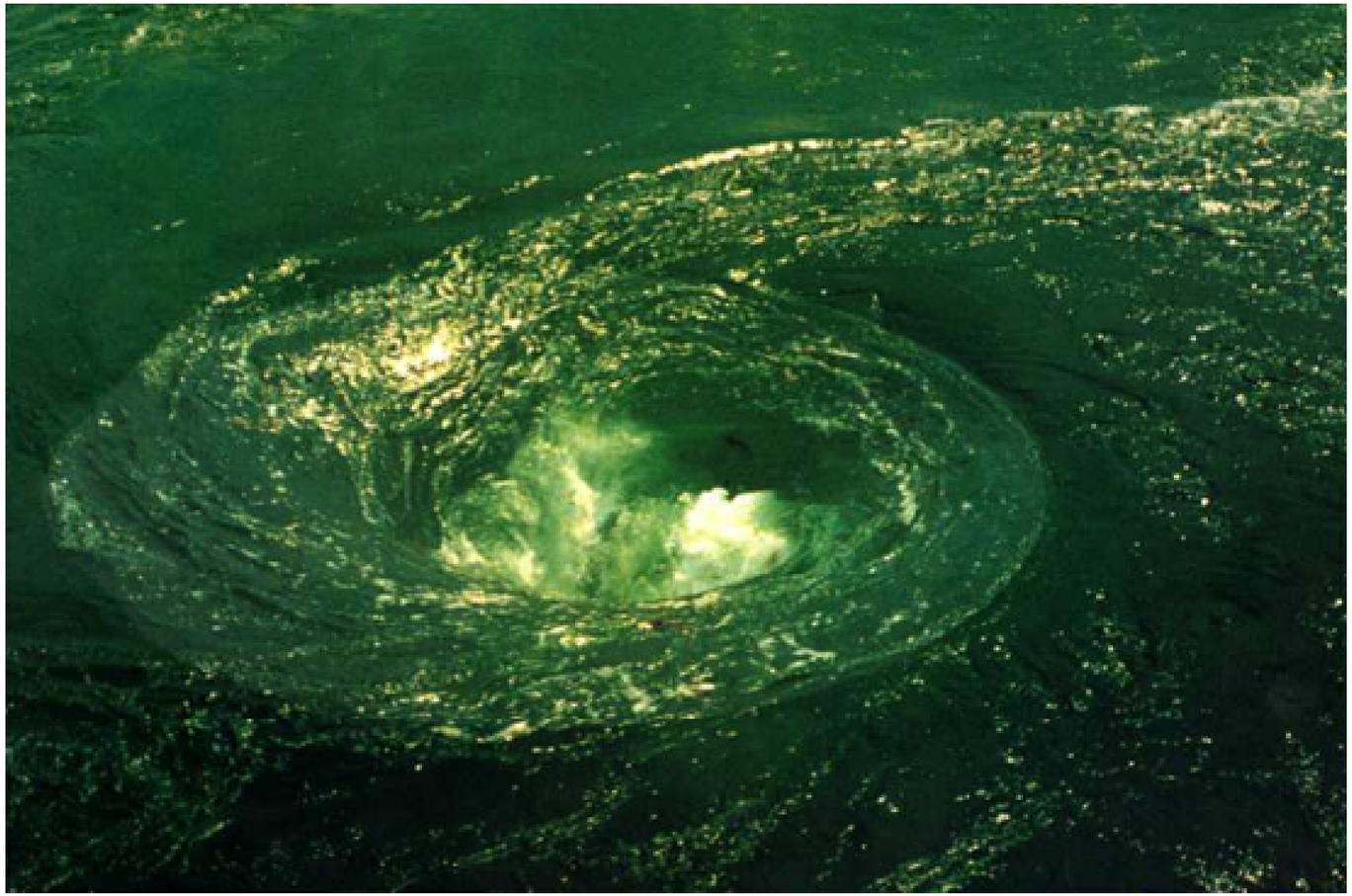
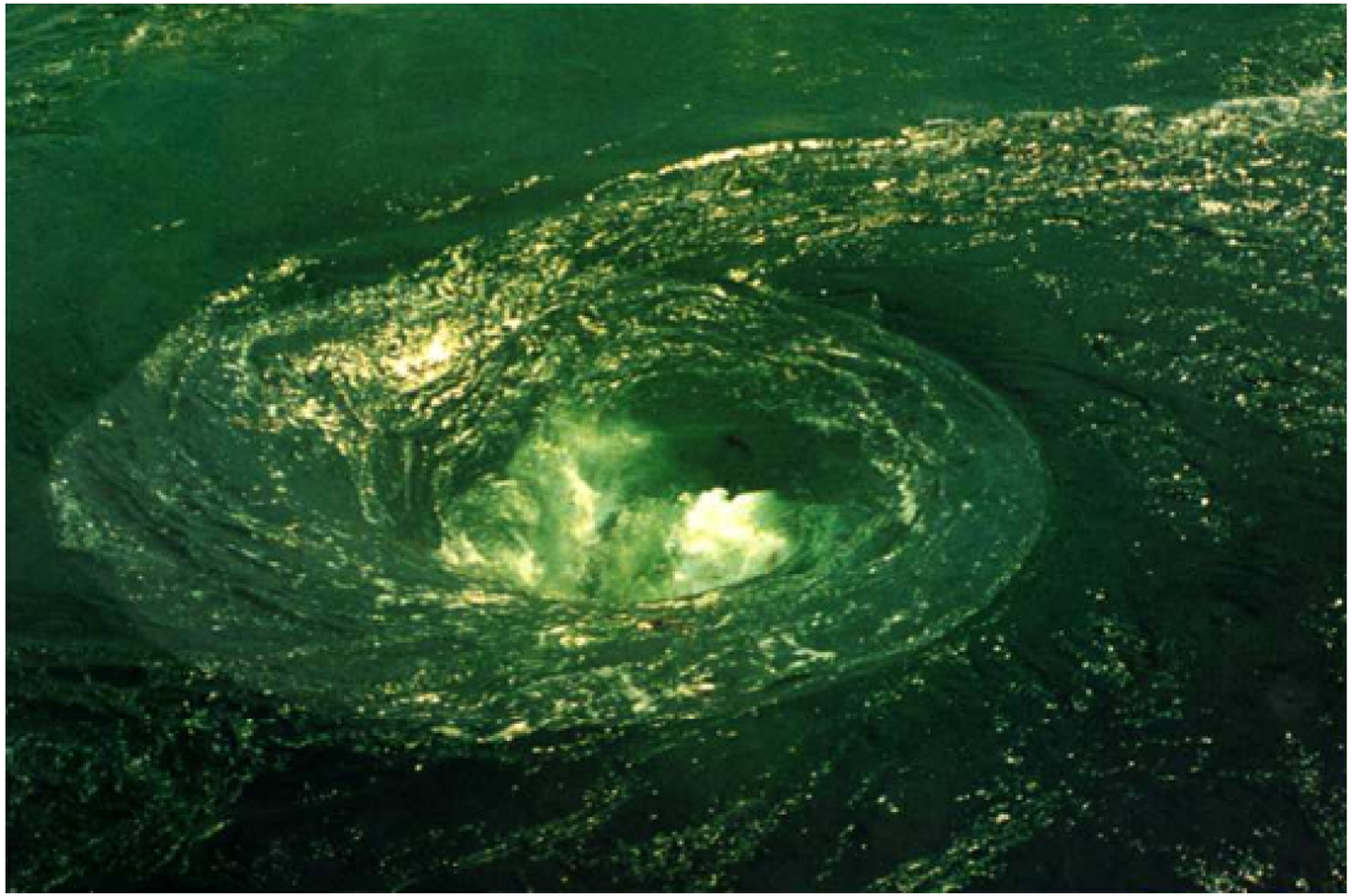


[Post et al. 2003]

Line Integral Convolution

- Basic idea
  - Global visualization
    - Compute an integral curve for each point of the domain



[Post et al. 2003]

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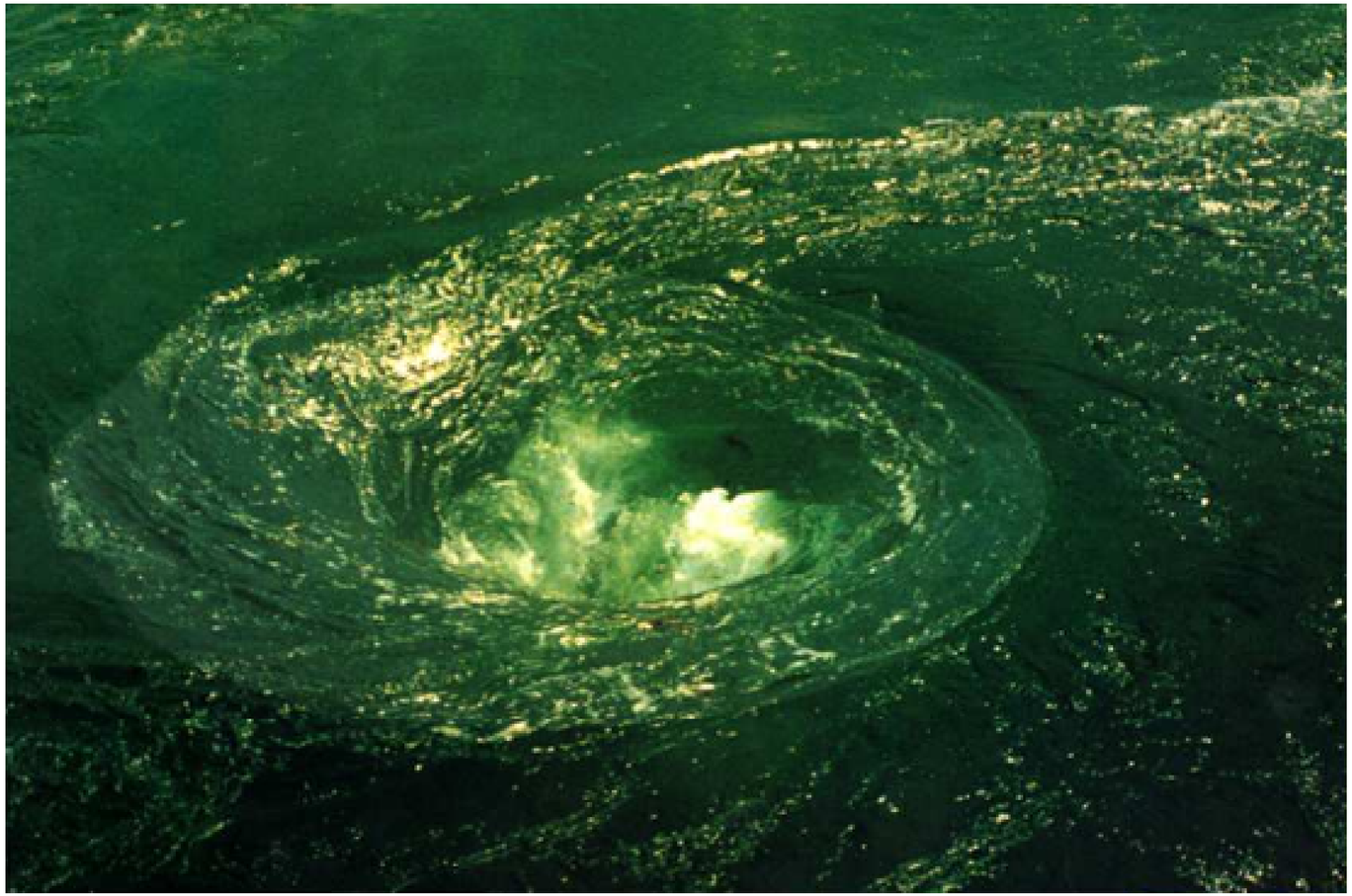
  - Problem
    - Hard to see anything



[Post et al. 2003]

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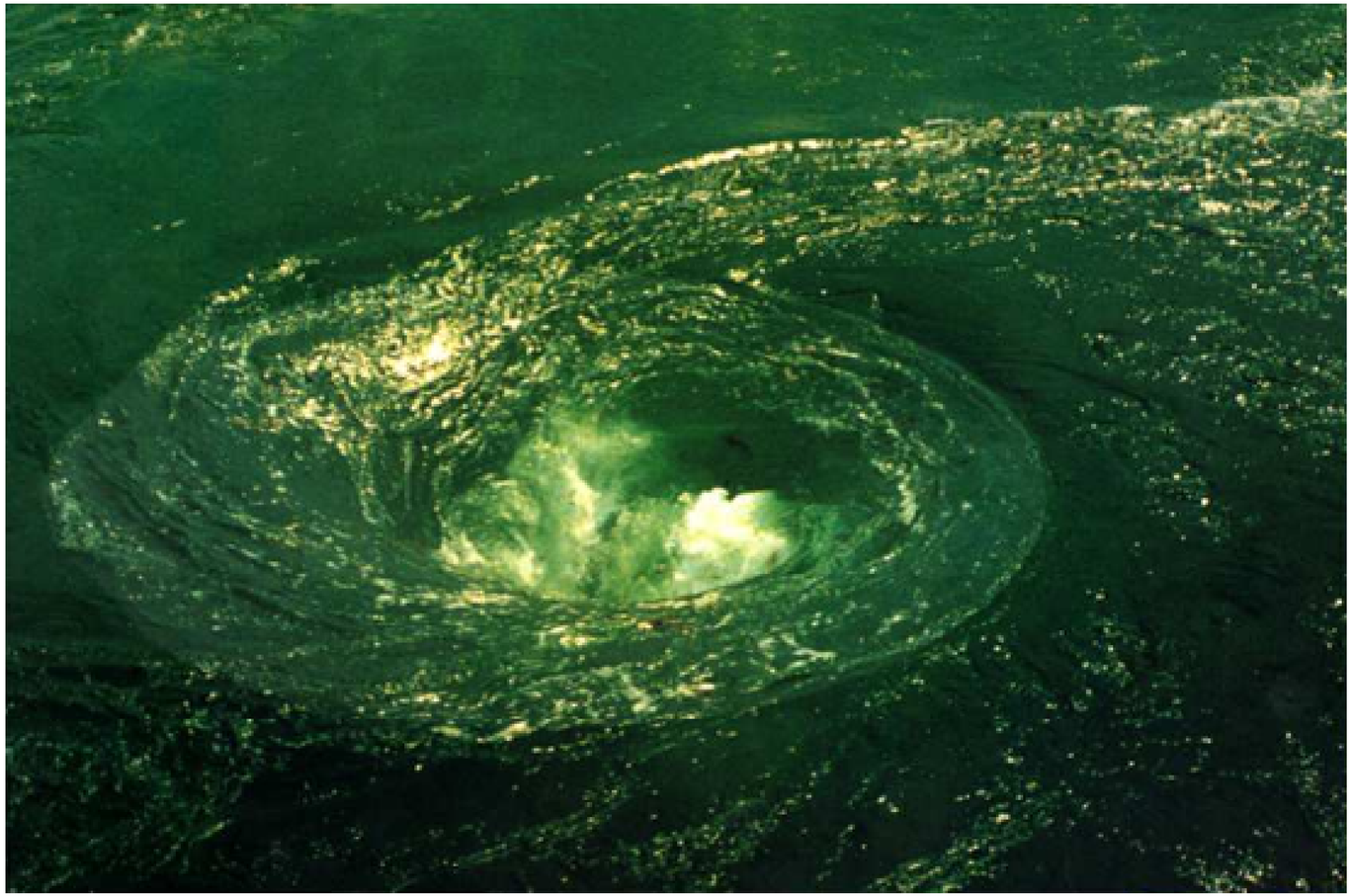
  - Key idea
    - Mimic light variation (noise)



[Post et al. 2003]

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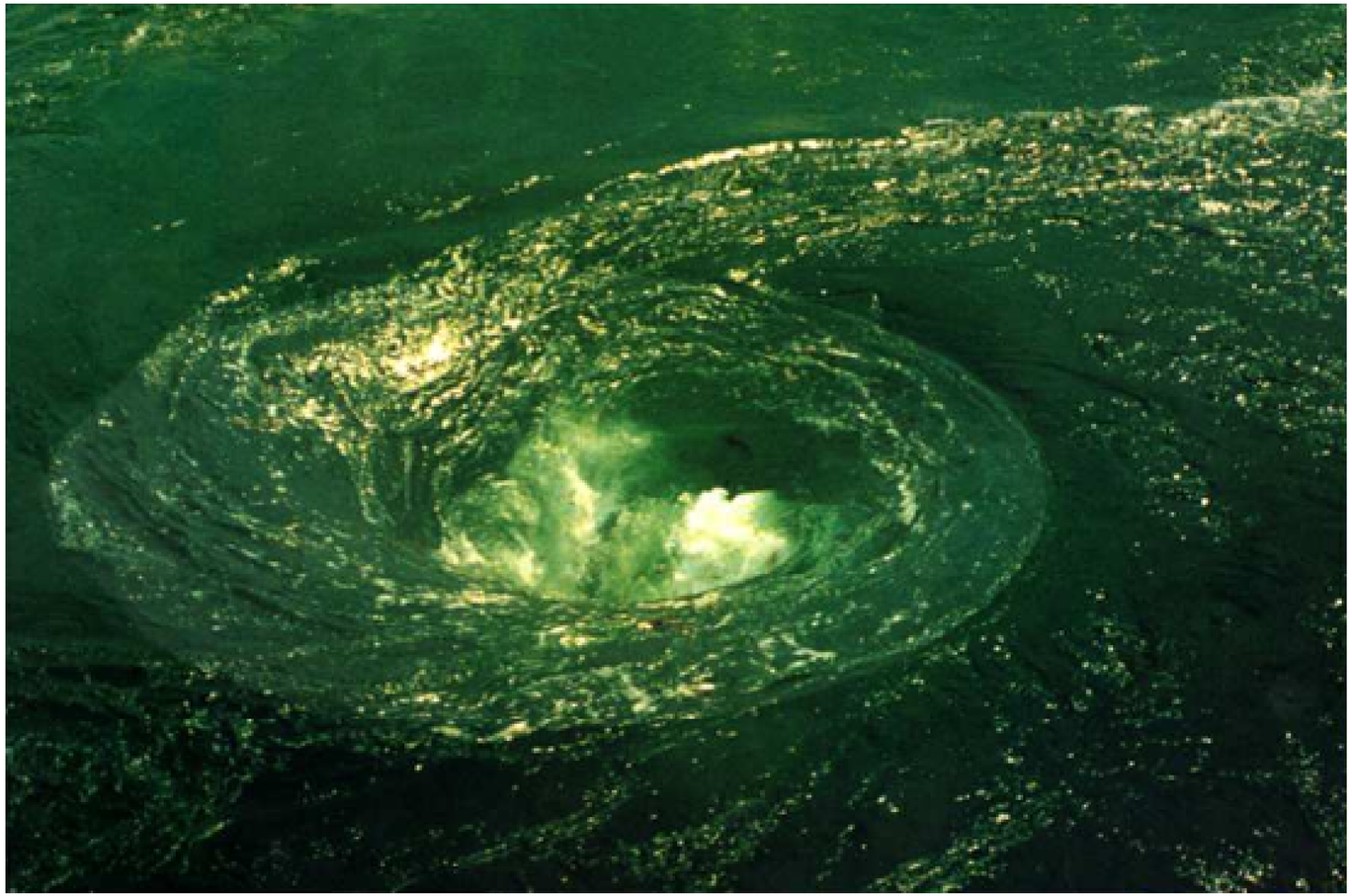

[Post et al. 2003]

Line Integral Convolution

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Line Integral Convolution

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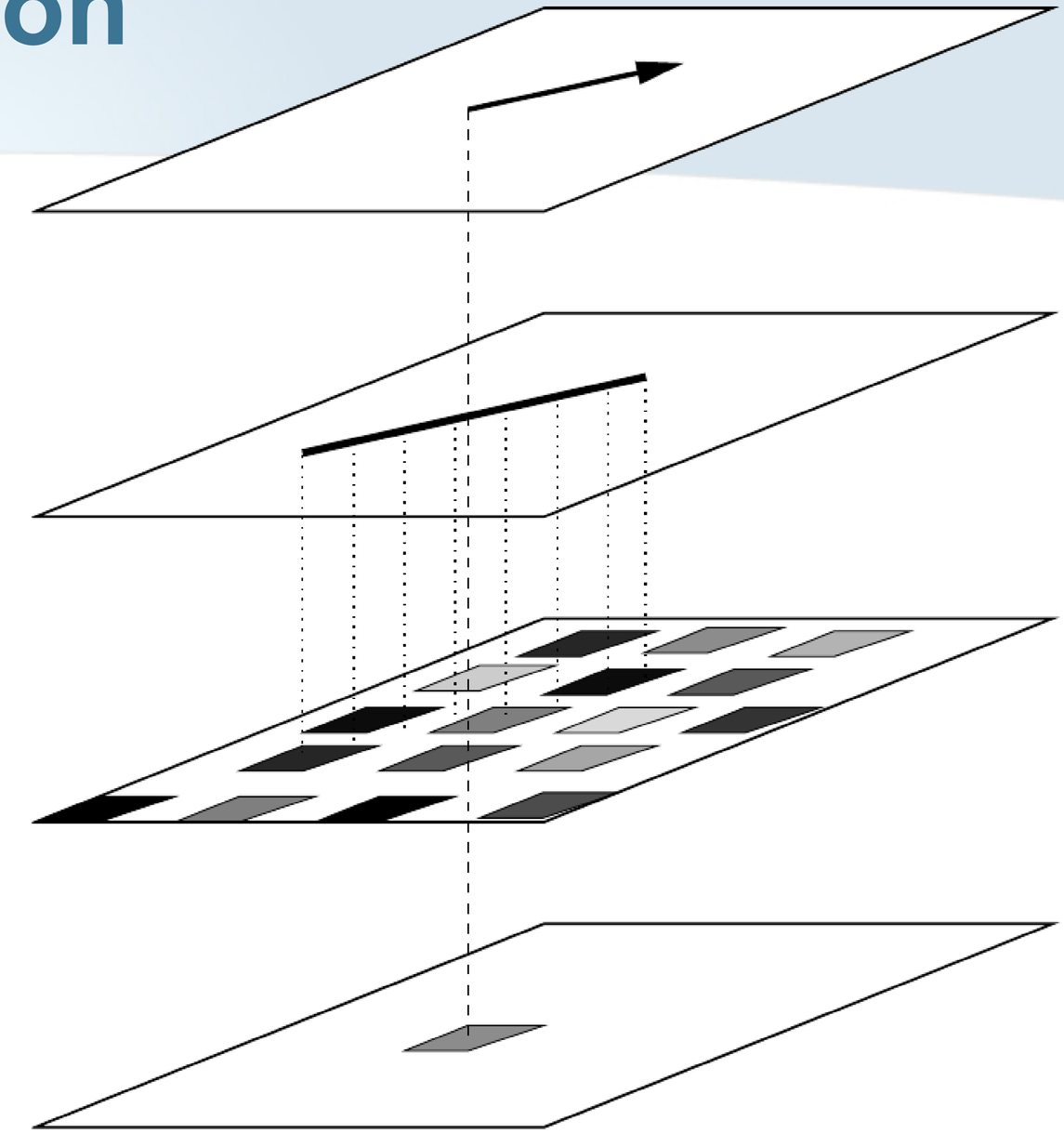

[Helgeland]

Line Integral Convolution

- Algorithm
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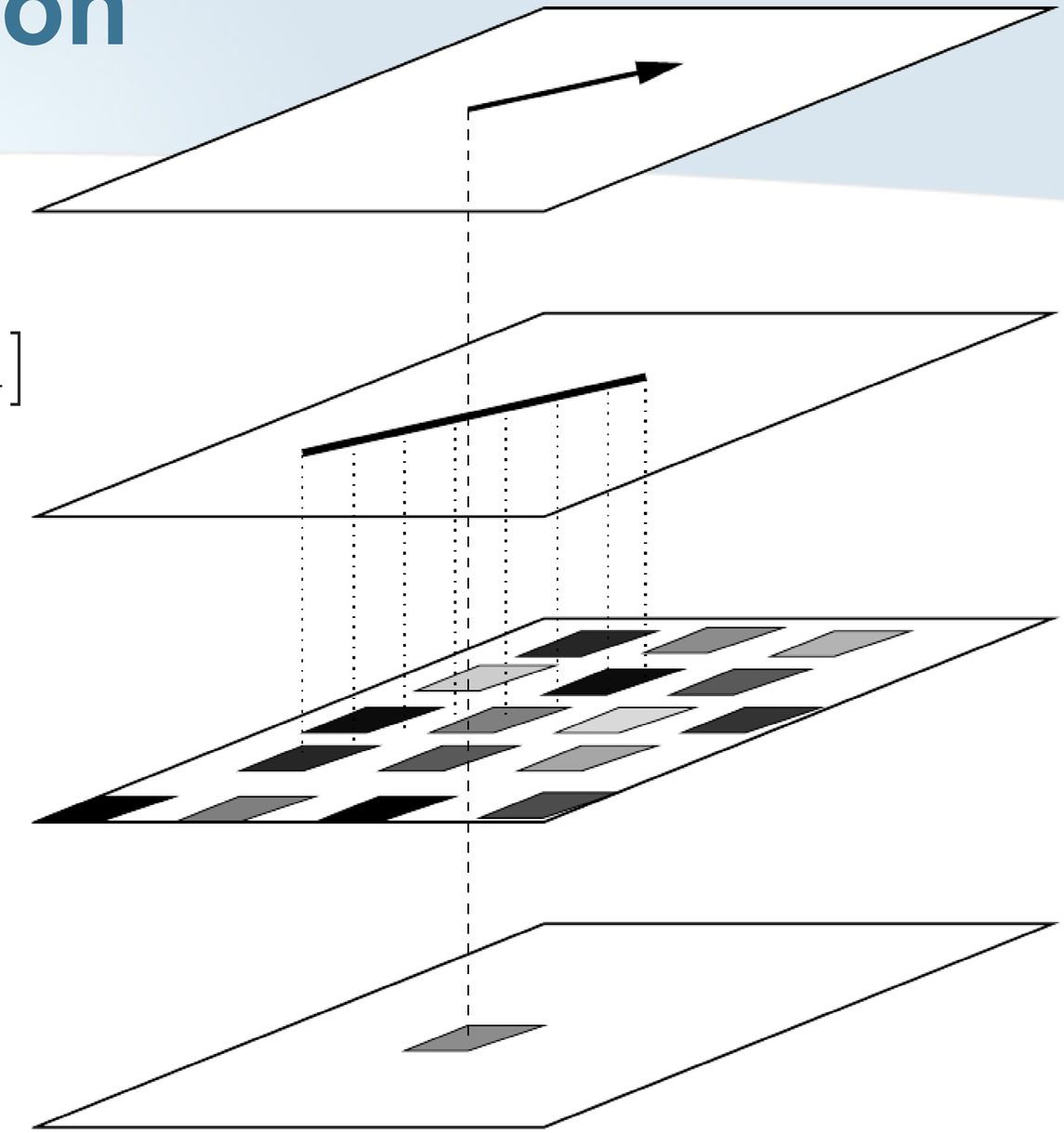

[Helgeland]

Line Integral Convolution

- Algorithm
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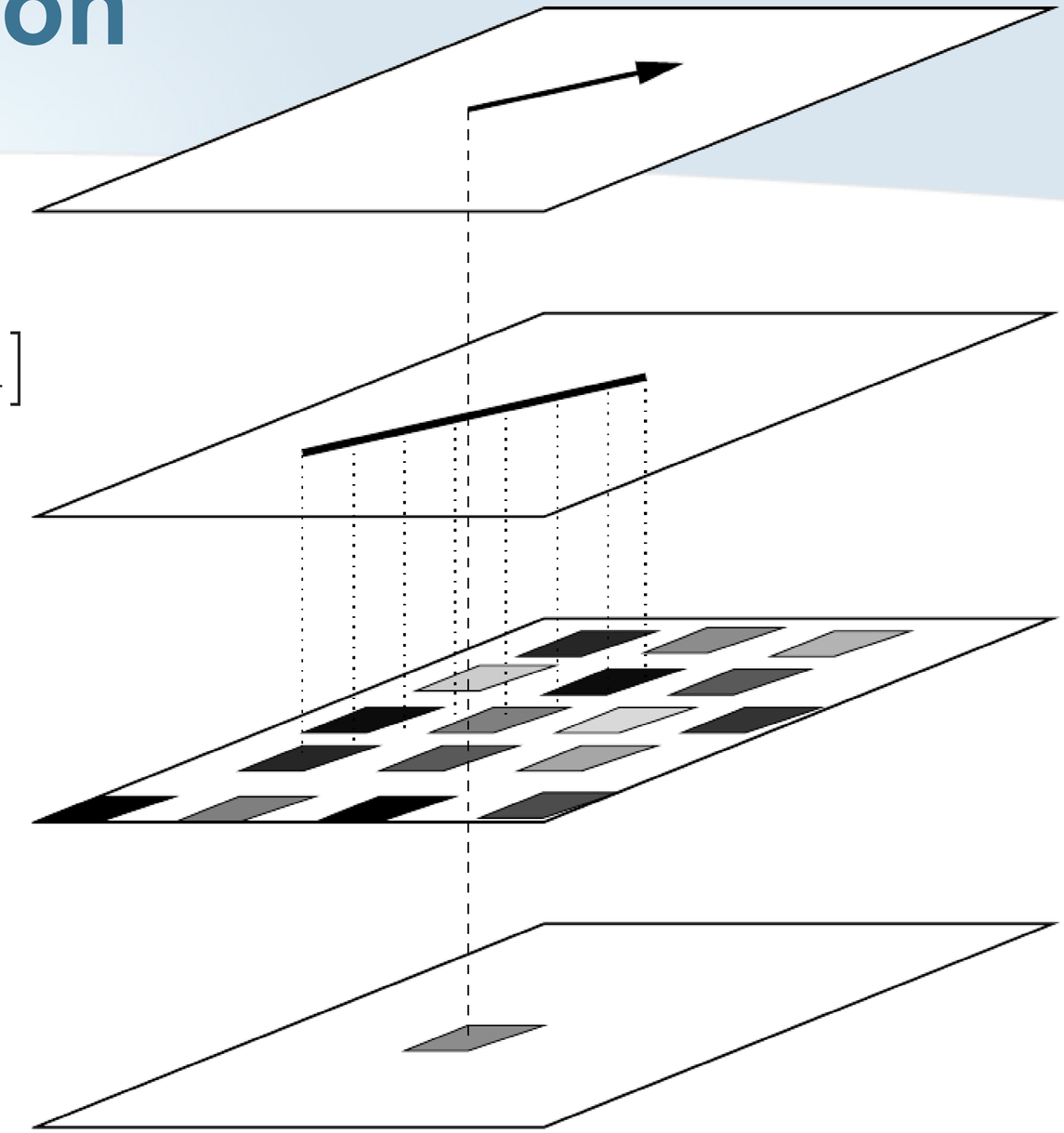
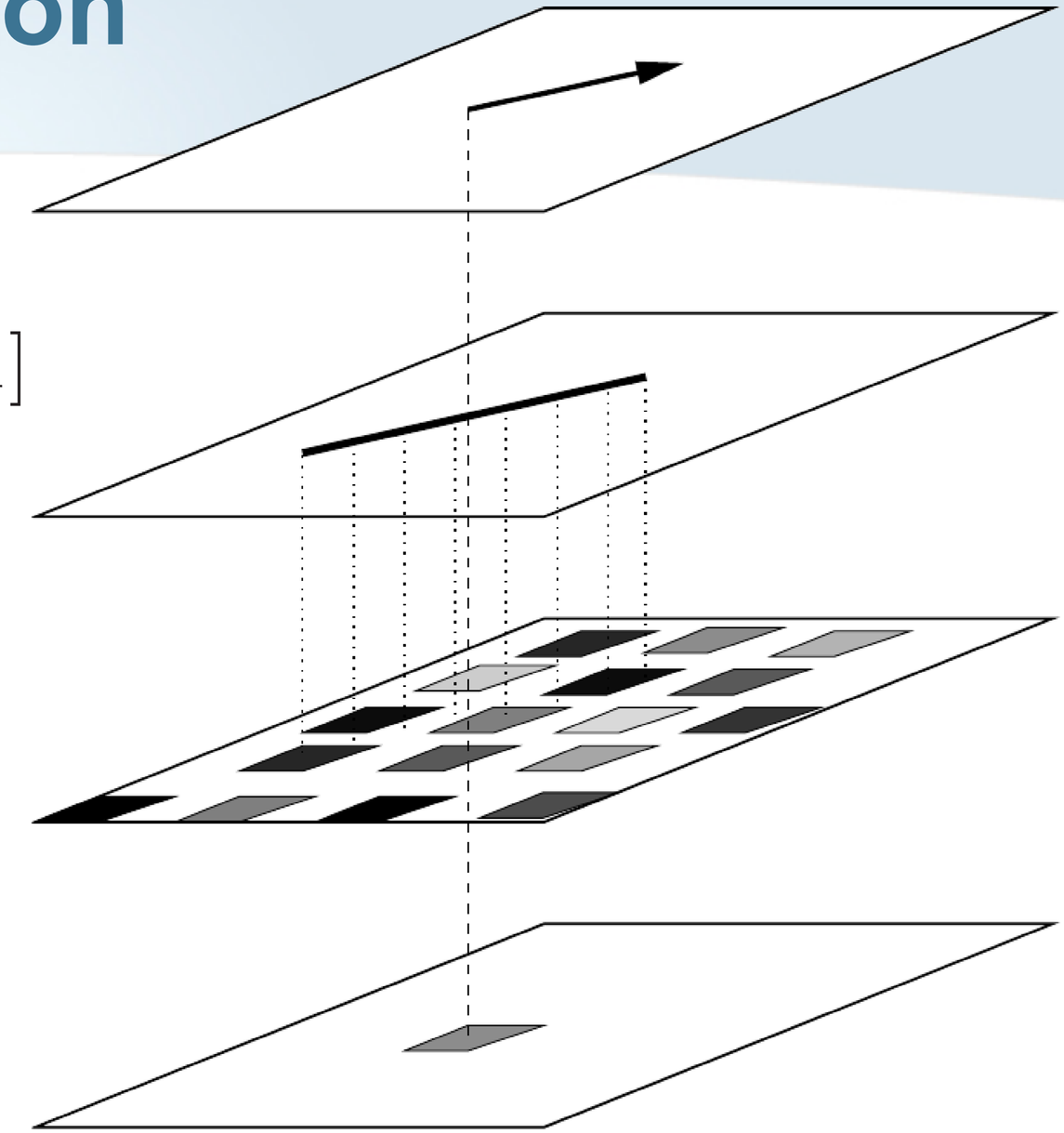
    - Random intensity for each vertex



[Helgeland]

Line Integral Convolution

- Algorithm
  - Compute a noise field $\mathcal{N} : \mathcal{D} \rightarrow [0, 1]$
    - Random intensity for each vertex
  - For each vertex of the domain
    - Compute its integral curve $\mathcal{C}$
      - Backwards and forwards



[Helgeland]

Line Integral Convolution

- Algorithm
  - Compute a noise field $\mathcal{N} : \mathcal{D} \rightarrow [0, 1]$
    - Random intensity for each vertex
  - For each vertex of the domain
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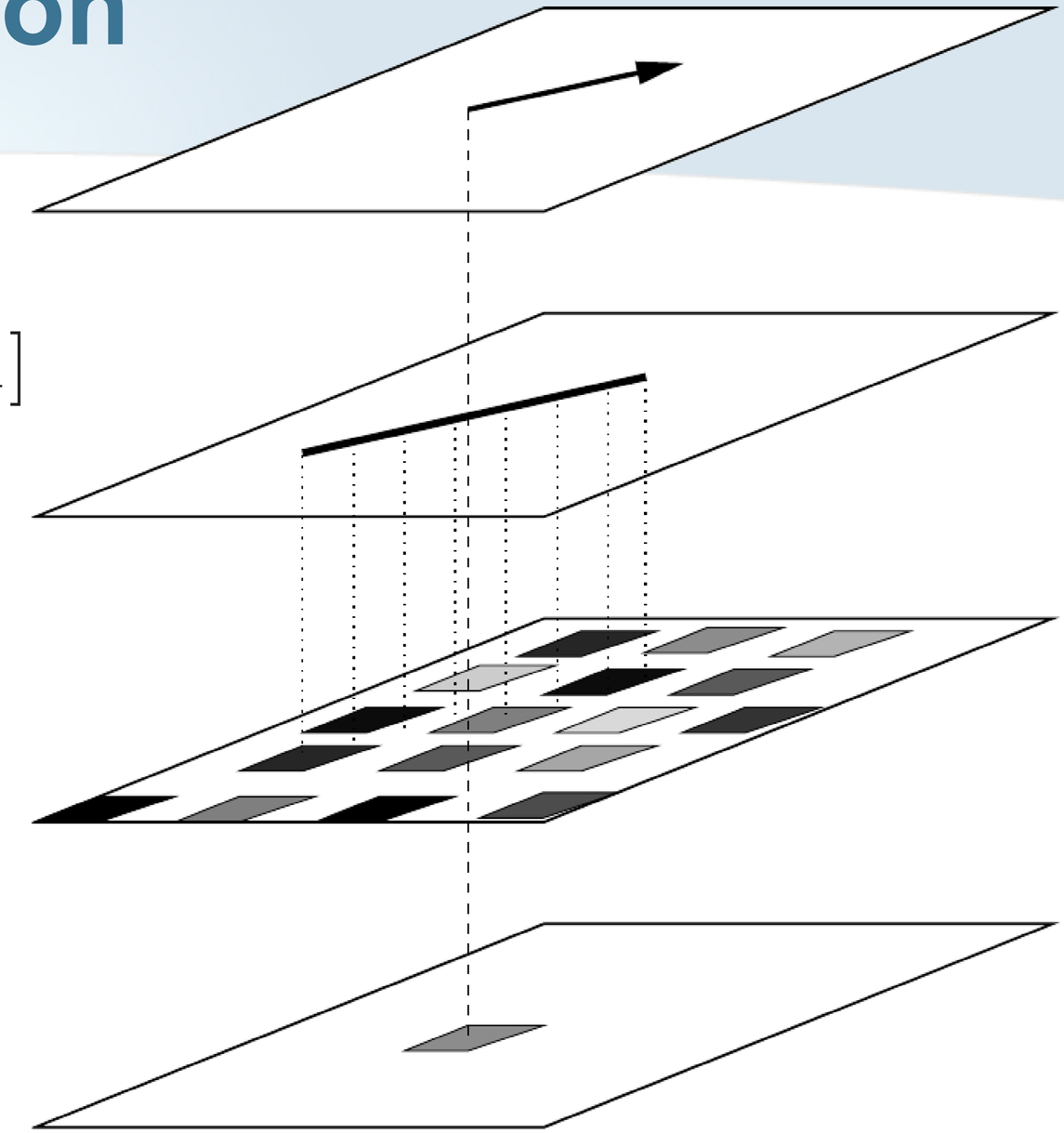
    - Convolution with the noise field



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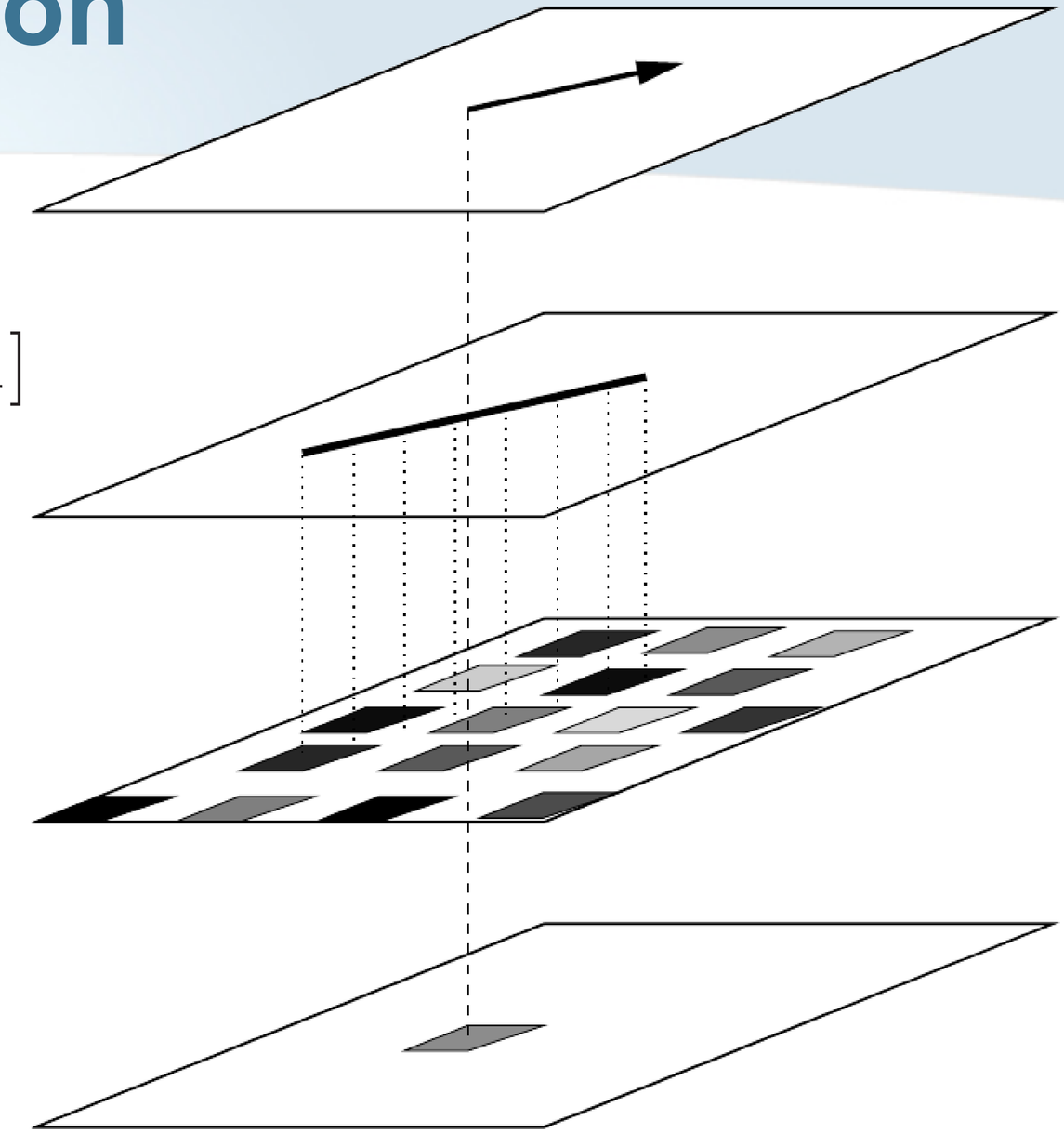
    - Convolution with the noise field

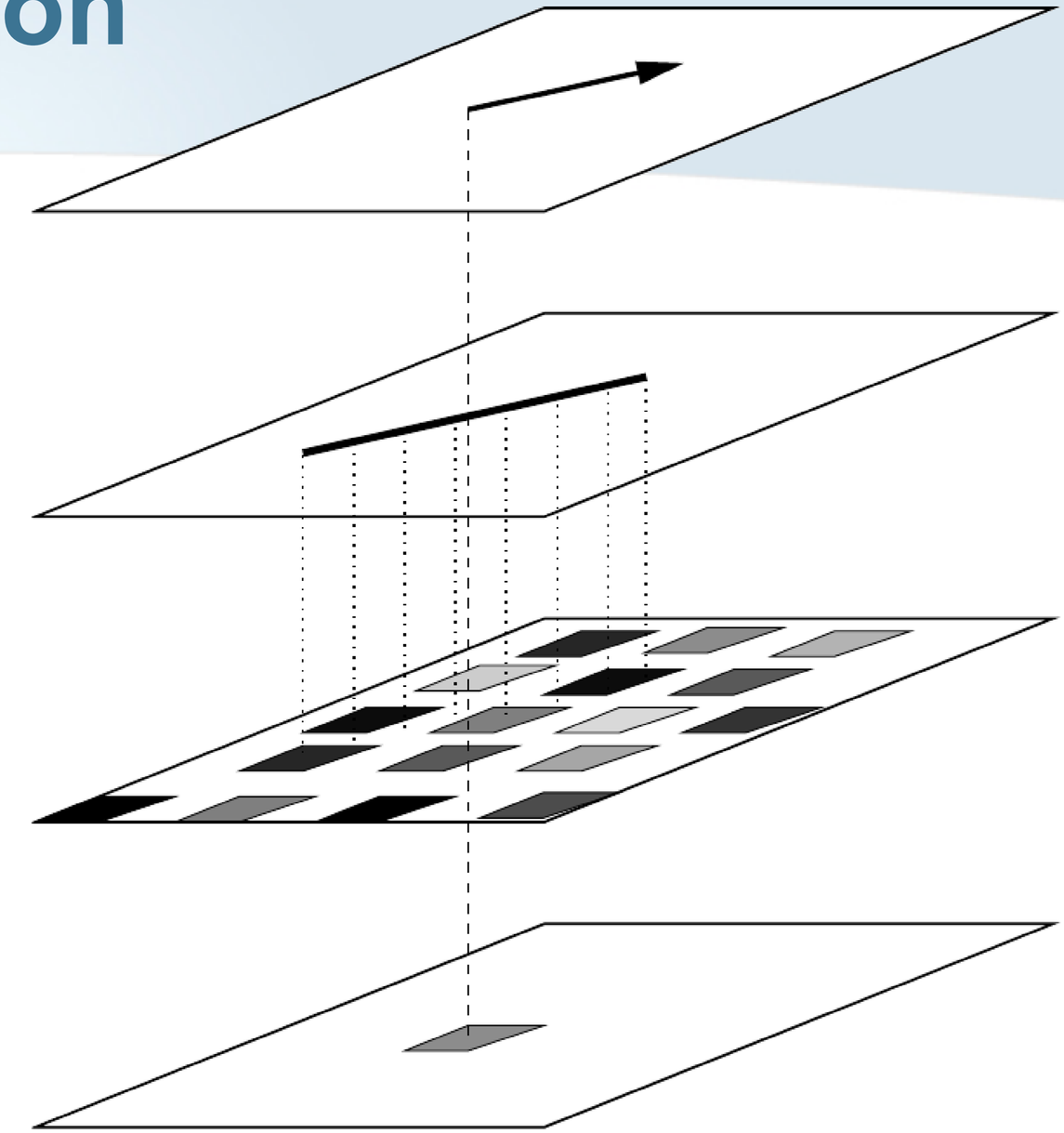
$$LIC(p) = \int_{c(p)-L}^{c(p)+L} k(u) \; . \; \mathcal{N}(c^{-1}(u))du$$



[Helgeland]

Line Integral Convolution

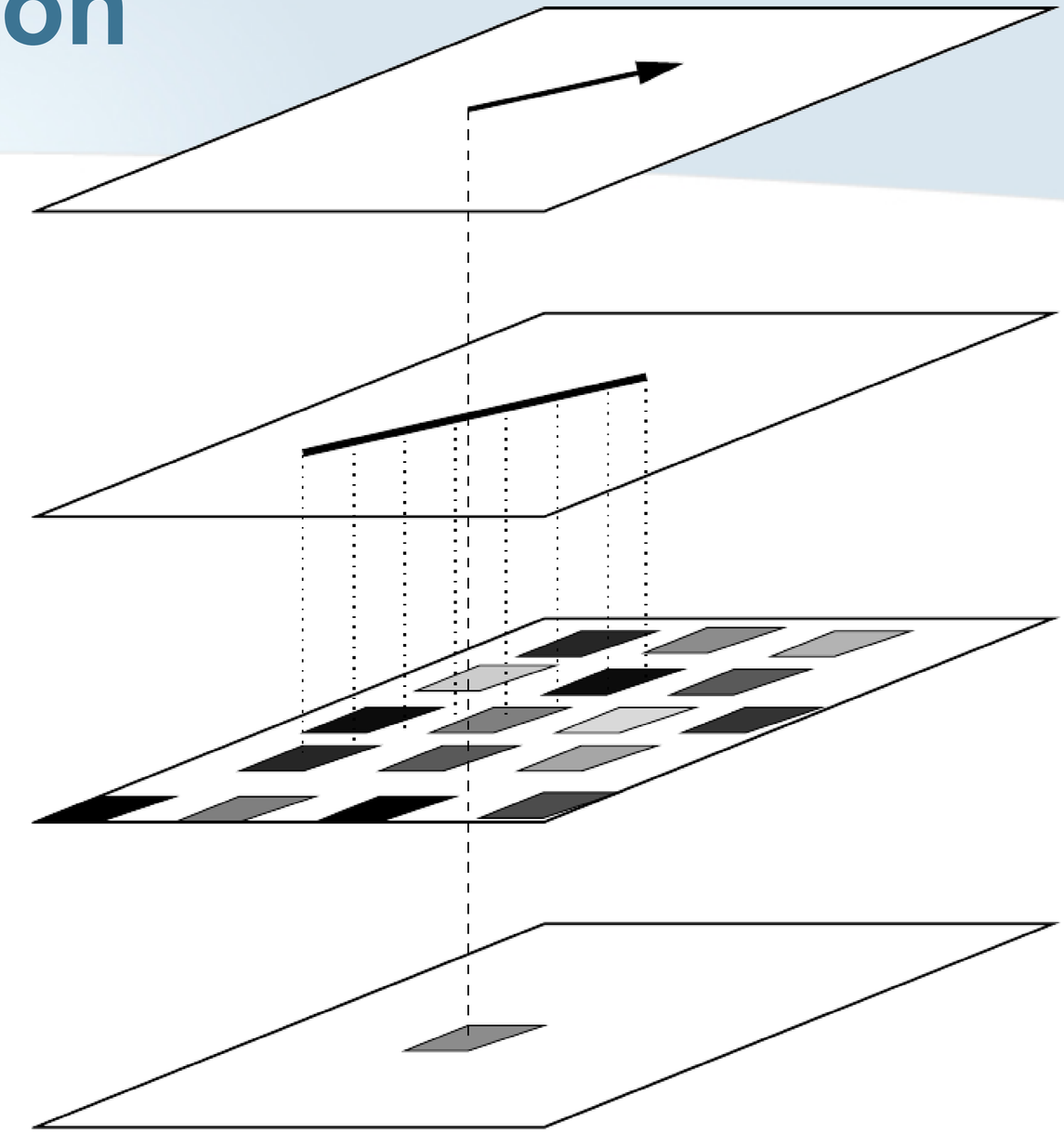
- Convolution kernel $k(u)$



[Helgeland]

Line Integral Convolution

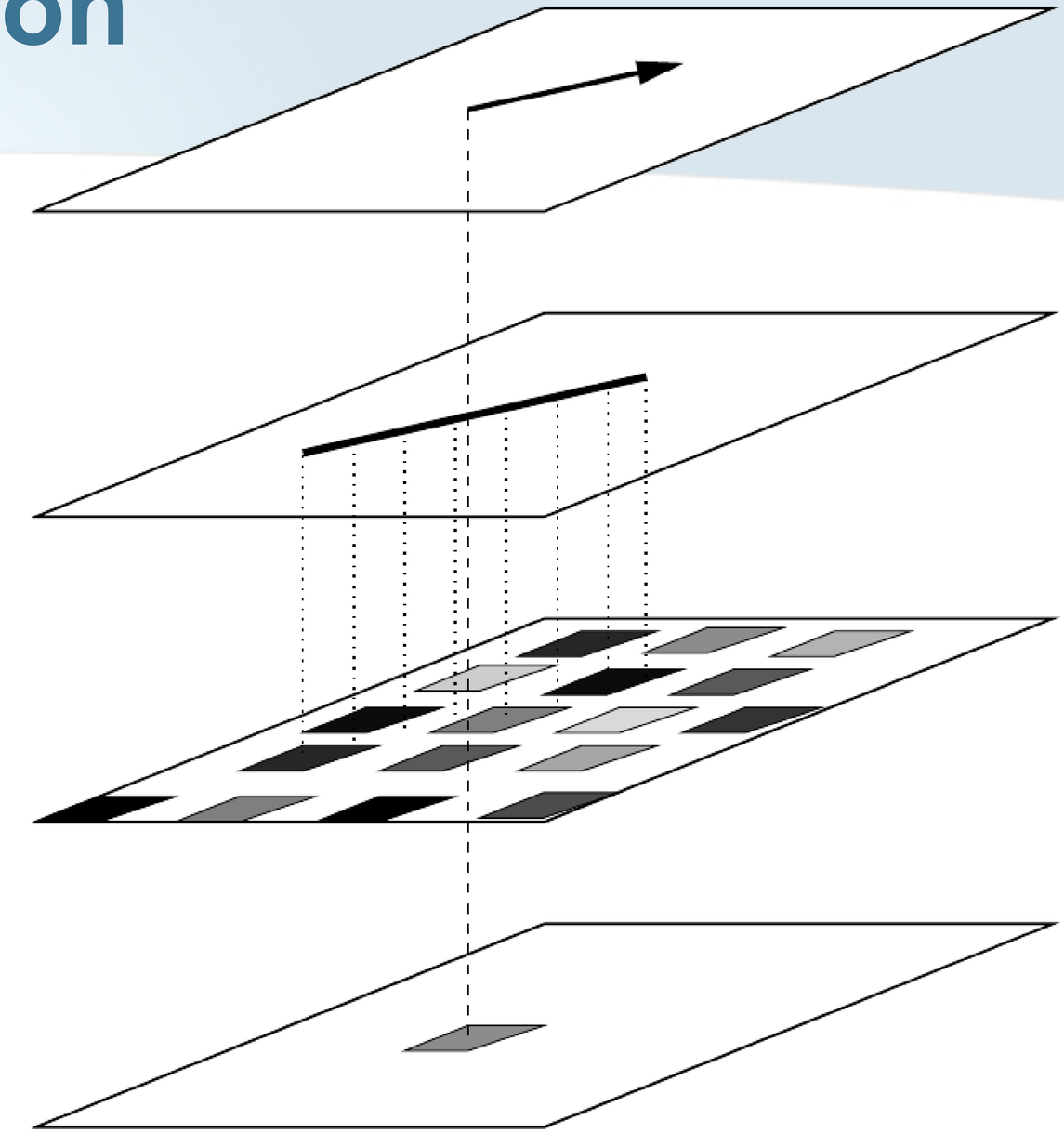
- Convolution kernel $k(u)$
  - Gaussian kernel



[Helgeland]

Line Integral Convolution

- Convolution kernel $k(u)$
  - Gaussian kernel
  - Finite support
    $[c(p) - L, c(p) + L]$

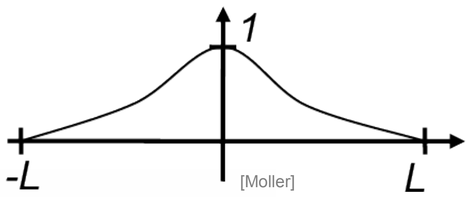


[Helgeland]

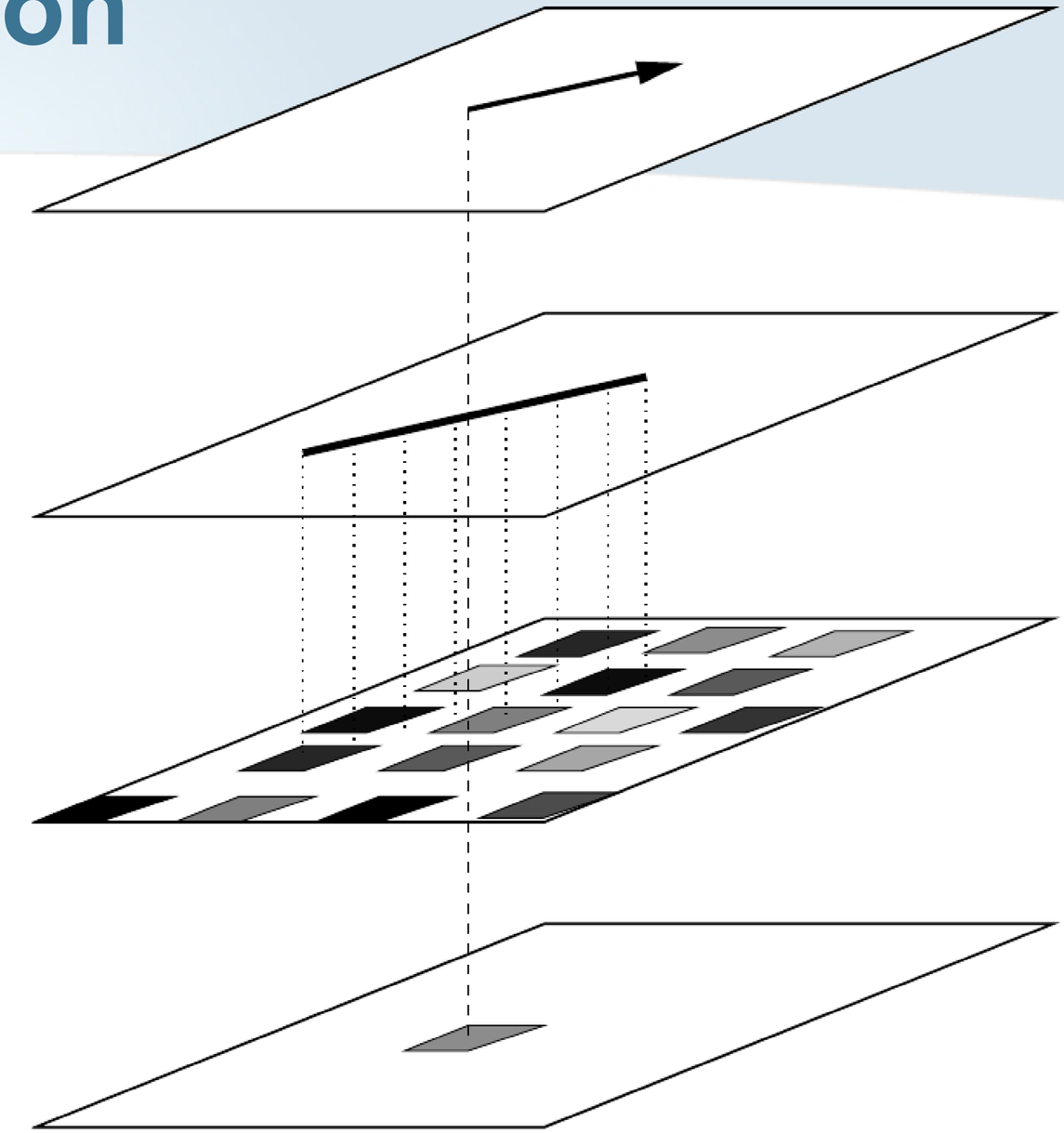
Line Integral Convolution

- Convolution kernel $k(u)$
  - Gaussian kernel
  - Finite support
    $[c(p) - L, c(p) + L]$
  - Normalized

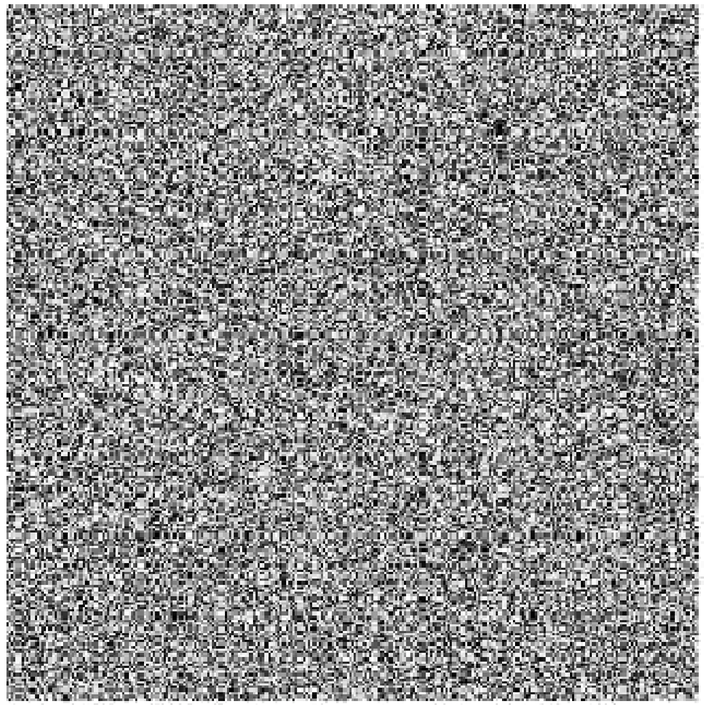
$$\int_{c(p)-L}^{c(p)+L} k(u)du = 1$$

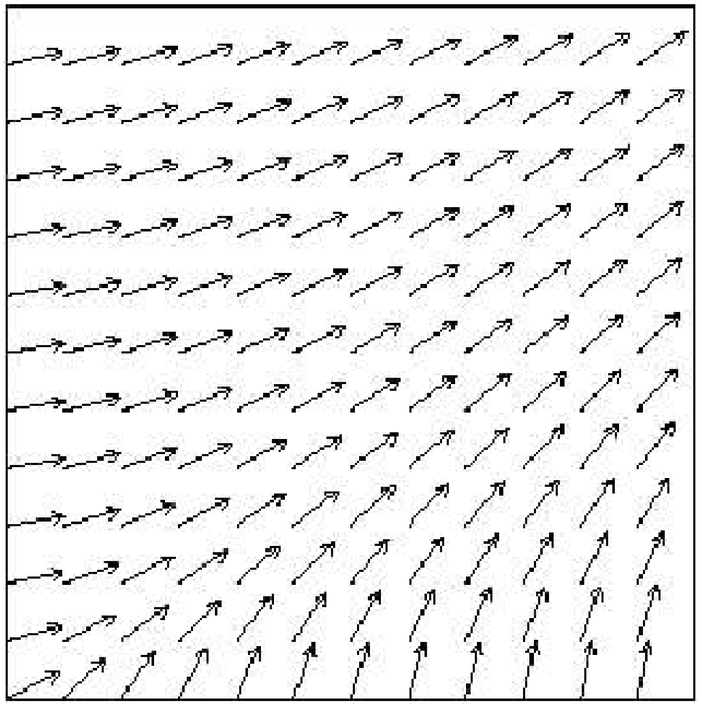


[Moller]


[Helgeland]

Line Integral Convolution





[Helgeland]

LIC for volumetric domains

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- Easy way
  - Slice the volume

LIC for volumetric domains

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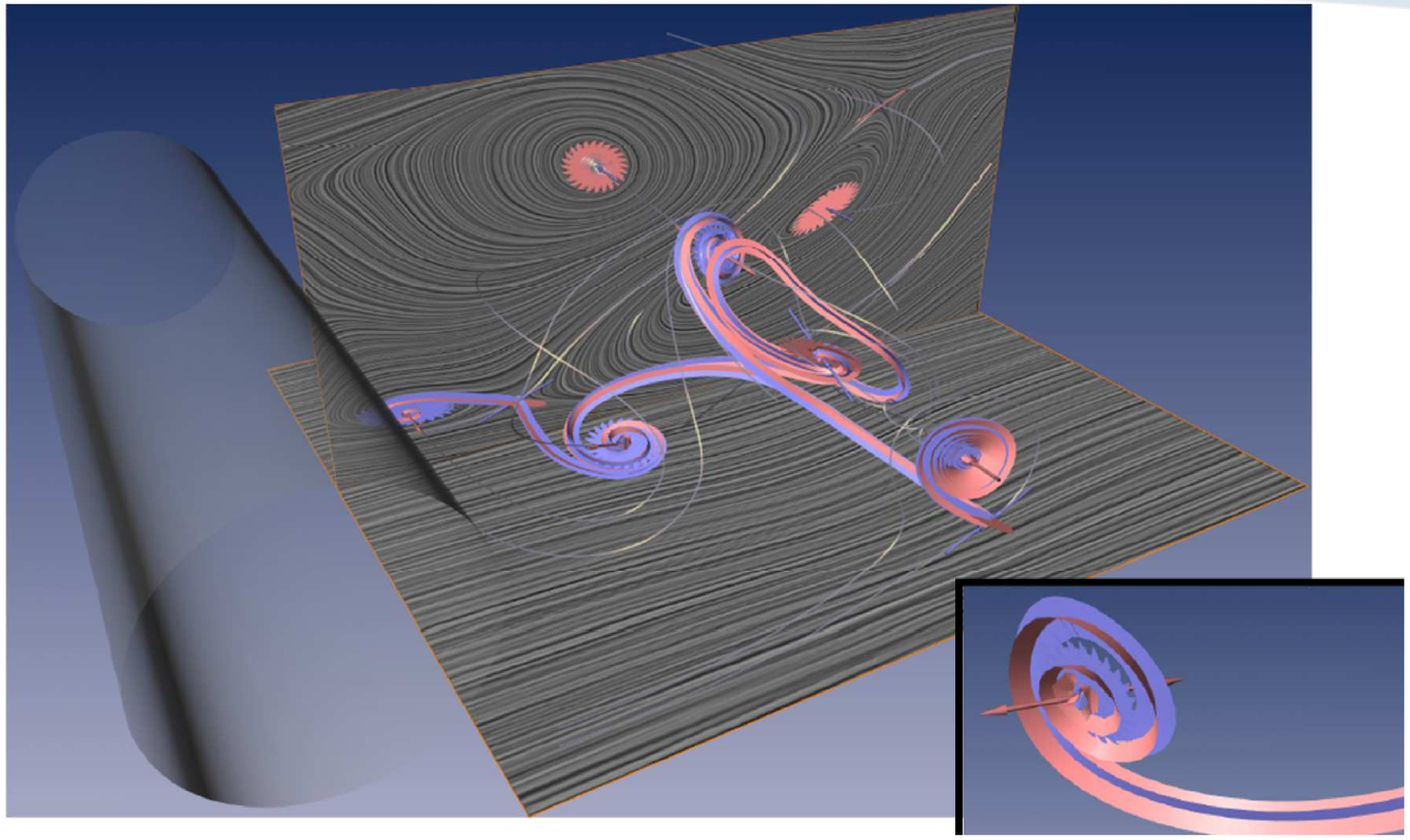

[Theisel 03]

LIC for volumetric domains

- Easy way
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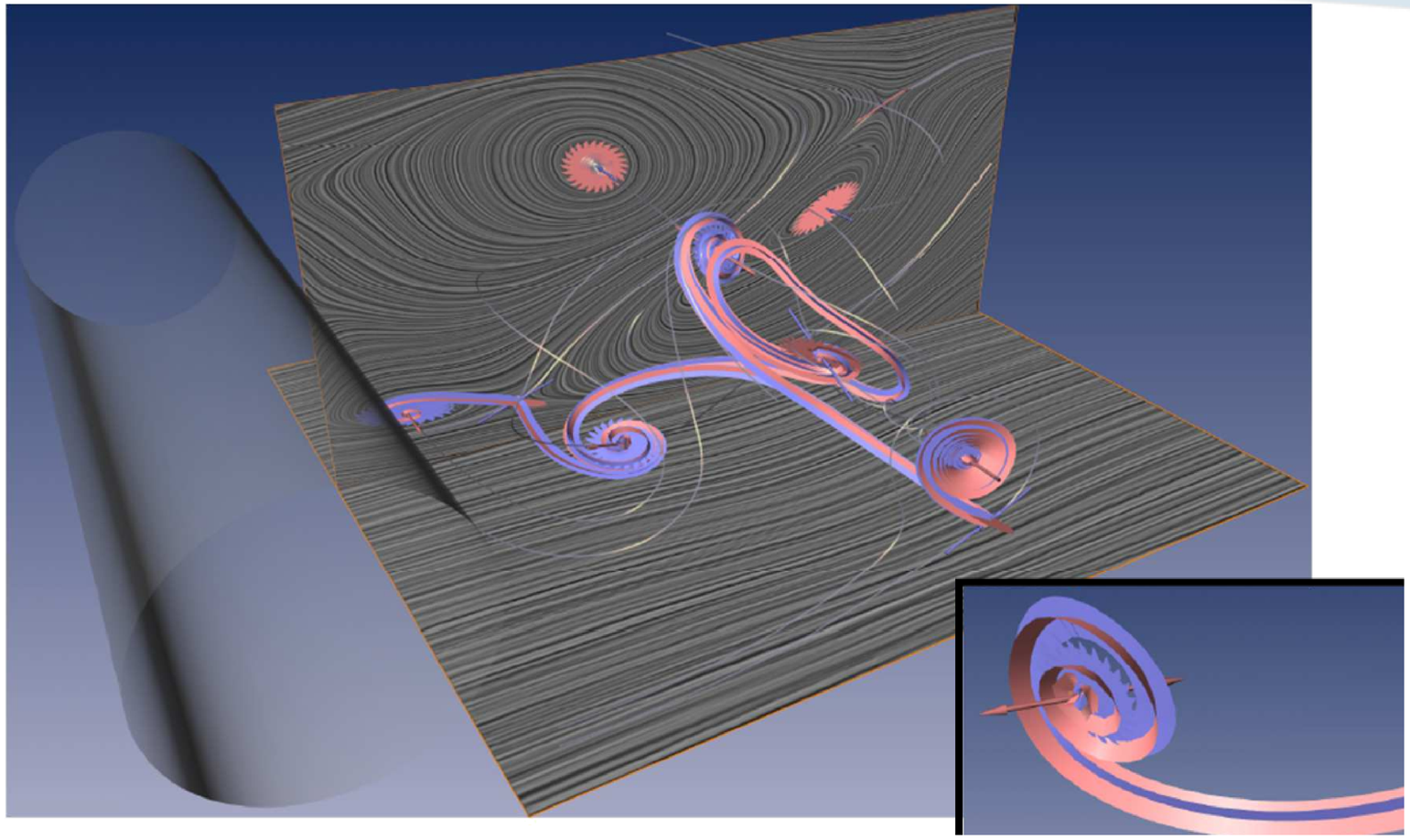
  - LIC for each slice
- Also,
  - What is LIC in the end?



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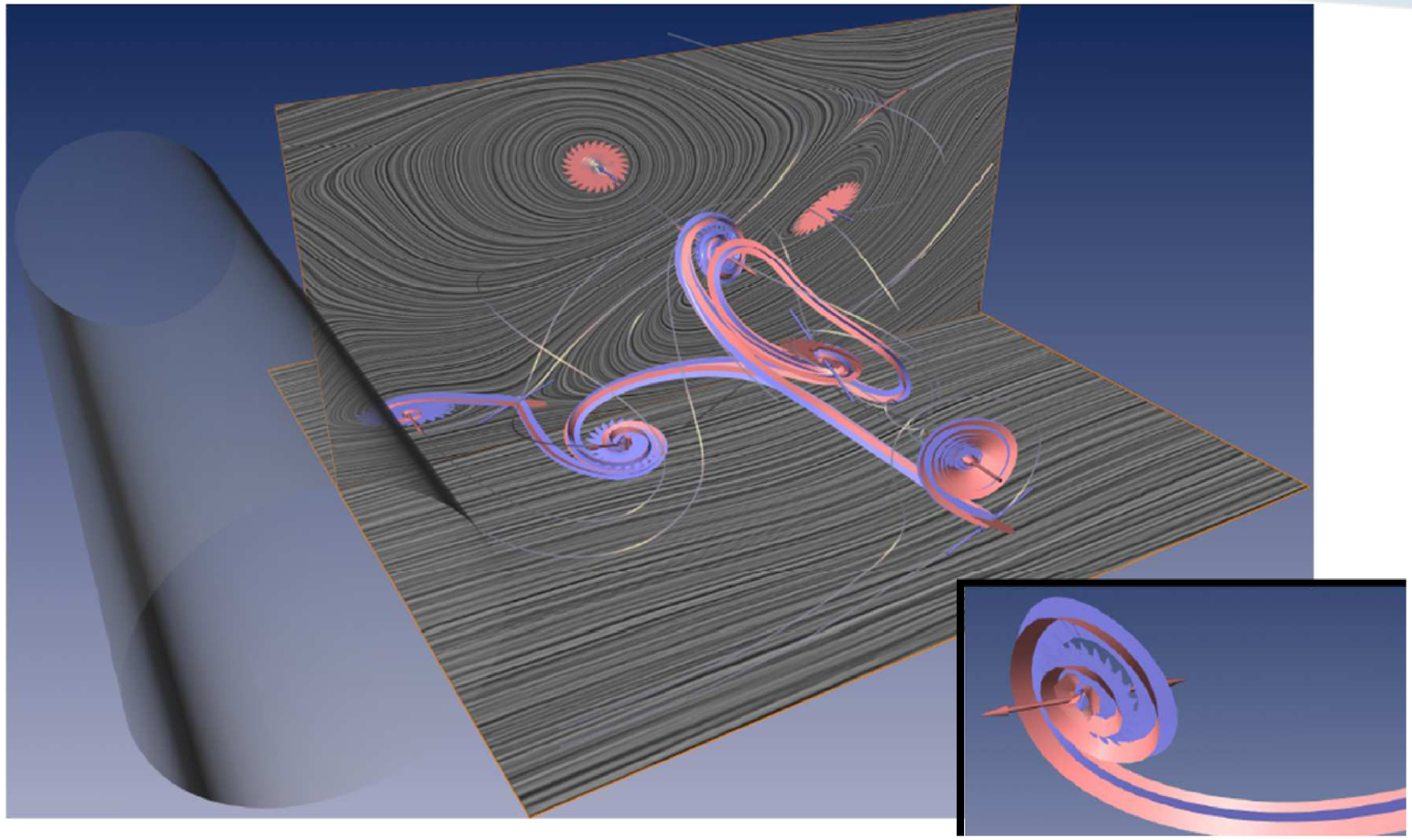

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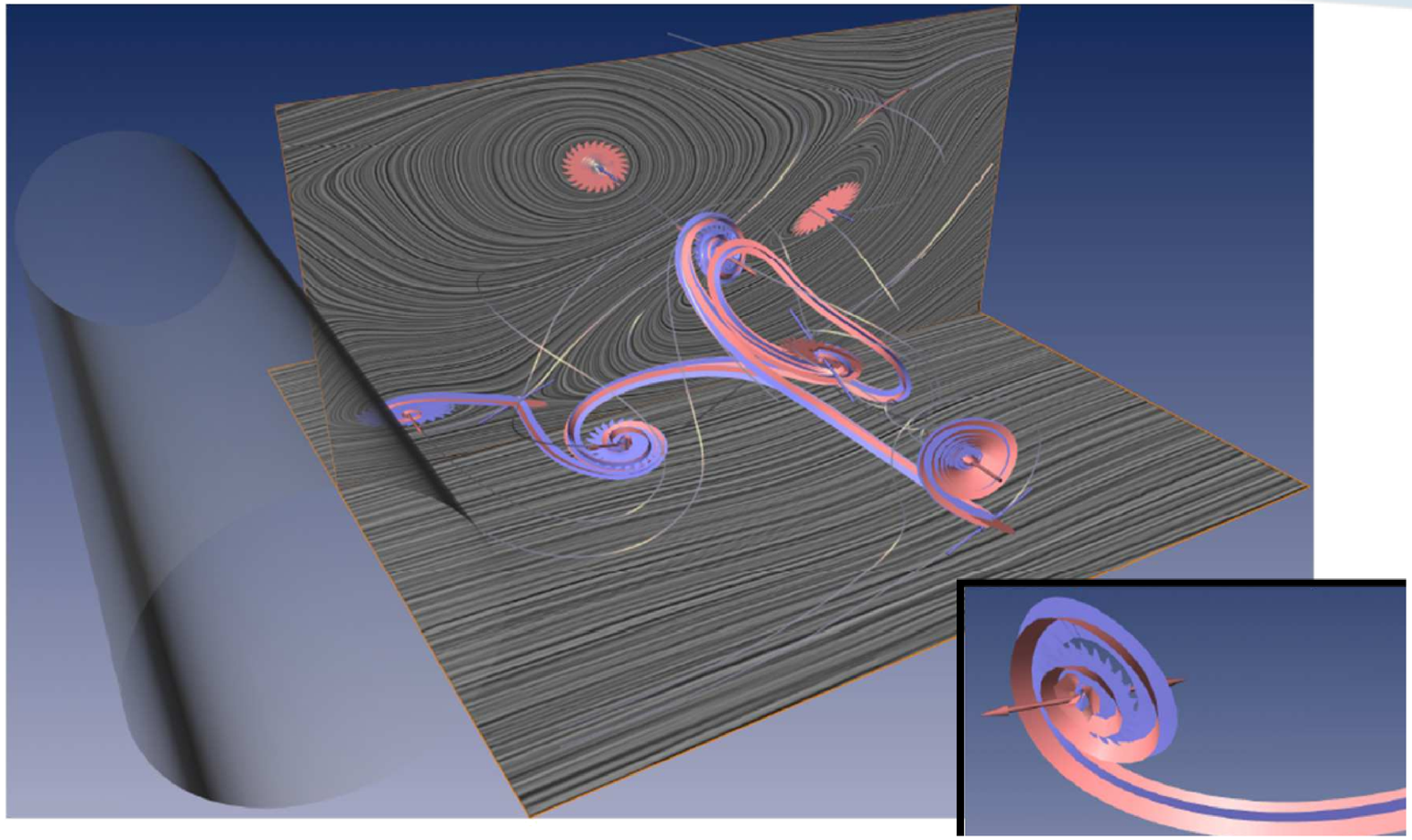
    - Volume rendering!



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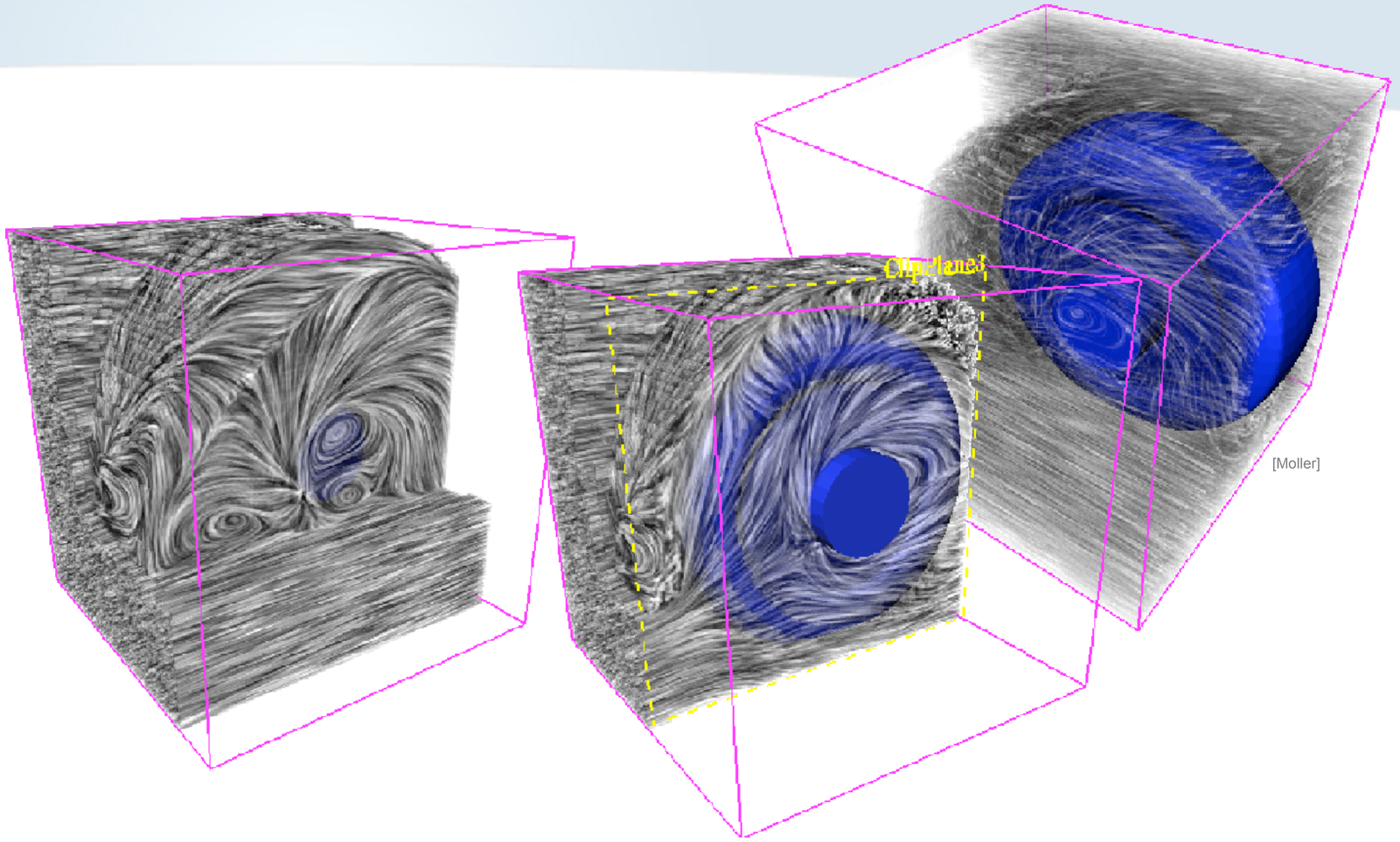

[Moller]

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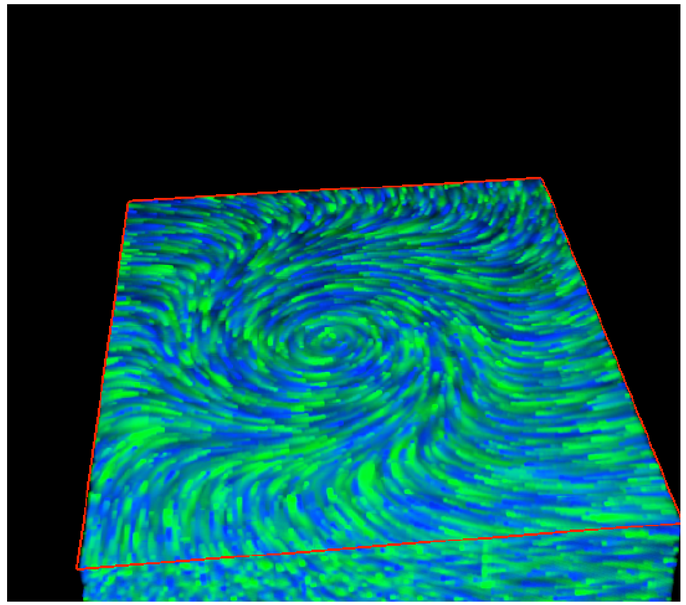
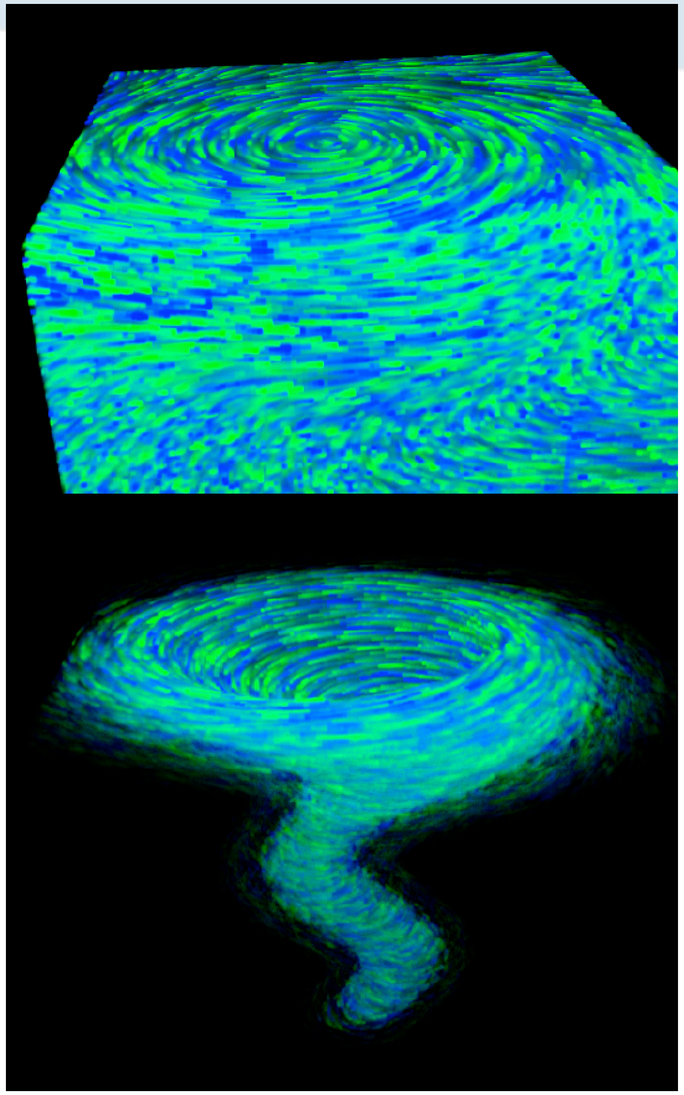
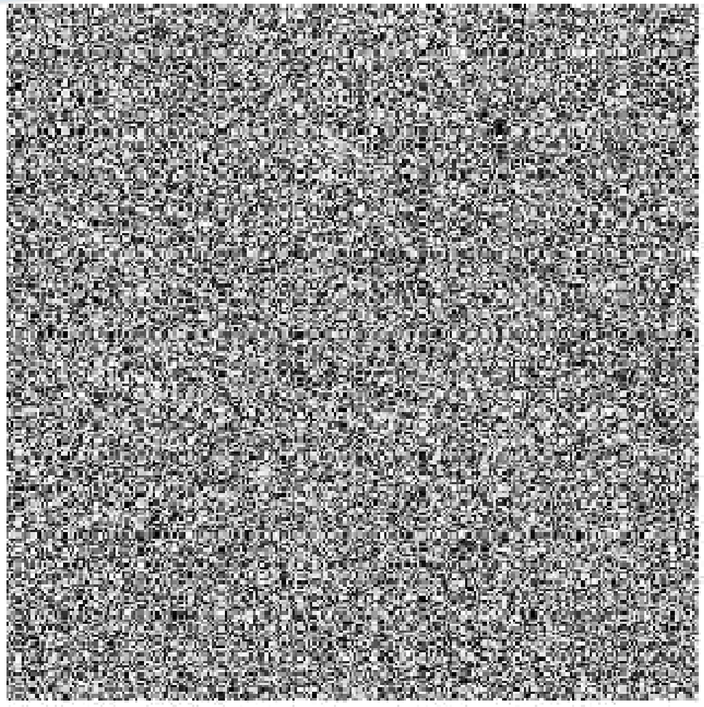
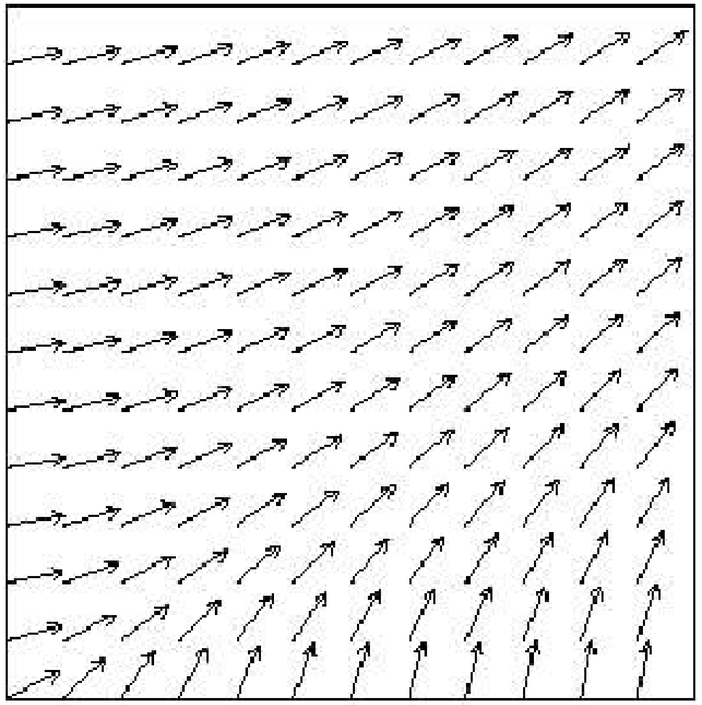




[Moller]

Line Integral Convolution

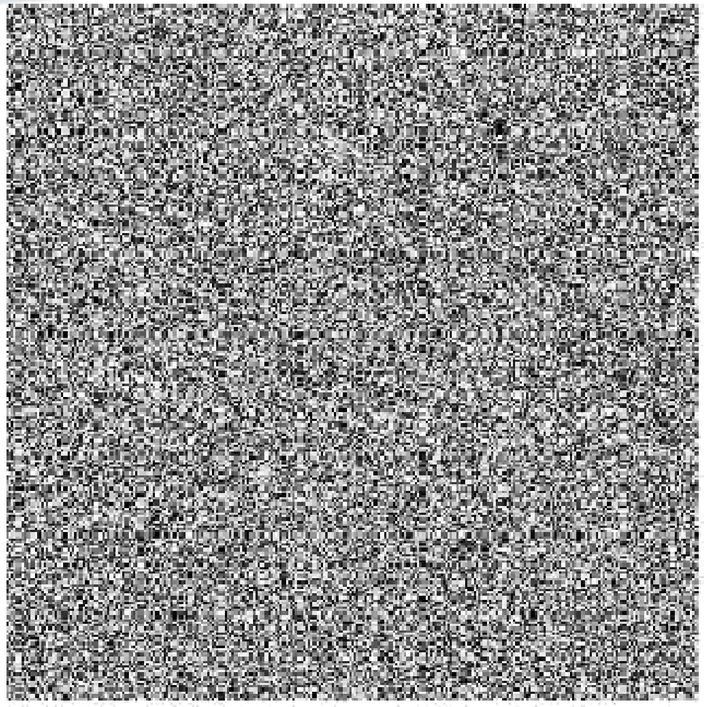


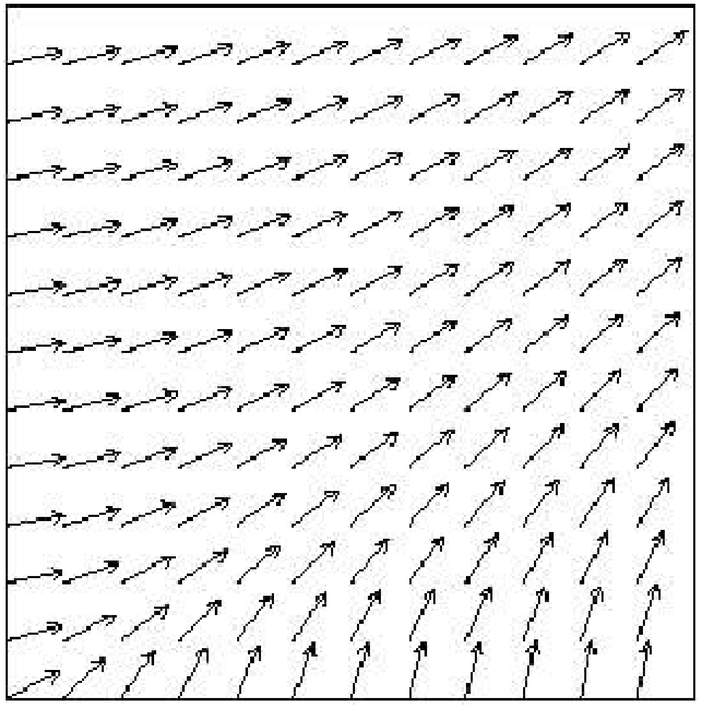


[Helgeland]

- Global visualization of the direction of the flow

Line Integral Convolution

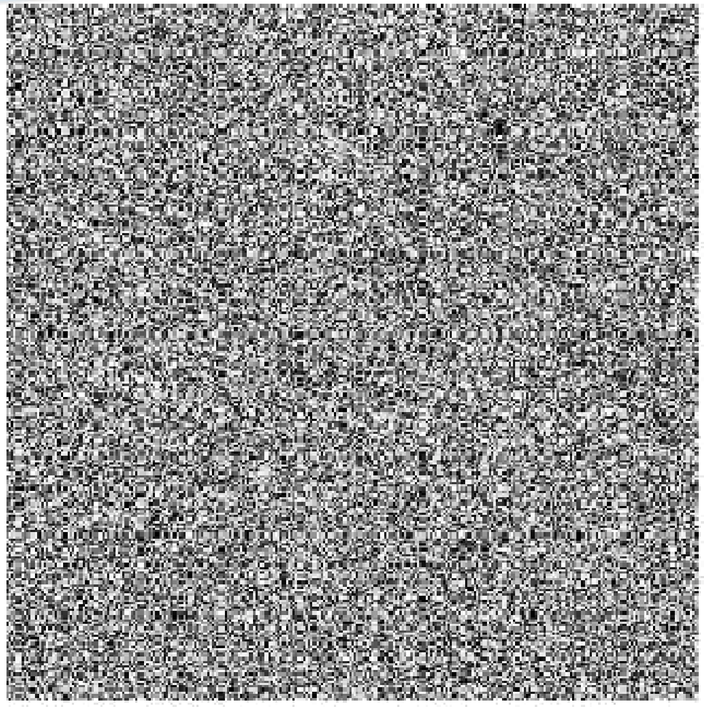


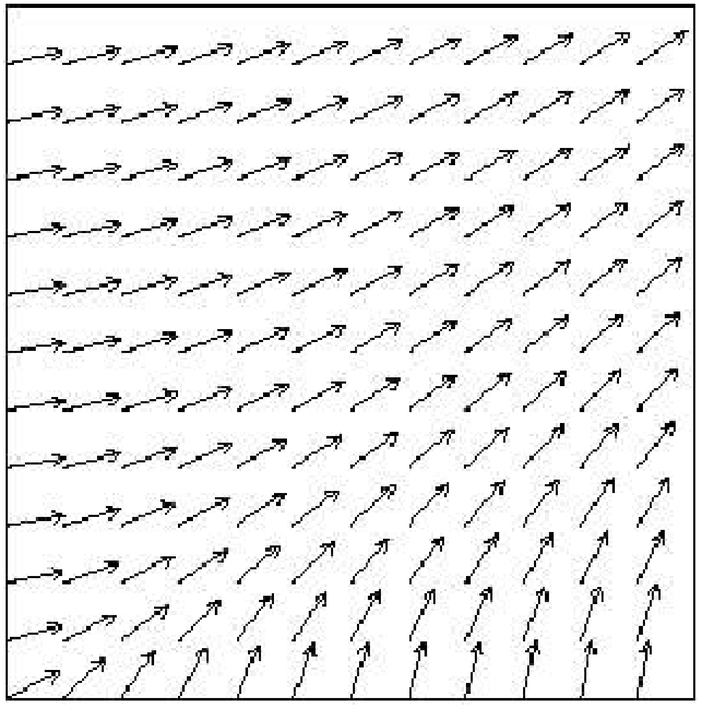


[Helgeland]

- Global visualization of the direction of the flow
  - What about its magnitude?

Line Integral Convolution



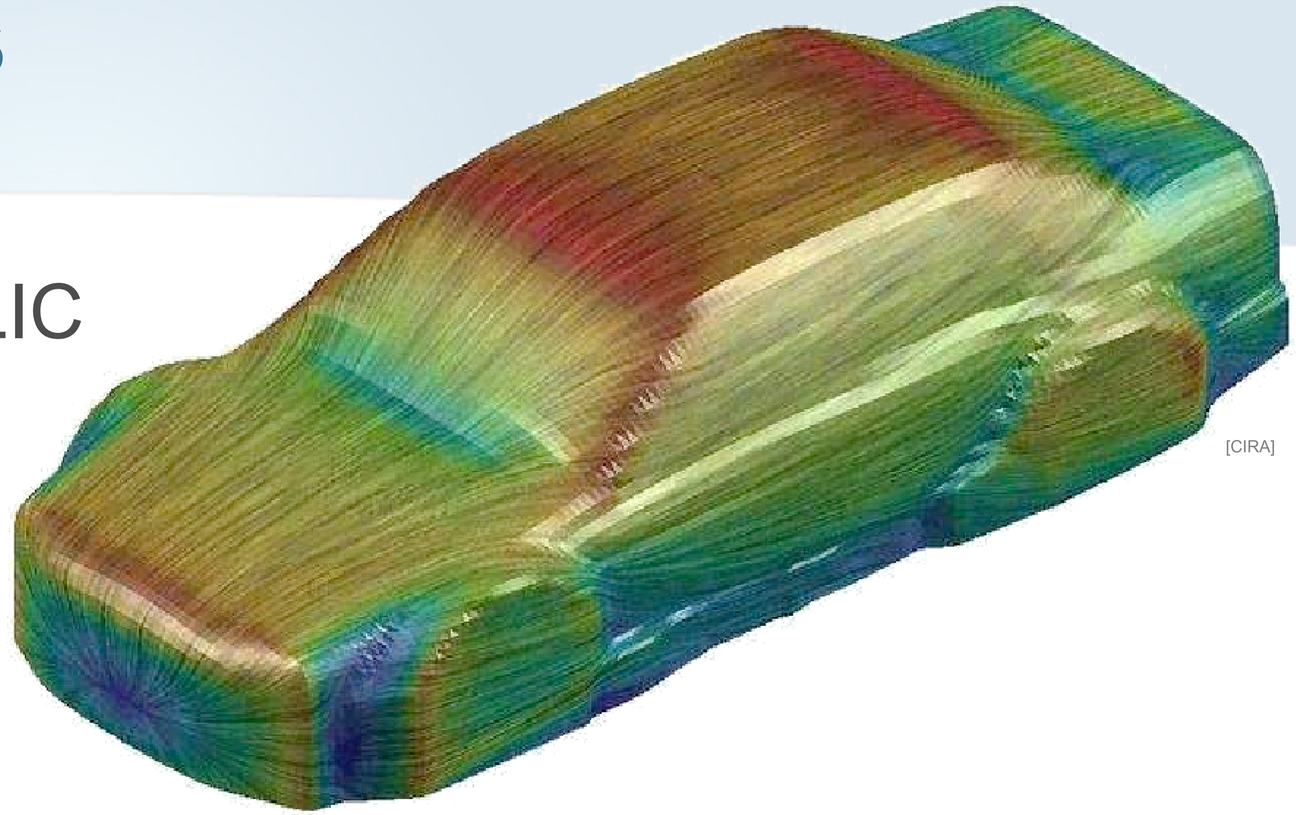


[Helgeland]

- Global visualization of the direction of the flow
  - What about its magnitude? Its orientation?

Derived scalar fields

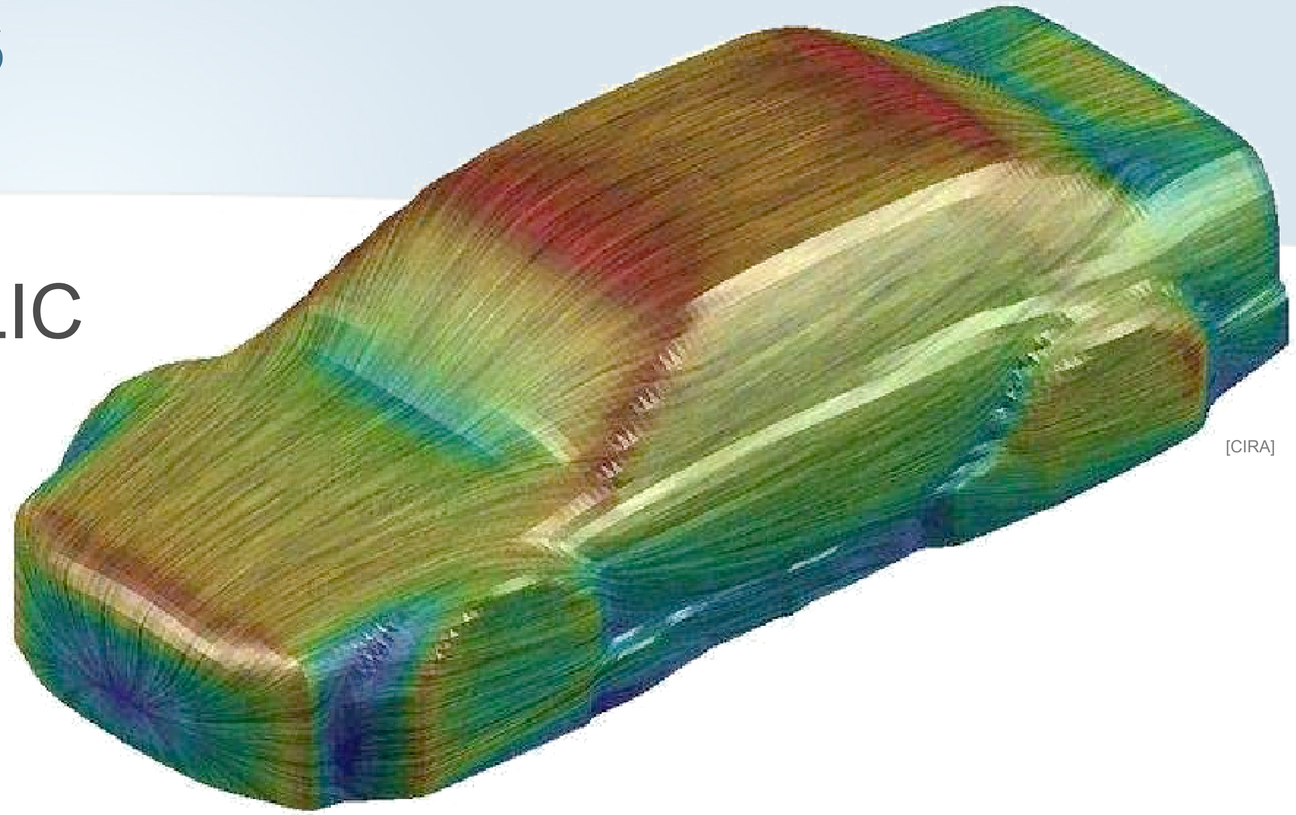
- Combine the magnitude with the LIC



[CIRA]

Derived scalar fields

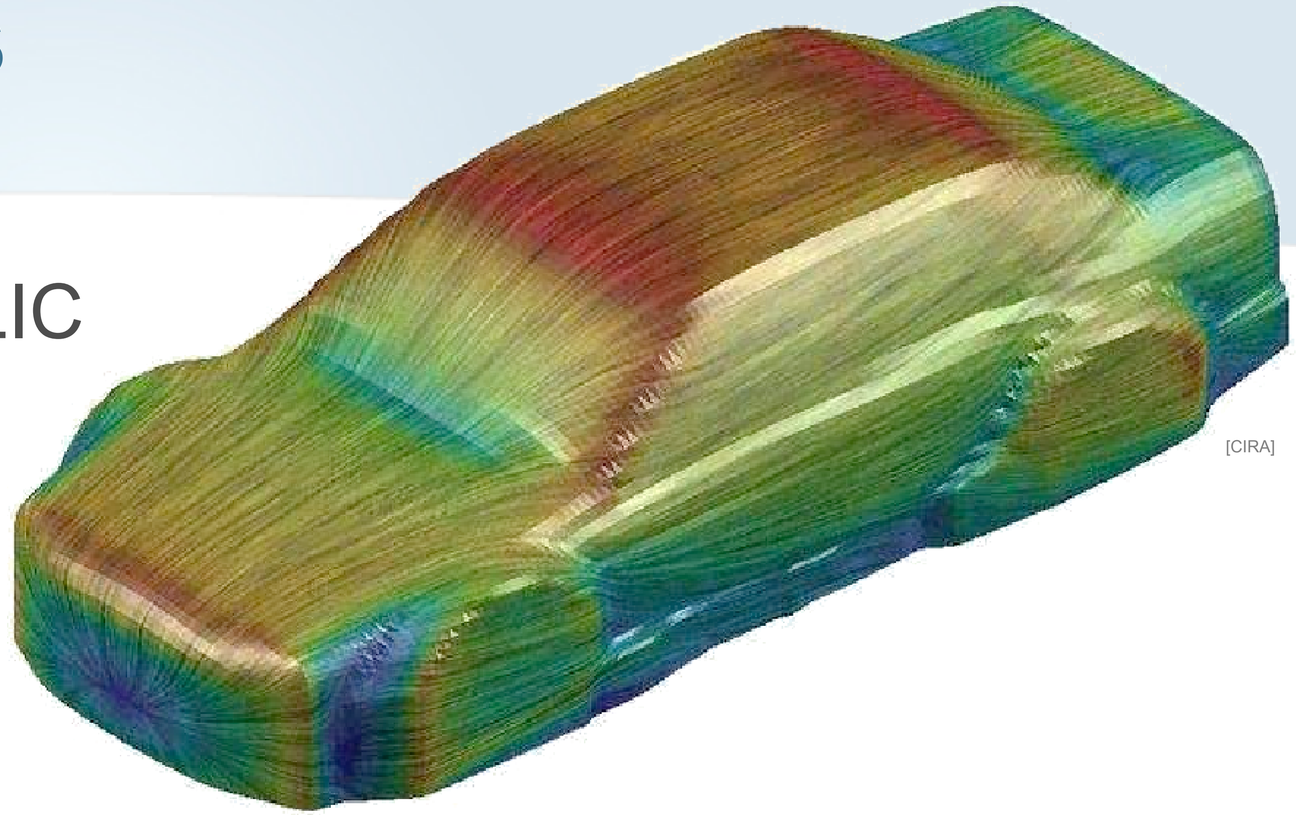
- Combine the magnitude with the LIC
  - Color map of the magnitude



[CIRA]

Derived scalar fields

- Combine the magnitude with the LIC
  - Color map of the magnitude
  - For each channel (RGB)
    - Multiply by the LIC value



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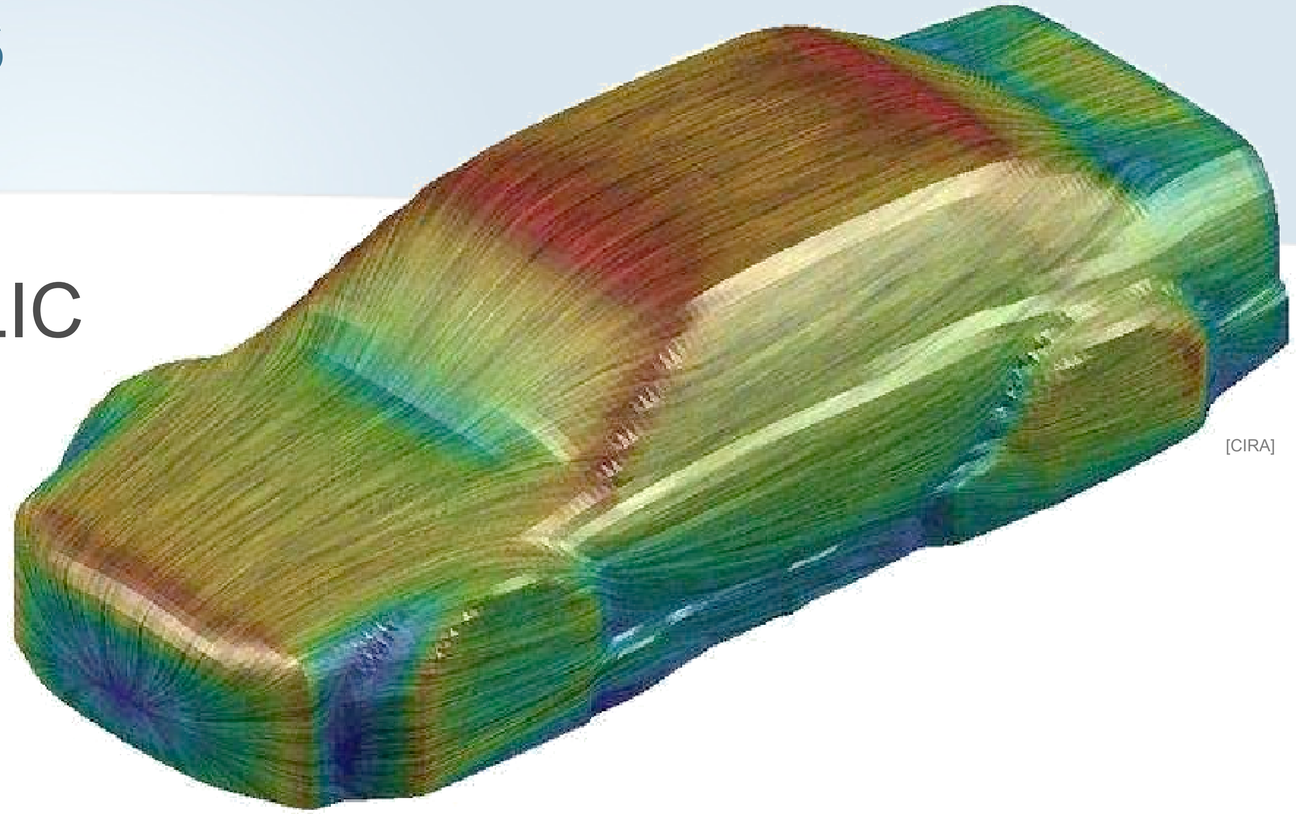

[CIRA]

- What other derived scalar fields would be interesting?

Derived scalar fields

- Combine the magnitude with the LIC
  - Color map of the magnitude
  - For each channel (RGB)
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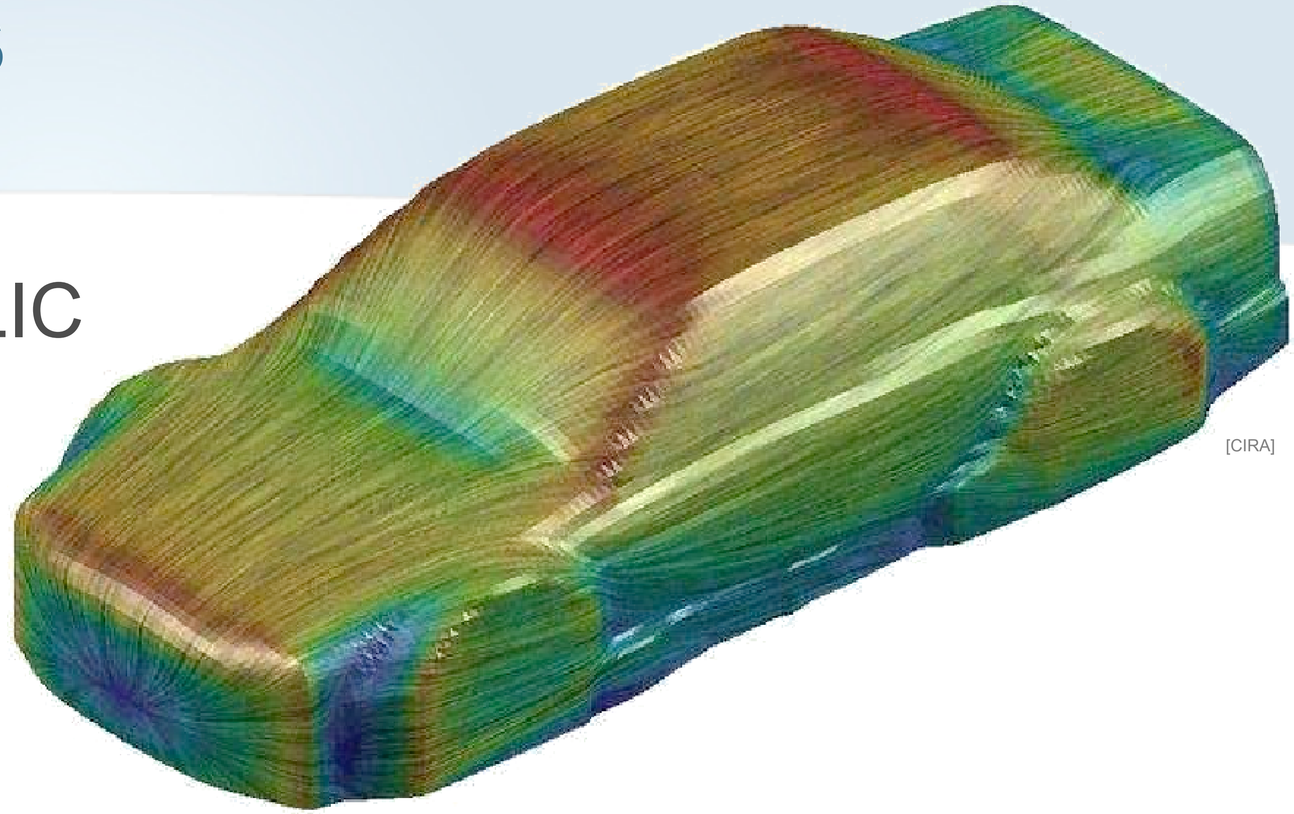
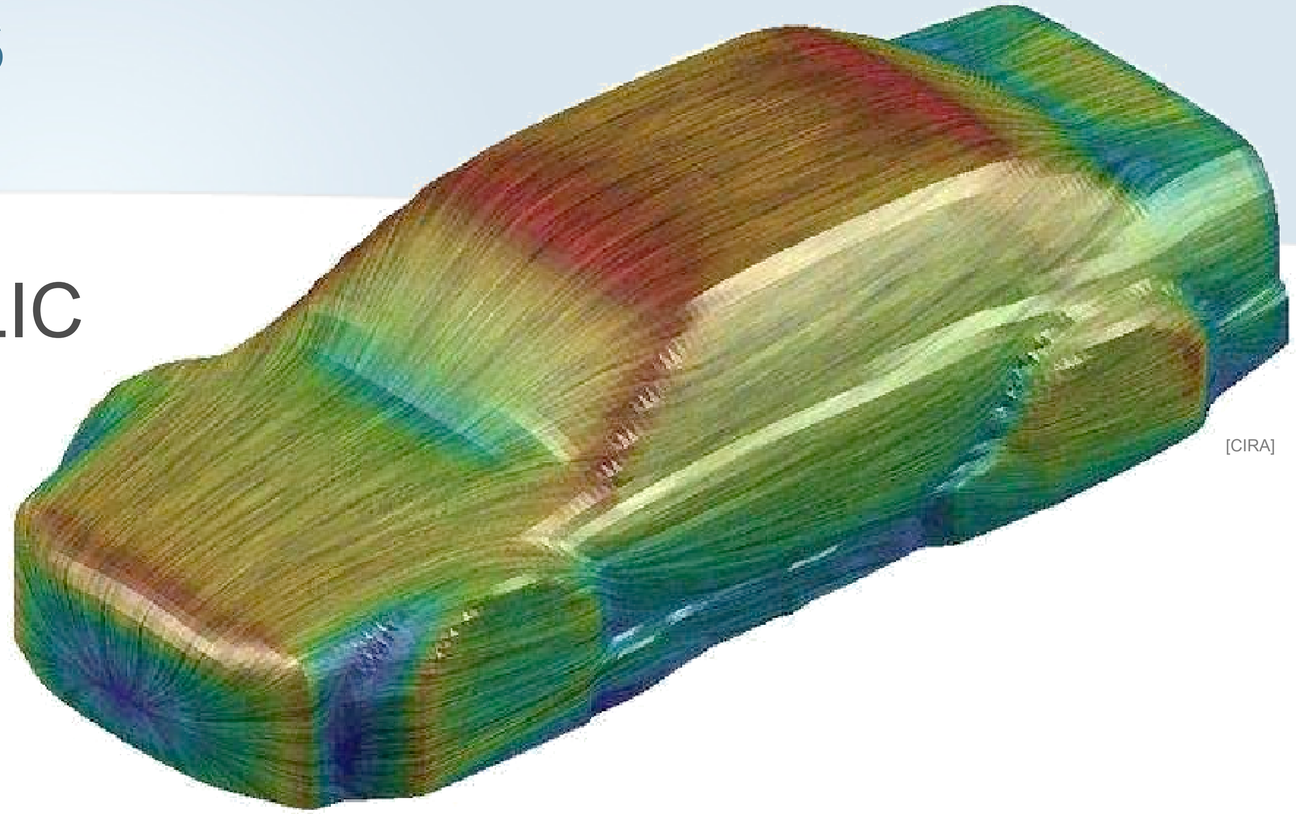

[CIRA]

- What other derived scalar fields would be interesting?
  - Flow orientation

Derived scalar fields

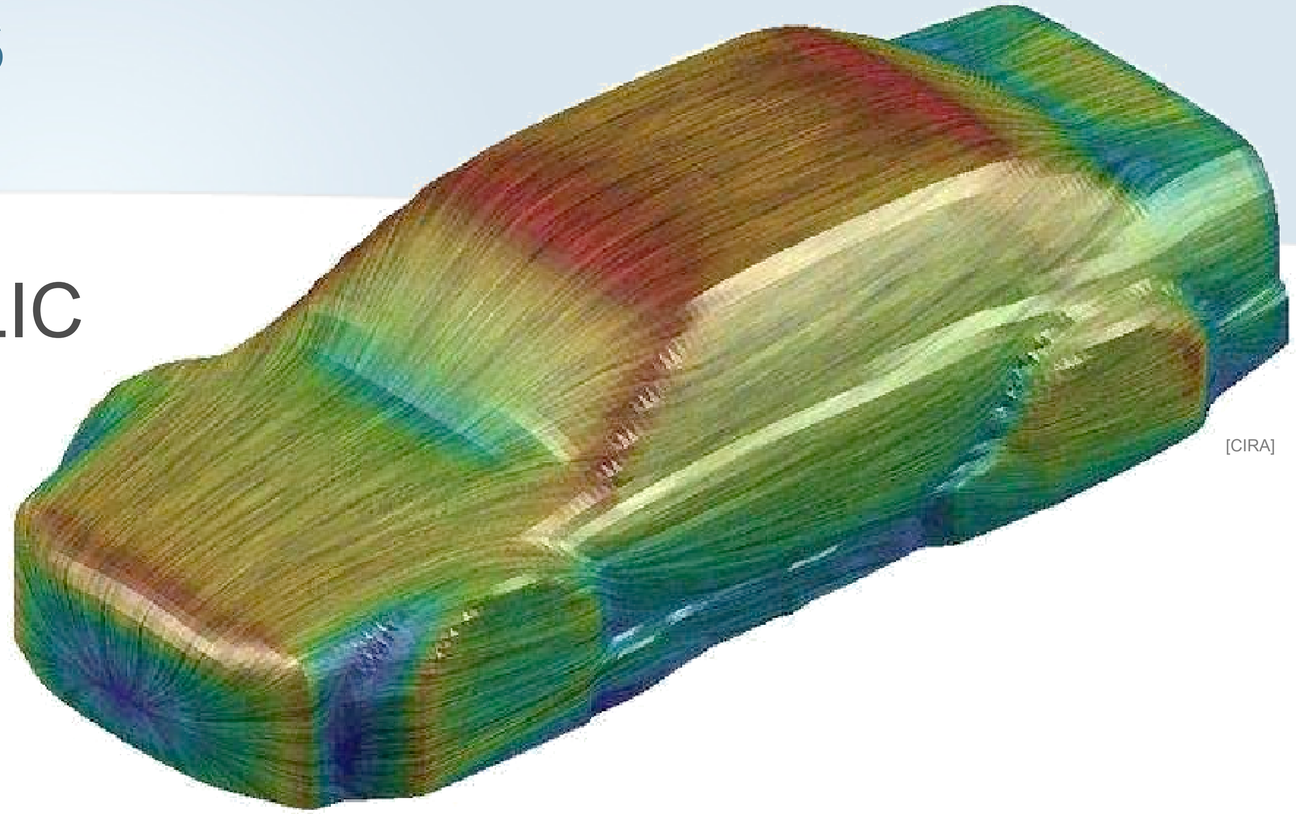
- Combine the magnitude with the LIC
  - Color map of the magnitude
  - For each channel (RGB)
    - Multiply by the LIC value


[CIRA]

- What other derived scalar fields would be interesting?
  - Flow orientation: divergence

Derived scalar fields

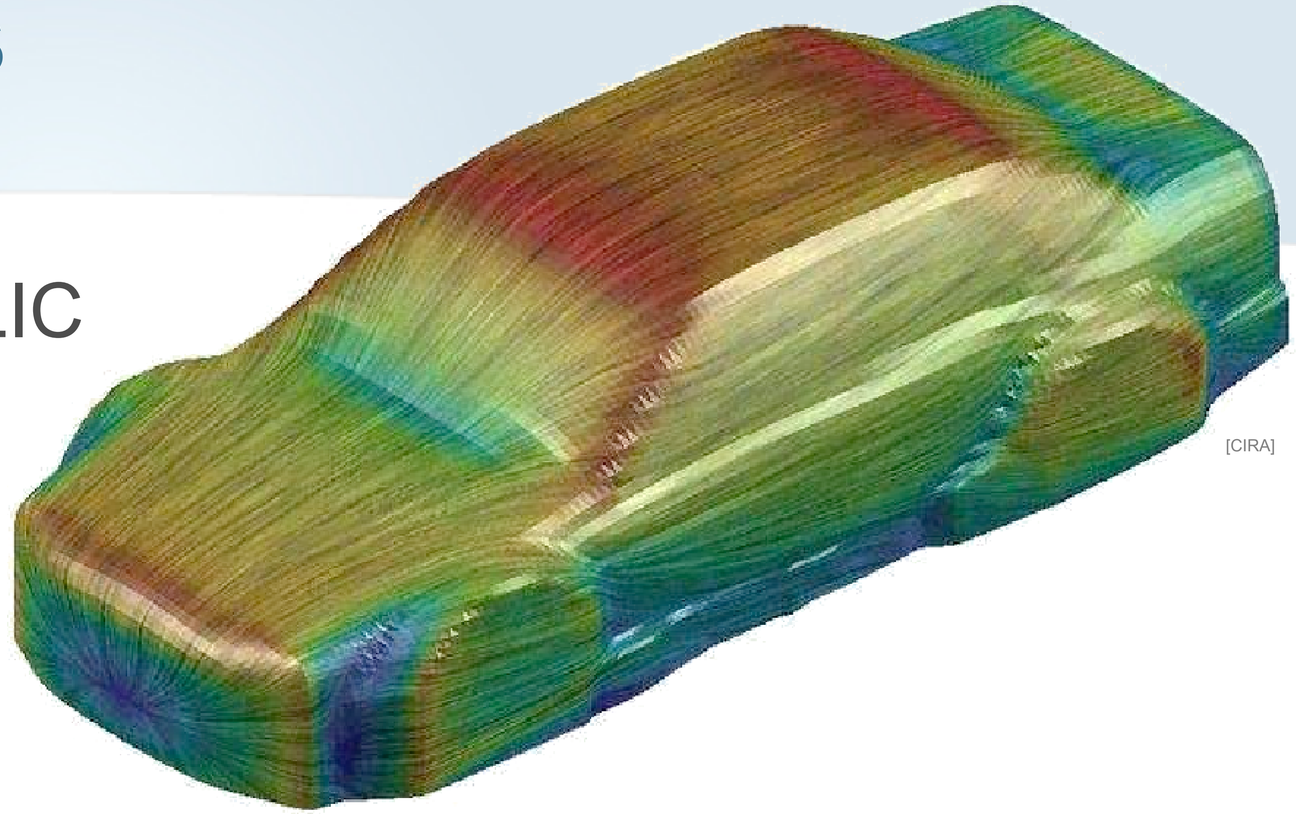
- Combine the magnitude with the LIC
  - Color map of the magnitude
  - For each channel (RGB)
    - Multiply by the LIC value


[CIRA]

- What other derived scalar fields would be interesting?
  - Flow orientation: divergence
  - Angular speed

Derived scalar fields

- Combine the magnitude with the LIC
  - Color map of the magnitude
  - For each channel (RGB)
    - Multiply by the LIC value

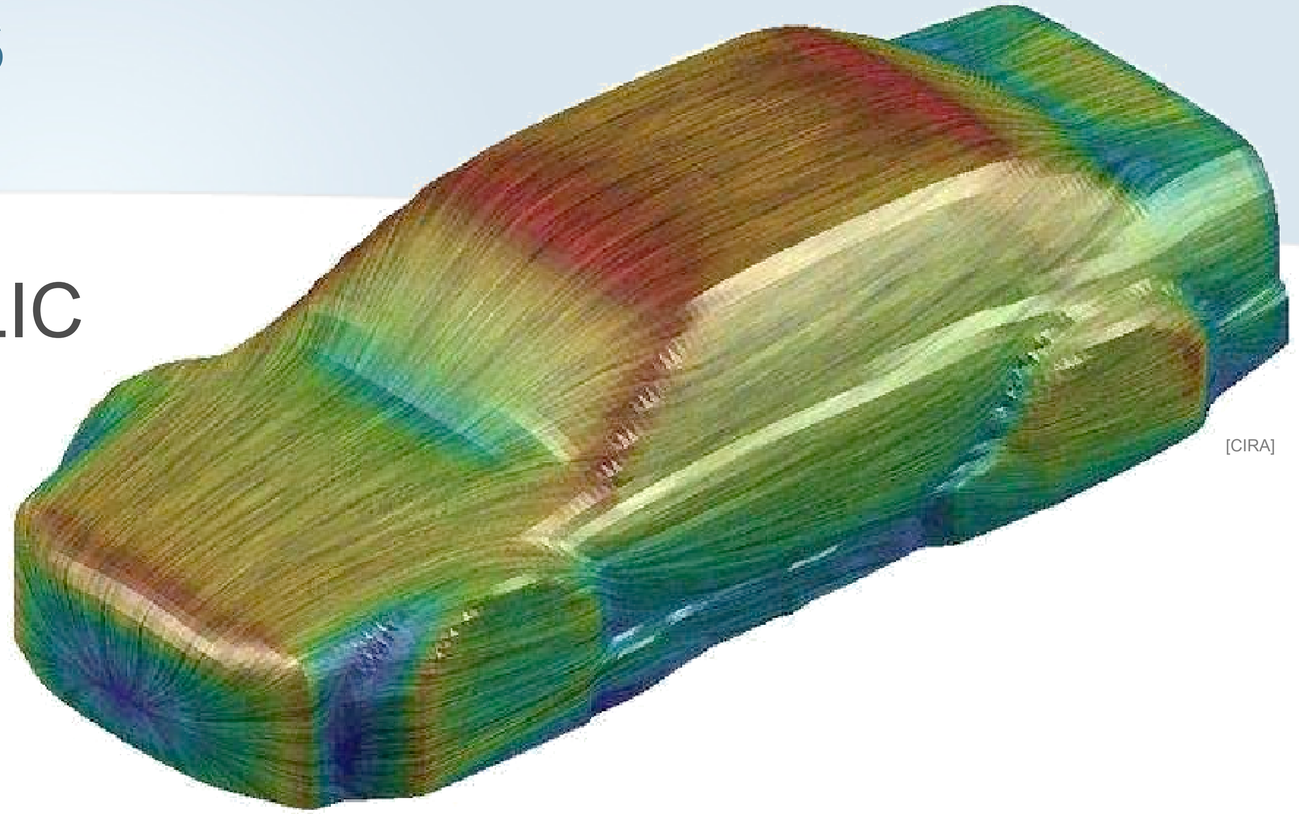


[CIRA]

- What other derived scalar fields would be interesting?
  - Flow orientation: divergence
  - Angular speed: magnitude of the curl

Derived scalar fields

- Combine the magnitude with the LIC
  - Color map of the magnitude
  - For each channel (RGB)
    - Multiply by the LIC value


[CIRA]

- What other derived scalar fields would be interesting?
  - Flow orientation: divergence
  - Angular speed: magnitude of the curl
  - Flow distortion

Derived scalar fields

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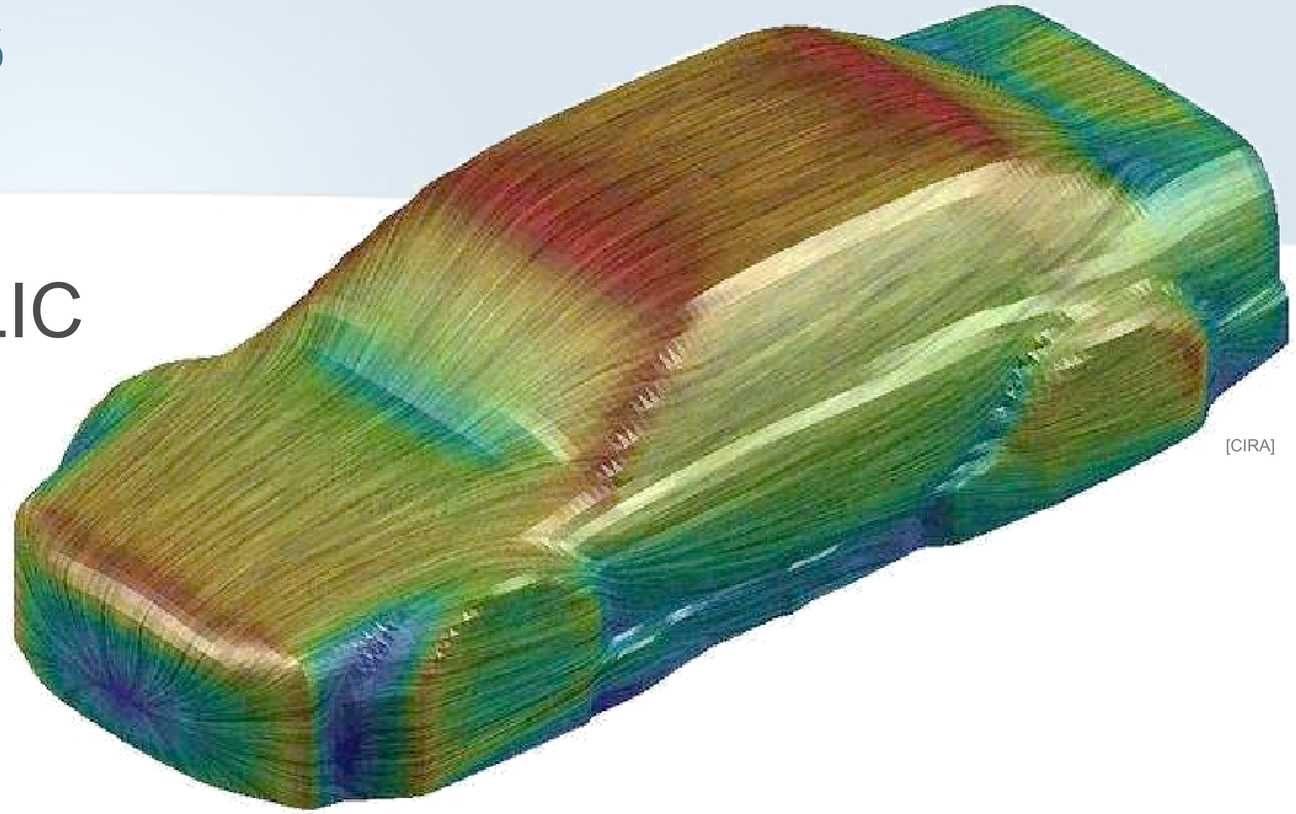

[CIRA]

- What other derived scalar fields would be interesting?
  - Flow orientation: divergence
  - Angular speed: magnitude of the curl
  - Flow distortion: Finite Time Lyaponov Exponent

Flow orientation

Flow orientation

- Divergence of the flow, for instance
  - $f : \mathbb{R}^3 \to \mathbb{R}^3$

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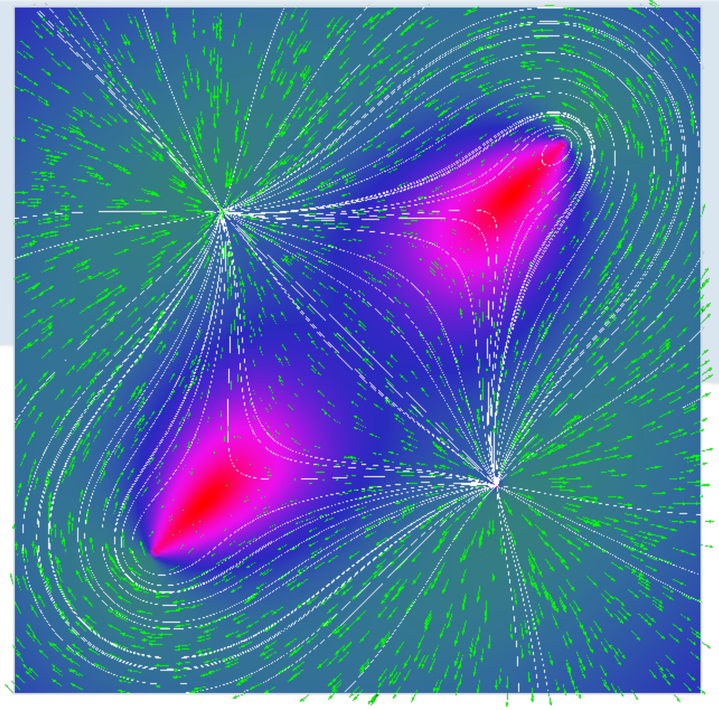
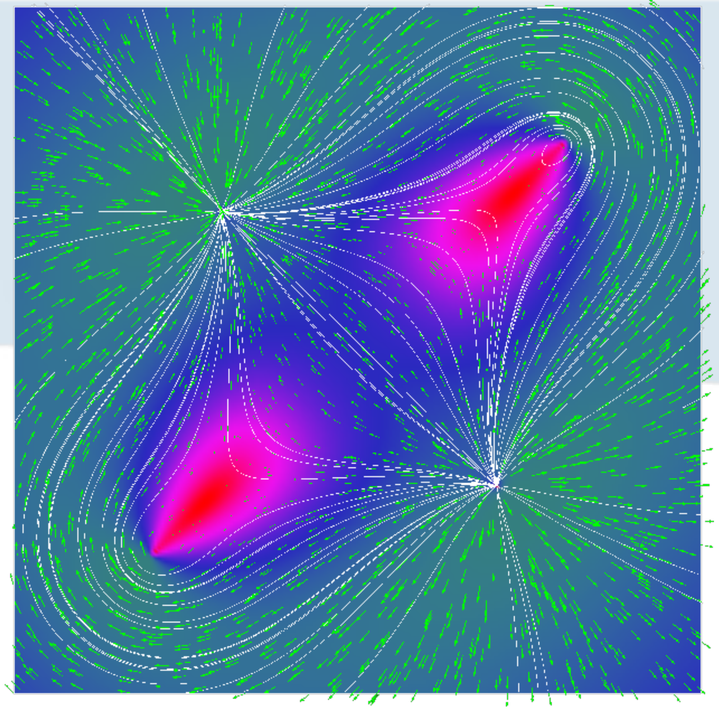


[http://www.deepakantony.com/]

Flow orientation



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  - $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$

$$div f = \nabla . f = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$

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Flow orientation

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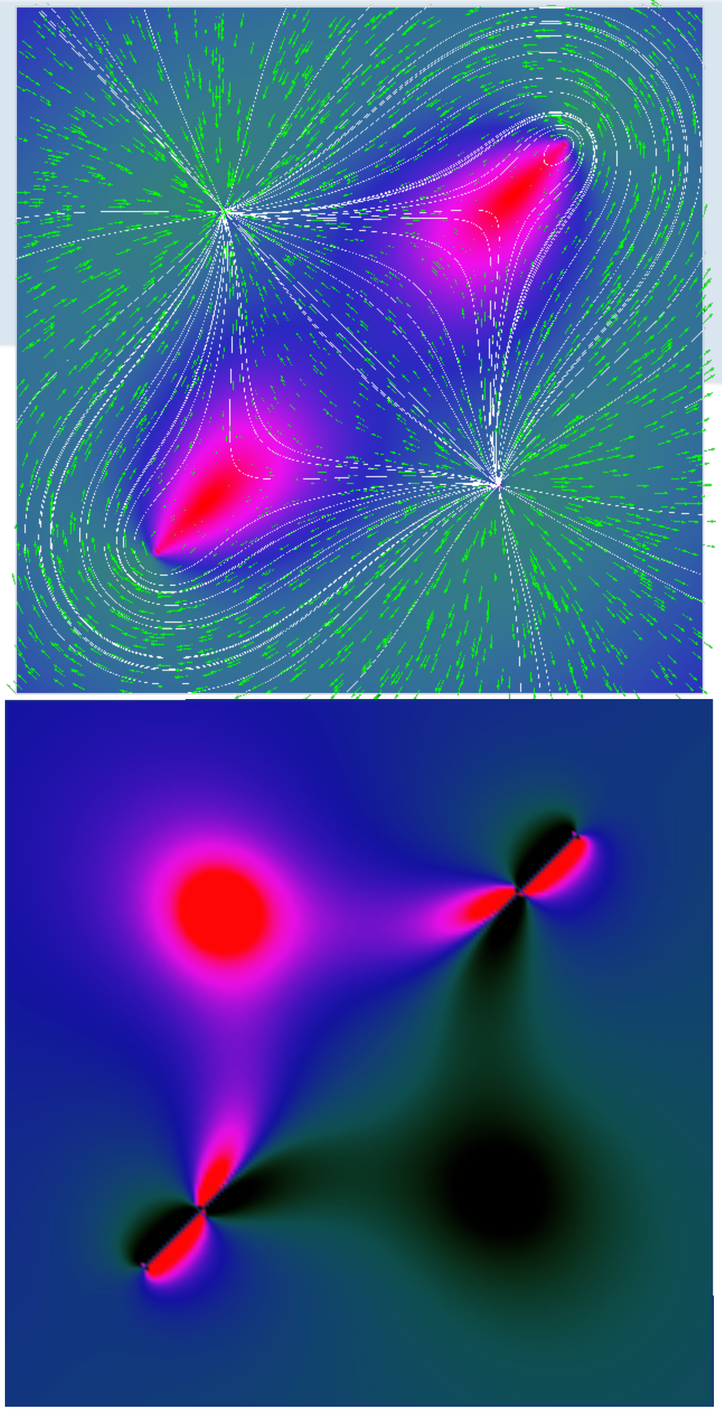
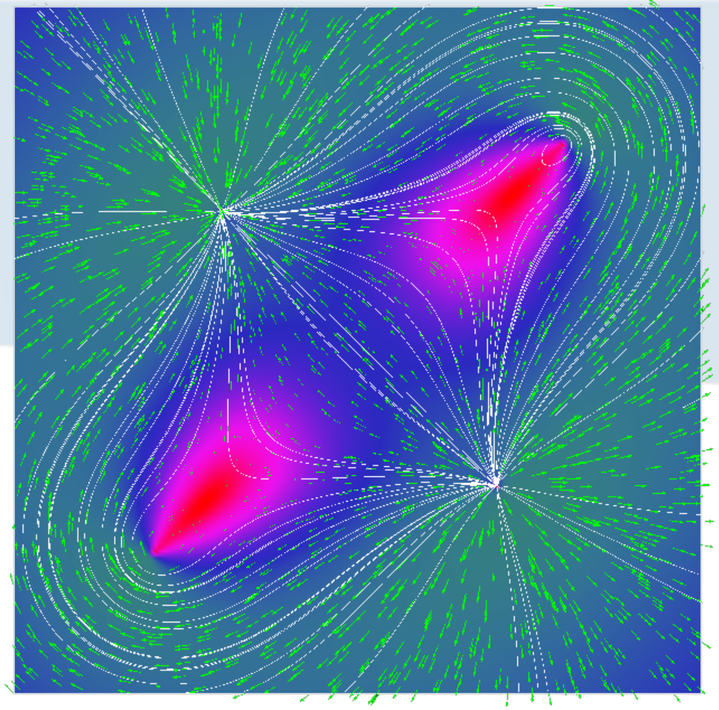


[http://www.deepakantony.com/]

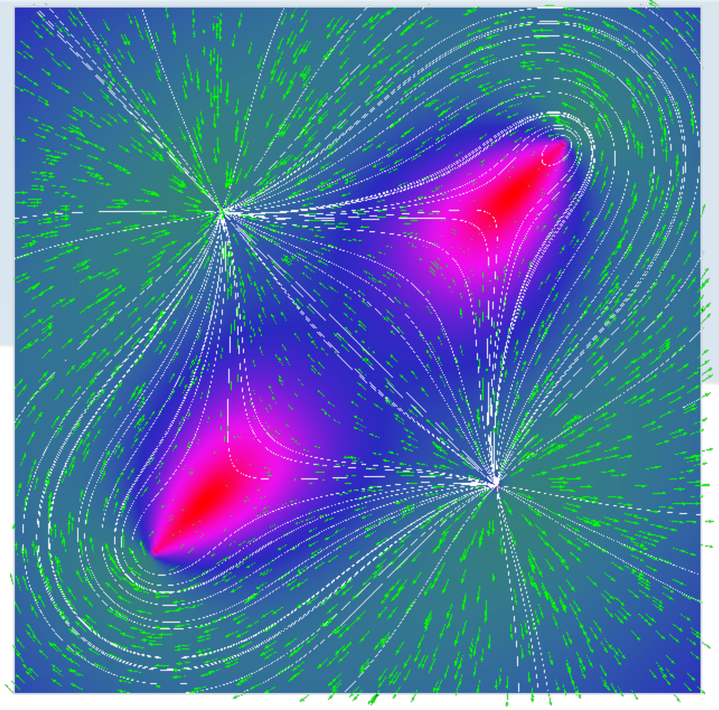
Angular speed



[http://www.deepakantony.com/]

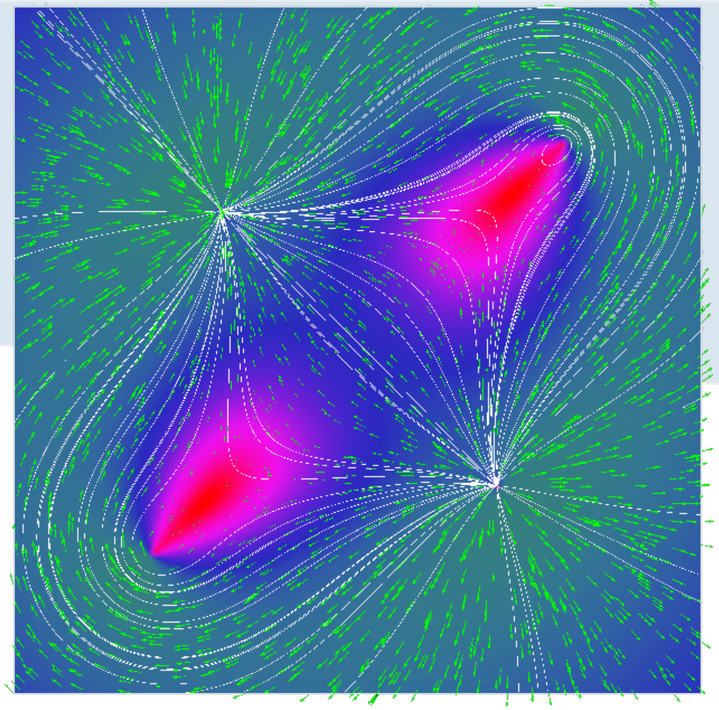
Angular speed

- Magnitude of the curl, for instance
  - $f : \mathbb{R}^3 \to \mathbb{R}^3$



[http://www.deepakantony.com/]

Angular speed



- Magnitude of the curl, for instance
  - $f : \mathbb{R}^3 \to \mathbb{R}^3$

$$\nabla \times f =$$

$$(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z})\vec{i} + (\frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x})\vec{j}$$

$$+(\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y})\vec{k}$$

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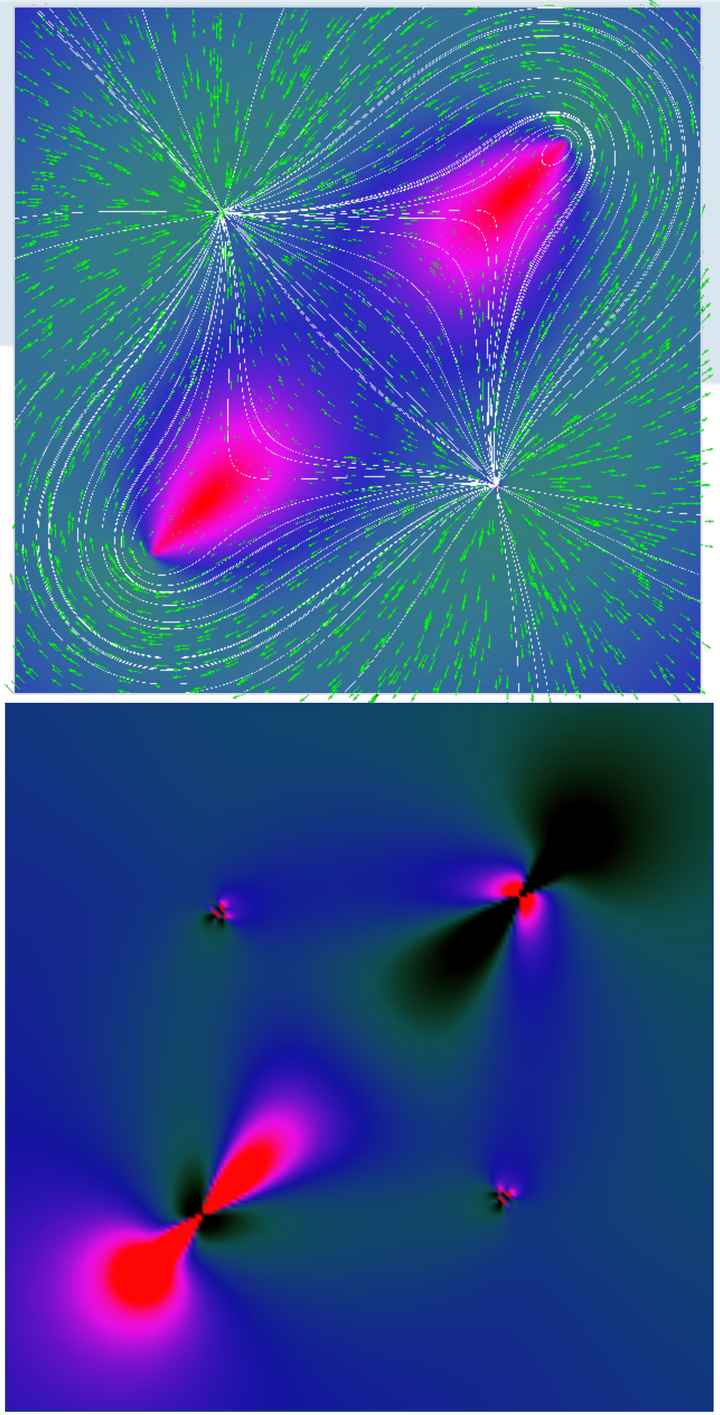

[http://www.deepakantony.com/]

Angular speed

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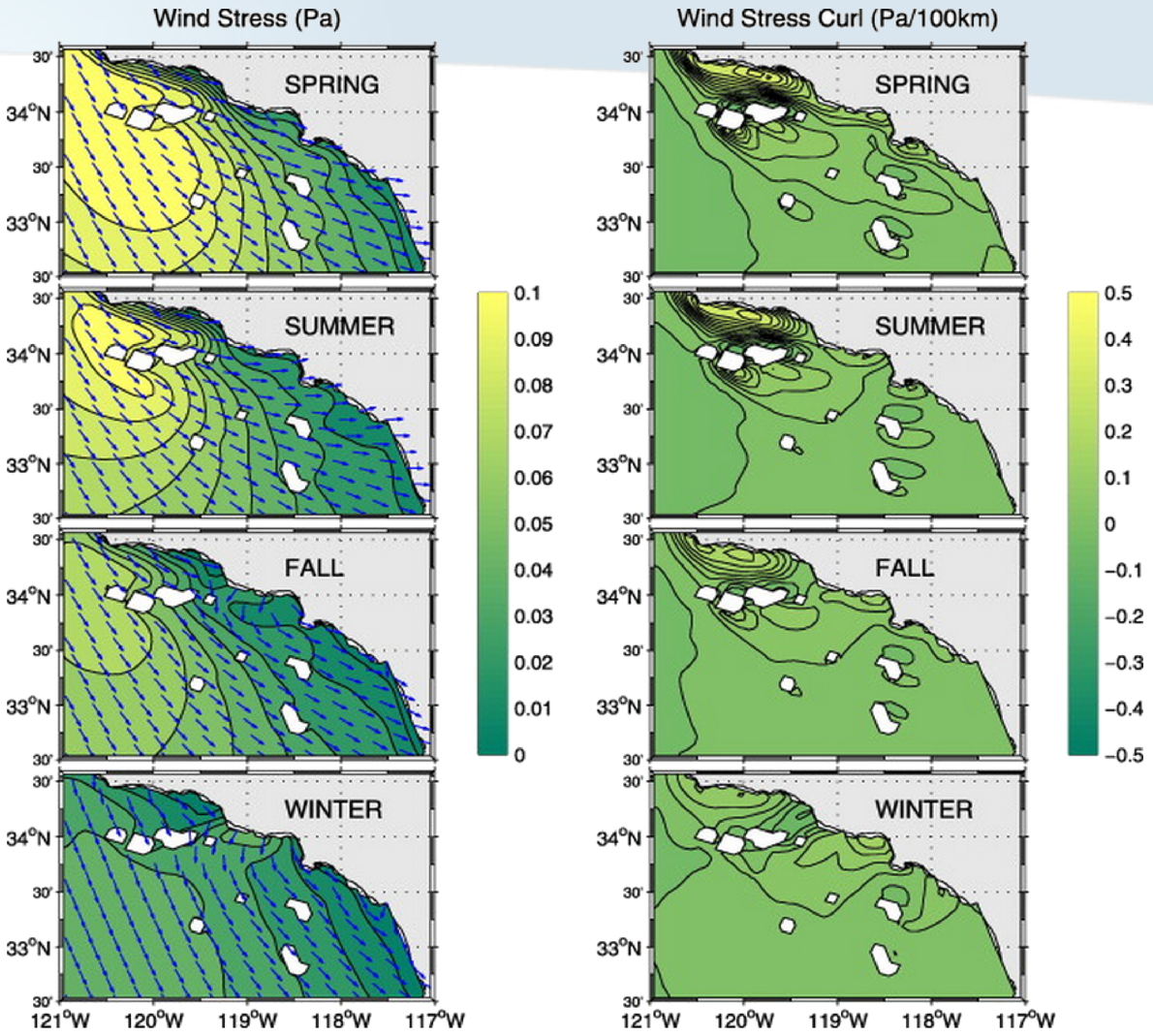

[Dong09]

Angular speed

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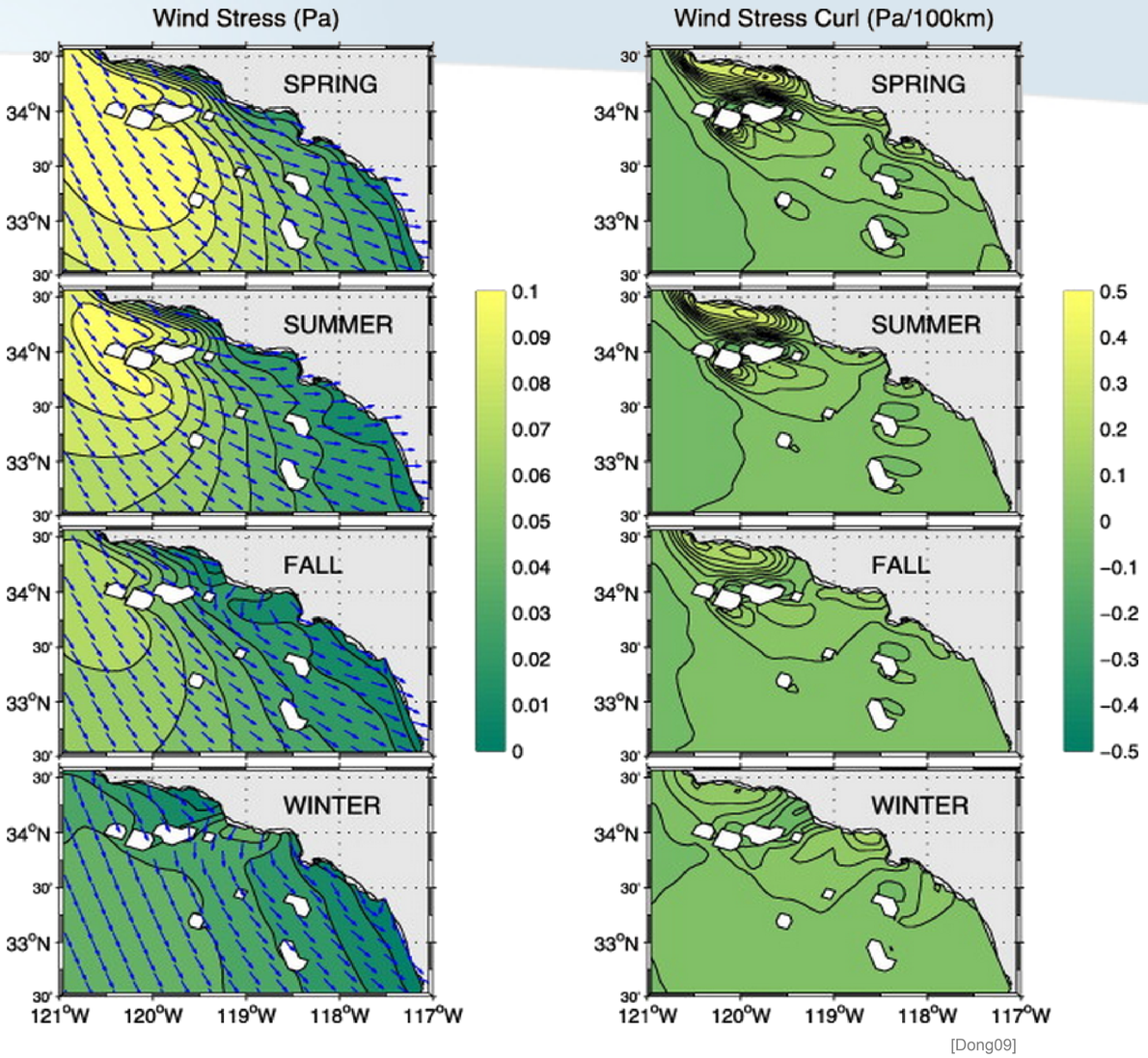
  - On PL-manifolds?



[Dong09]

So far

- Extract geometrical features
  - Streamlines

So far

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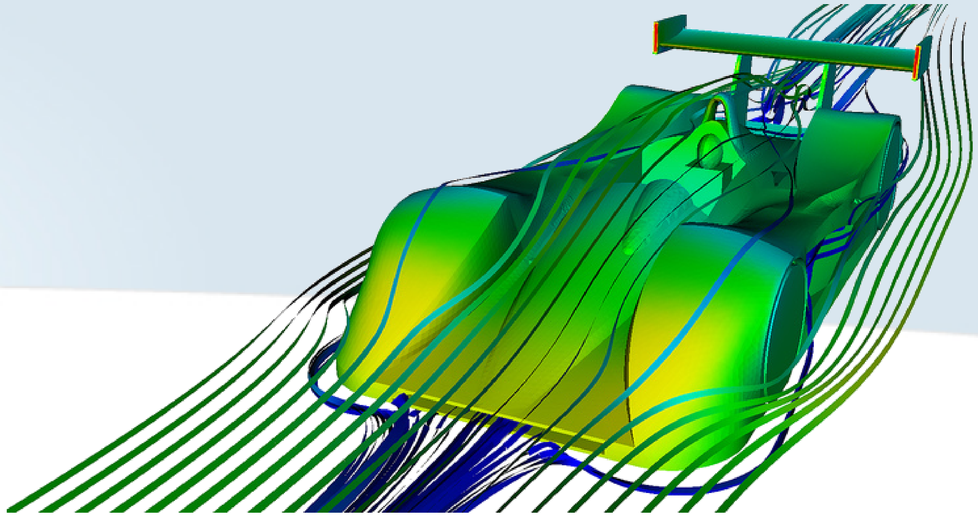
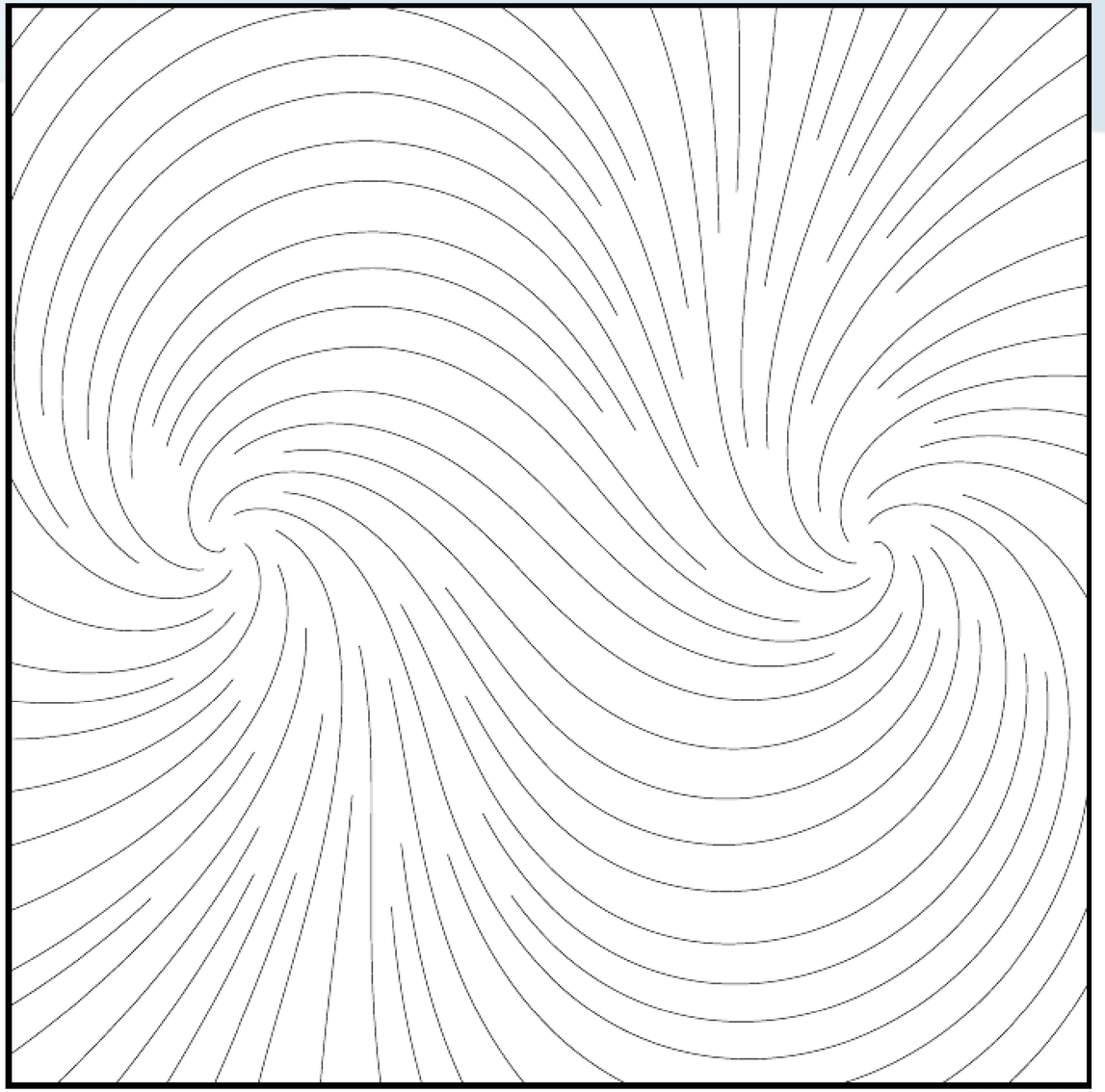
  - Streamlines



[Kitware]

So far

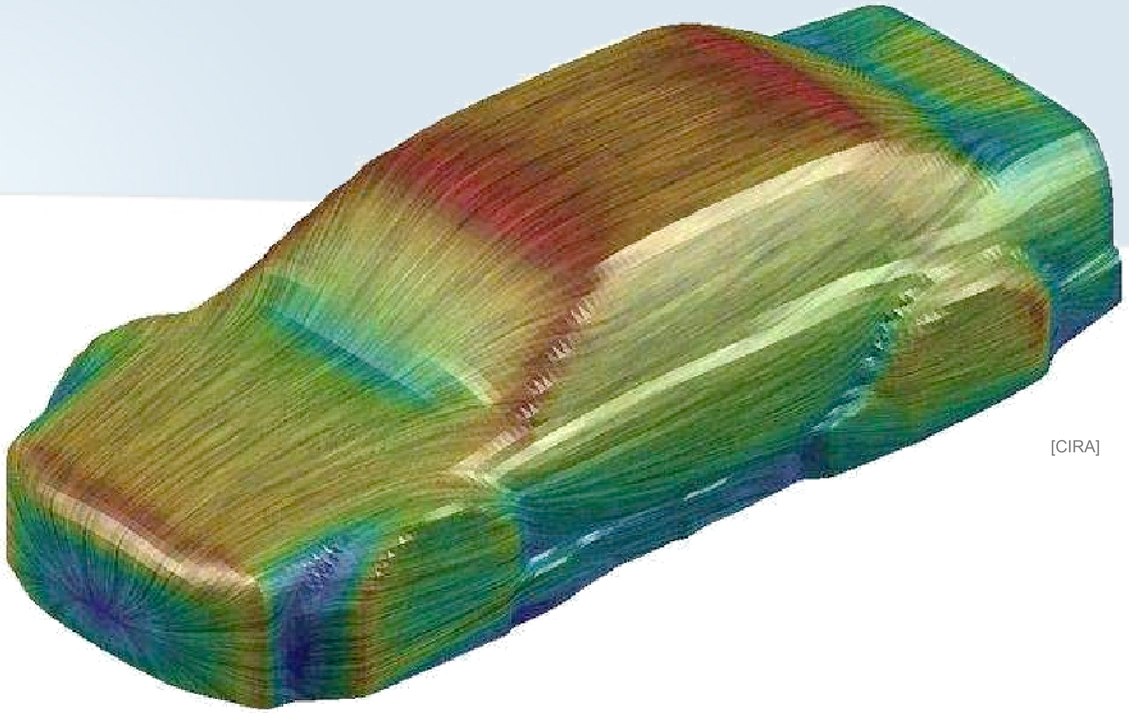
- Extract geometrical features
  - Streamlines
    - Seeding



[Jobard et al. 97]

So far

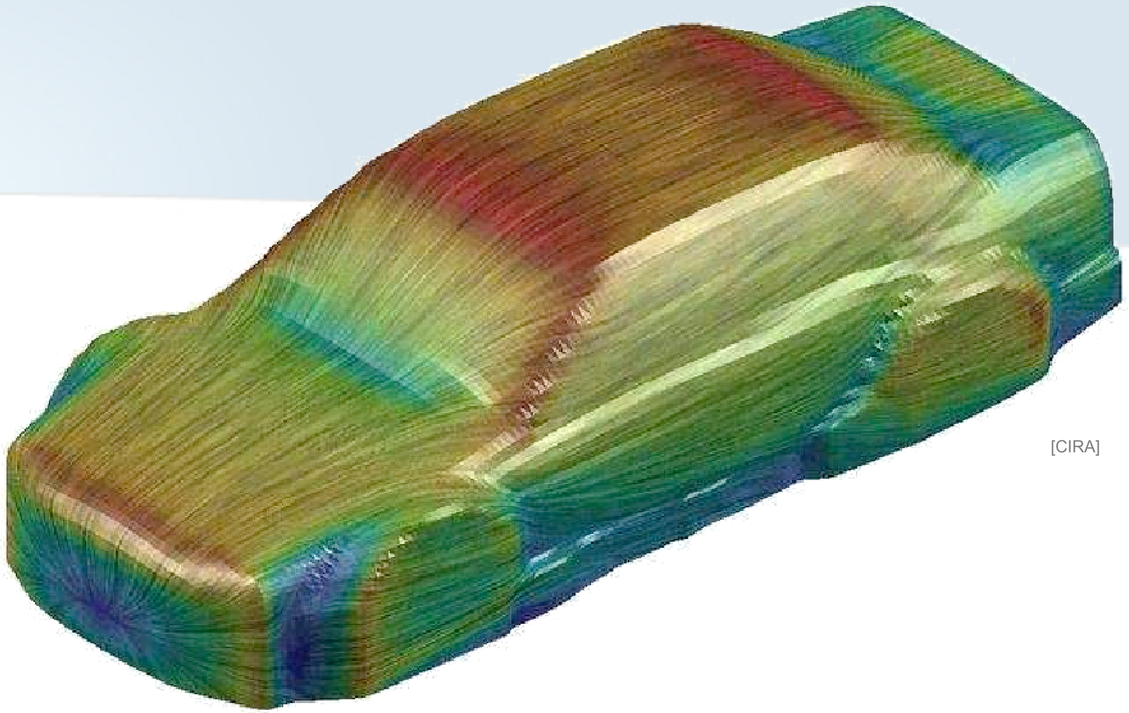
- Extract geometrical features
  - Streamlines
    - Seeding, Line Integral Convolution



[CIRA]

So far

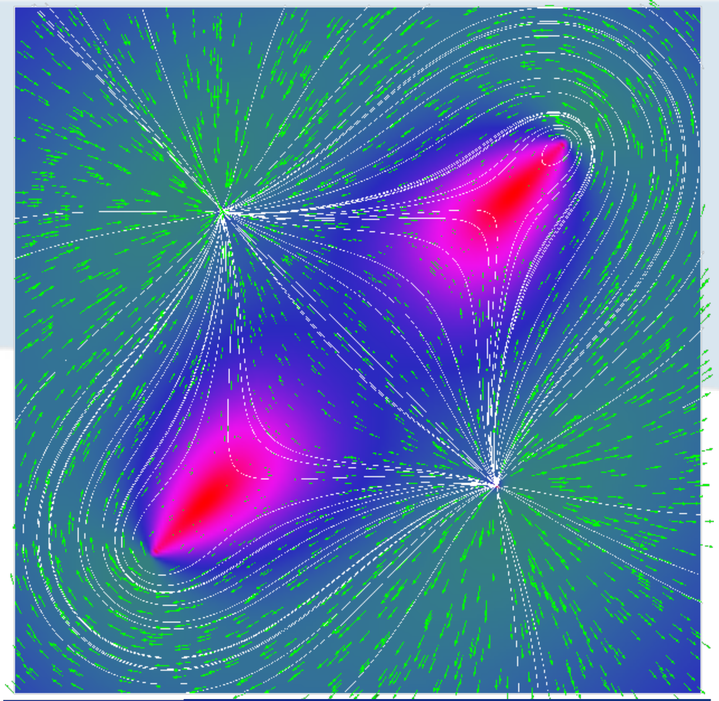
- Extract geometrical features
  - Streamlines
    - Seeding, Line Integral Convolution
- Extract geometrical measures

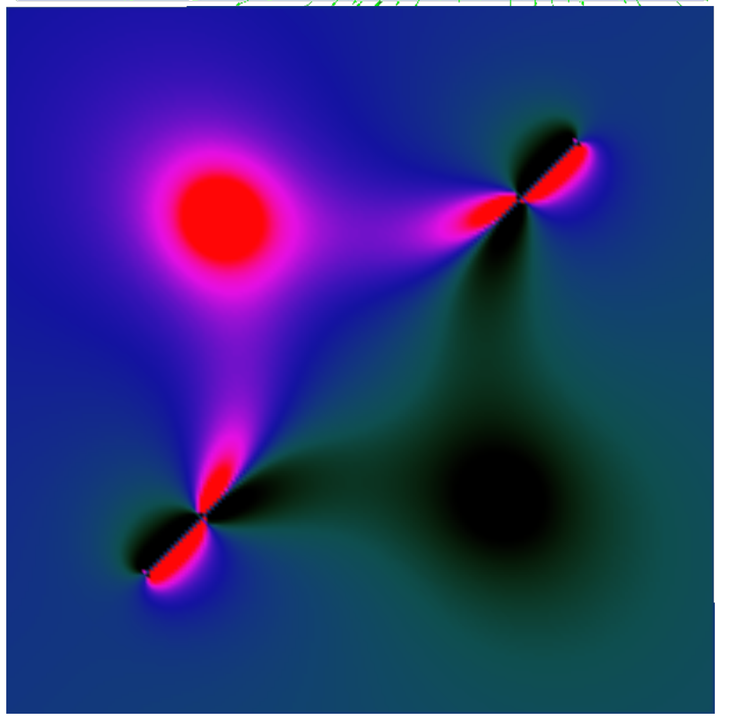


[CIRA]

So far

- Extract geometrical features
  - Streamlines
    - Seeding, Line Integral Convolution
- Extract geometrical measures
  - Magnitude, orientation, angular speed, distortion

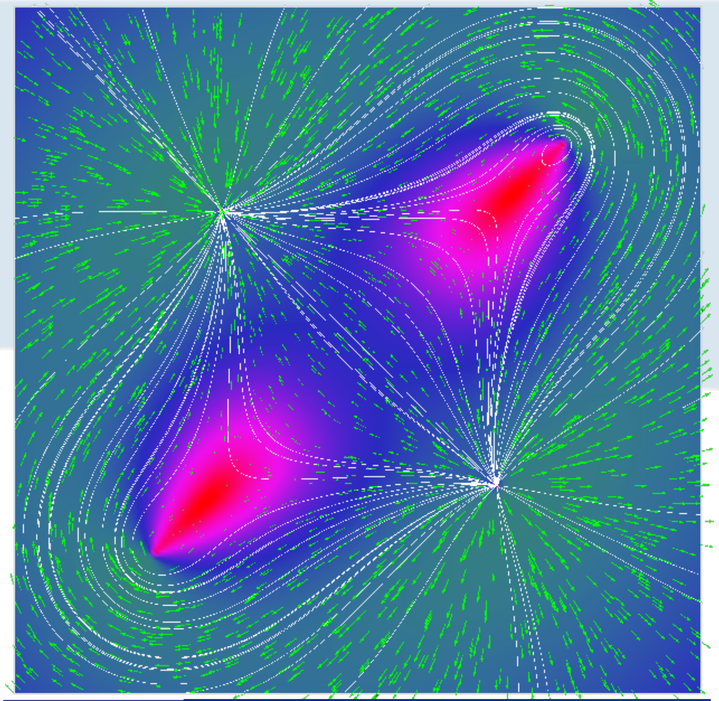


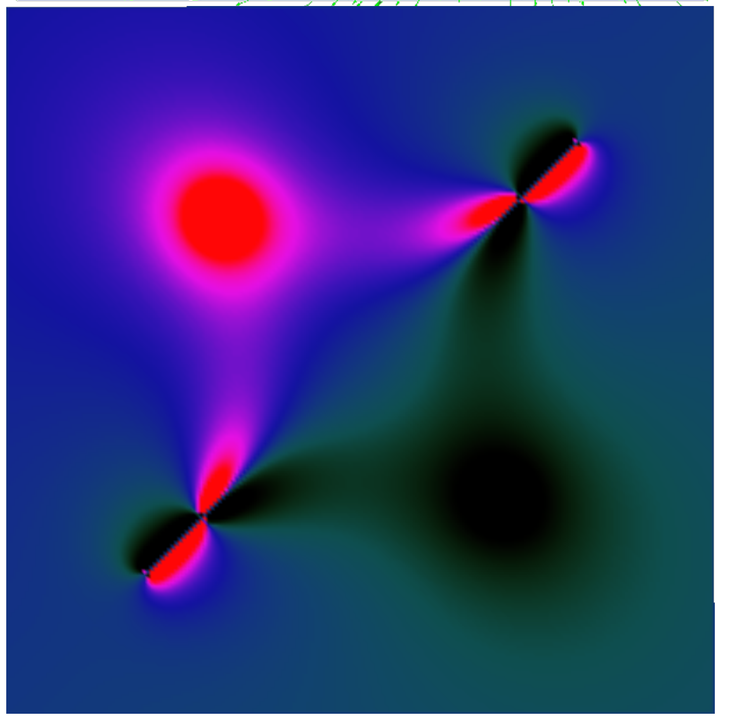


[http://www.deepakantony.com/]

So far

- Extract geometrical features
  - Streamlines
    - Seeding, Line Integral Convolution
    - **Where do they end/start?**
- Extract geometrical measures
  - Magnitude, orientation, angular speed, distortion





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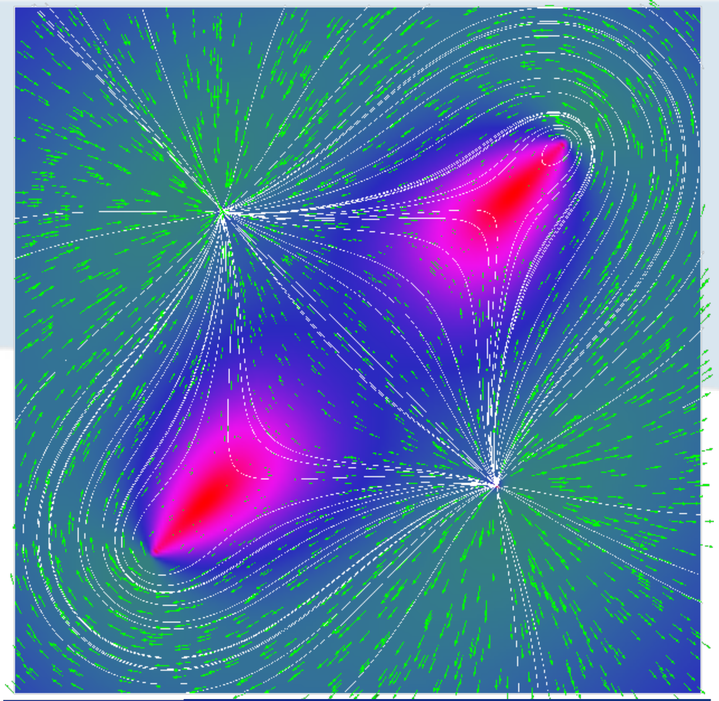
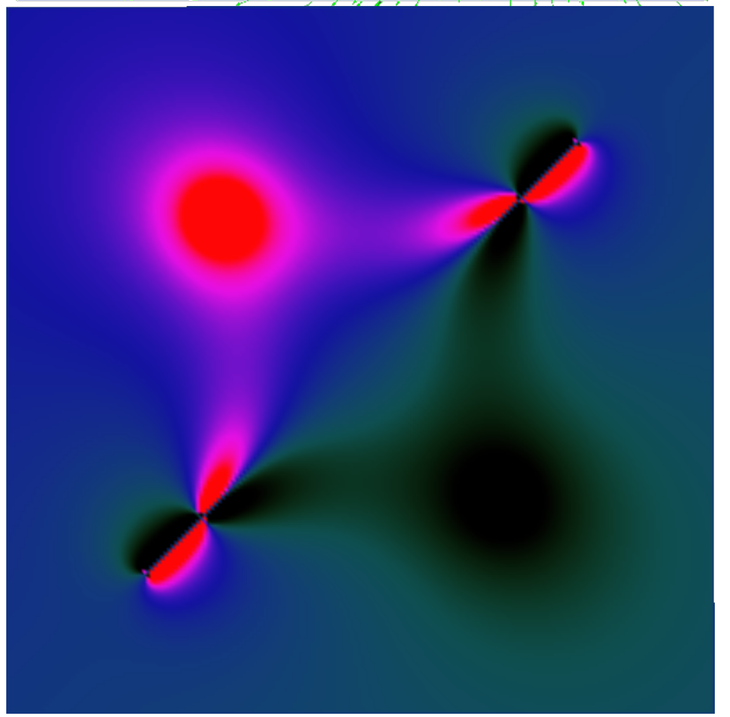
So far

- Extract geometrical features
  - Streamlines
    - Seeding, Line Integral Convolution
    - **Where do they end/start?**
- Extract geometrical measures
  - Magnitude, orientation, angular speed, distortion

- Vector field topology



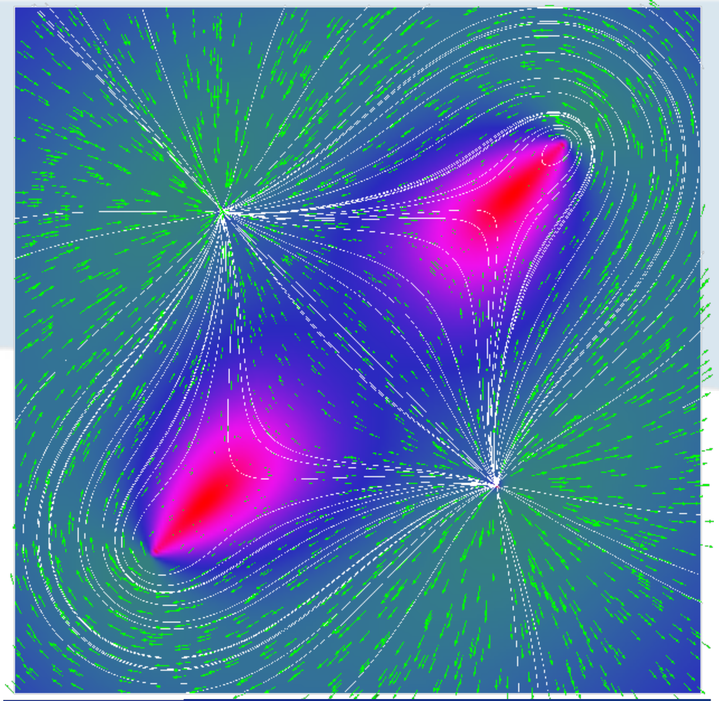


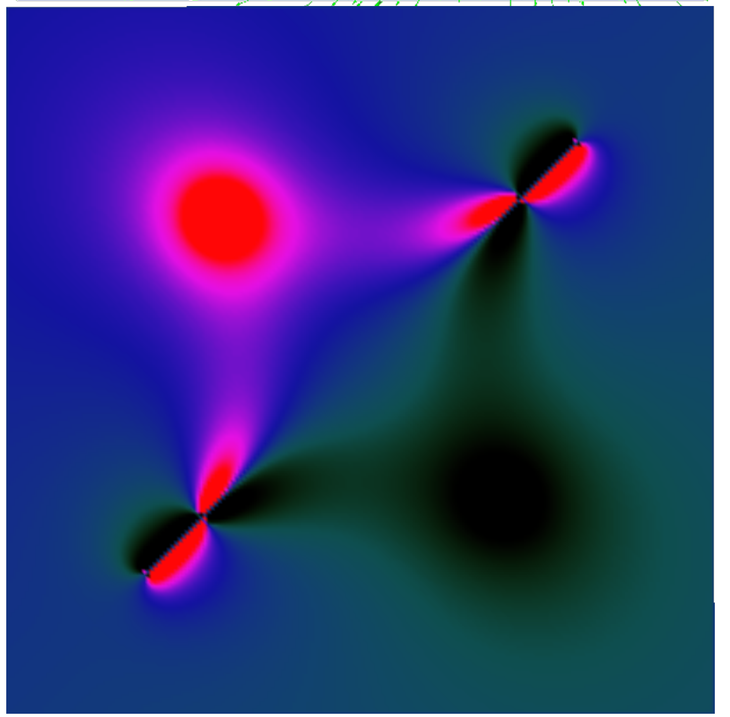
[http://www.deepakantony.com/]

So far

- Extract geometrical features
  - Streamlines
    - Seeding, Line Integral Convolution
    - **Where do they end/start?**
- Extract geometrical measures
  - Magnitude, orientation, angular speed, distortion

- Vector field topology
  - Summarizes all this information





[http://www.deepakantony.com/]

Gradient field topology

Gradient field topology

- For instance
  - $\mathcal{D} \subset \mathbb{R}^n$
    - $f : \mathbb{R}^n \to \mathbb{R}^n$

Gradient field topology

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  - Gradient field, example
    - $g : \mathbb{R}^n \to \mathbb{R}$
    - $f = \nabla g = (\frac{\partial g}{\partial x_1}, \frac{\partial g}{\partial x_2}, \ldots, \frac{\partial g}{\partial x_n})$

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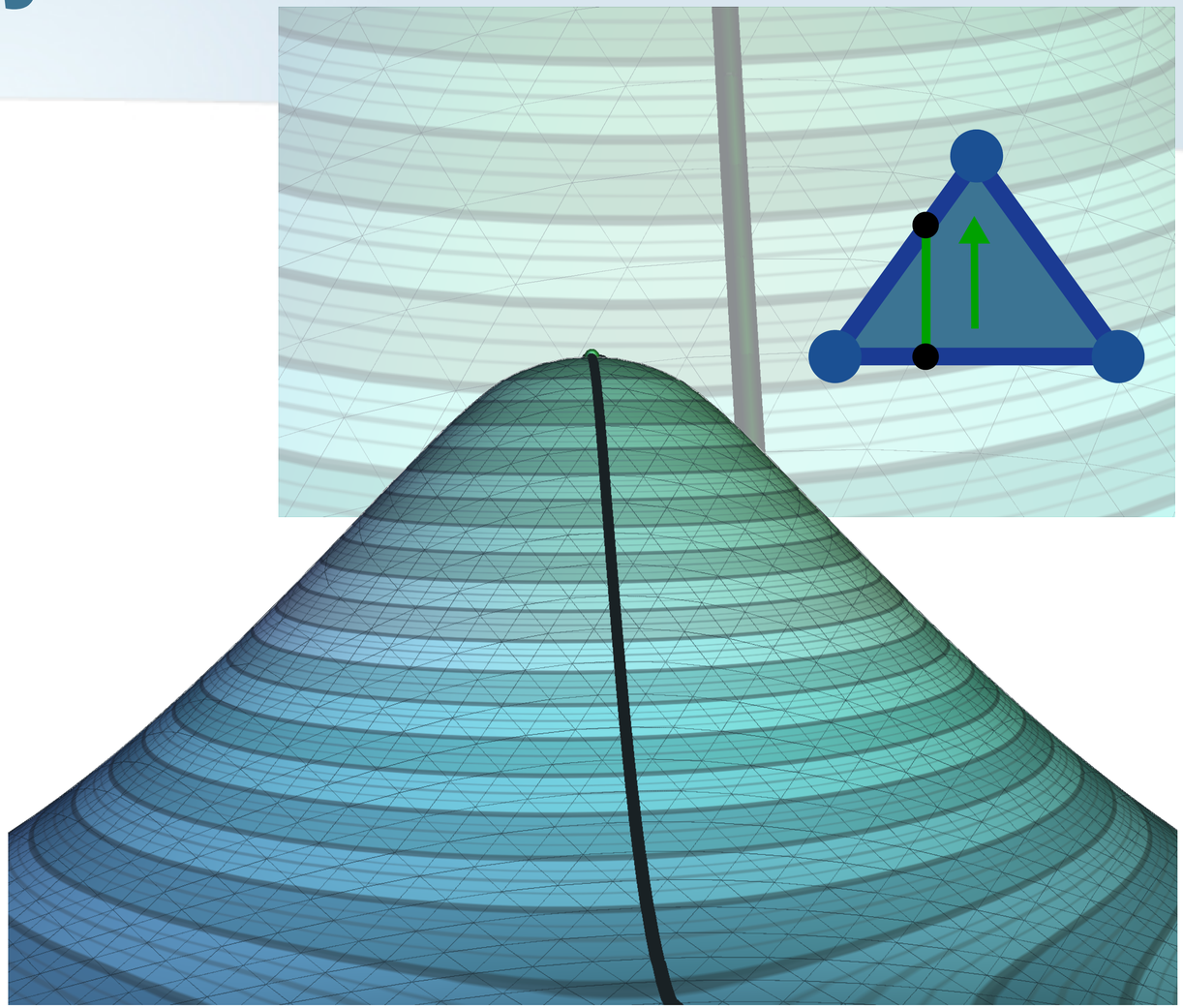
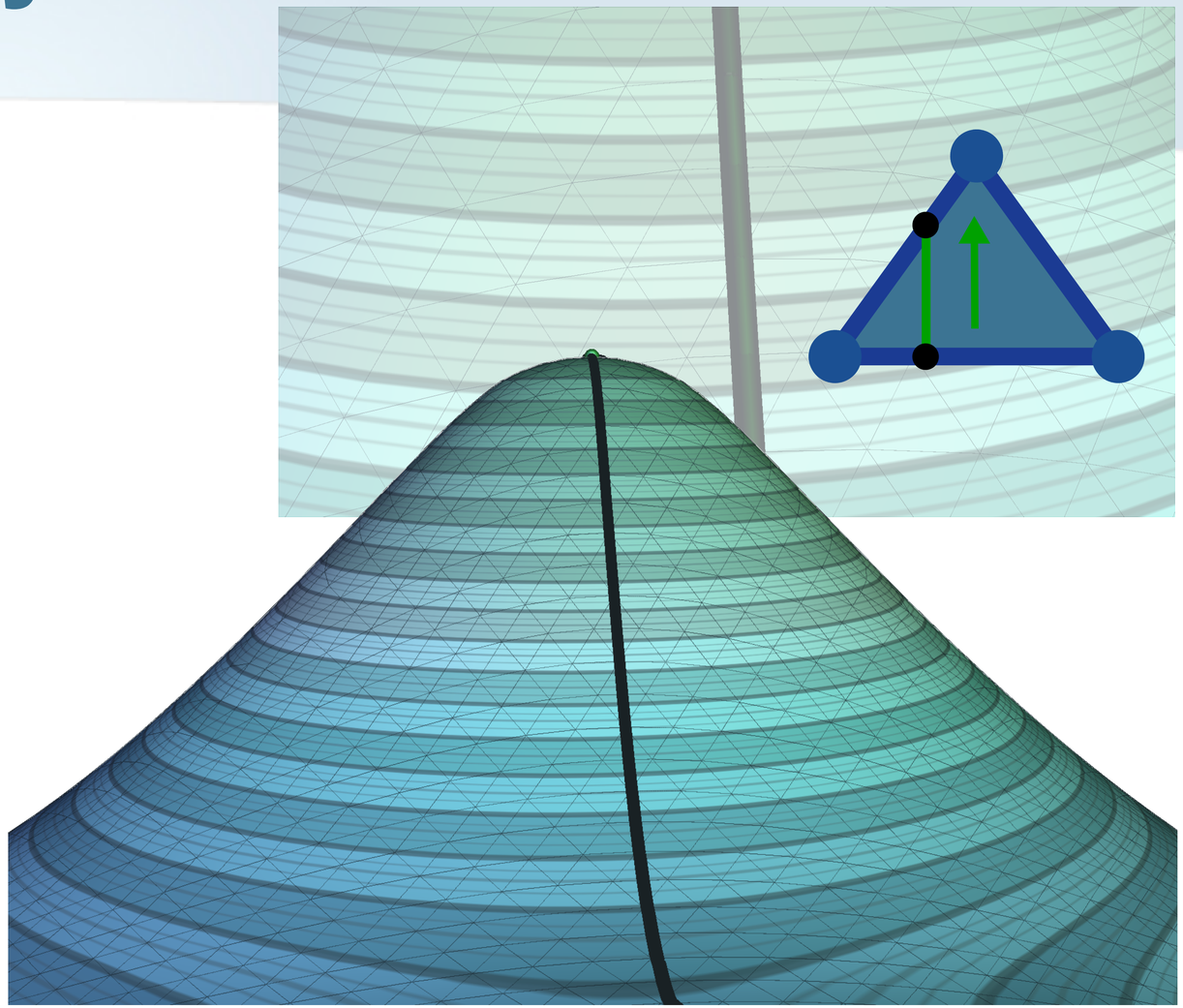


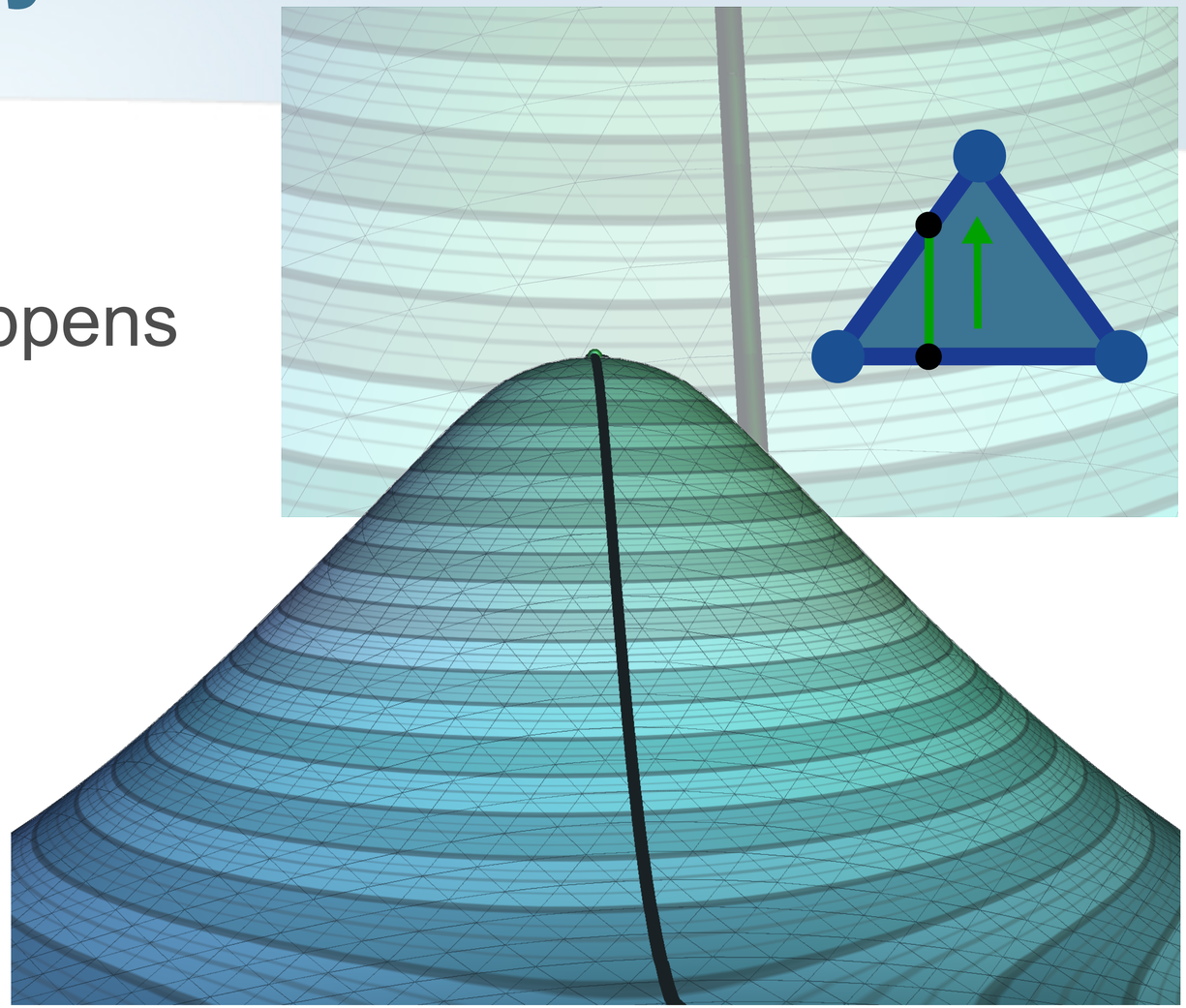
Gradient field topology

- Intuition of critical points



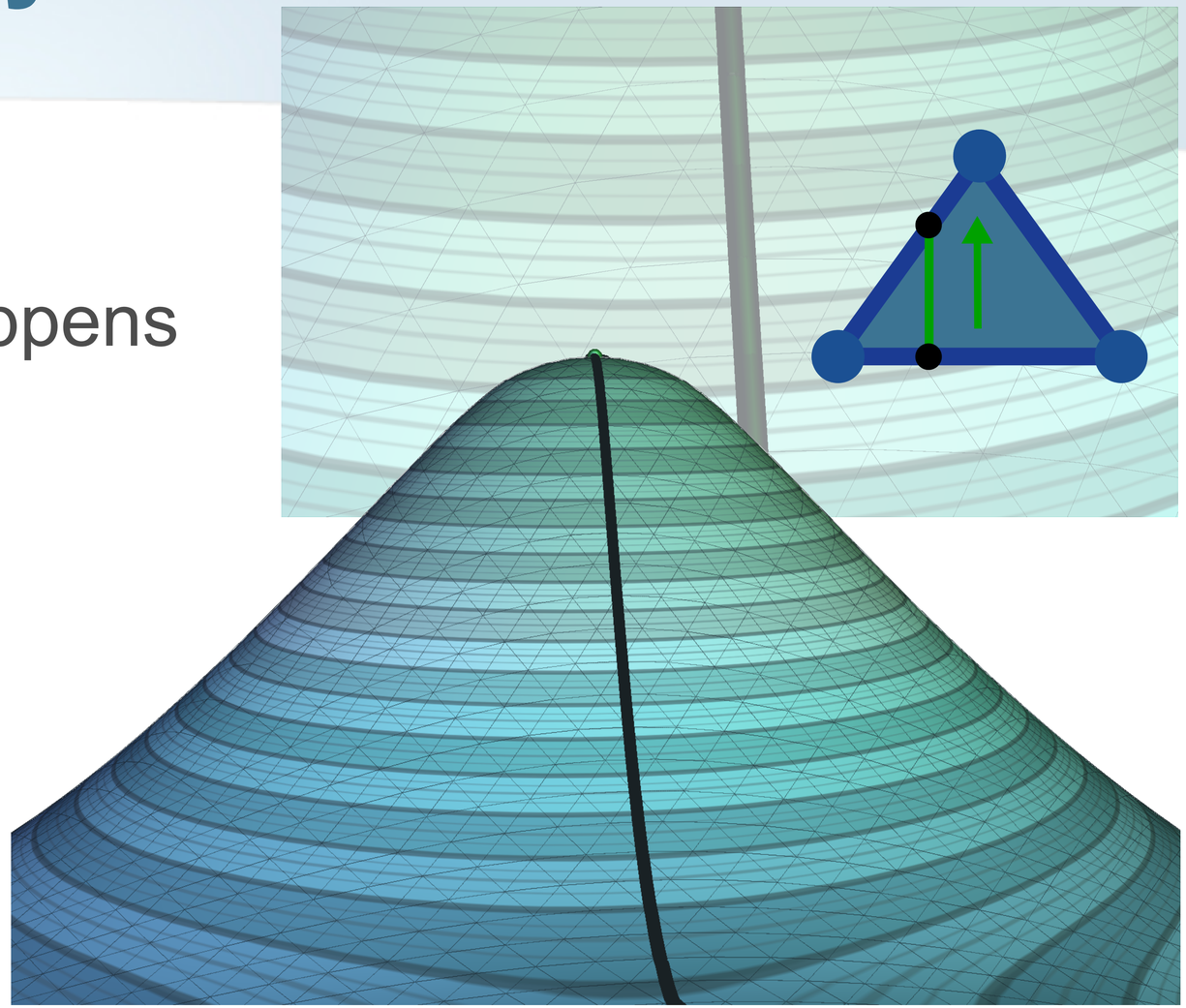
Gradient field topology

- Intuition of critical points
  - Points where something critical happens



Gradient field topology

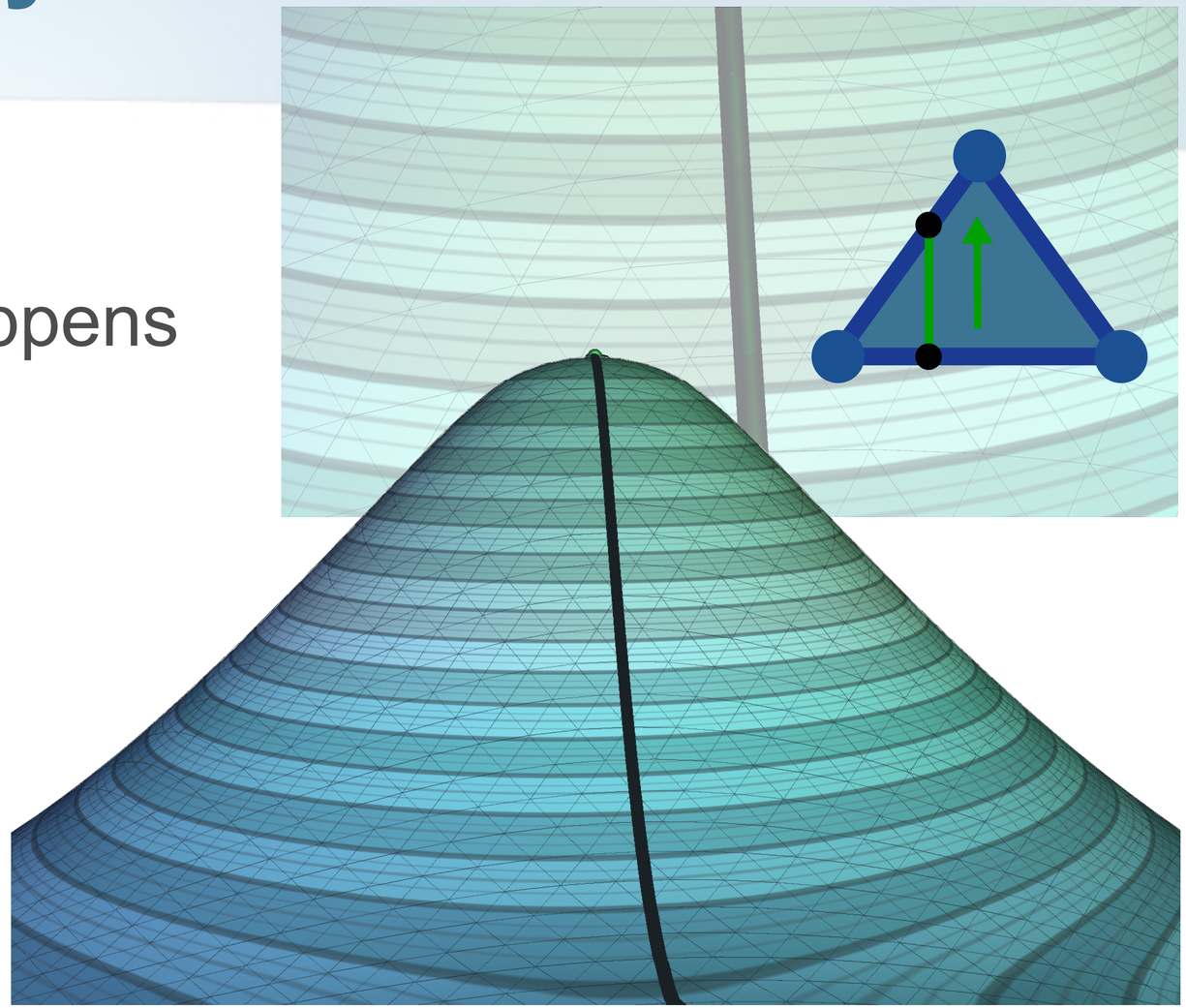
- Intuition of critical points
  - Points where something critical happens
  - Where the flow stops



Gradient field topology

- Intuition of critical points
  - Points where something critical happens
  - Where the flow stops

- Where does the gradient stops?



Gradient field topology

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- Where does the gradient stops?
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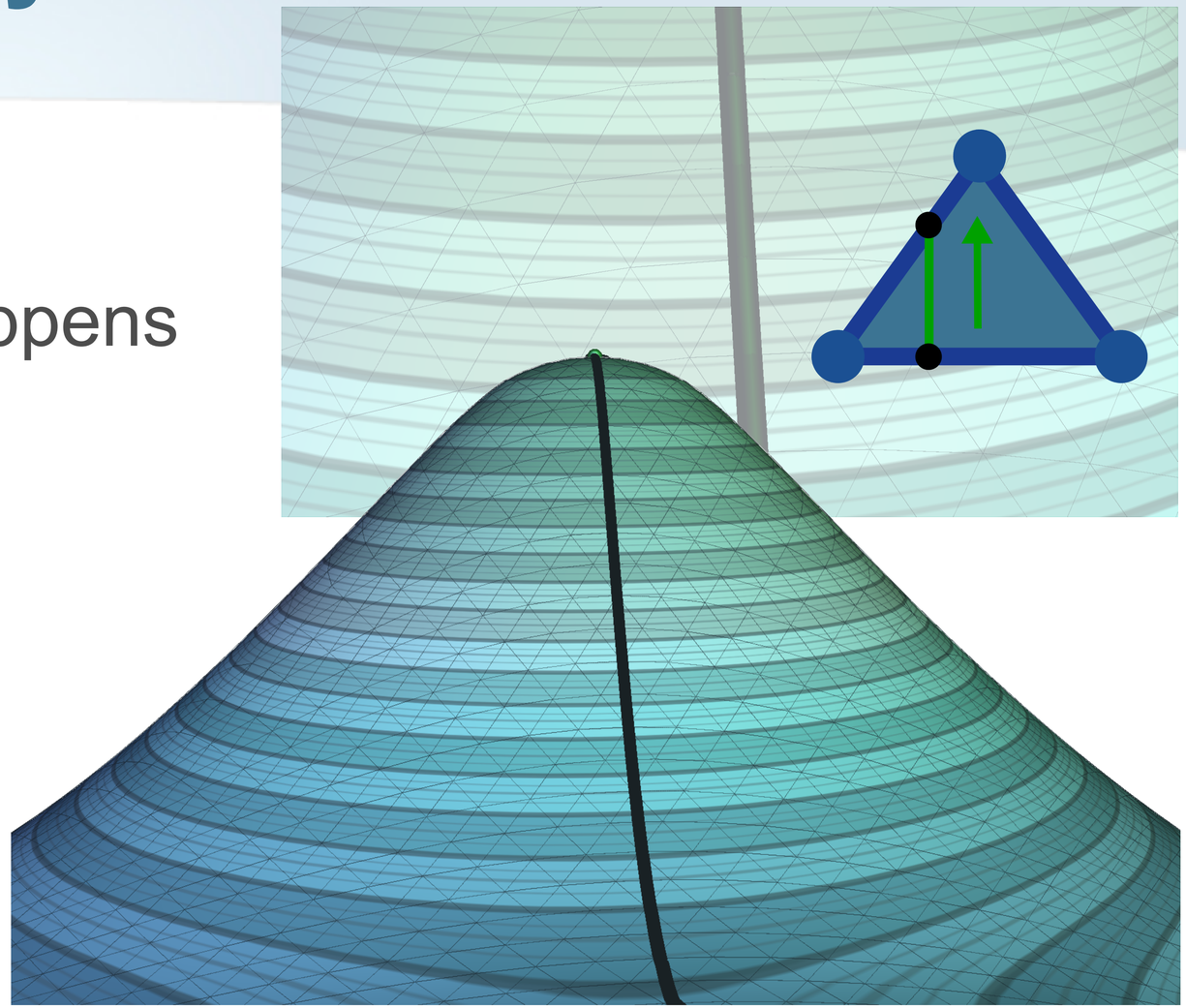

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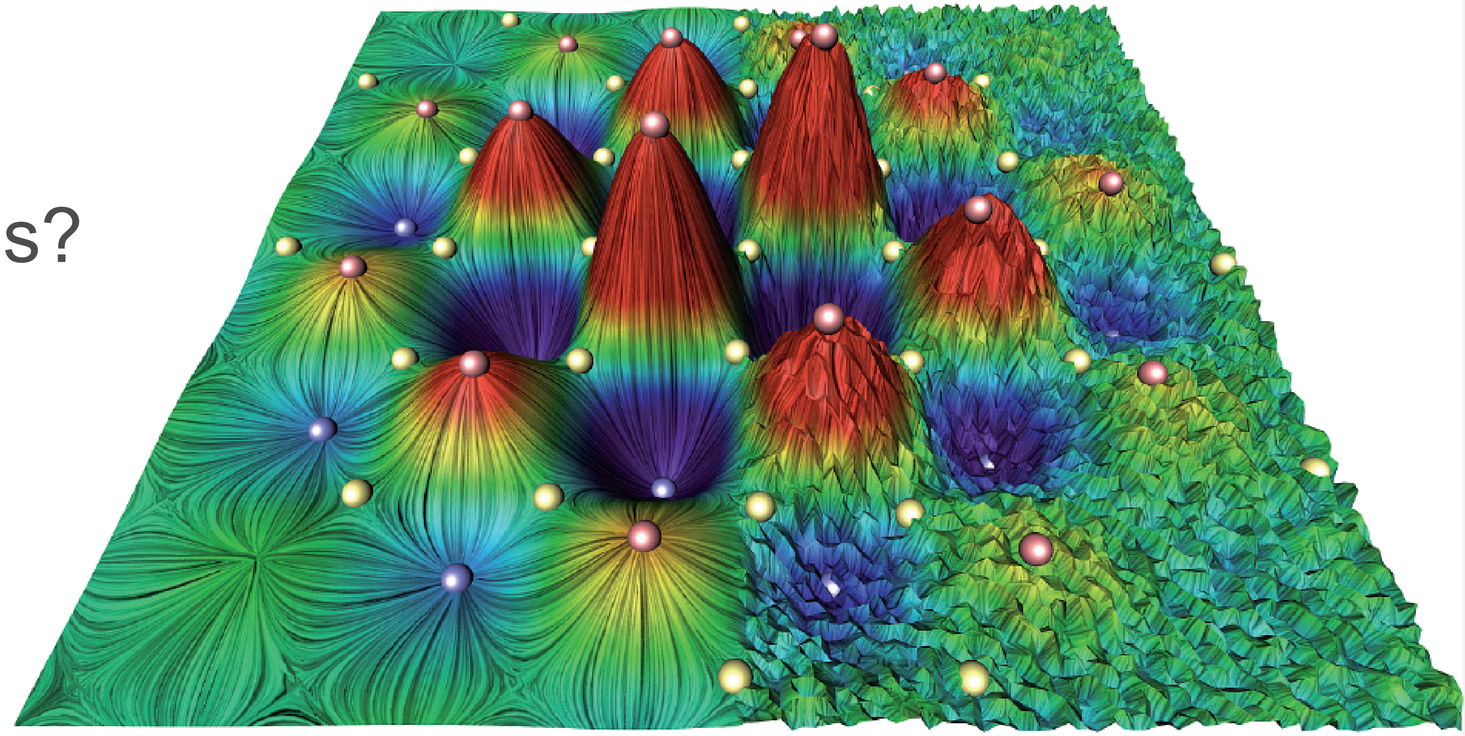
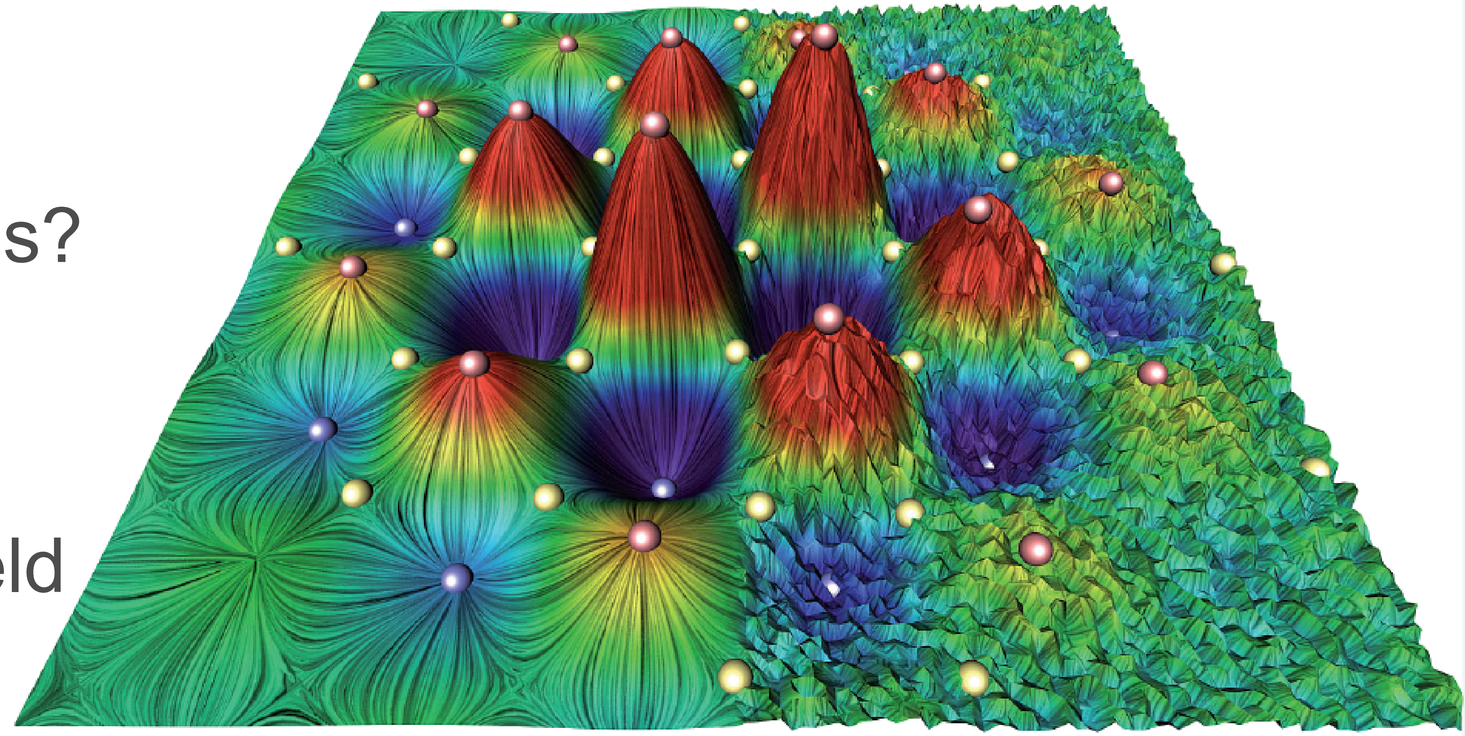


[Reininghaus11]

Gradient field topology

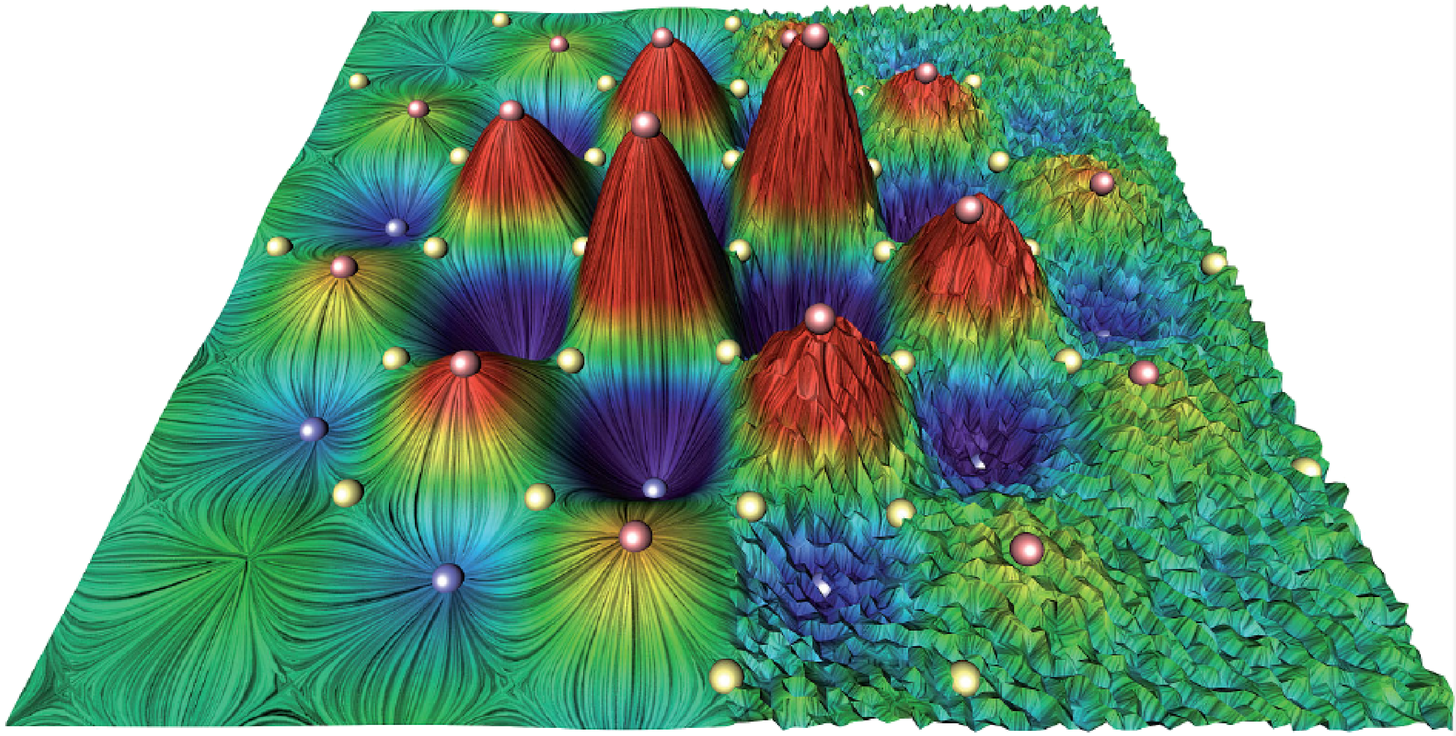
- Intuition of critical points
  - Points where something critical happens
  - Where the flow stops
- Where does the gradient stops?
  - At the critical points of g
- Critical points of a gradient field
  - Same as for scalar fields



[Reininghaus11]

Gradient field topology

- Critical points of a vector field
  - Points where the magnitude vanishes



[Reininghaus11]

Gradient field topology

- Critical points of a vector field
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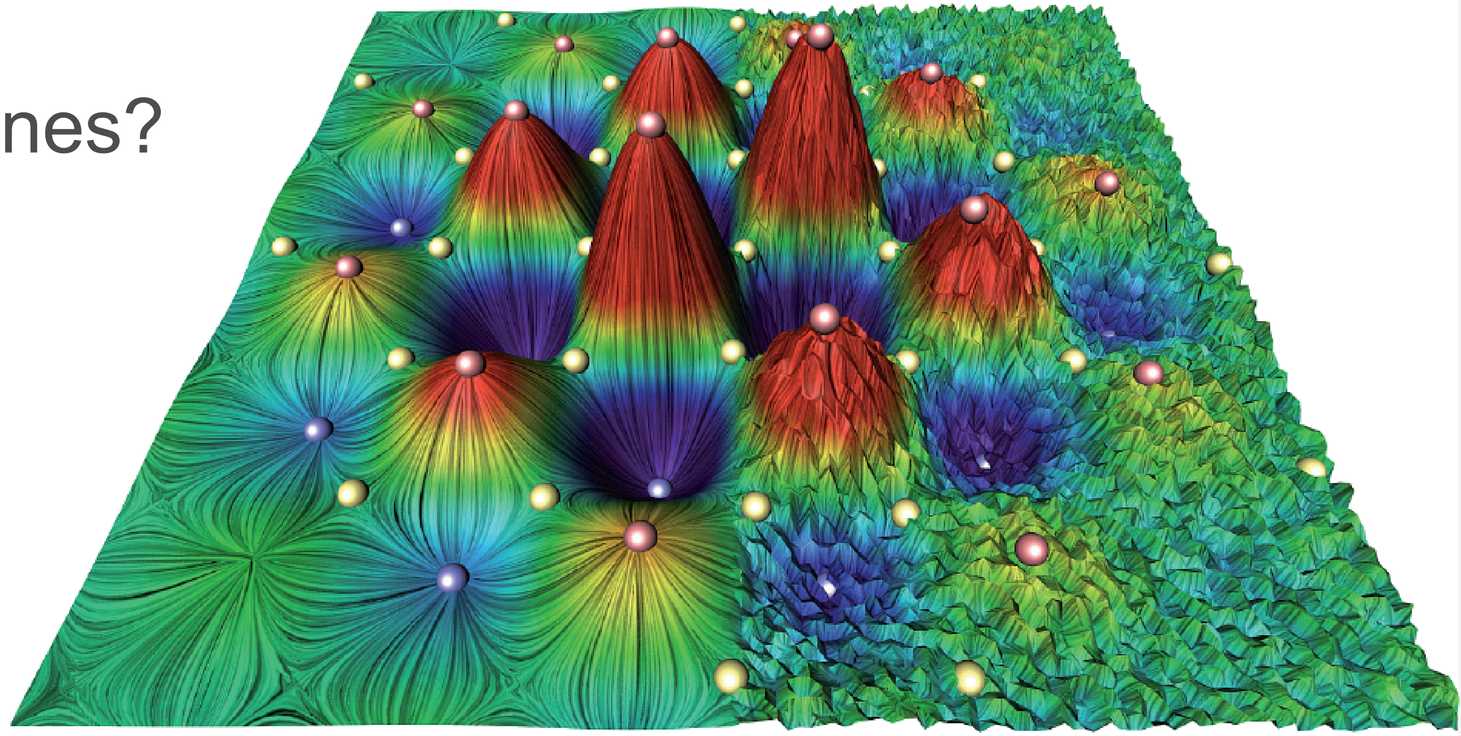
- What's the relation to streamlines?



[Reininghaus11]

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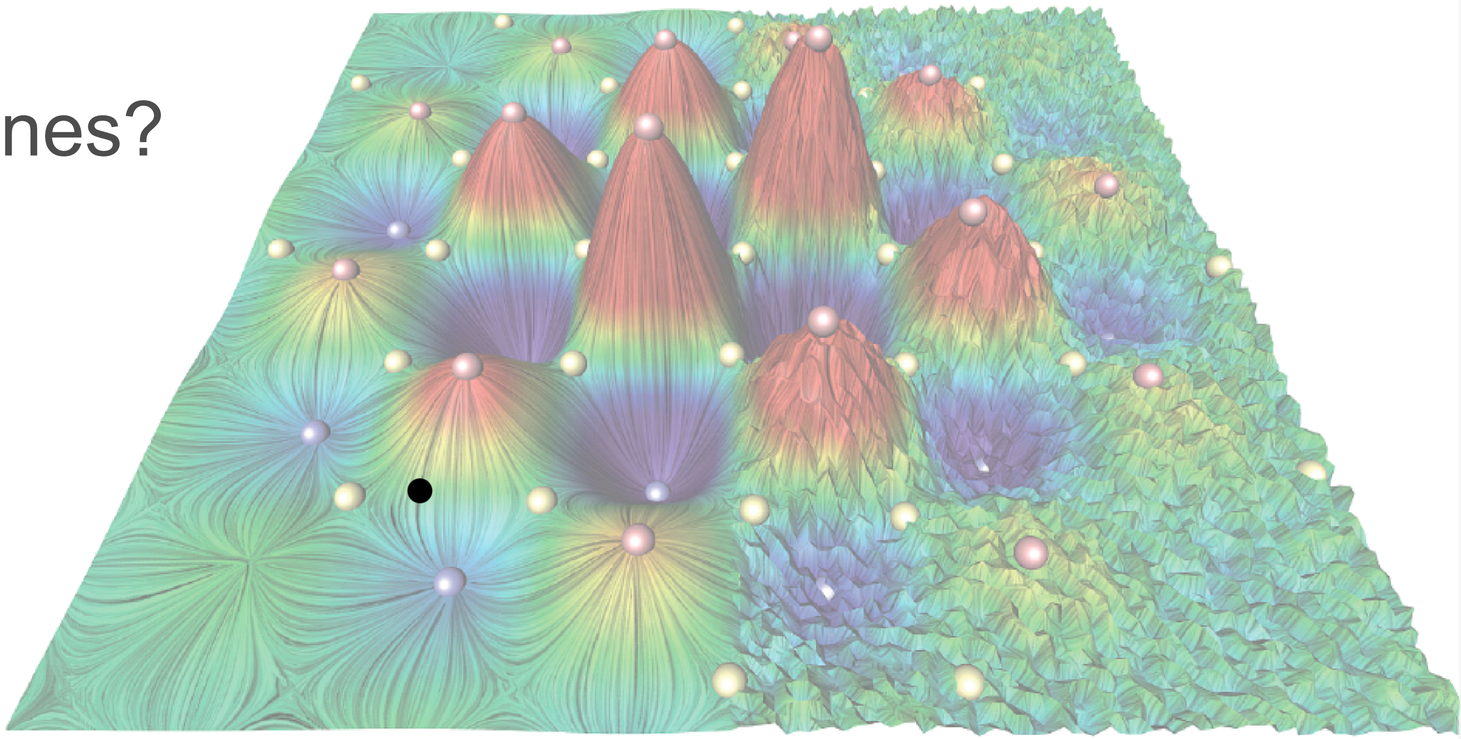

[Reininghaus11]

Gradient field topology

- Critical points of a vector field
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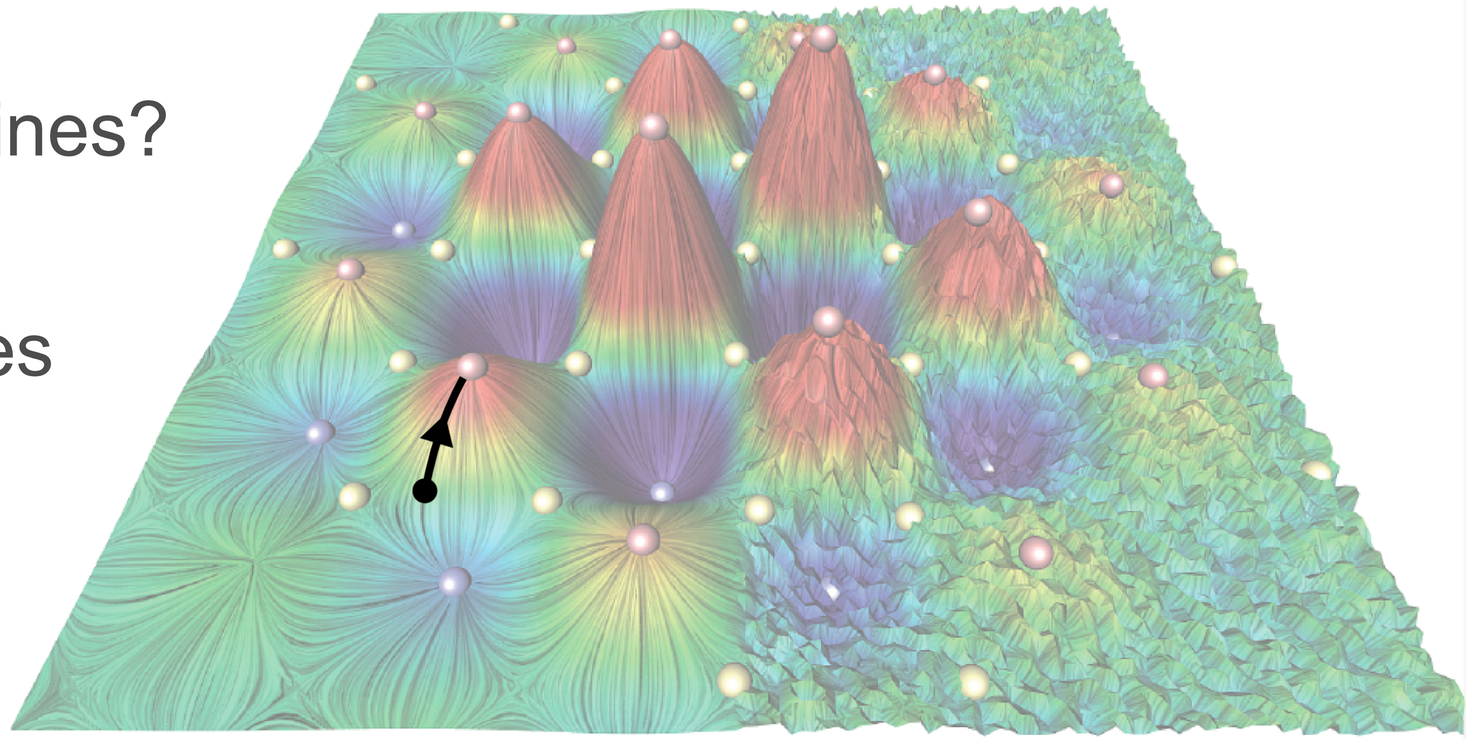
  - On closed domains
  - Critical points are streamlines extremities



[Reininghaus11]

Gradient field topology

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  - Critical points are streamlines extremities
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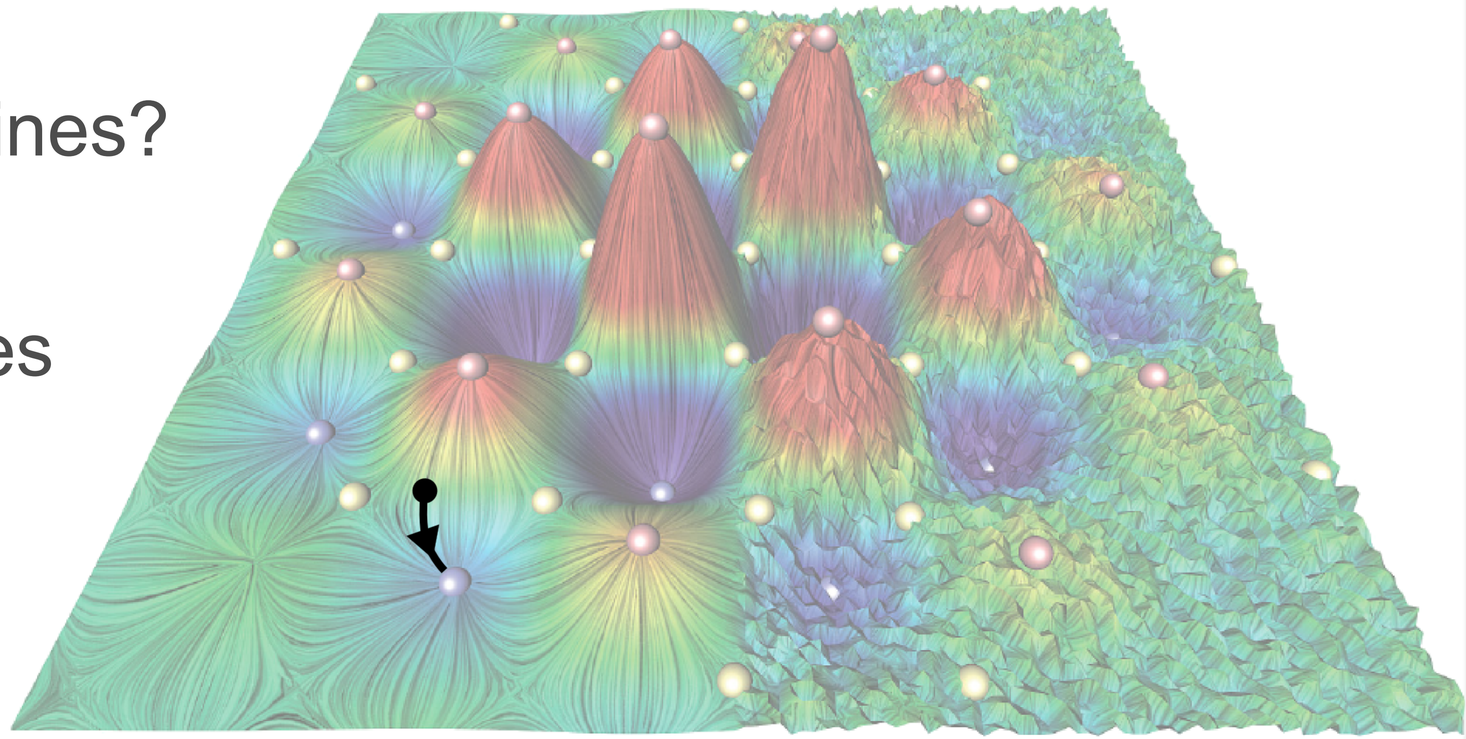

[Reininghaus11]

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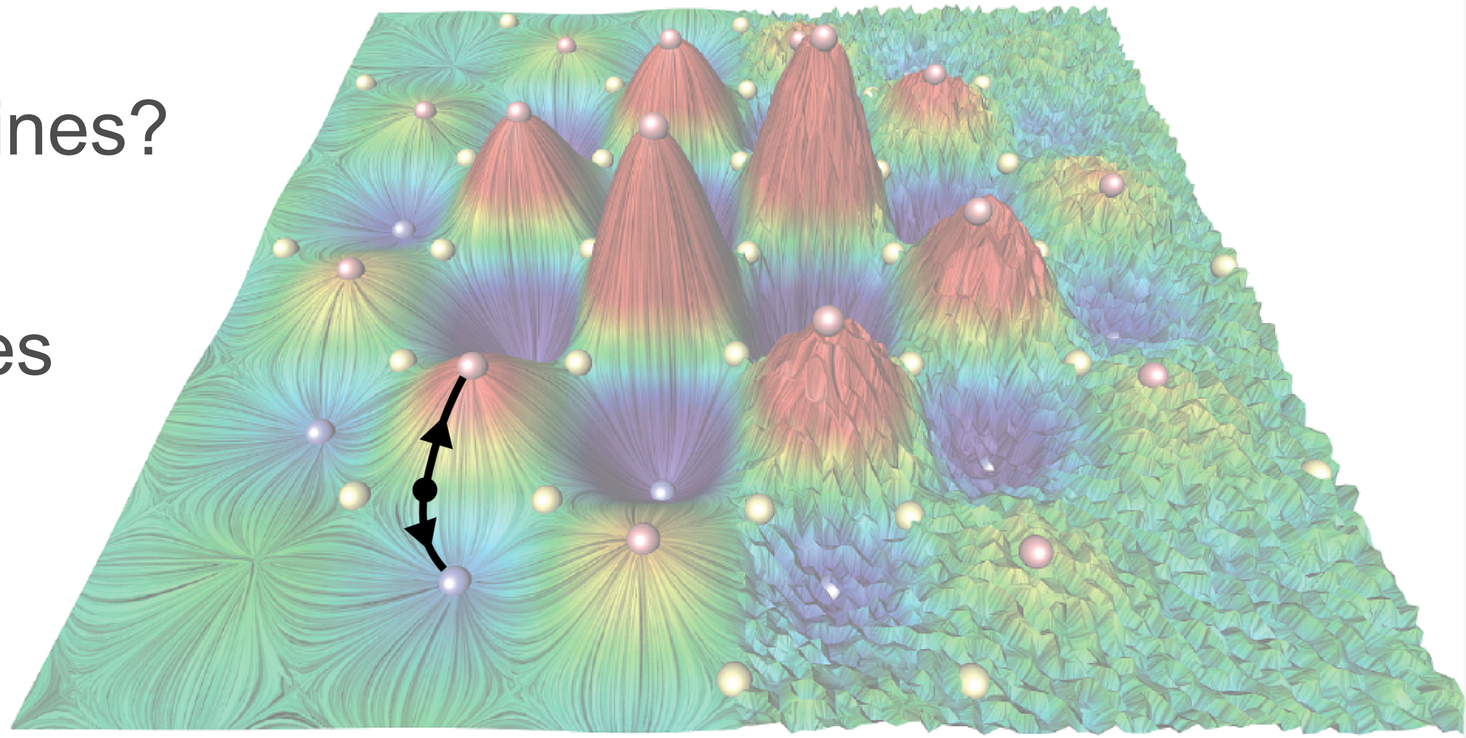
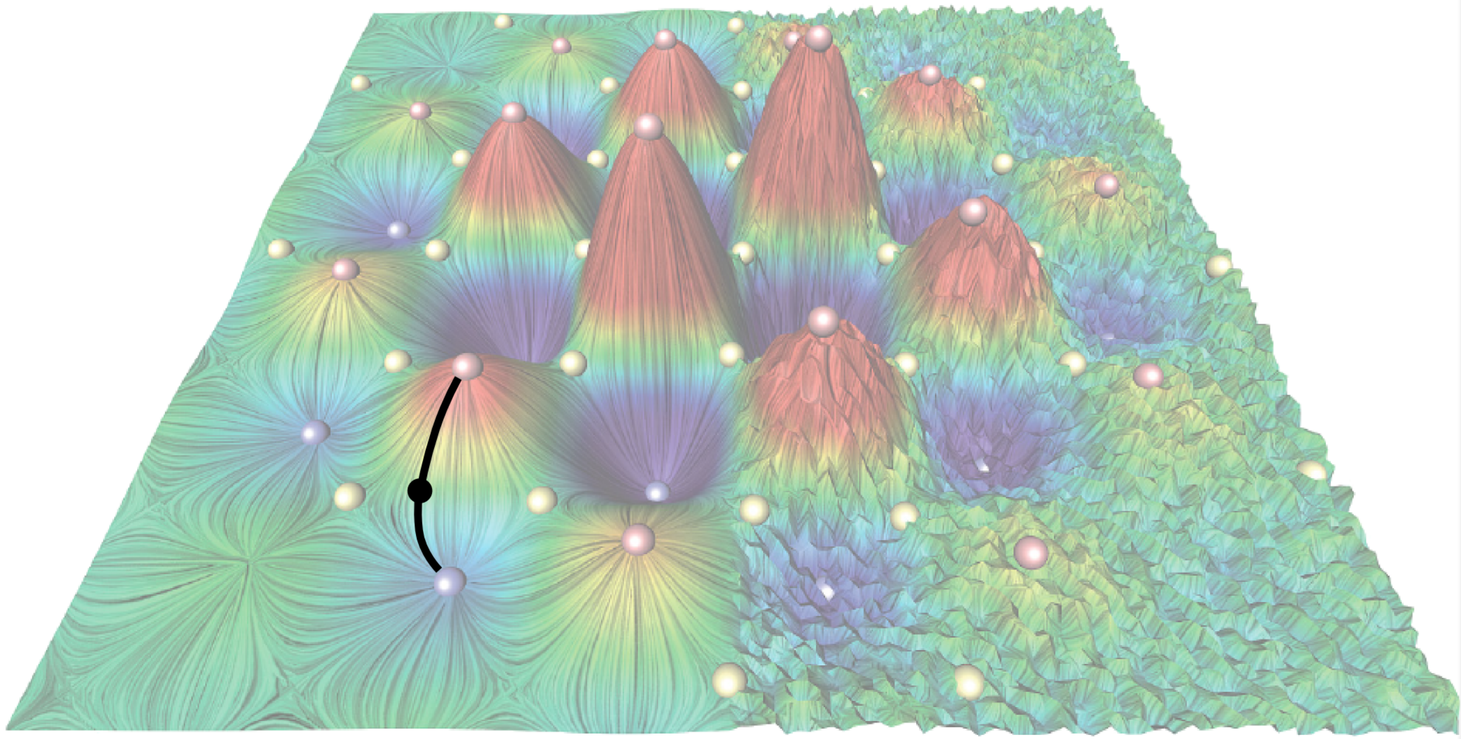


[Reininghaus11]

Gradient field topology

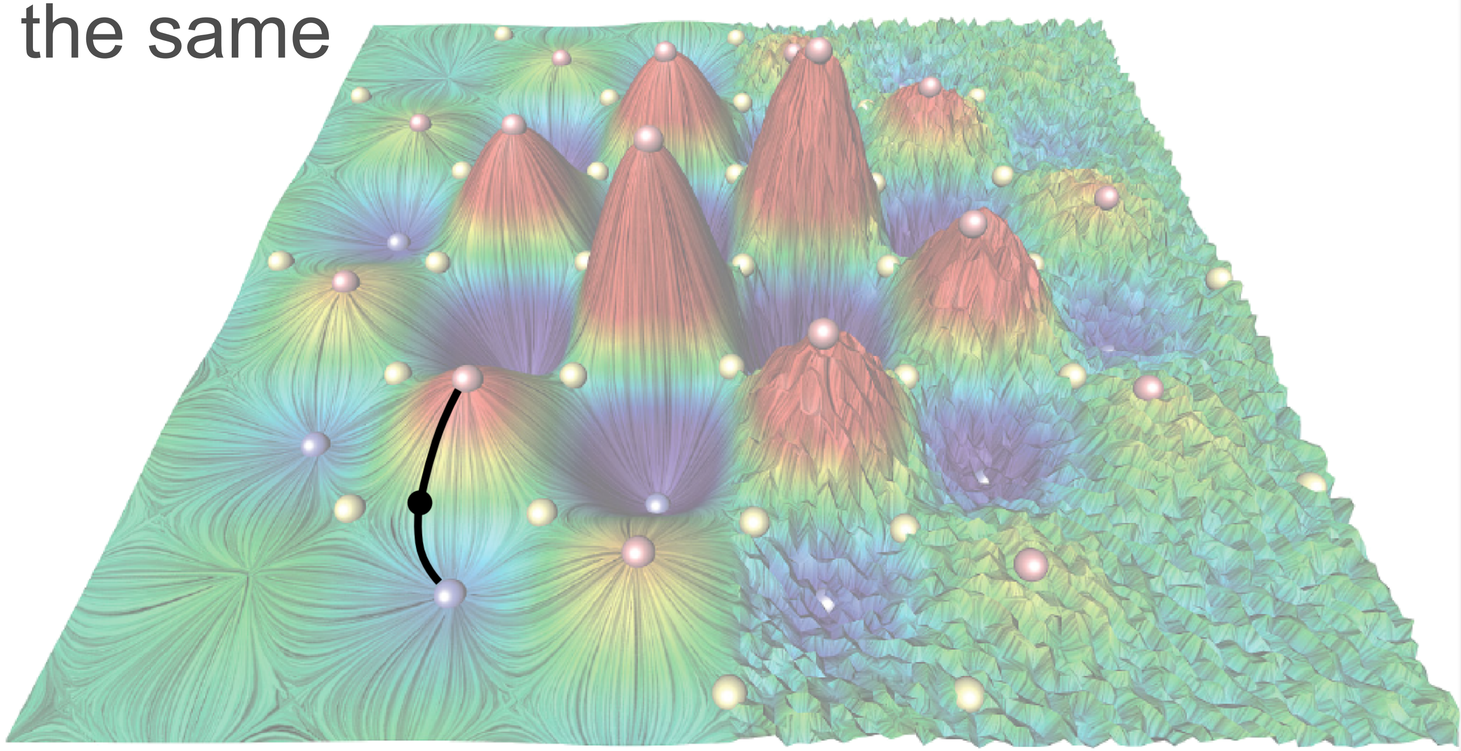
- Understanding the structure of the critical points



[Reininghaus11]

Gradient field topology

- Understanding the structure of the critical points
- Several streamlines can have the same extremities



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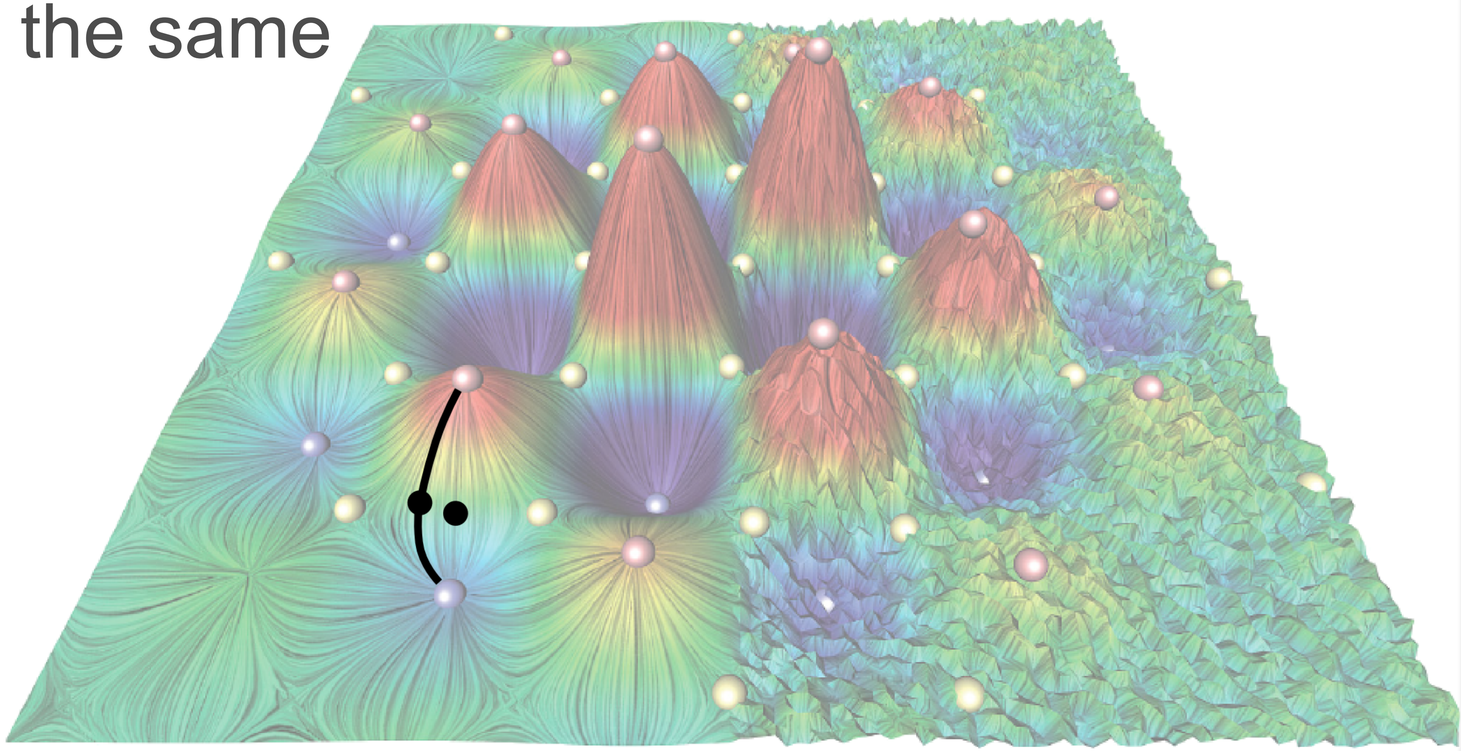

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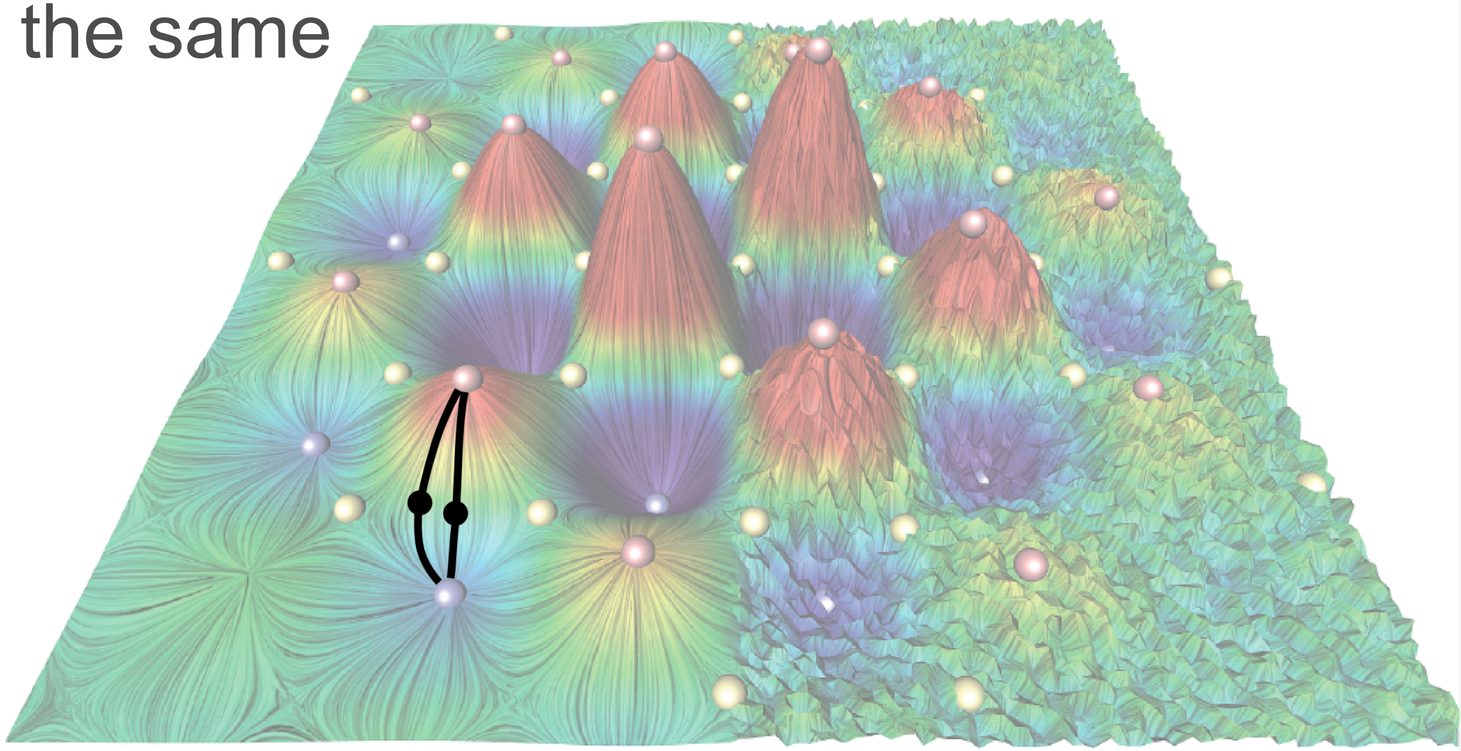

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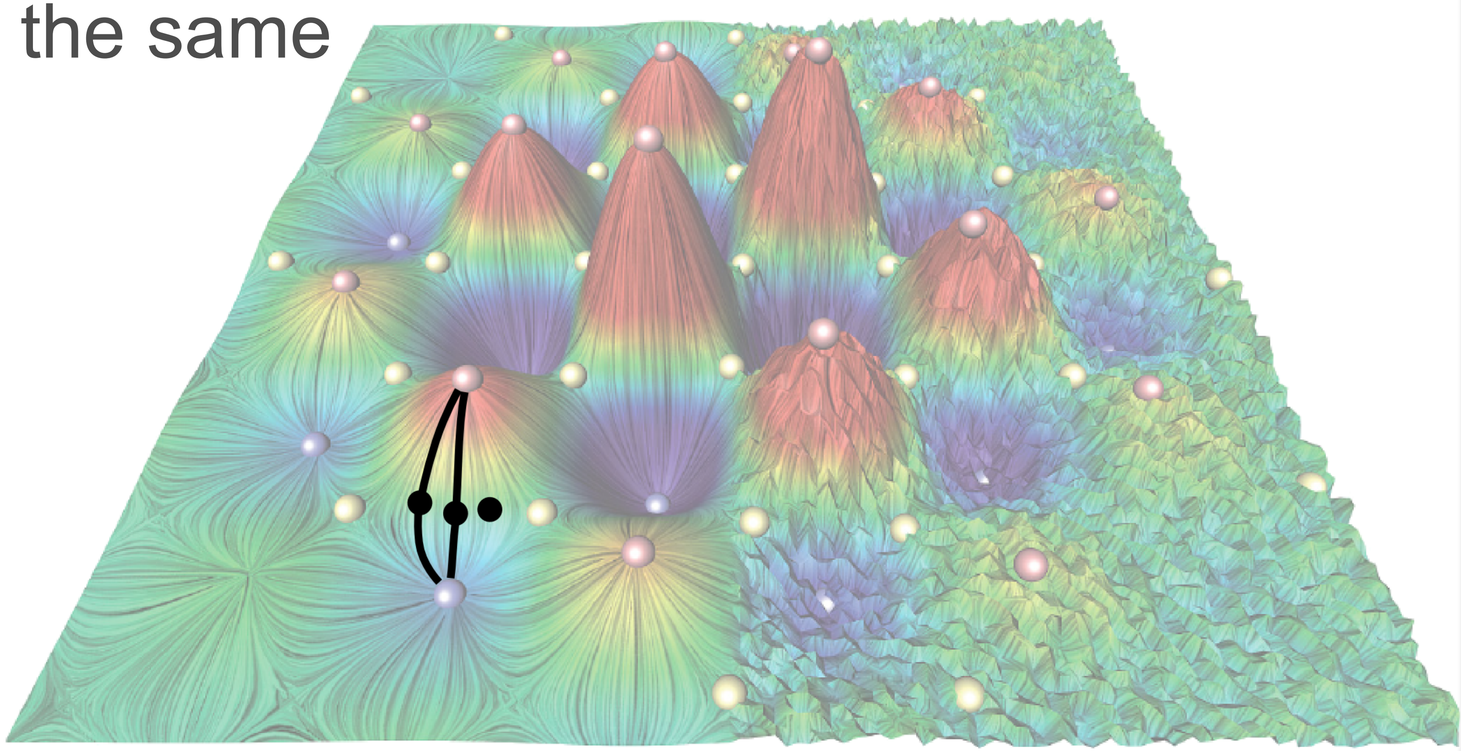

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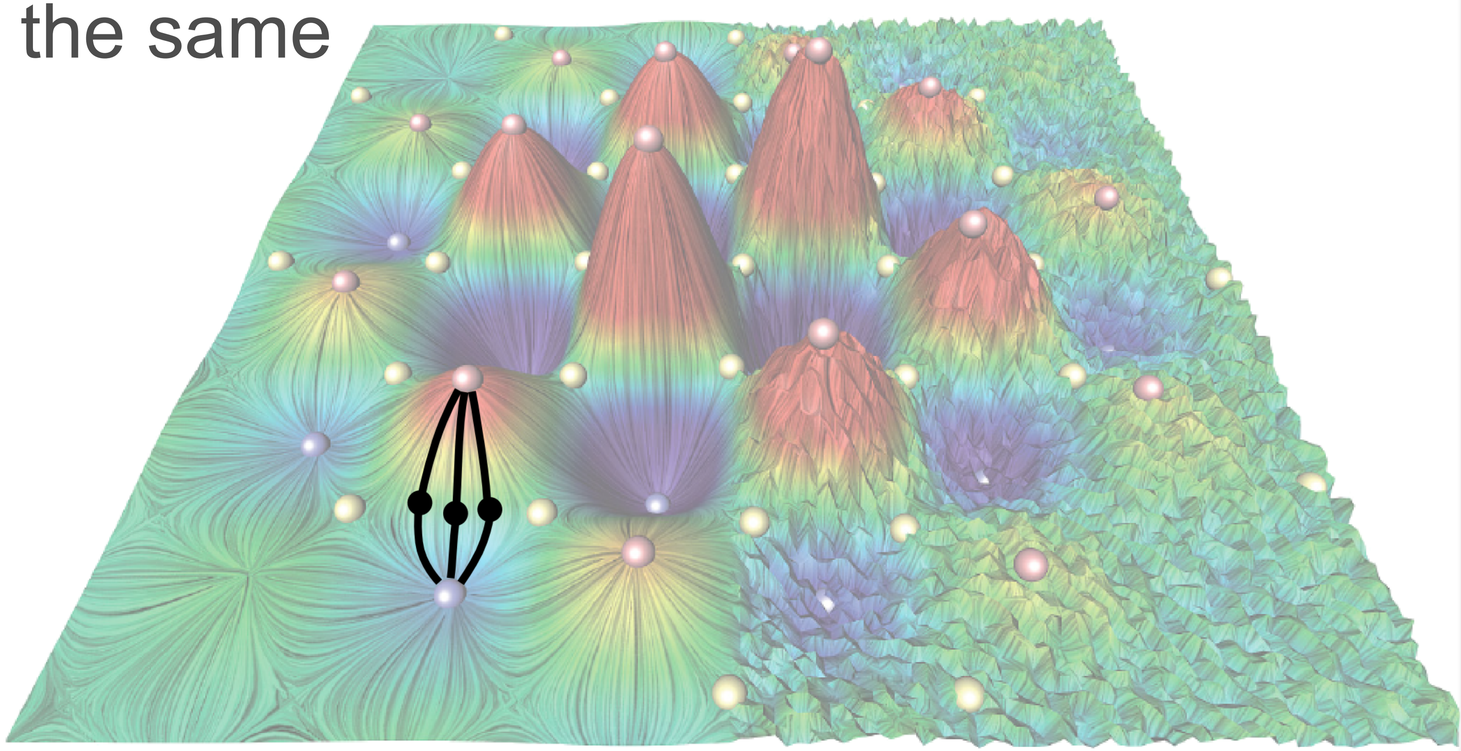
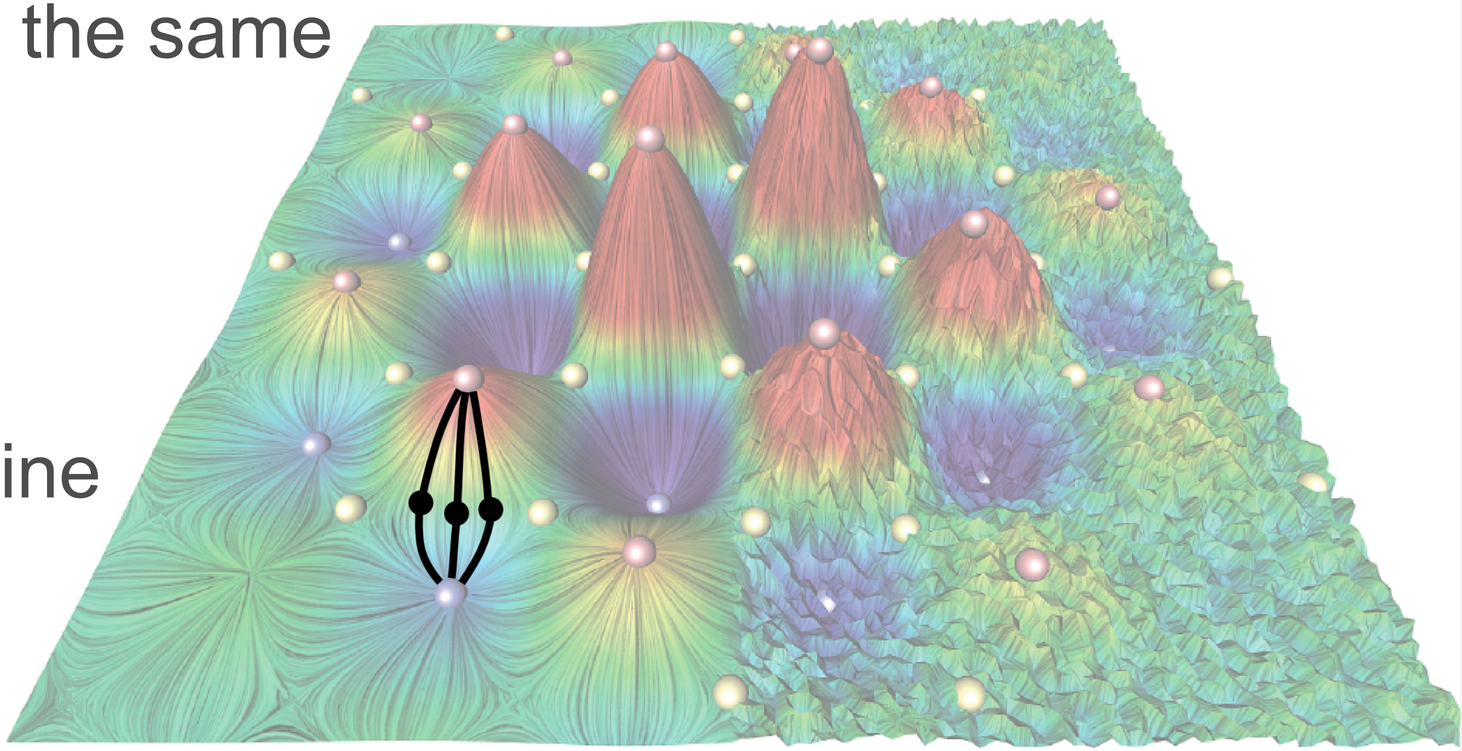


[Reininghaus11]

Gradient field topology

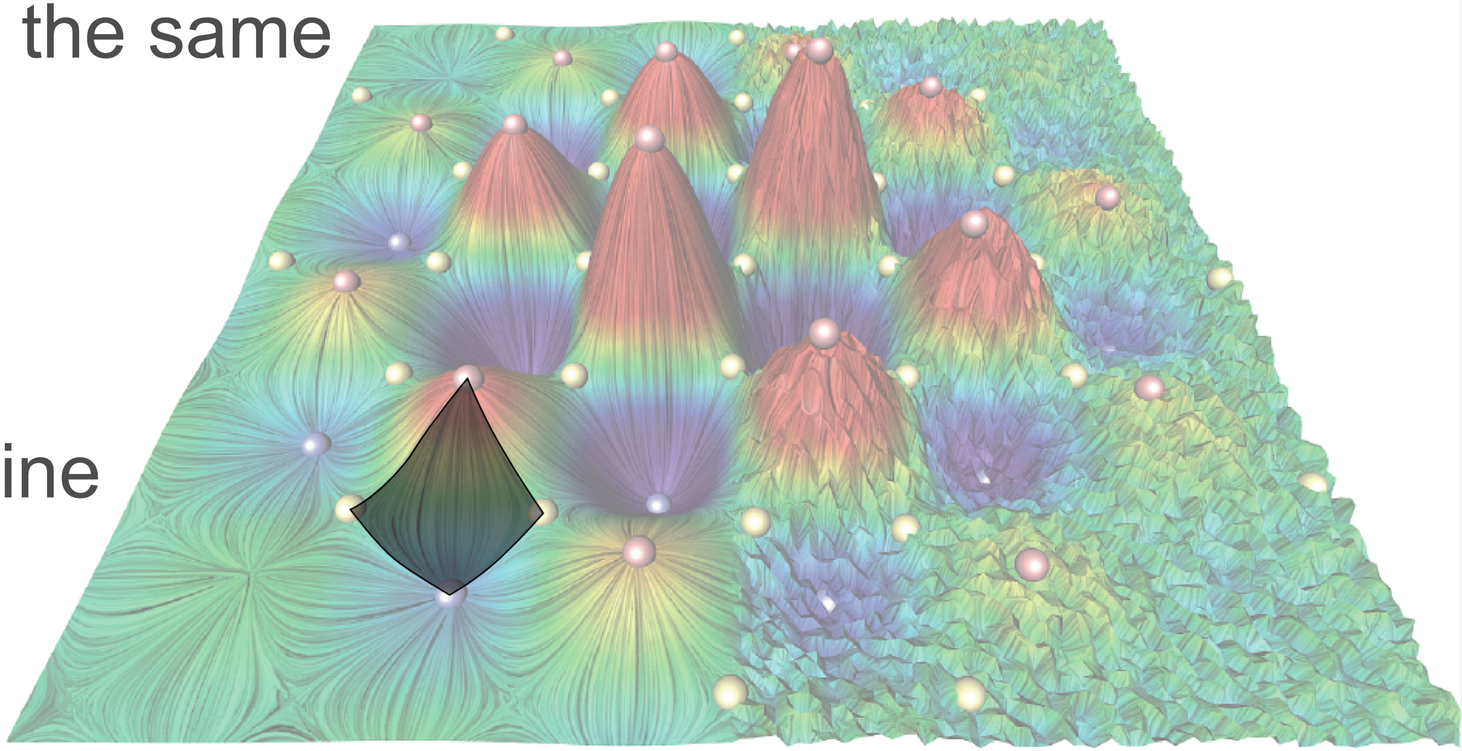
- Understanding the structure of the critical points
- Several streamlines can have the same extremities
- Equivalence relation
  - All the points whose streamline shares identical extremities



[Reininghaus11]

Gradient field topology

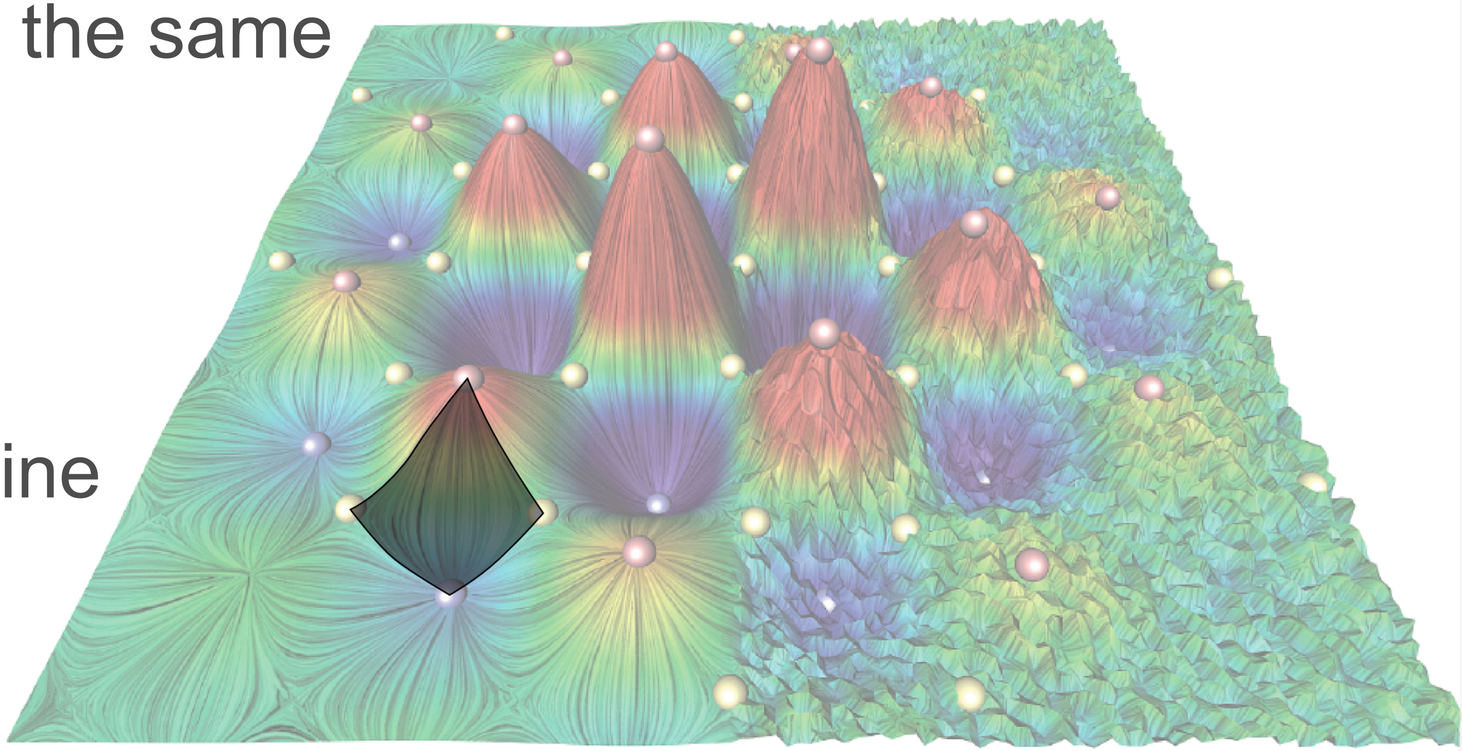
- Understanding the structure of the critical points
- Several streamlines can have the same extremities
- Equivalence relation
  - All the points whose streamline shares identical extremities



[Reininghaus11]

Gradient field topology

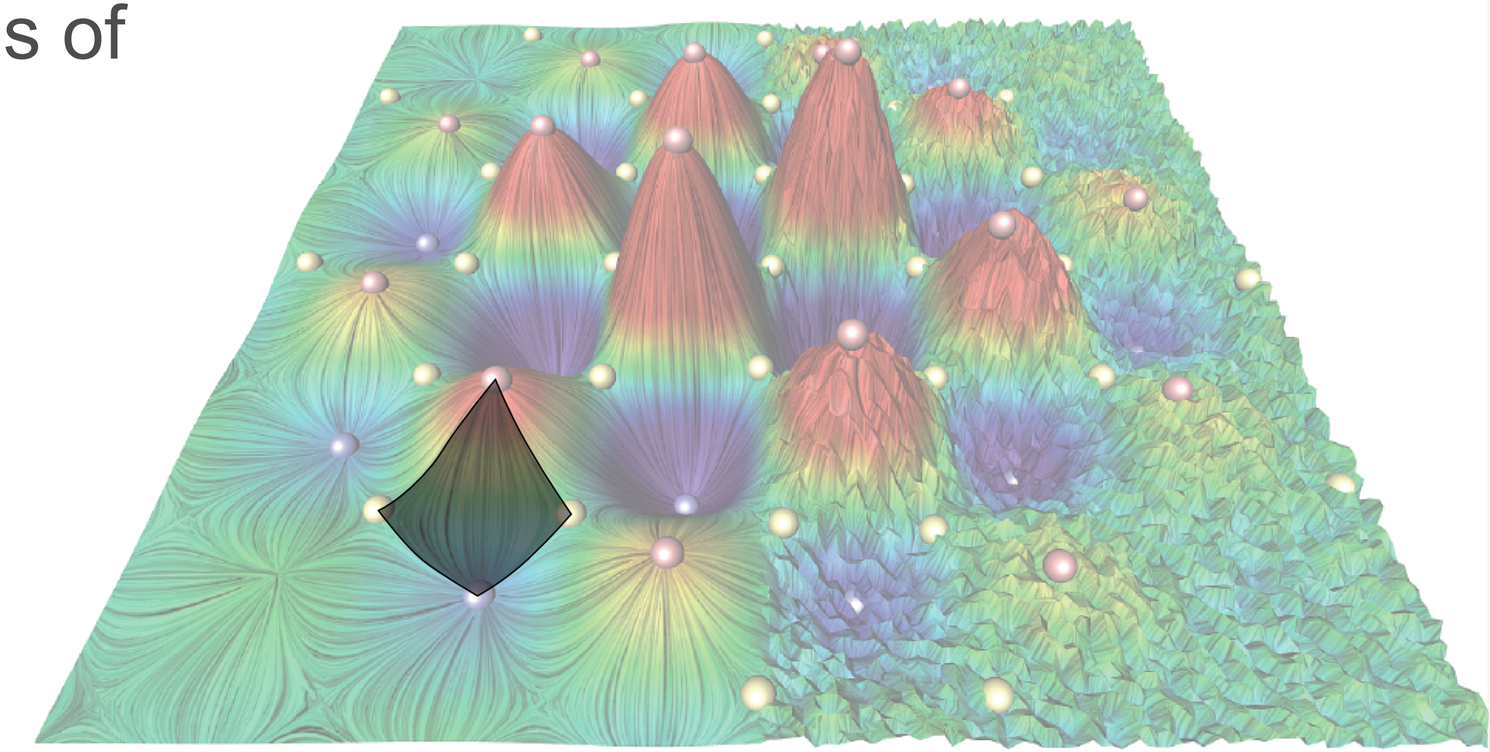
- Understanding the structure of the critical points
- Several streamlines can have the same extremities
- Equivalence relation
  - All the points whose streamline shares identical extremities
  - **Notion of flow cell**



[Reininghaus11]

Gradient field topology

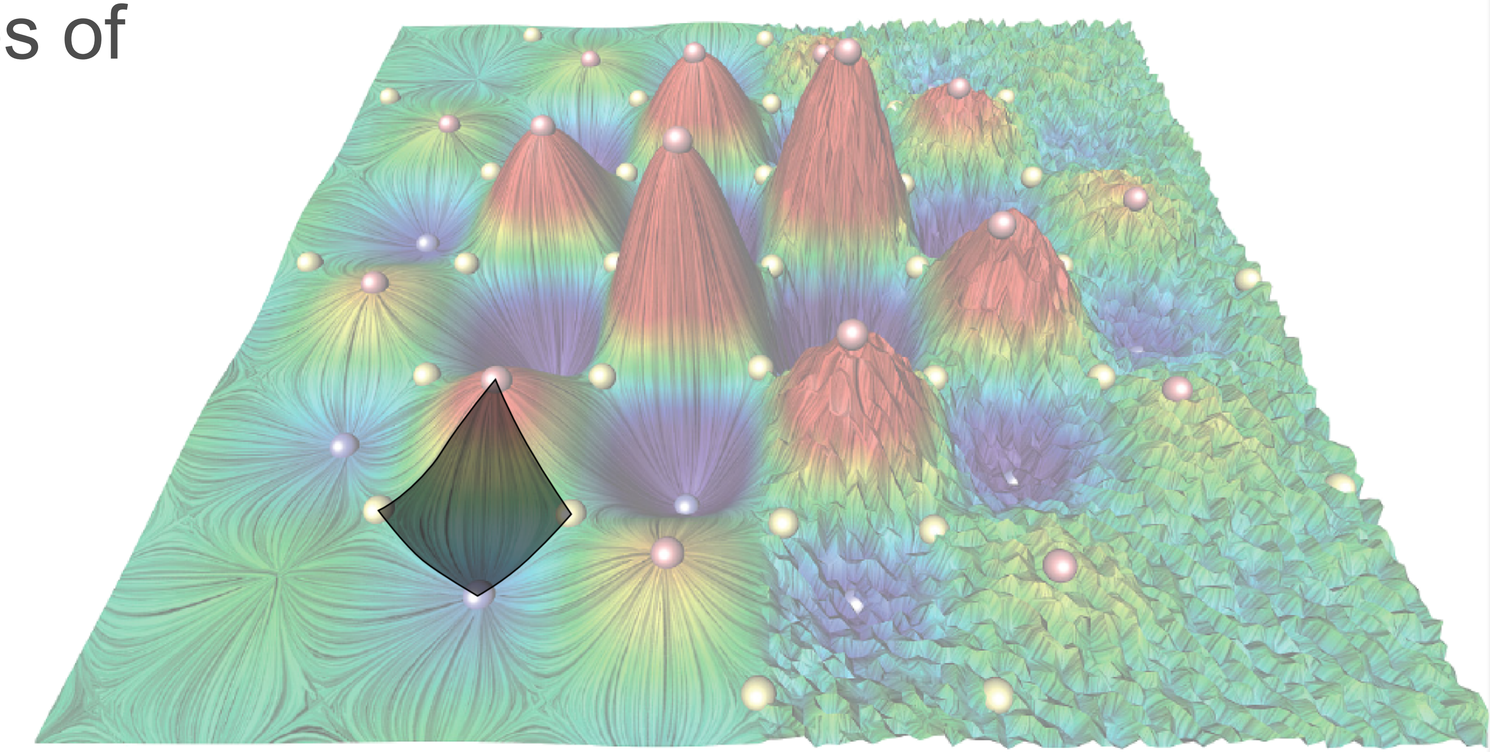
- Understanding the structure of the critical points
- Now, what are the boundaries of the flow cells?



[Reininghaus11]

Gradient field topology

- Understanding the structure of the critical points
- Now, what are the boundaries of the flow cells?
- Streamlines between critical points



[Reininghaus11]

Critical points of a vector field

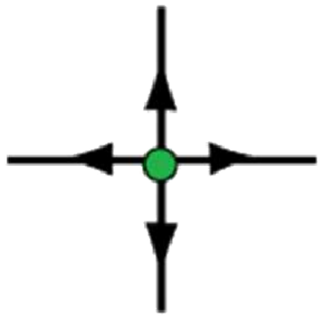
- For example
  - $f : \mathbb{R}^2 \to \mathbb{R}^2$

Critical points of a vector field

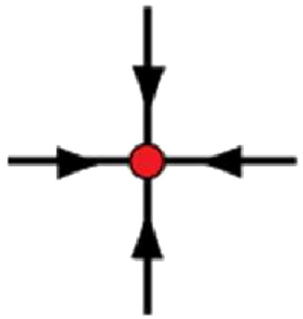
- For example
  - $f : \mathbb{R}^2 \to \mathbb{R}^2$
- Points where the magnitude vanishes
  - $f(p) = p$

Critical points of a vector field

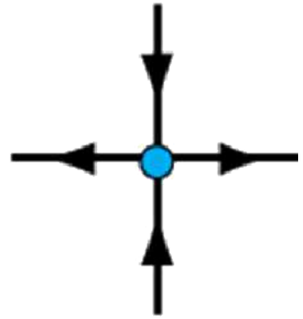
- For example
  - $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$
- Points where the magnitude vanishes
  - $f(p) = p$



Source



Sink

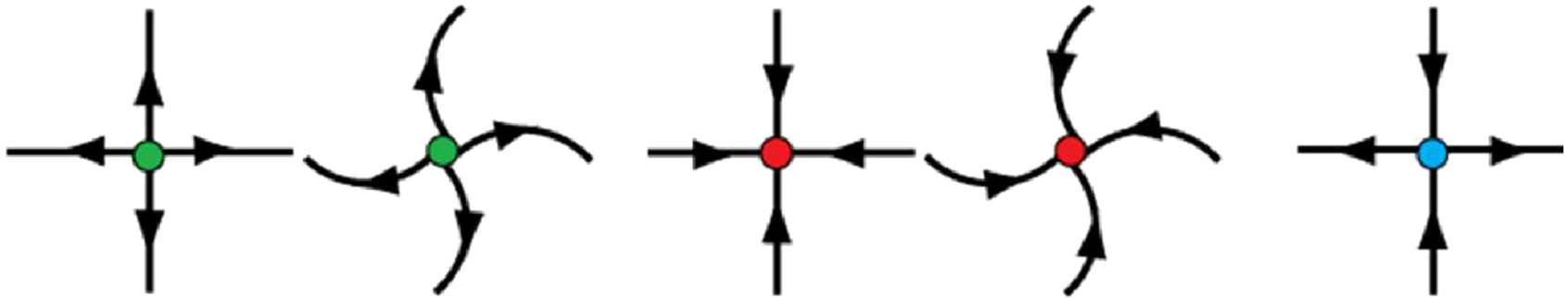


Saddle

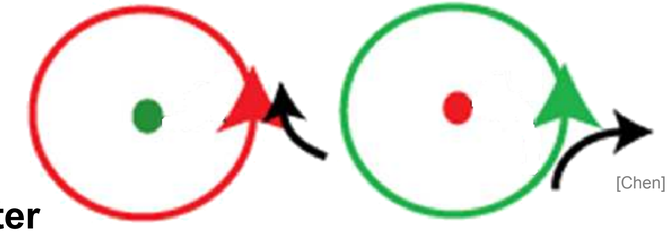
[Chen]

Critical points of a vector field

- For example
  - $f : \mathbb{R}^2 \to \mathbb{R}^2$
- Points where the magnitude vanishes
  - $f(p) = p$
- With curl
  - More critical points



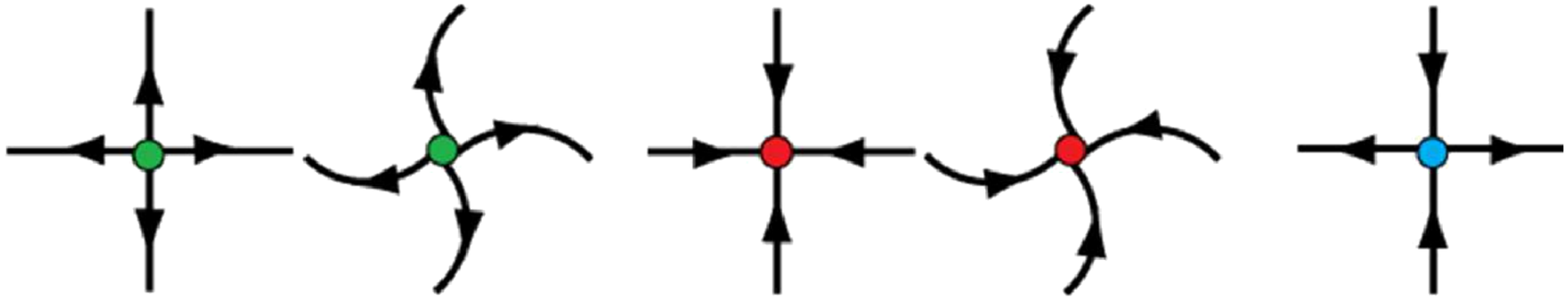
Source Sink Saddle



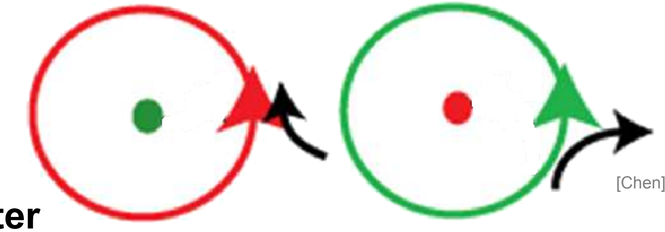
Center

[Chen]

Classifying critical points



Source Sink Saddle

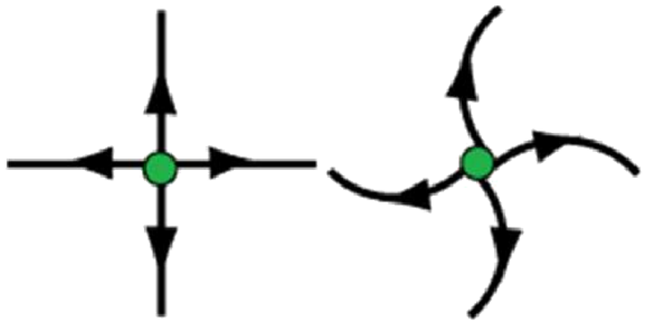


Center

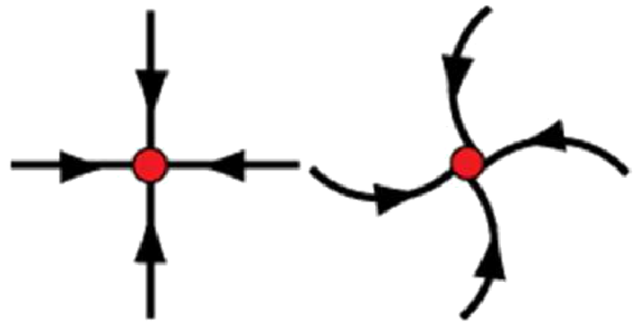
[Chen]

Classifying critical points

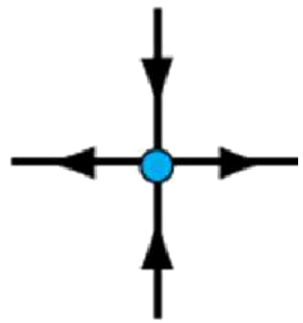
- Jacobian of the vector field



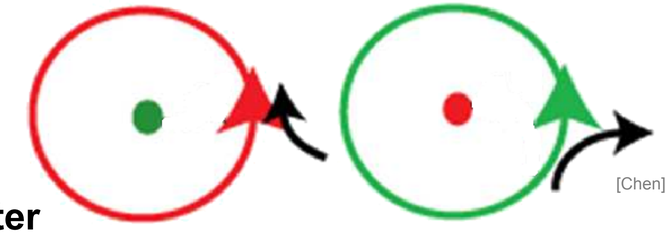
Source



Sink



Saddle



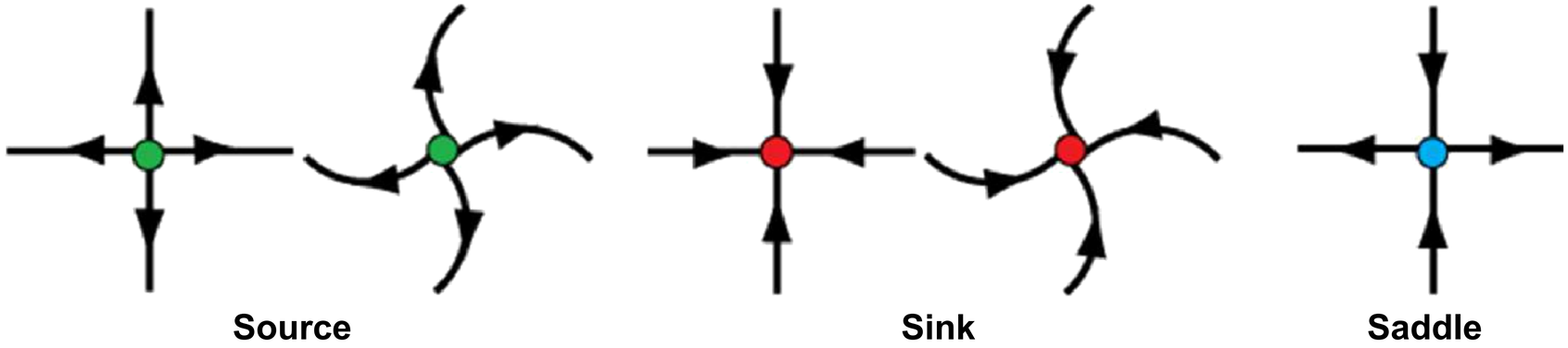
Center

[Chen]

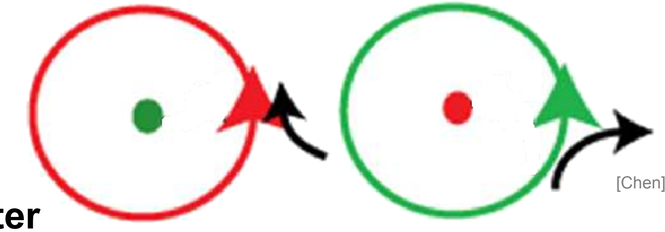
Classifying critical points

- Jacobian of the vector field

$$J = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{bmatrix}$$



Source Sink Saddle



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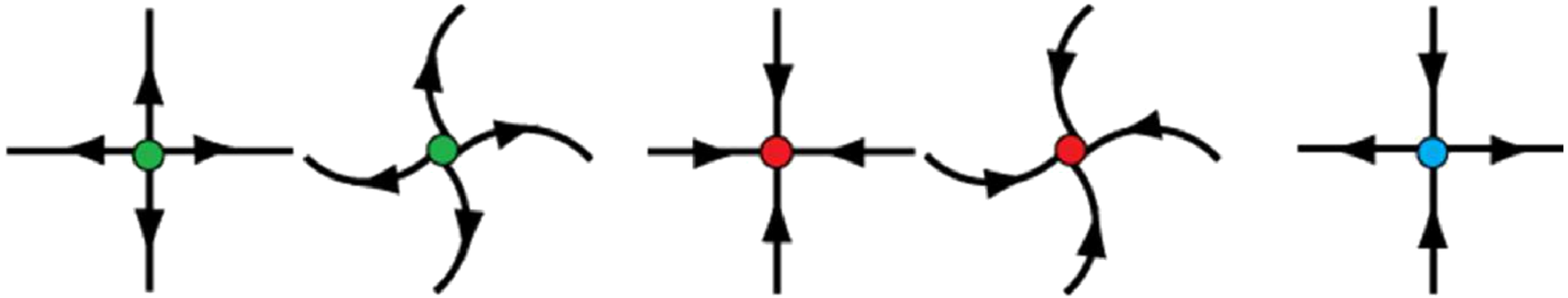
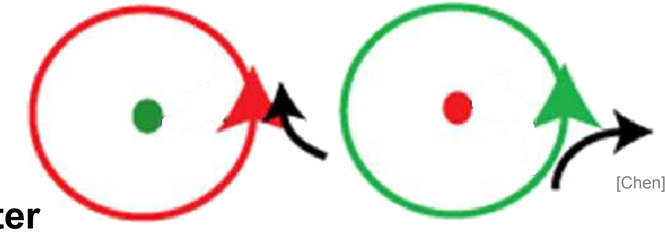
- Eigenvalues of J
  - 2 eigenvalues



Source Sink Saddle



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[Chen]

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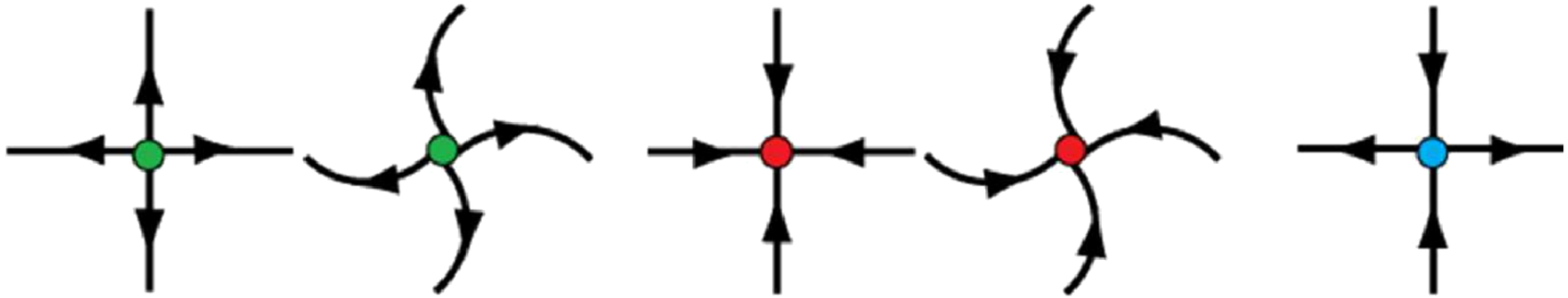
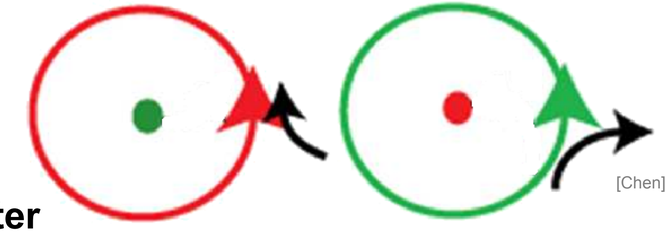
- Eigenvalues of J
  - 2 eigenvalues
  - For each



Source Sink Saddle



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[Chen]

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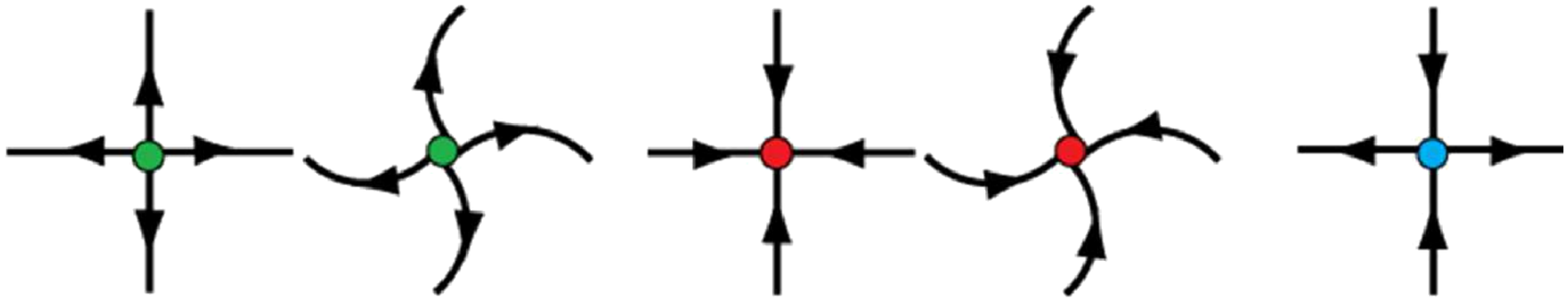
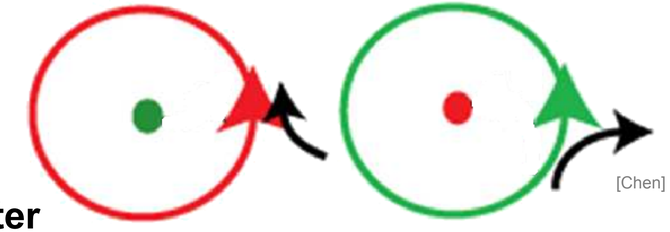
- Eigenvalues of J
  - 2 eigenvalues
  - For each
    - Real and imaginary parts
      - –Symmetric, antisymmetric



Source Sink Saddle



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[Chen]

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  - 2 eigenvalues
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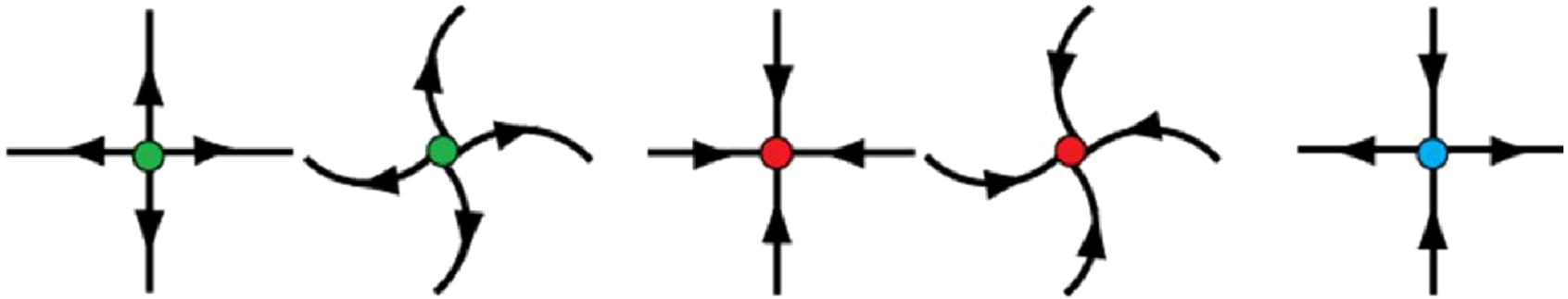
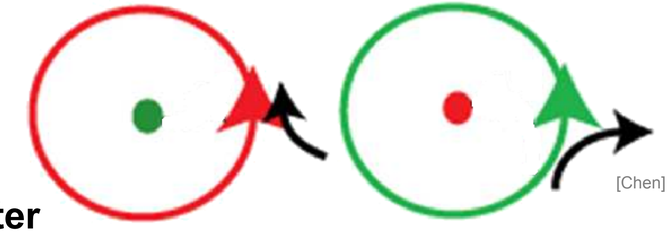
    - R1, I1 - R2, I2



Source Sink Saddle



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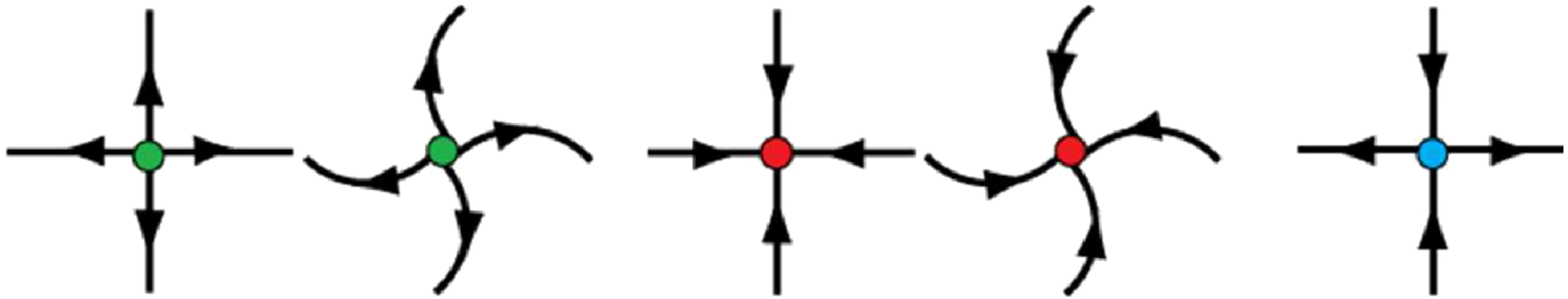
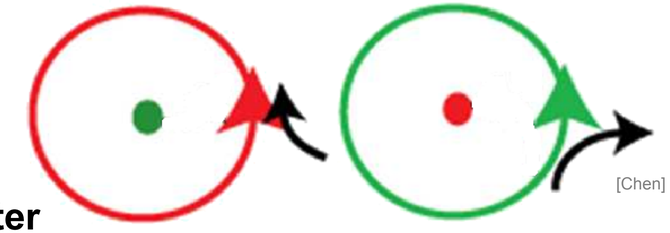
$$J = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{bmatrix}$$

- Eigenvalues of J
  - 2 eigenvalues
  - For each
    - Real and imaginary parts
      - –Symmetric, antisymmetric
    - R1, I1 - R2, I2

$$R_1 > 0, R_2 > 0$$
$$I_1 = 0, I_2 = 0$$



Source Sink Saddle



Center

[Chen]

Classifying critical points

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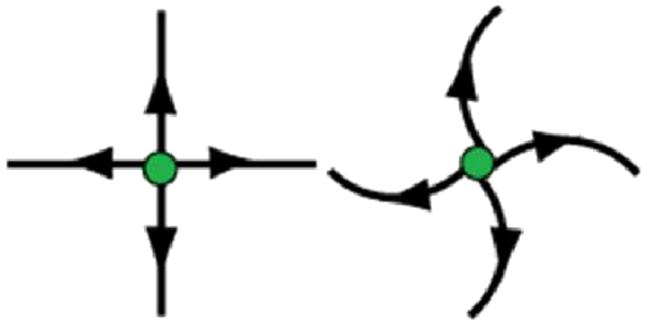
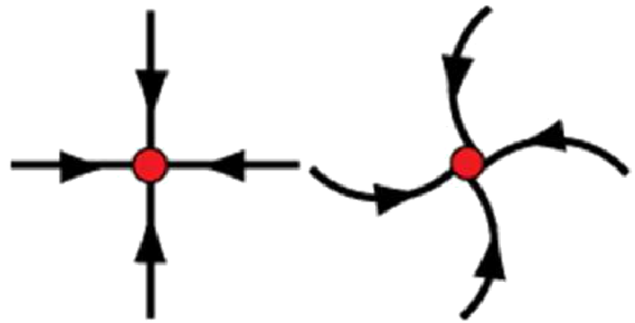
- Eigenvalues of J
  - 2 eigenvalues
  - For each
    - Real and imaginary parts
      - –Symmetric, antisymmetric
    - R1, I1 - R2, I2

$R_1 > 0, R_2 > 0$
$I_1 = 0, I_2 = 0$

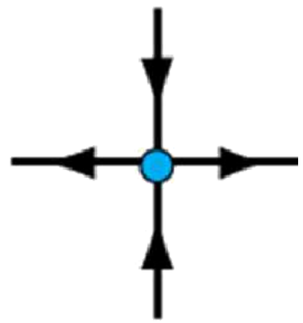


Source

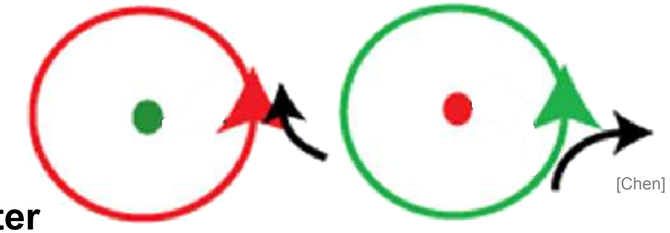
$R_1 < 0, R_2 < 0$
$I_1 = 0, I_2 = 0$



Sink



Saddle



Center

[Chen]

Classifying critical points

- Jacobian of the vector field

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    - Real and imaginary parts
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    - R1, I1 - R2, I2

$R_1 > 0, R_2 > 0$
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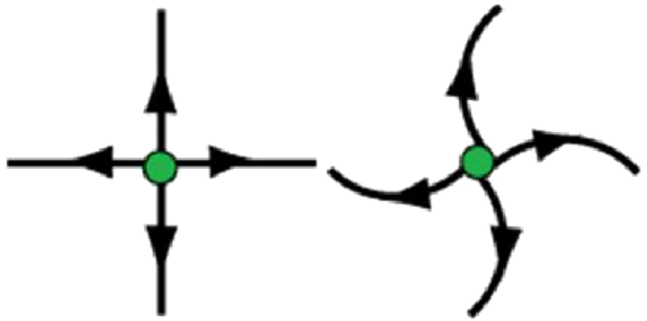
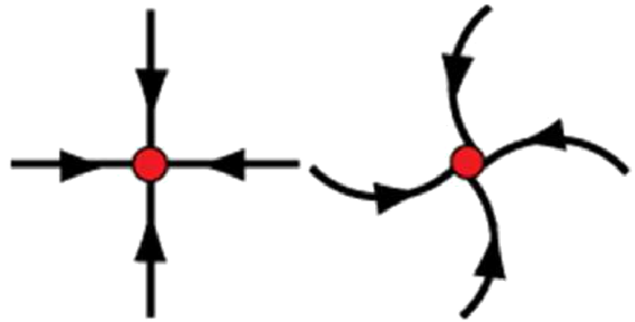


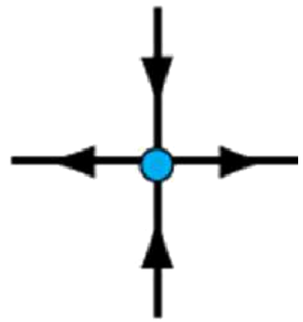
Source

$R_1 < 0, R_2 < 0$
$I_1 = 0, I_2 = 0$

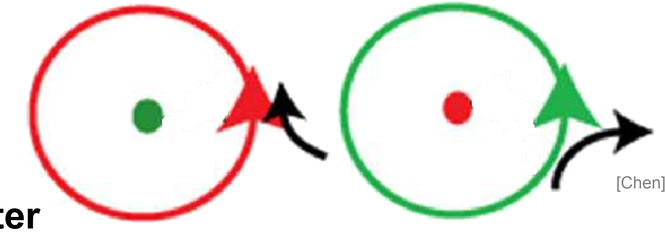


Sink

$R_1.R_2 < 0$
$I_1 = 0, I_2 = 0$



Saddle



Center

[Chen]

Classifying critical points

- Jacobian of the vector field

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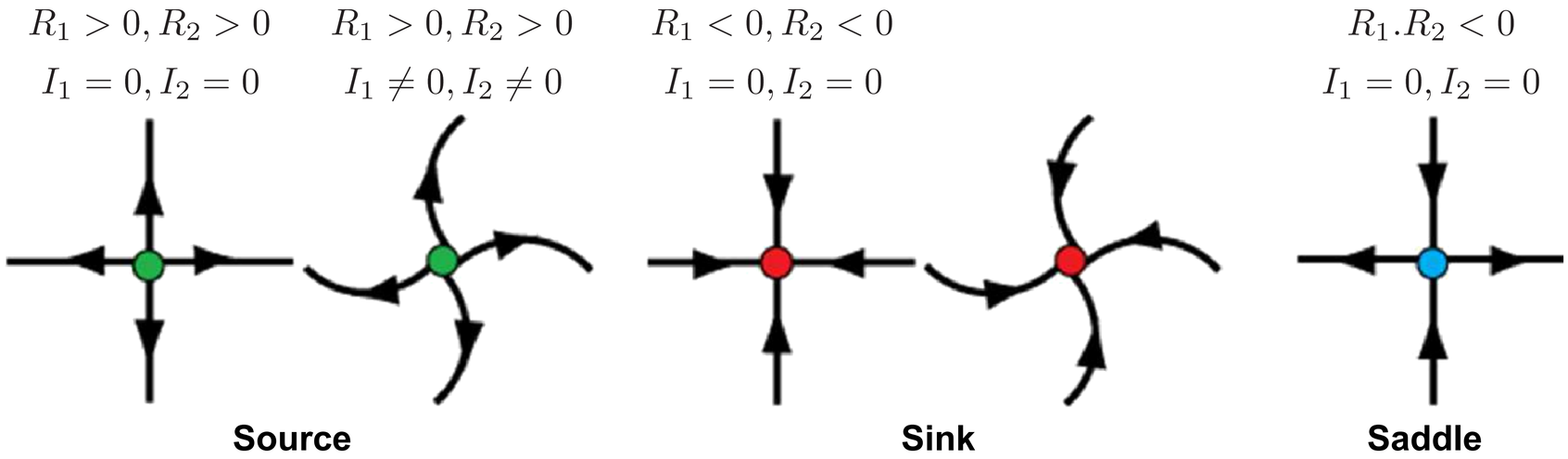

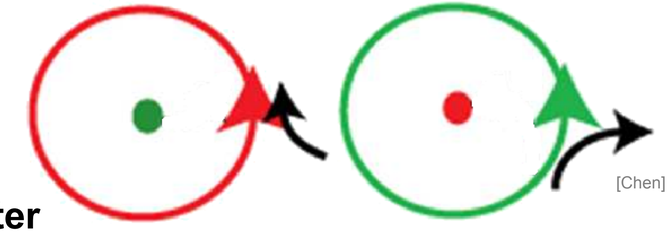

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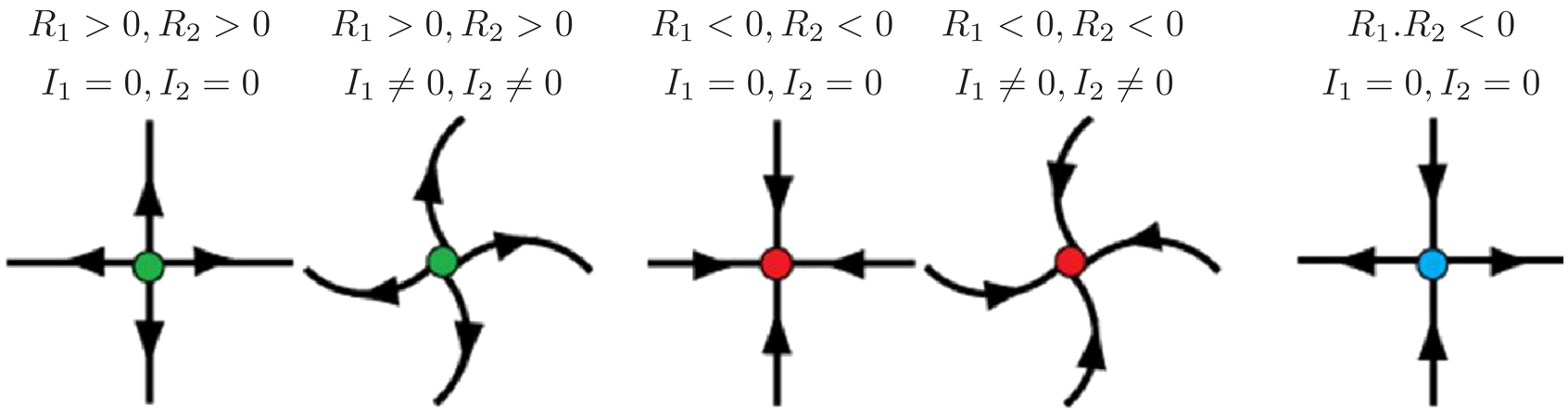
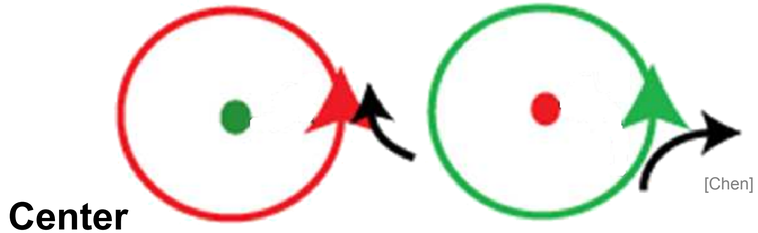


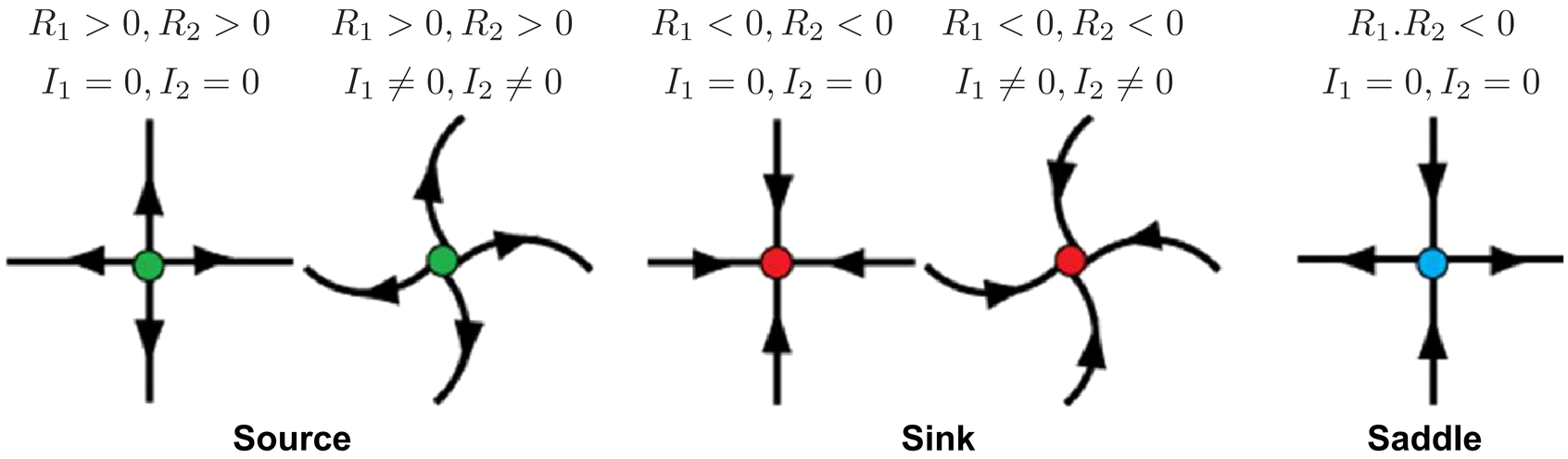


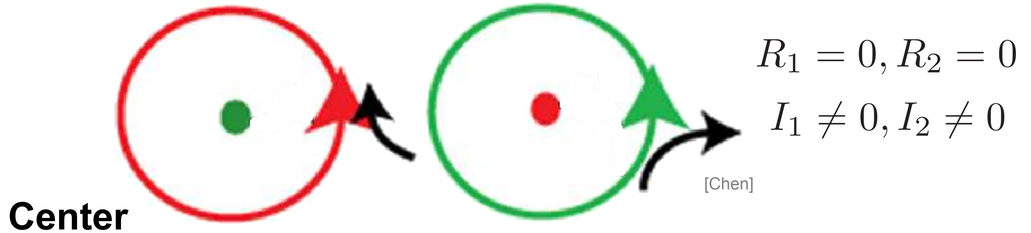
Classifying critical points

- Jacobian of the vector field

$$J = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{bmatrix}$$

- Eigenvalues of J
  - 2 eigenvalues
  - For each
    - Real and imaginary parts
      - –Symmetric, antisymmetric
    - R1, I1 - R2, I2





Vector field decomposition

Vector field decomposition

- Critical point extraction
  - Identify the cells of the domain containing critical points

Vector field decomposition

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  - Identify the cells of the domain containing critical points
  - Sub-sampling for accurate locations

Vector field decomposition

- Critical point extraction
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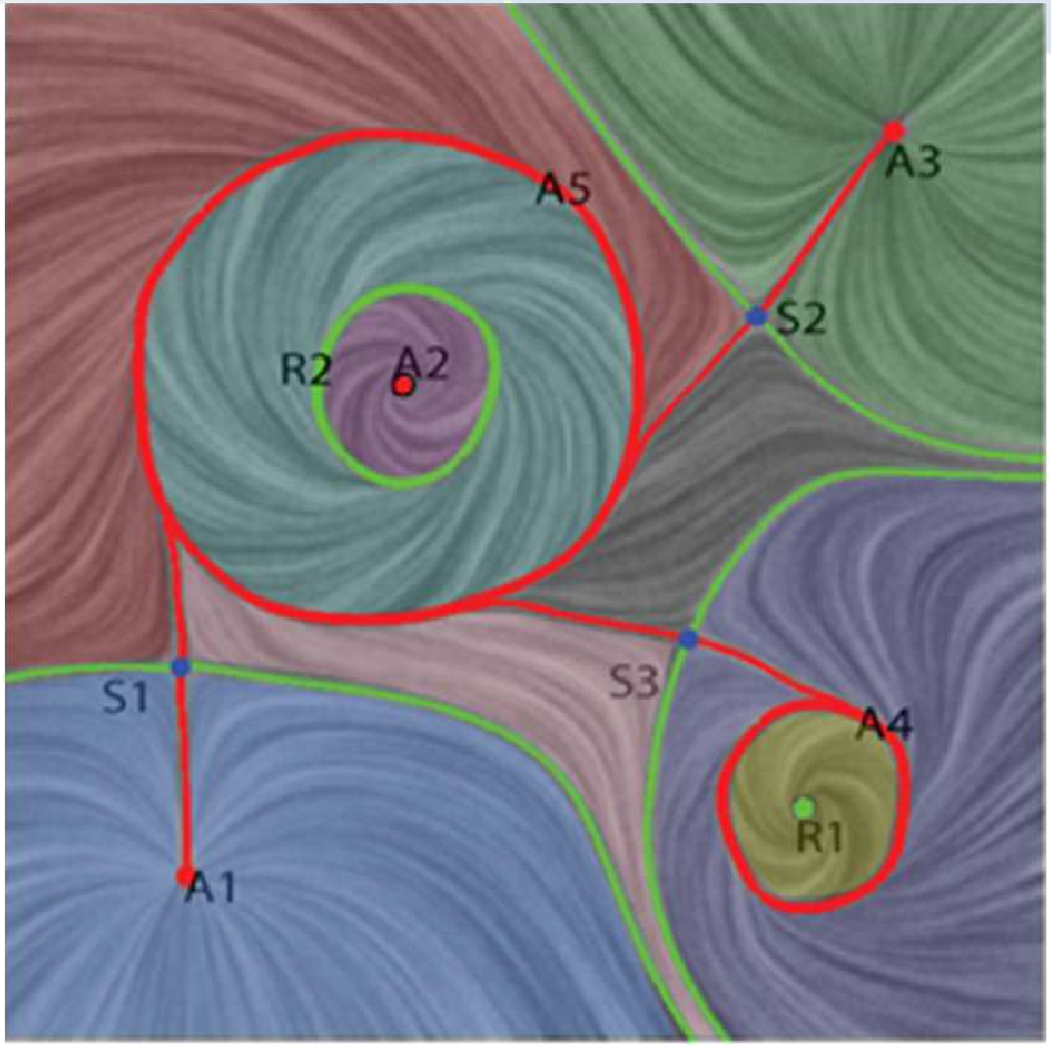
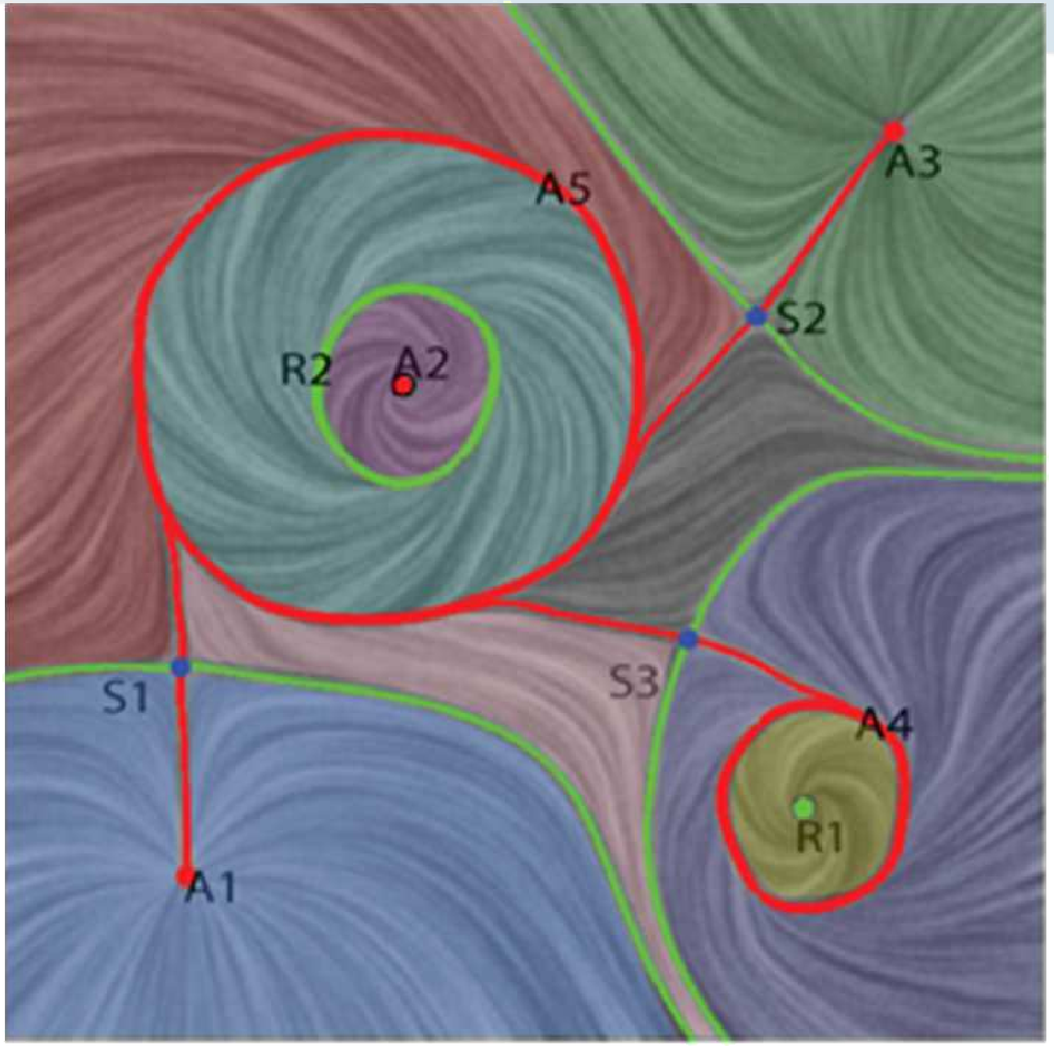
- Decomposition



[Chen]

Vector field decomposition

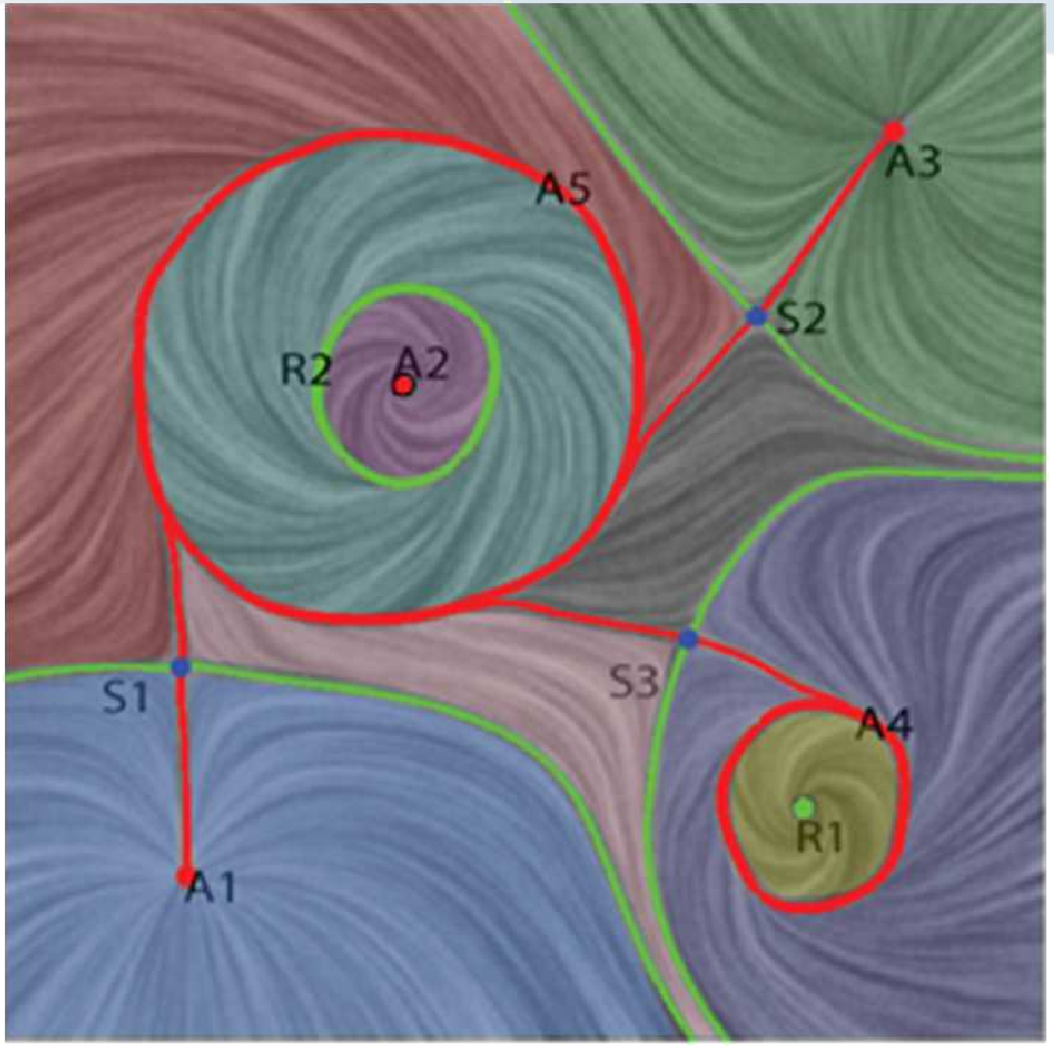
- Critical point extraction
  - Identify the cells of the domain containing critical points
  - Sub-sampling for accurate locations
- Decomposition
  - Backward and forward streamlines from the critical points



[Chen]

Vector field decomposition

- Critical point extraction
  - Identify the cells of the domain containing critical points
  - Sub-sampling for accurate locations
- Decomposition
  - Backward and forward streamlines from the critical points
  - Periodic orbits!



[Chen]

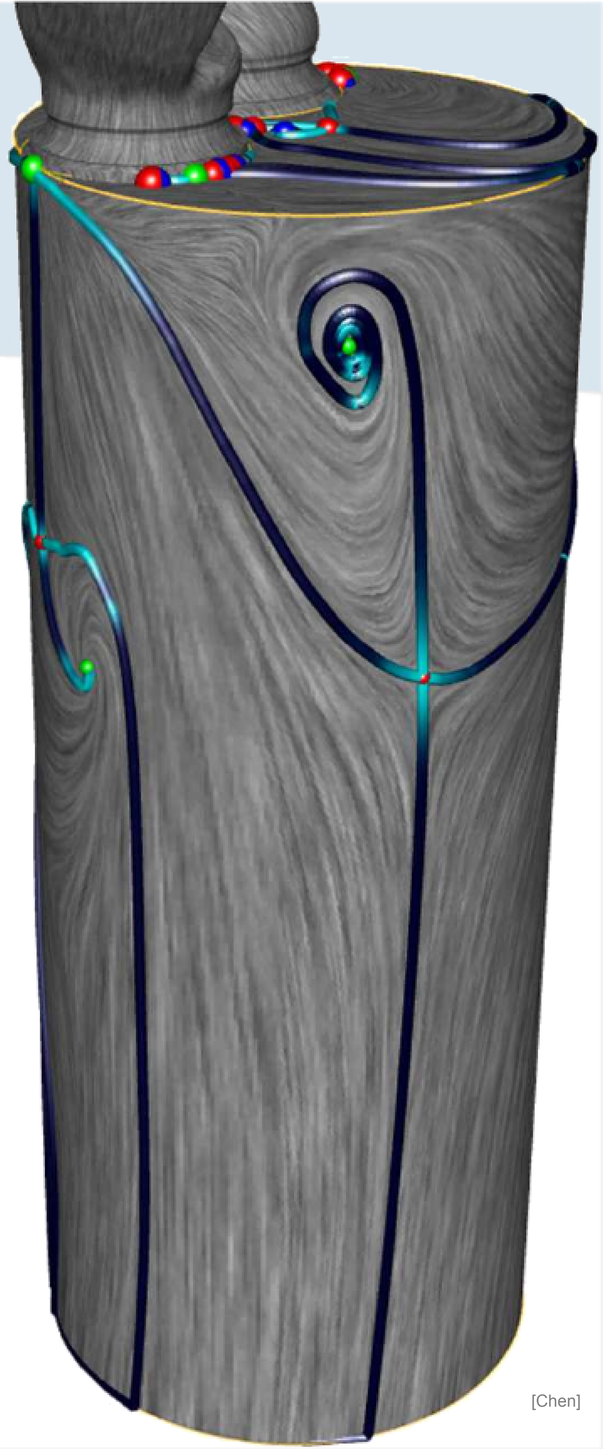
Non planar domains

Non planar domains

- PL 2-manifolds in $\mathbb{R}^3$

Non planar domains

- PL 2-manifolds in $\mathbb{R}^3$
  - Trivial extension
  - Numerical evaluations slightly more involved



[Chen]

Non planar domains

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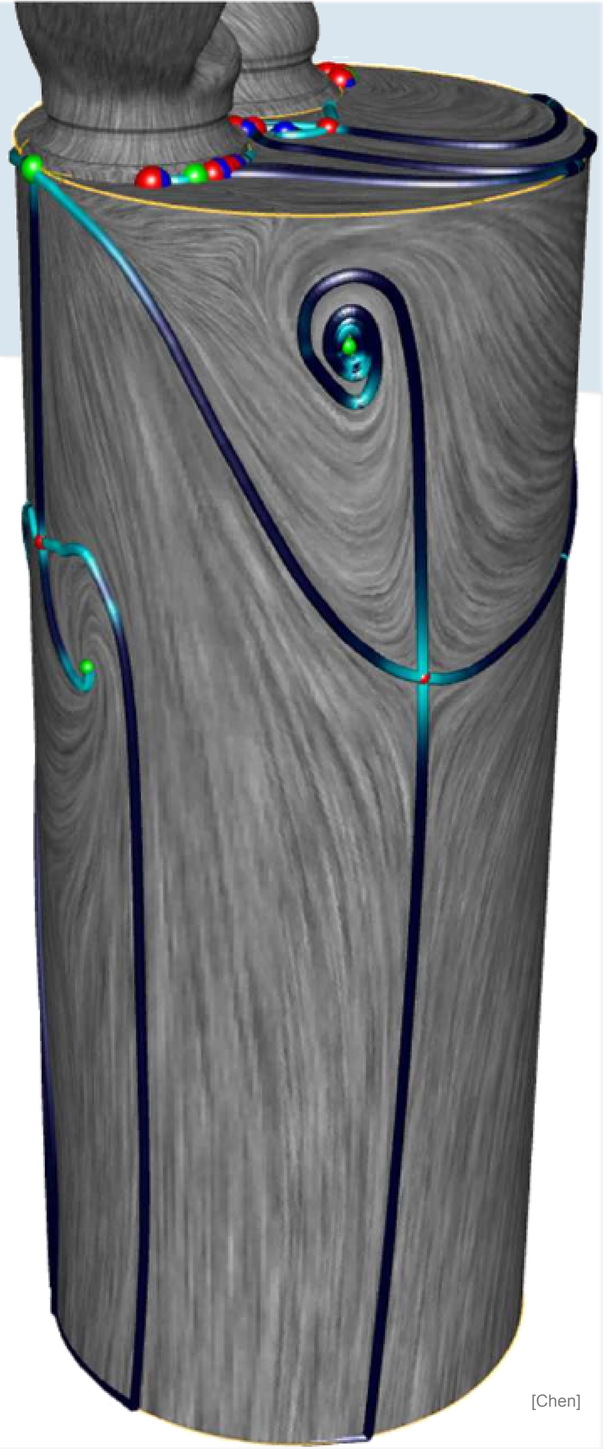
  - Trivial extension
  - Numerical evaluations slightly more involved
- Volumetric domains
  - Similar process



[Chen]

Non planar domains

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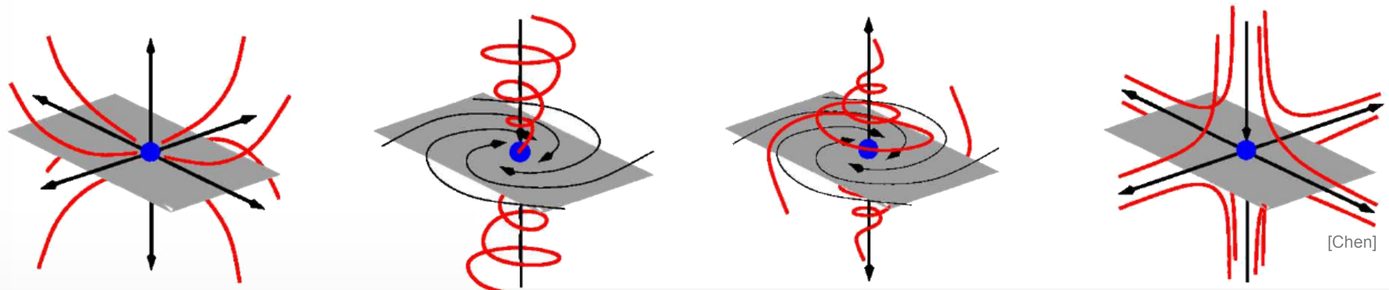
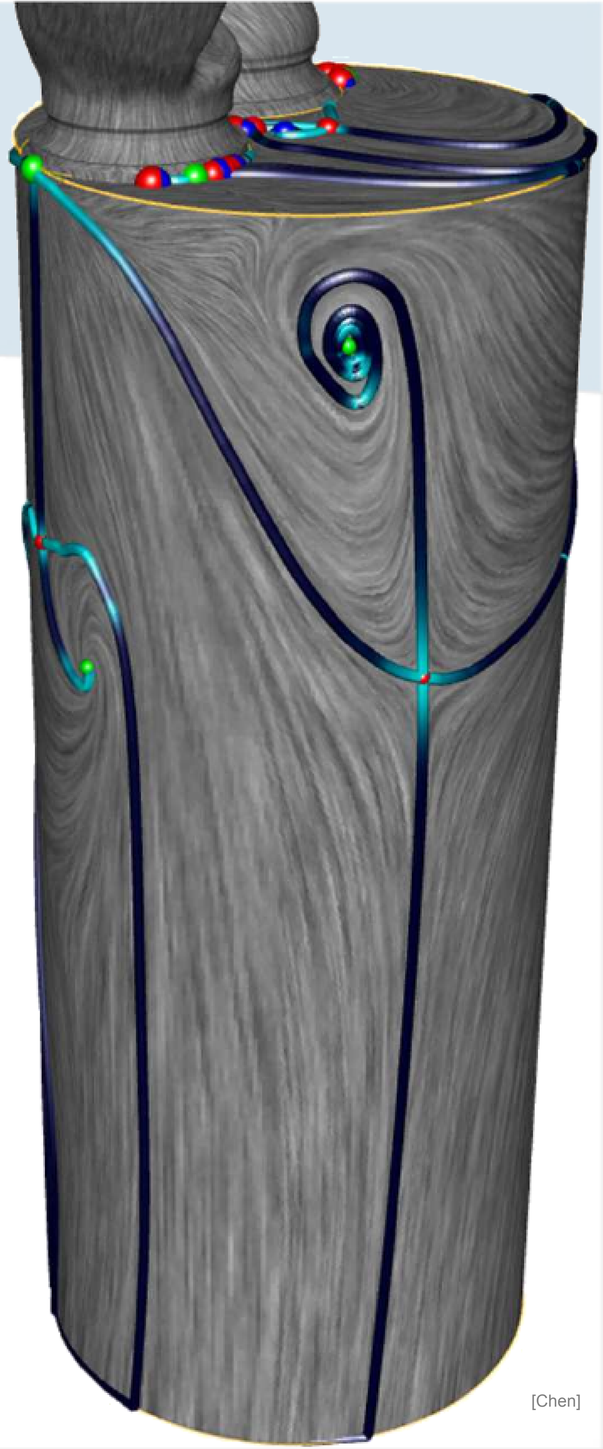
- Volumetric domains
  - Similar process
    - Critical points: spiral effects



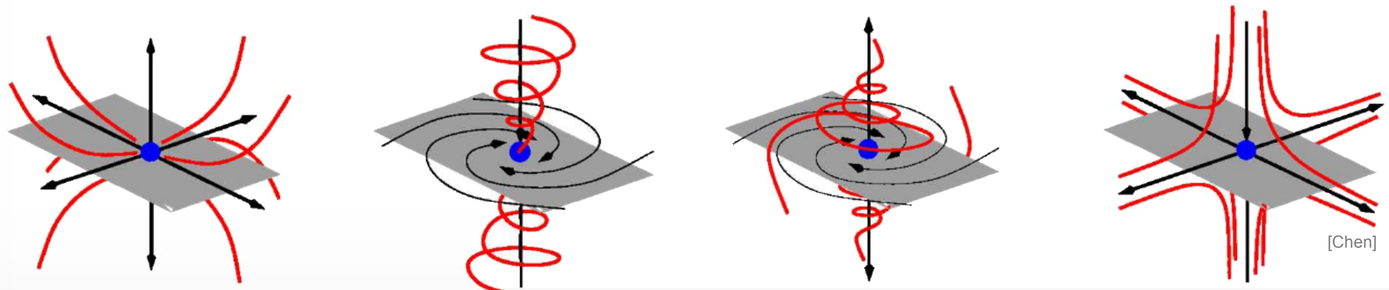
[Chen]



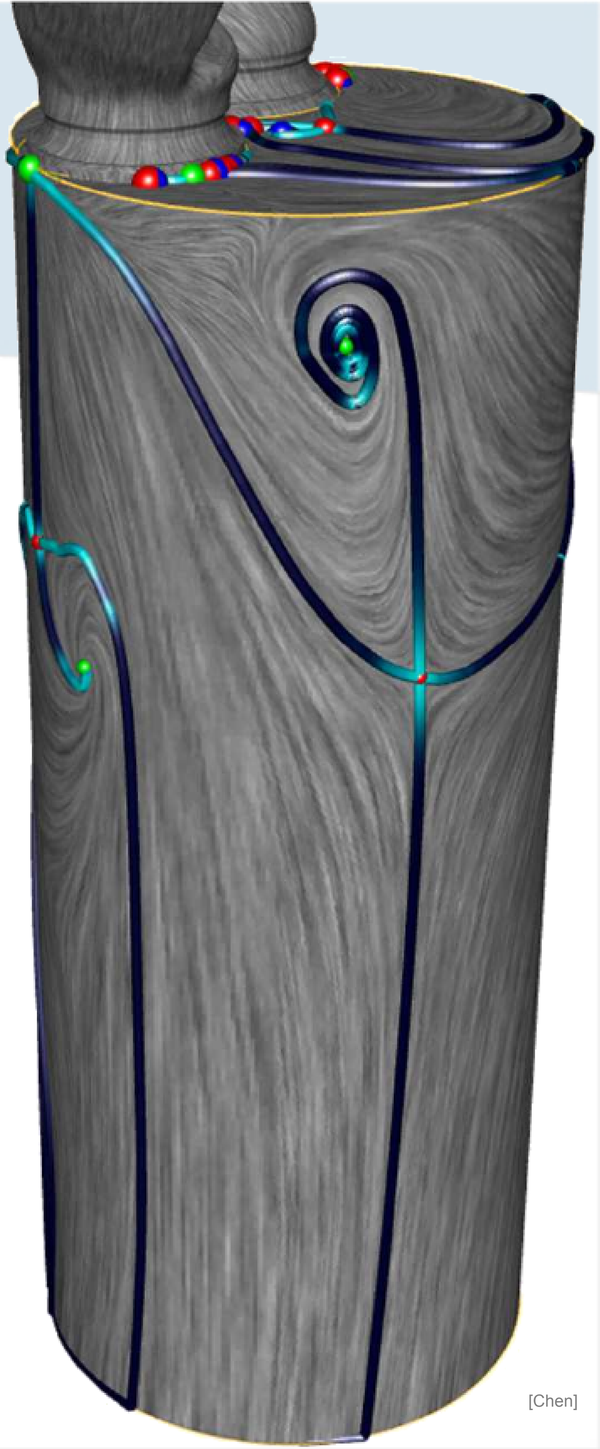
[Chen]

Non planar domains

- PL 2-manifolds in $\mathbb{R}^3$
  - Trivial extension
  - Numerical evaluations slightly more involved
- Volumetric domains
  - Similar process
    - Critical points: spiral effects
    - Streamlines and streamsurfaces as separatrices

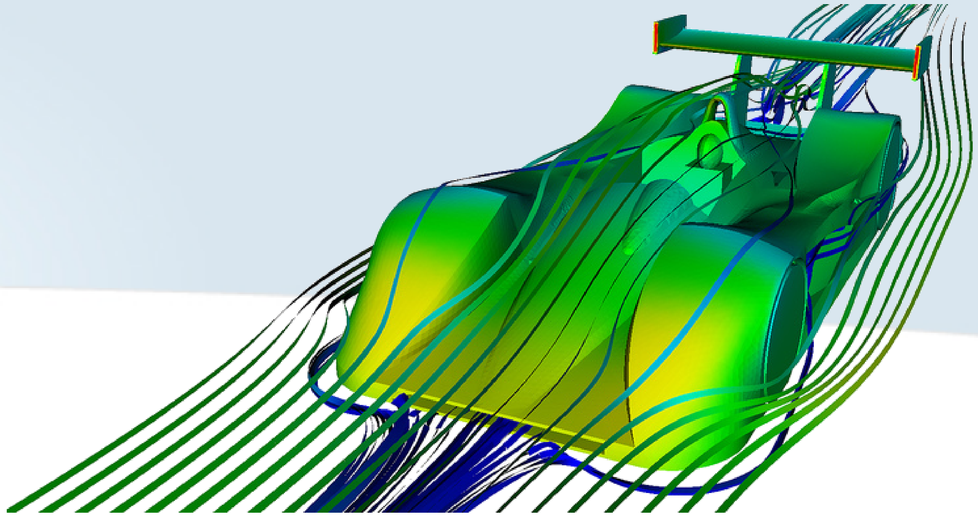


[Chen]

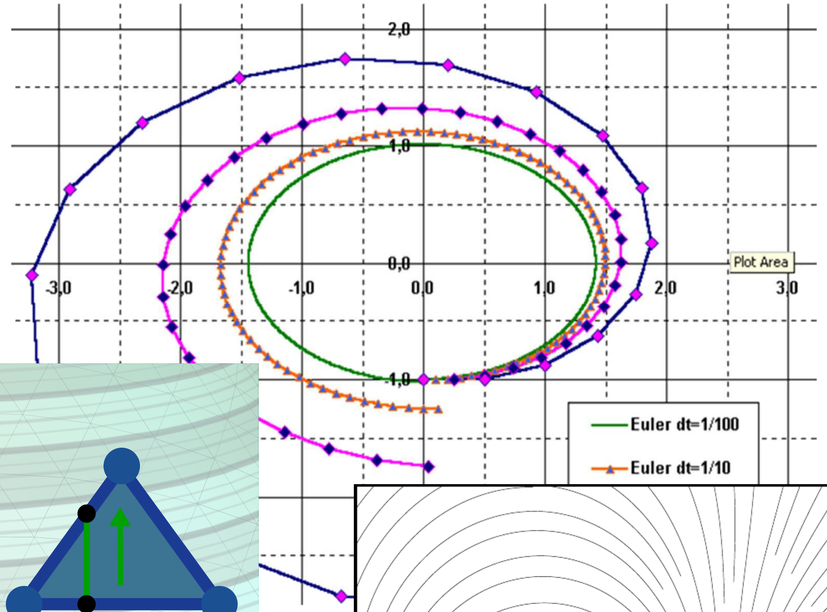


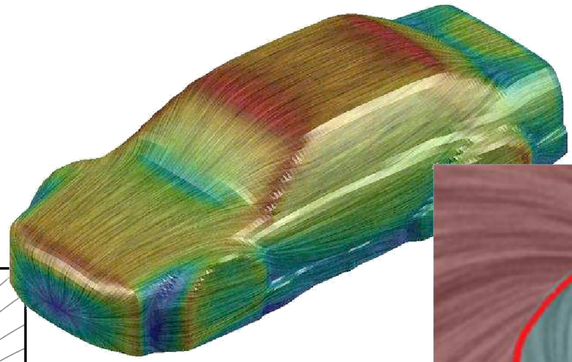
[Chen]

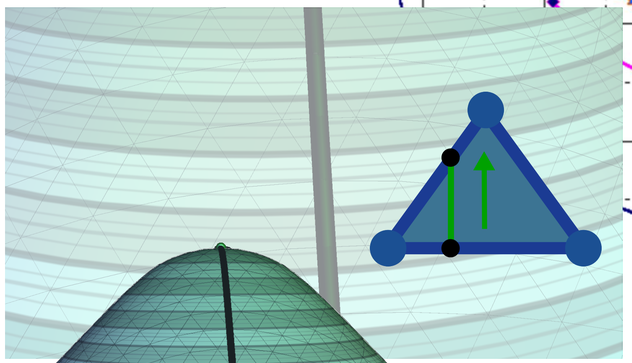
Vector fields

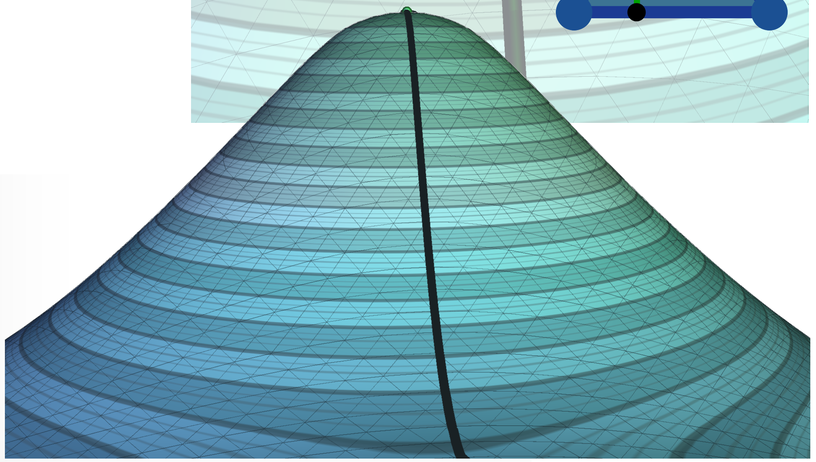


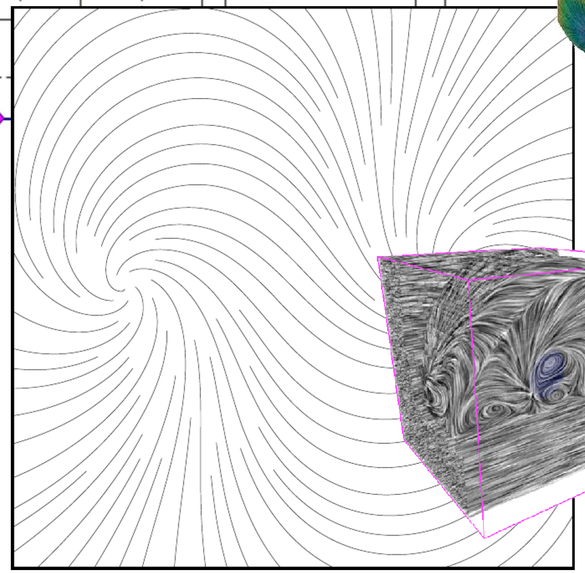
[Kitware]

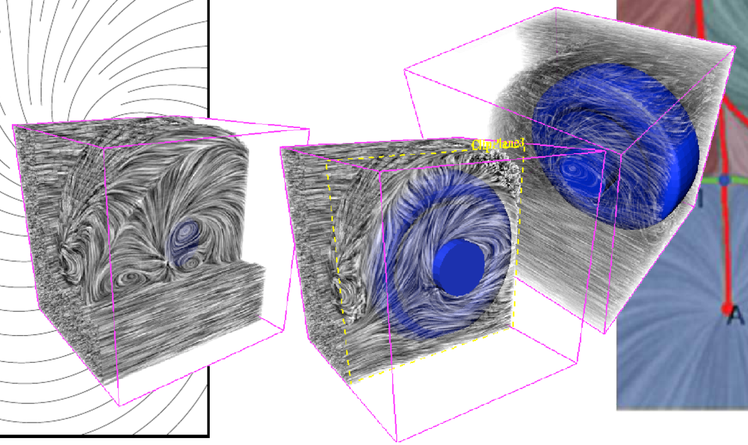


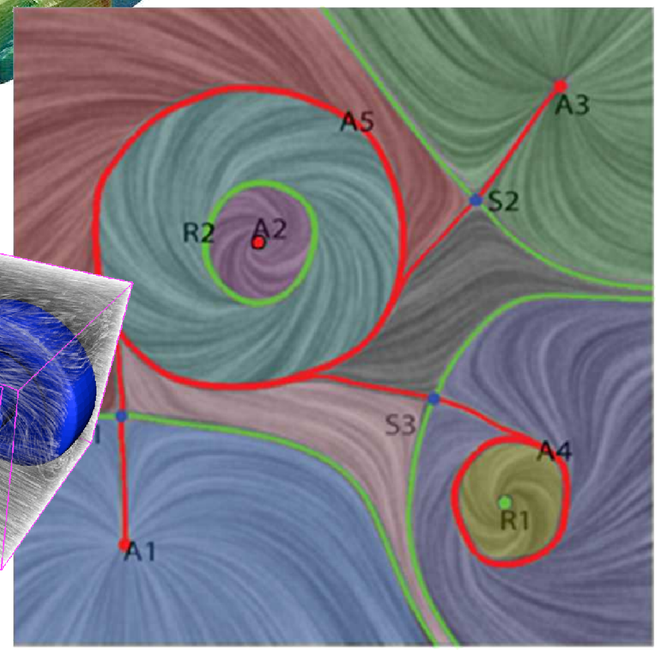


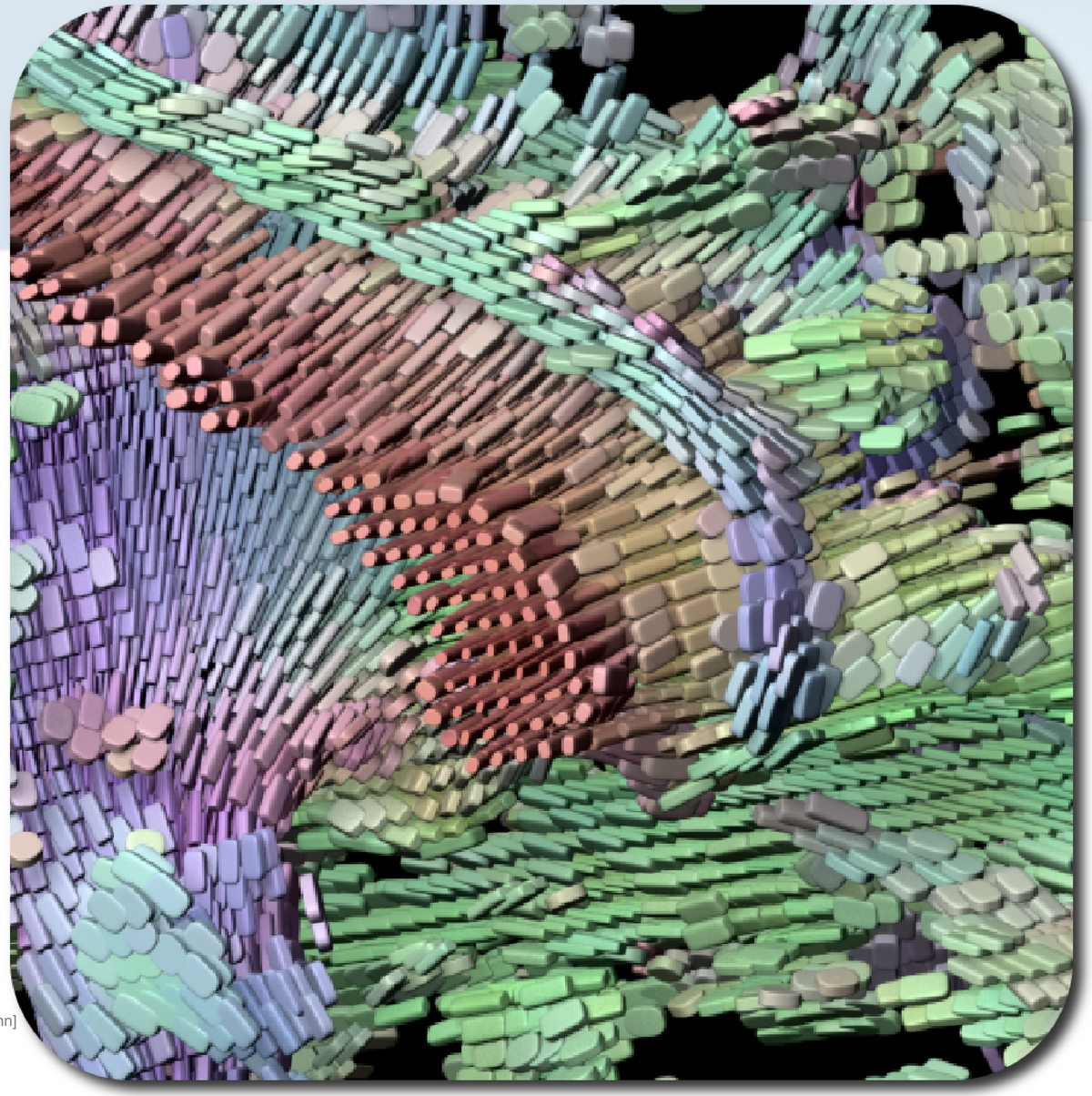












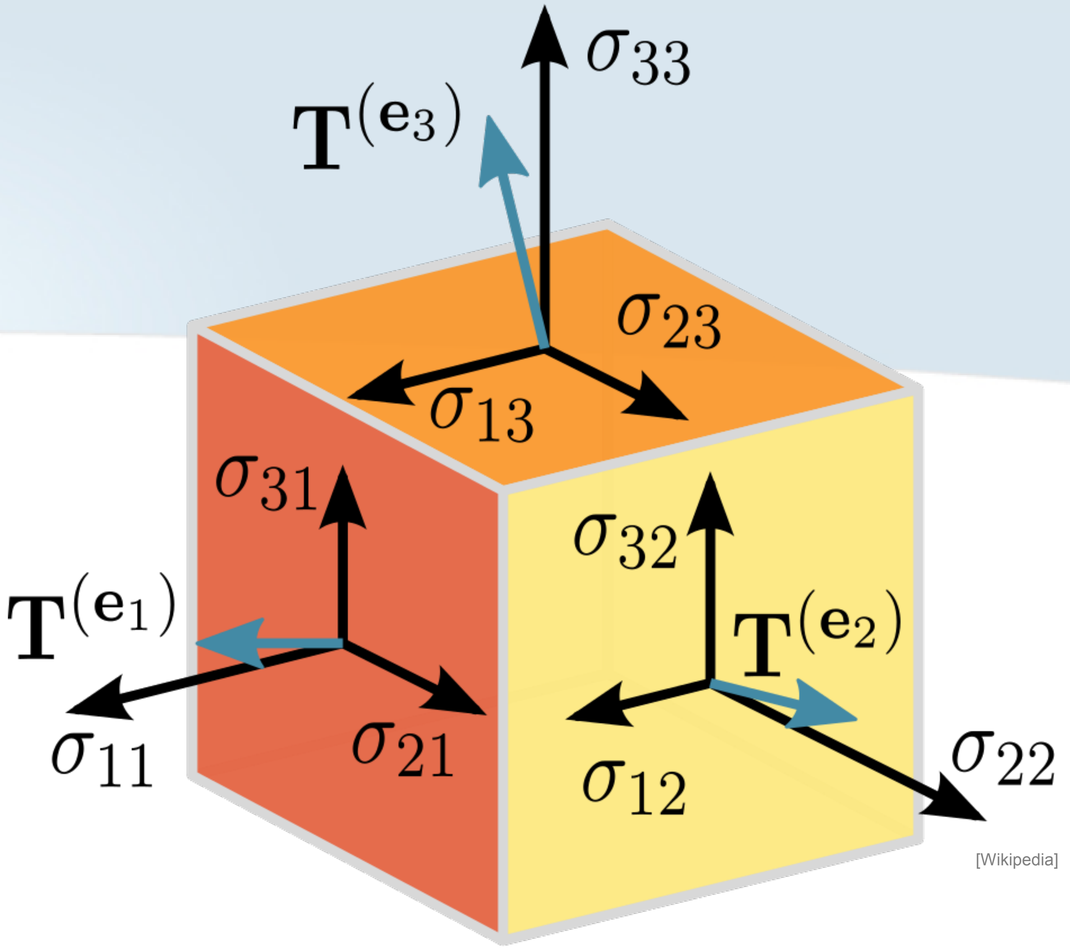
[Kindlmann]

Tensor Field Visualization

Notion of tensor

Notion of tensor

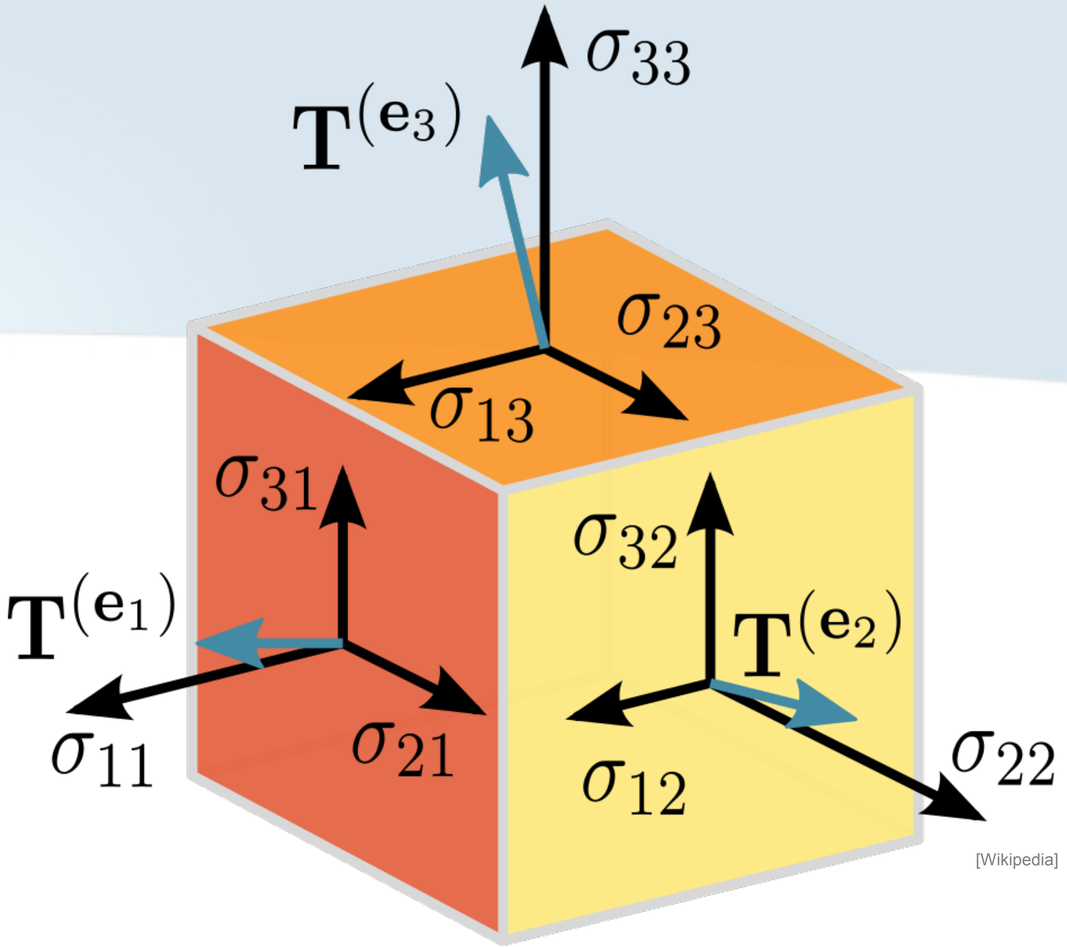
- Generalization of scalars and vectors



[Wikipedia]

Notion of tensor

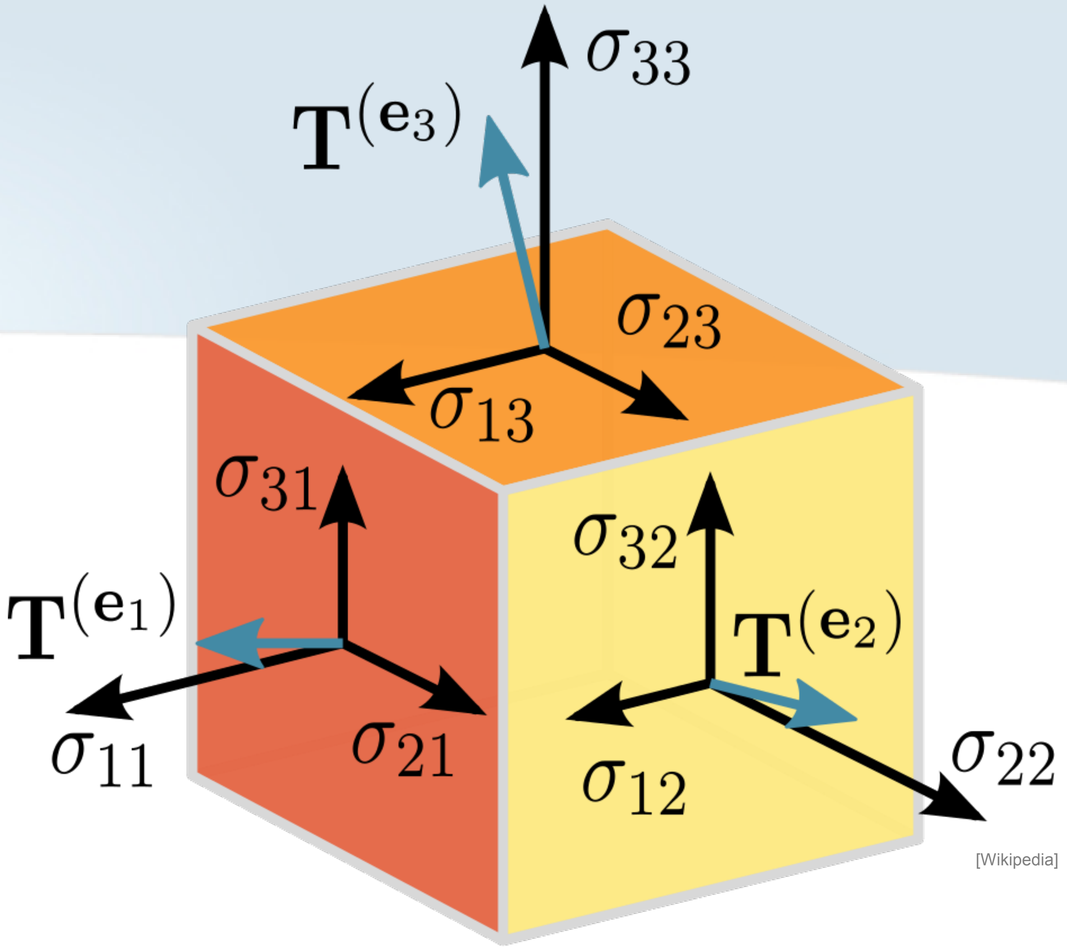
- Generalization of scalars and vectors
- Describes linear relations between



[Wikipedia]

Notion of tensor

- Generalization of scalars and vectors
- Describes linear relations between
  - Scalars
  - Vectors
  - Or other tensors



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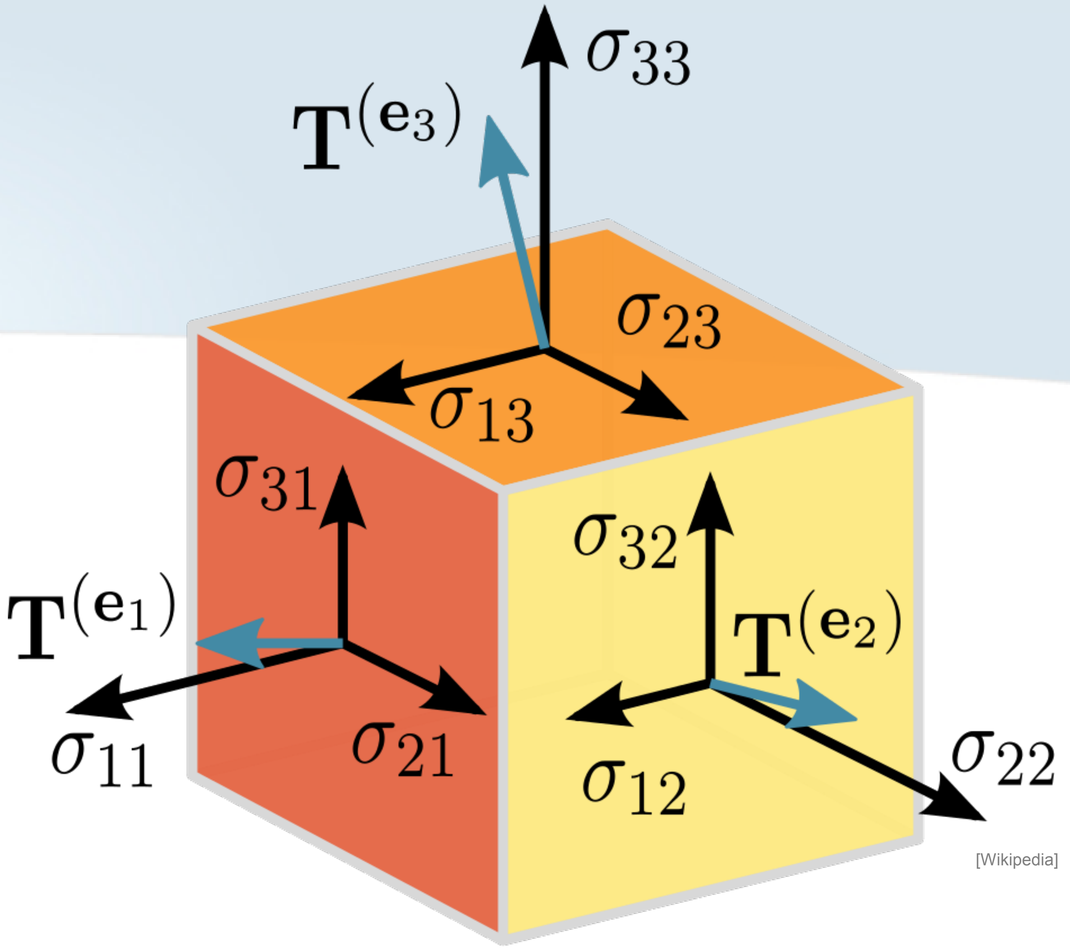

[Wikipedia]

- Multi-dimensional array of numerical values

Notion of tensor

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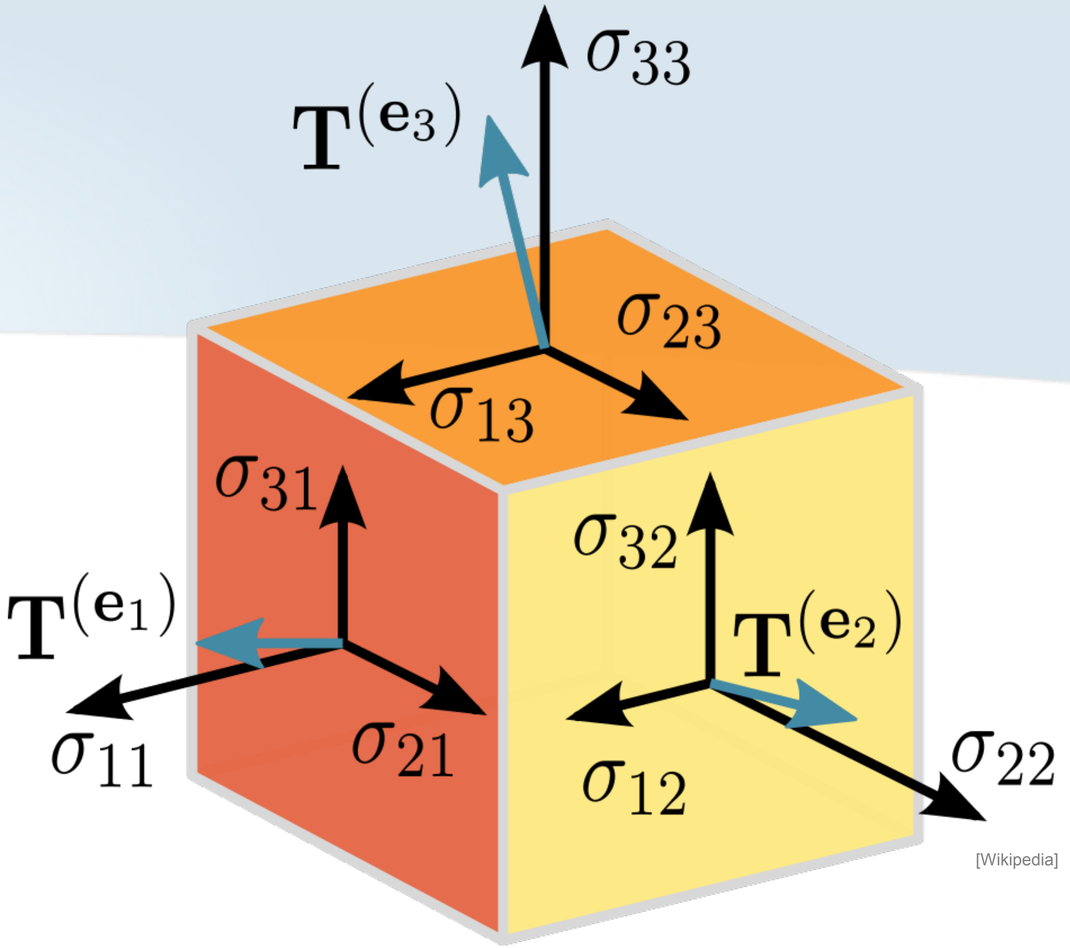

[Wikipedia]

- Multi-dimensional array of numerical values
  - Order of a tensor

Notion of tensor

- Generalization of scalars and vectors
- Describes linear relations between
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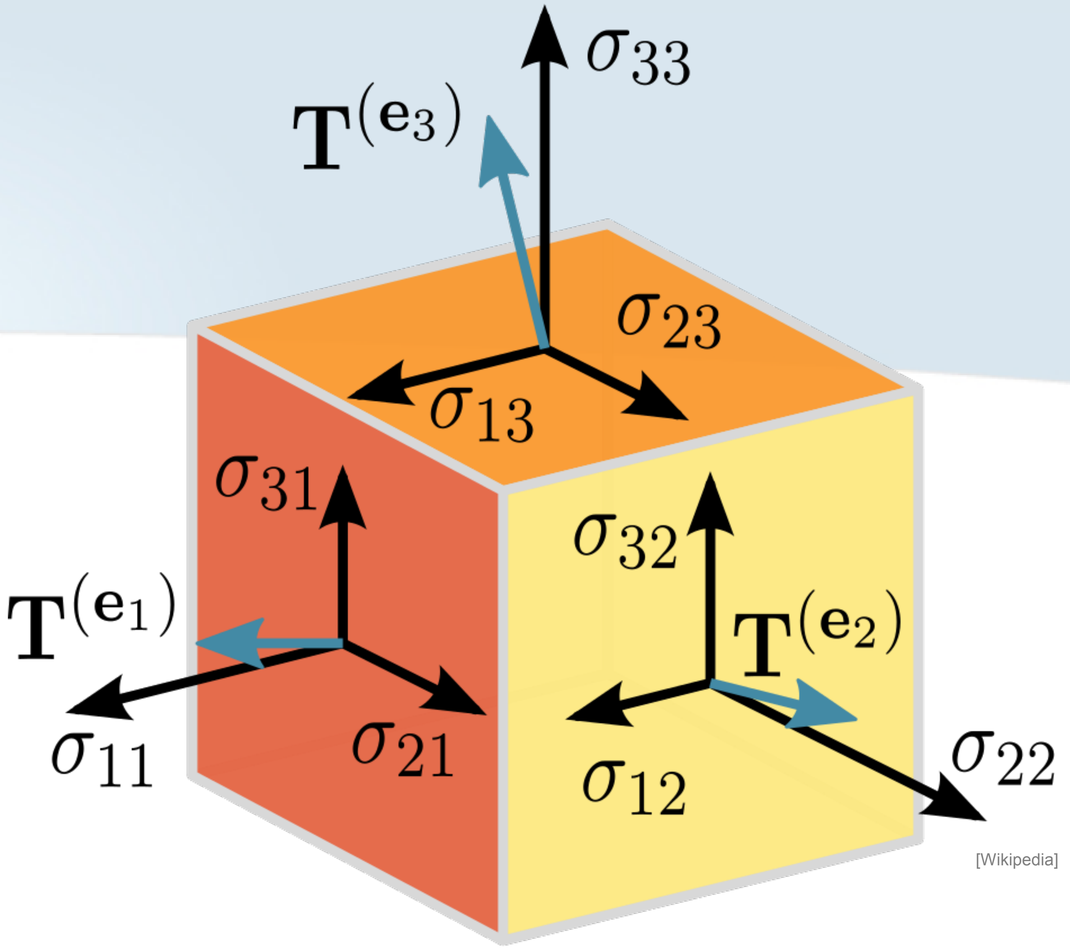
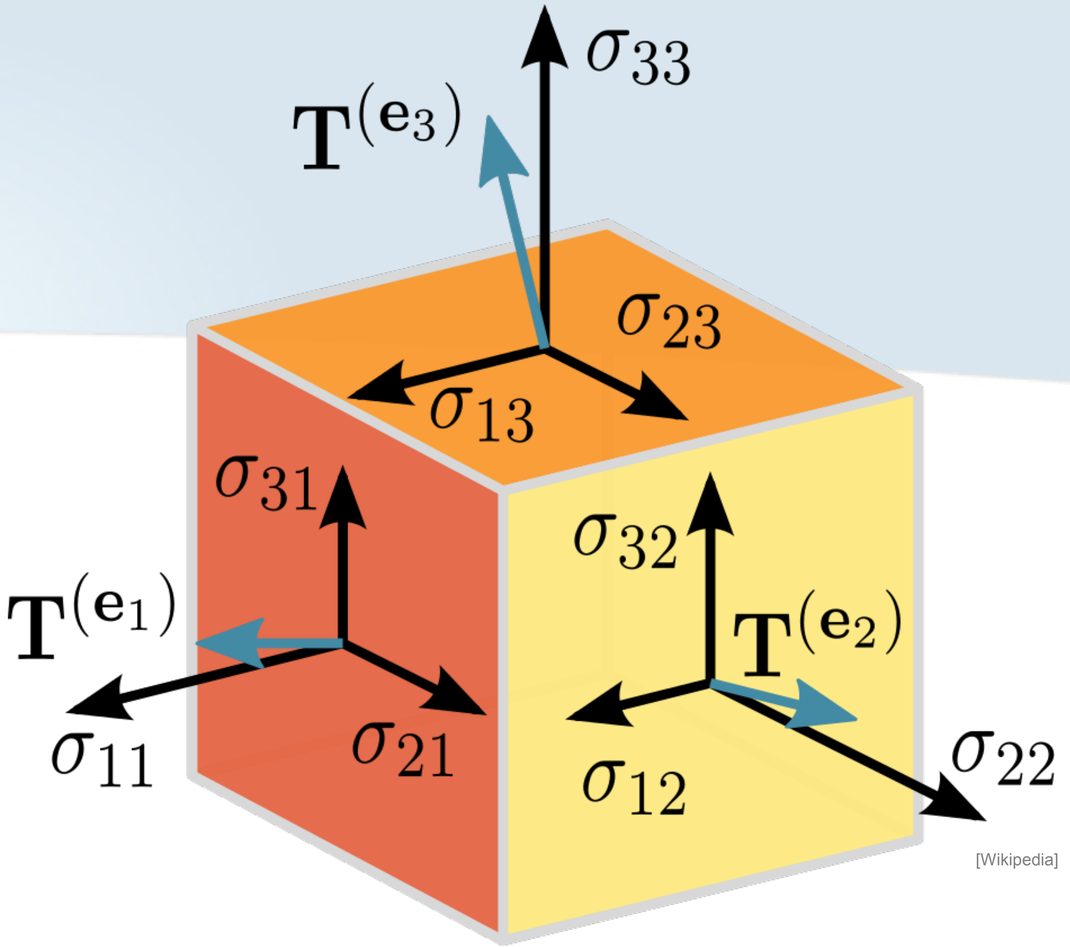


[Wikipedia]

- Multi-dimensional array of numerical values
  - Order of a tensor
    - Number of dimensions

Notion of tensor

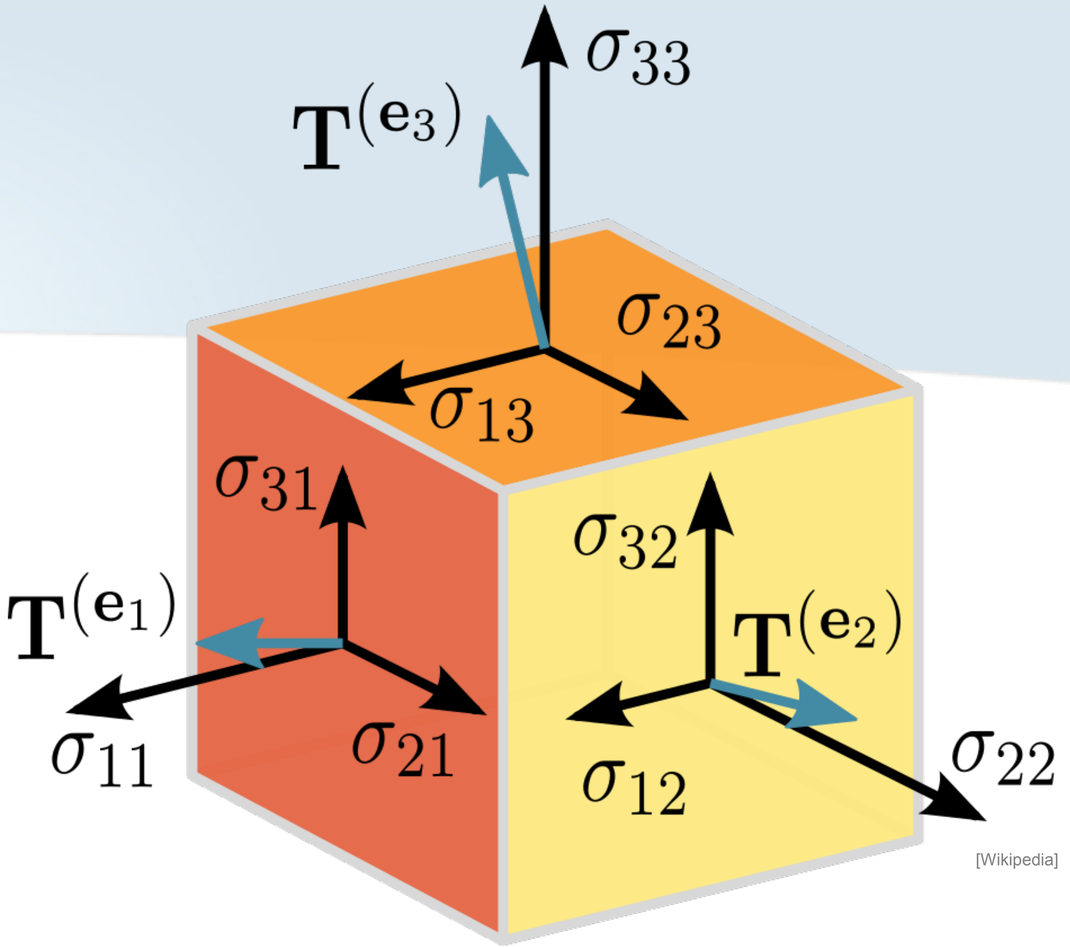
- Order of a tensor



[Wikipedia]

Notion of tensor

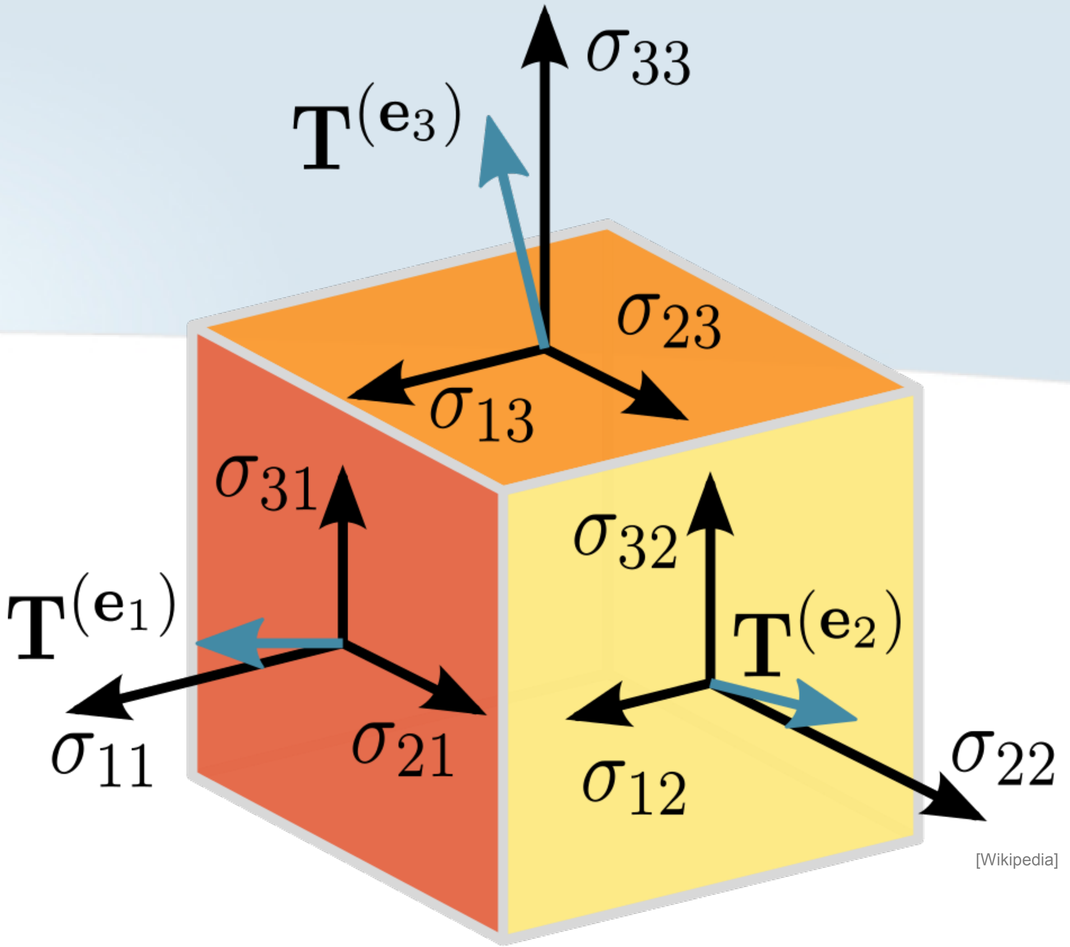
- Order of a tensor
  - Scalar value: 0^{th} order tensor



[Wikipedia]

Notion of tensor

- Order of a tensor
  - Scalar value: 0th order tensor
  - Vector field: 1st order tensor



[Wikipedia]

Notion of tensor

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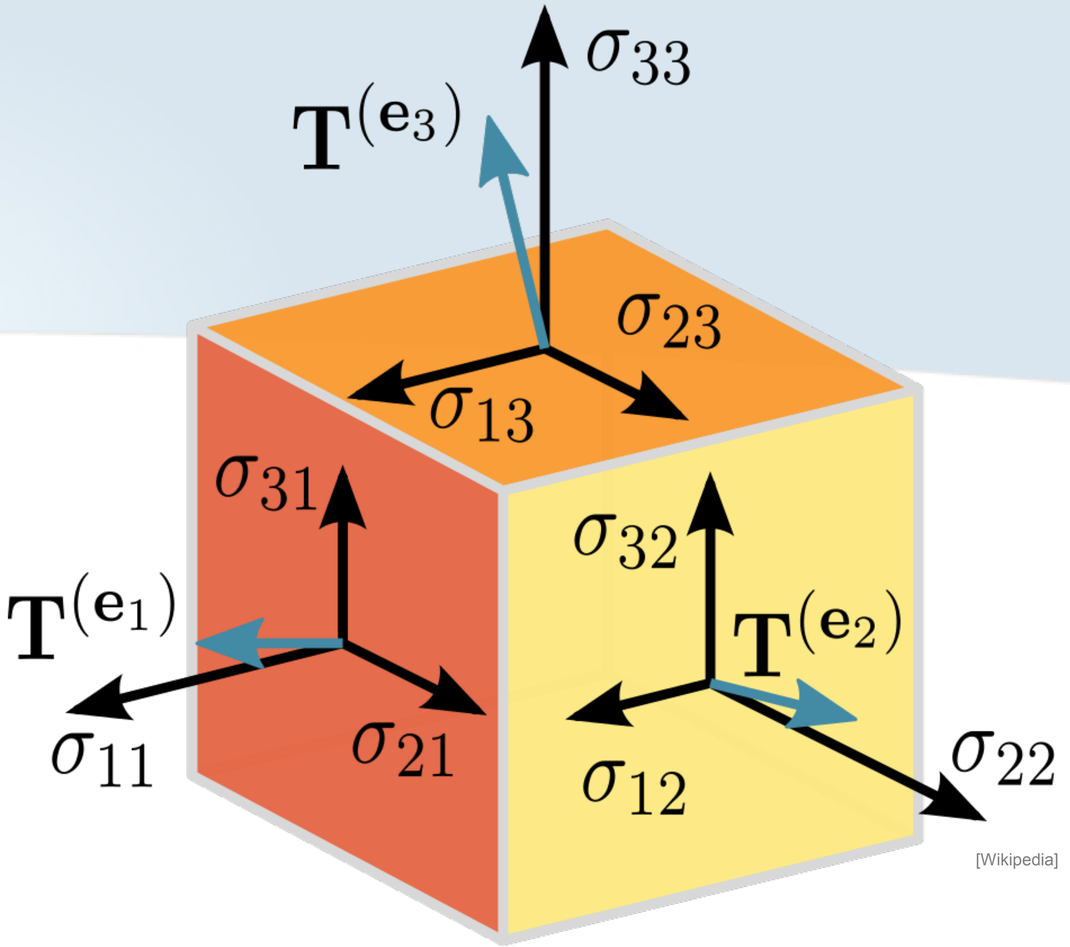
  - Vector field: 1st order tensor
  - (dxd)-matrix: 2nd order tensor



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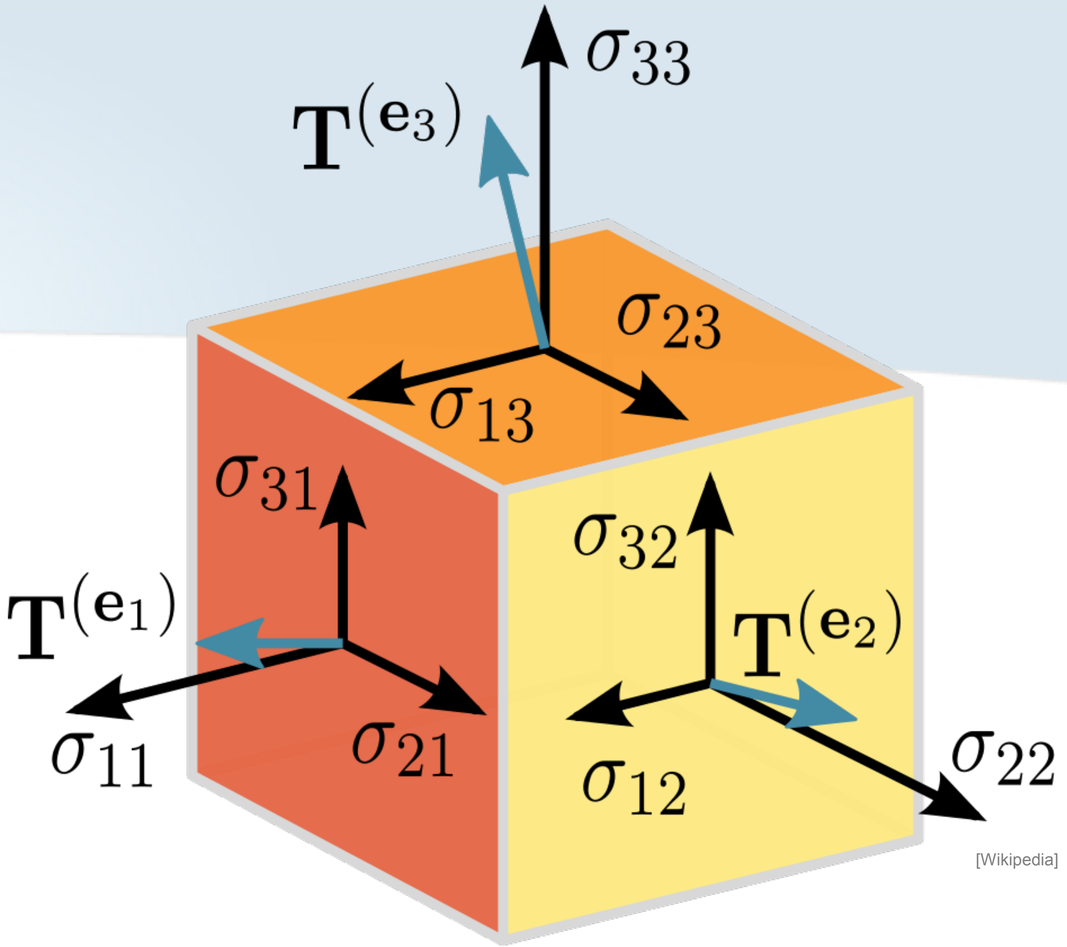


[Wikipedia]

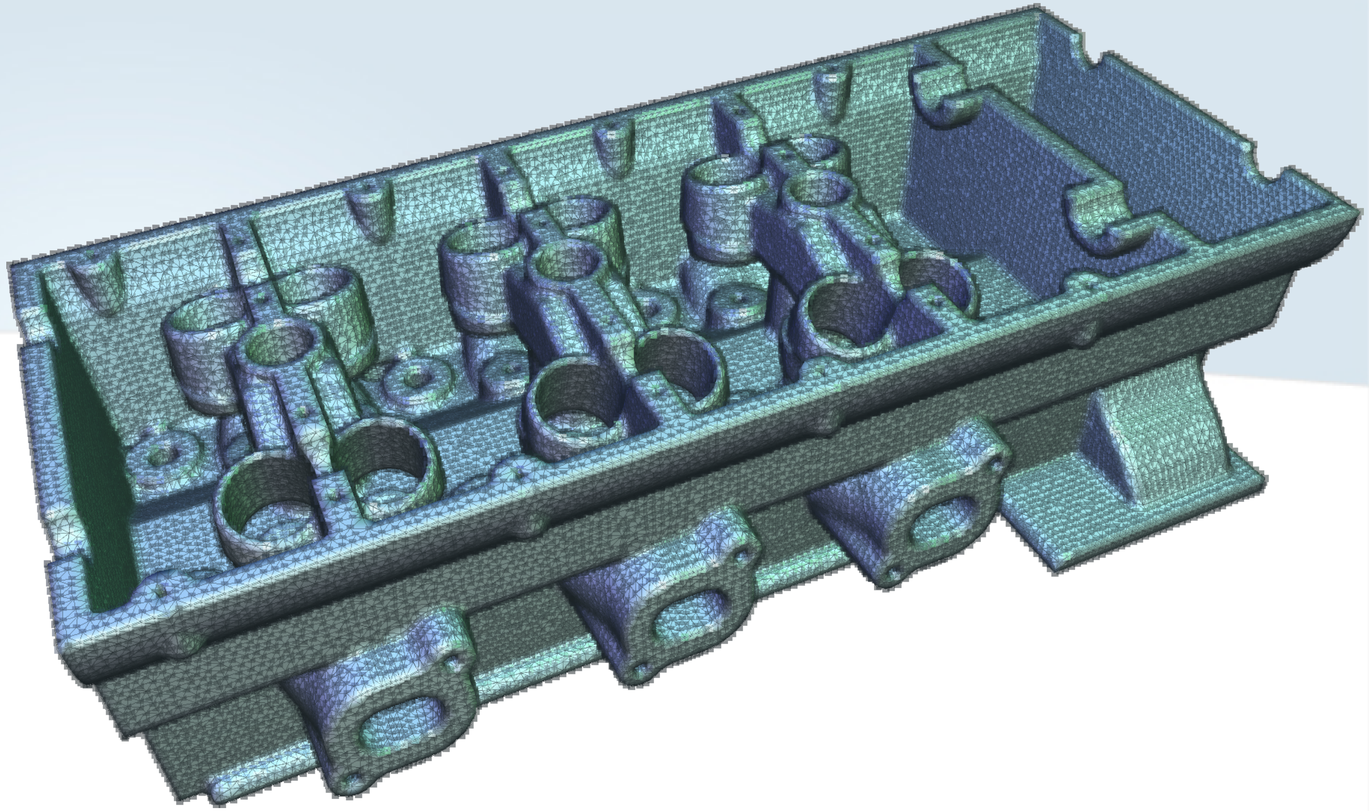
- Here, **mostly symmetric 2nd order tensors**

In practice

- Given a domain $\mathcal{D}$

In practice

- Given a domain $\mathcal{D}$



In practice

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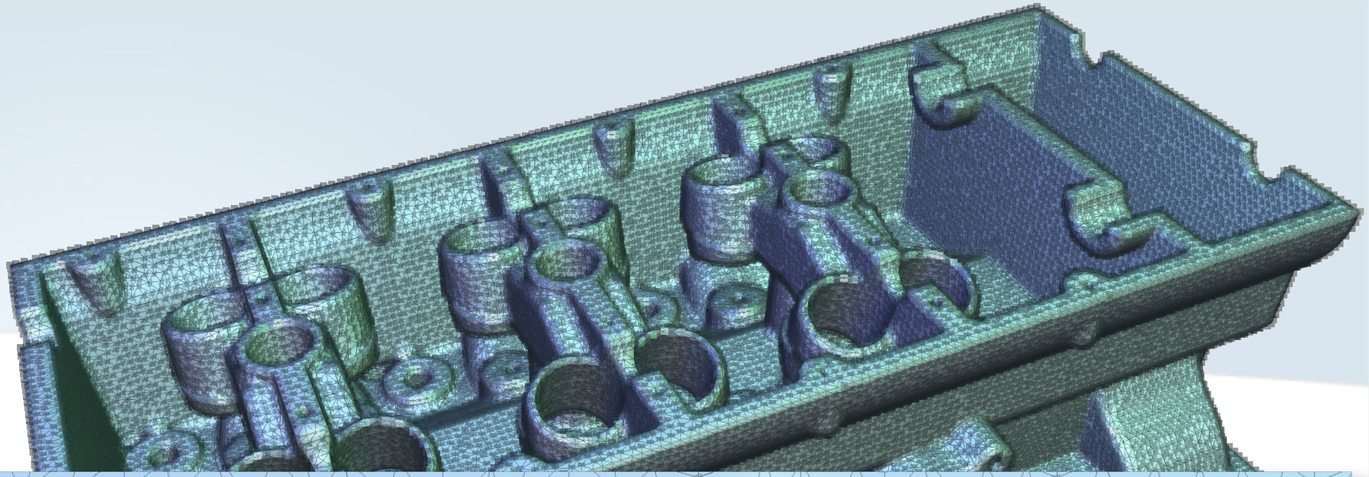
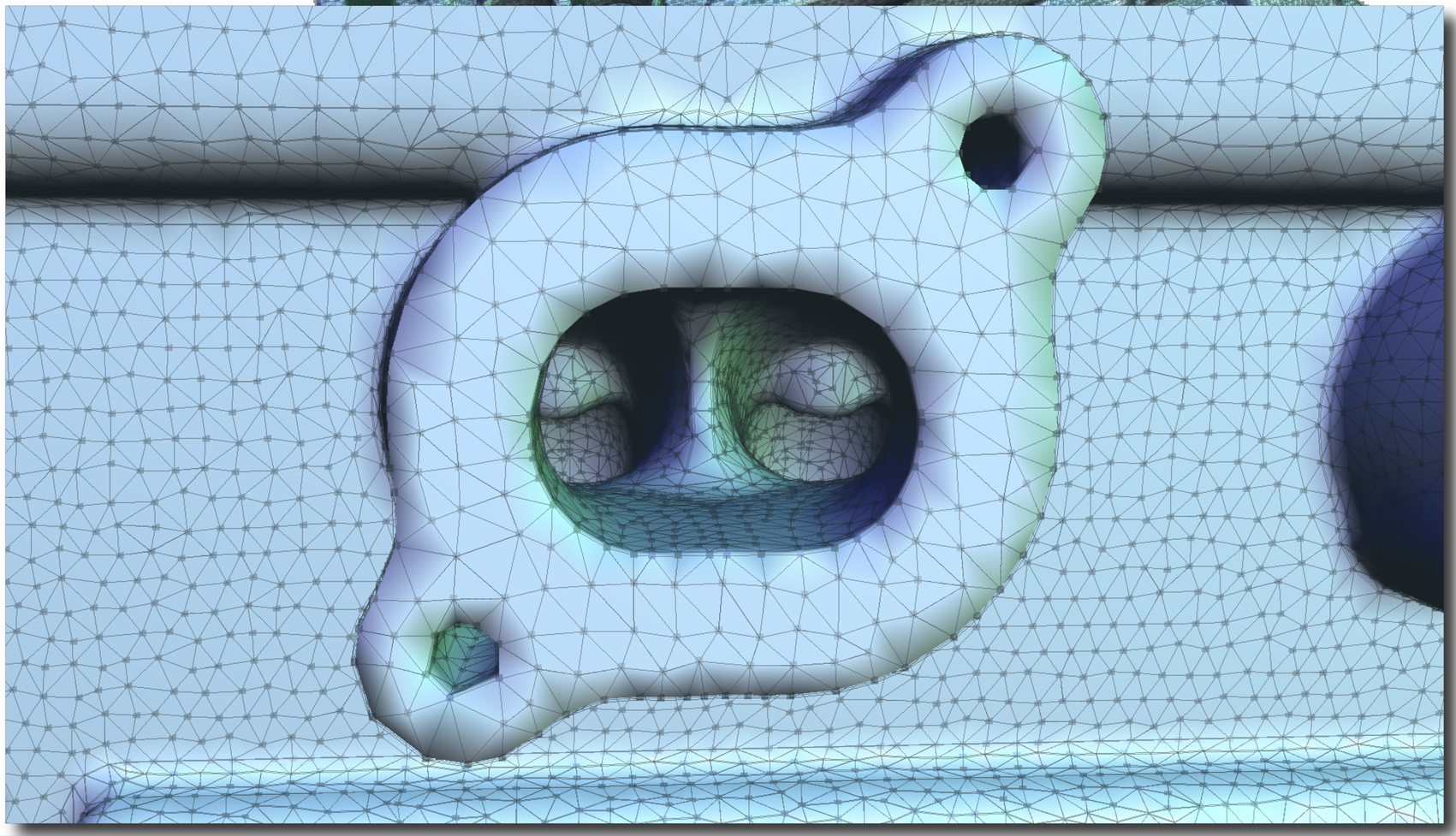
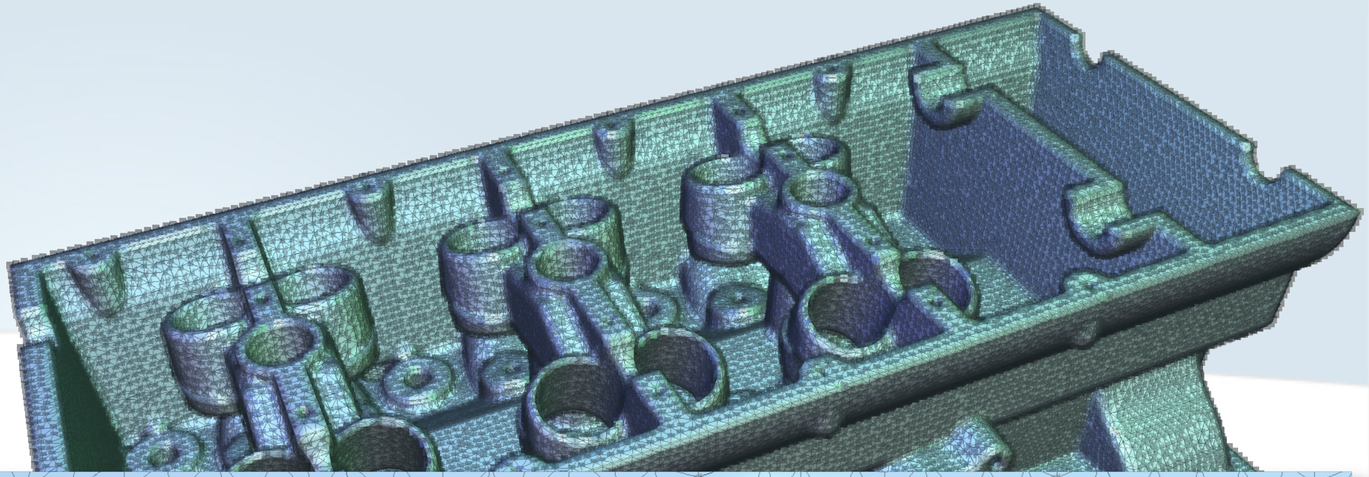
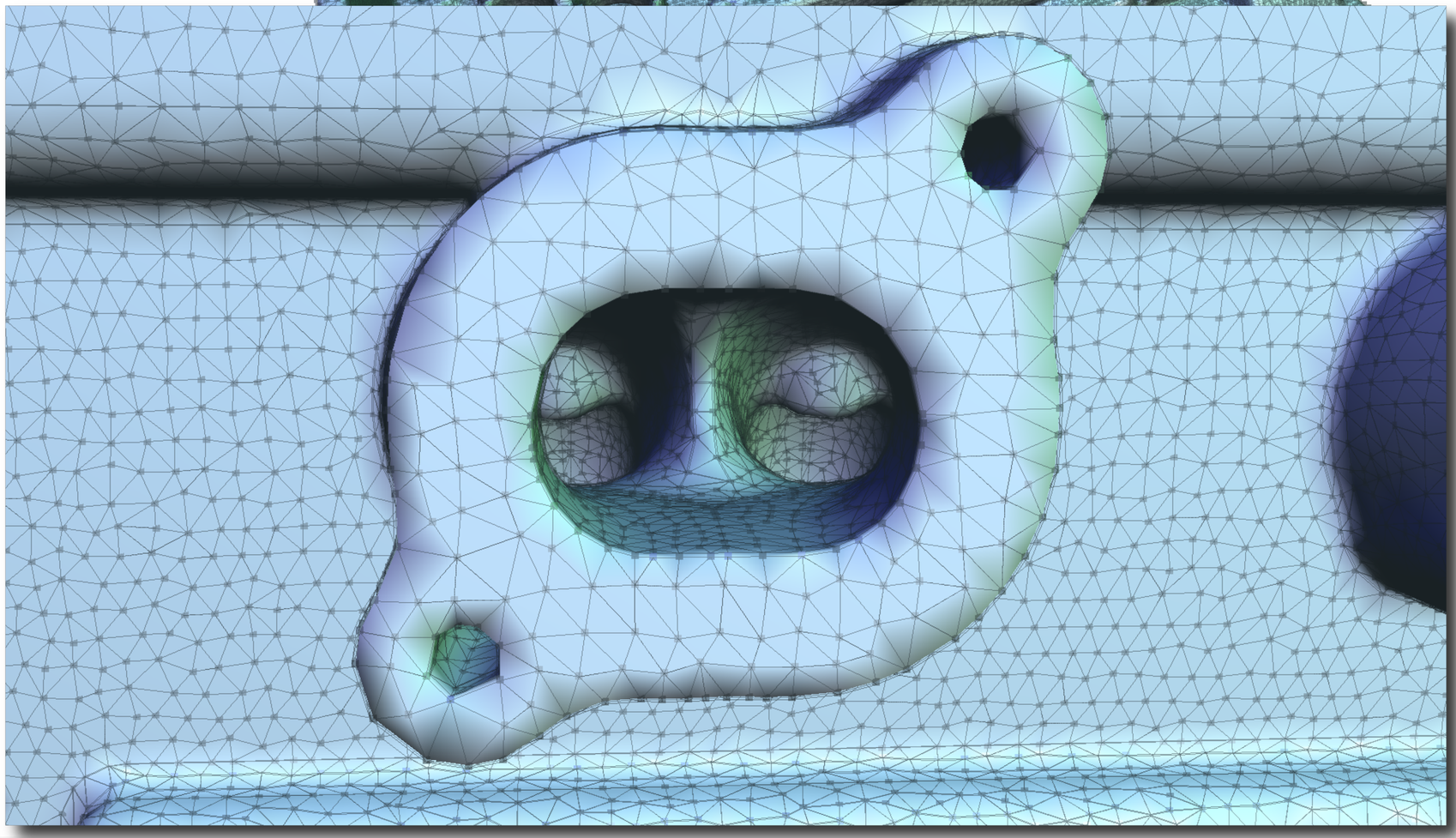




In practice

- Given a domain $\mathcal{D}$
- For each vertex v





In practice

- Given a domain $\mathcal{D}$
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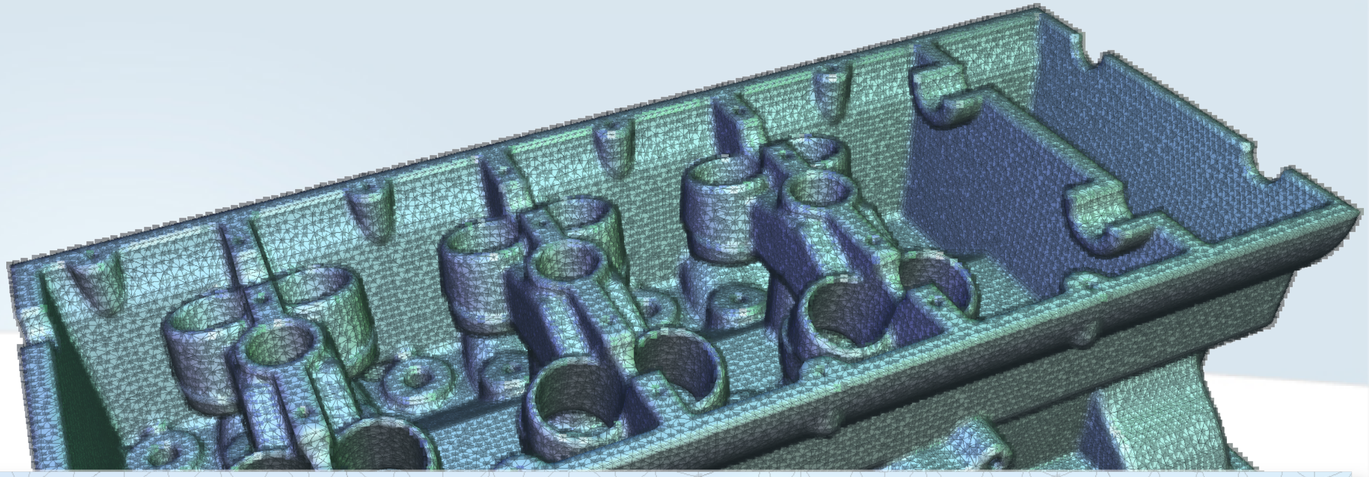
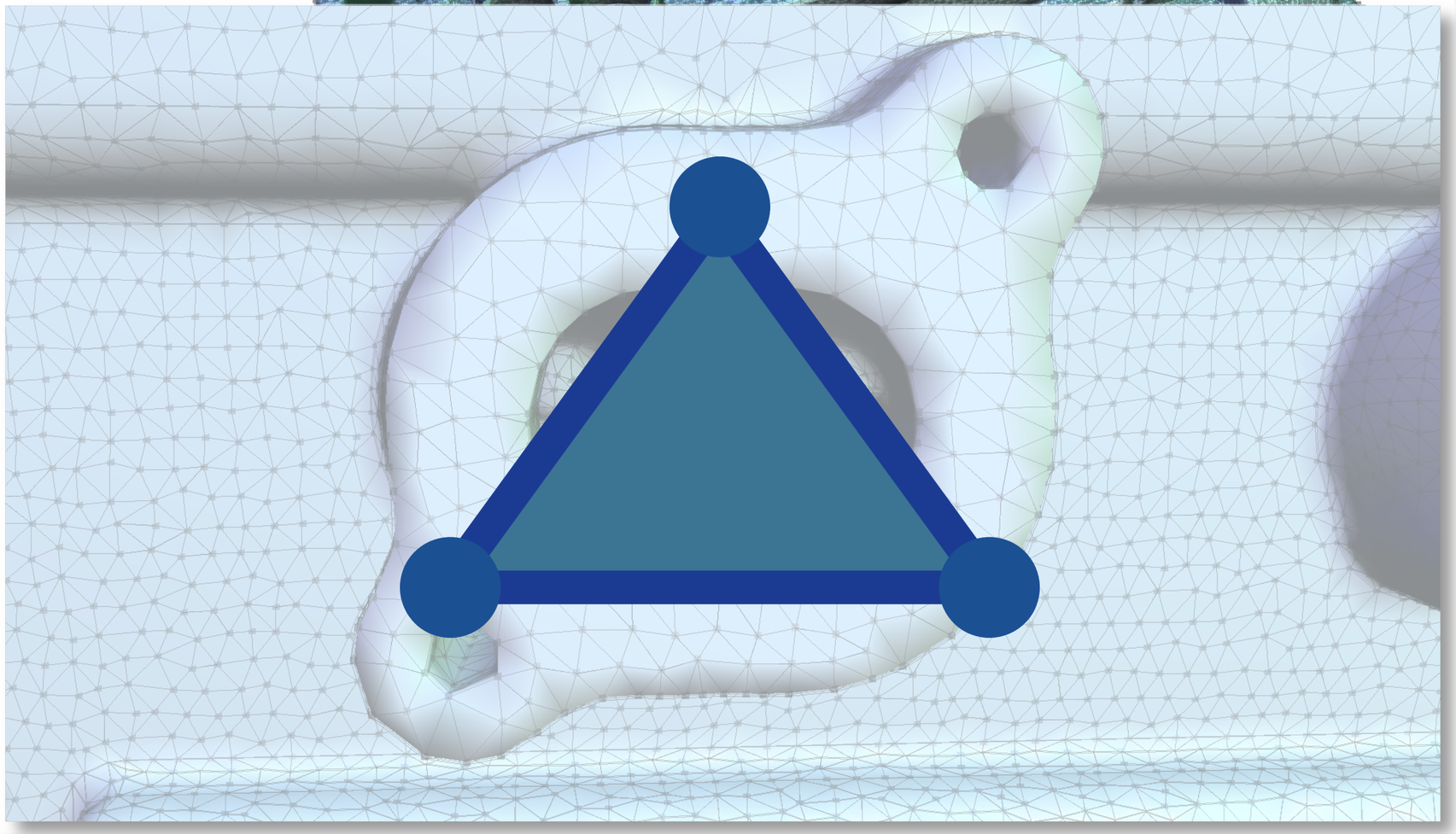
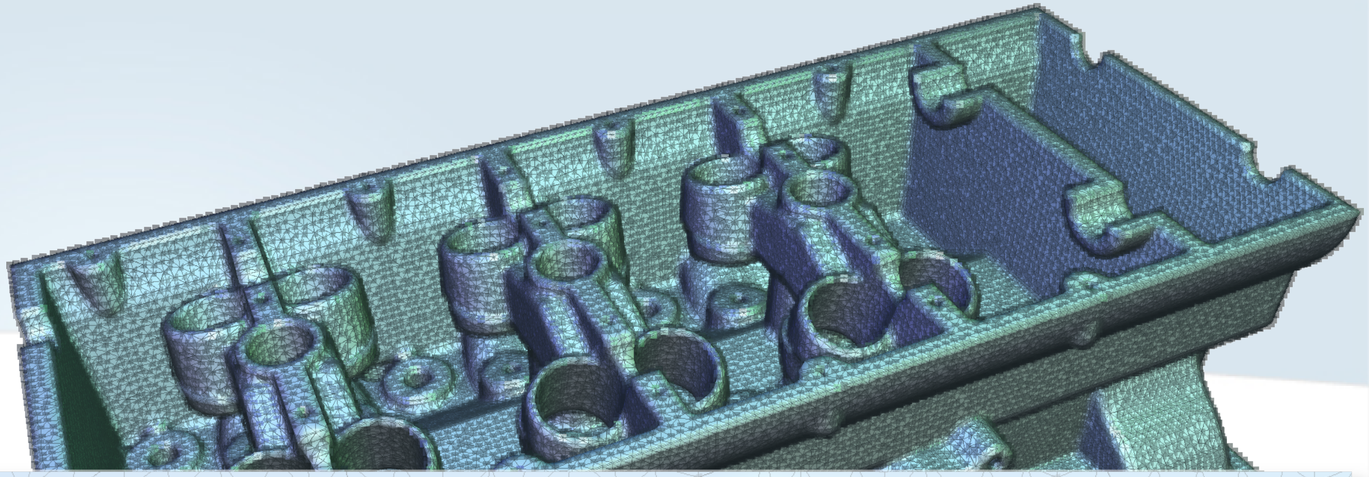
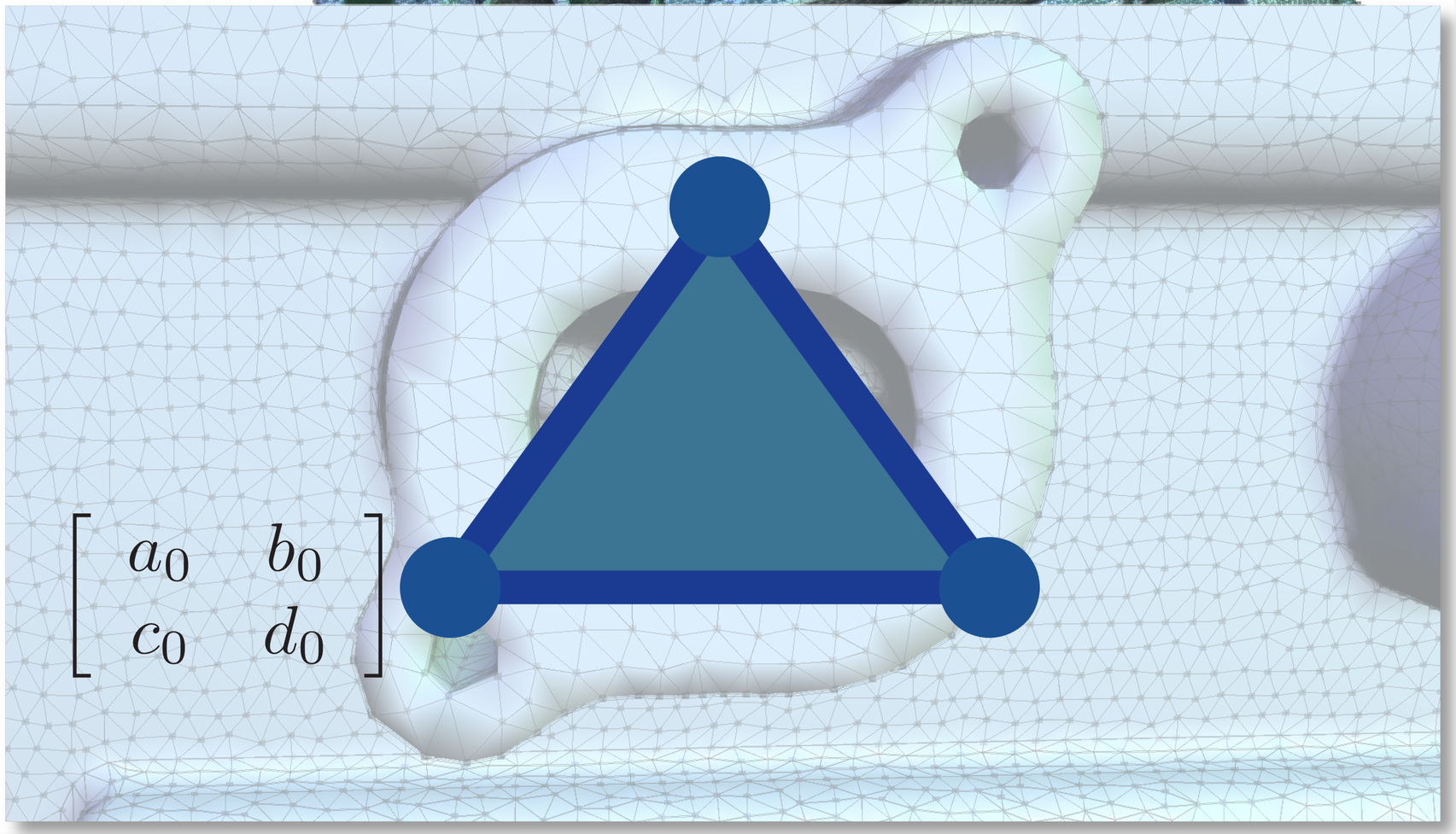




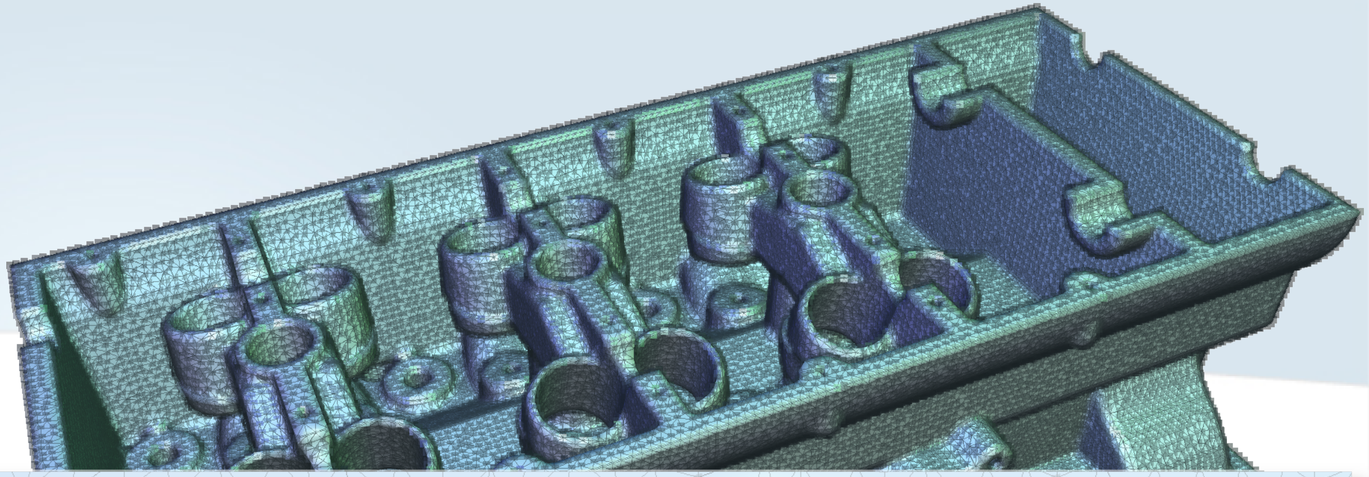
In practice

- Given a domain $\mathcal{D}$
- For each vertex v
- One matrix $f(v)$





In practice



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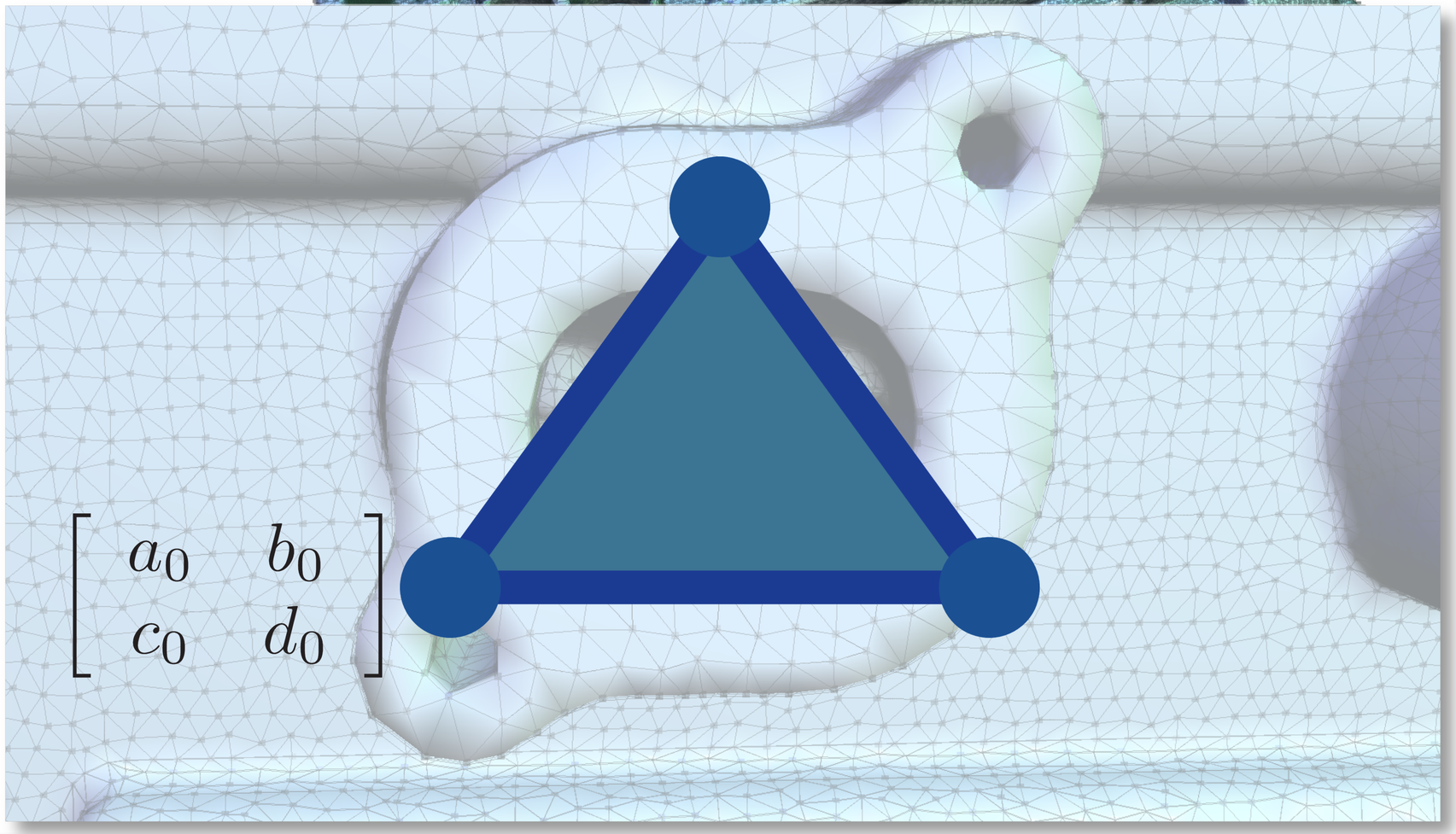
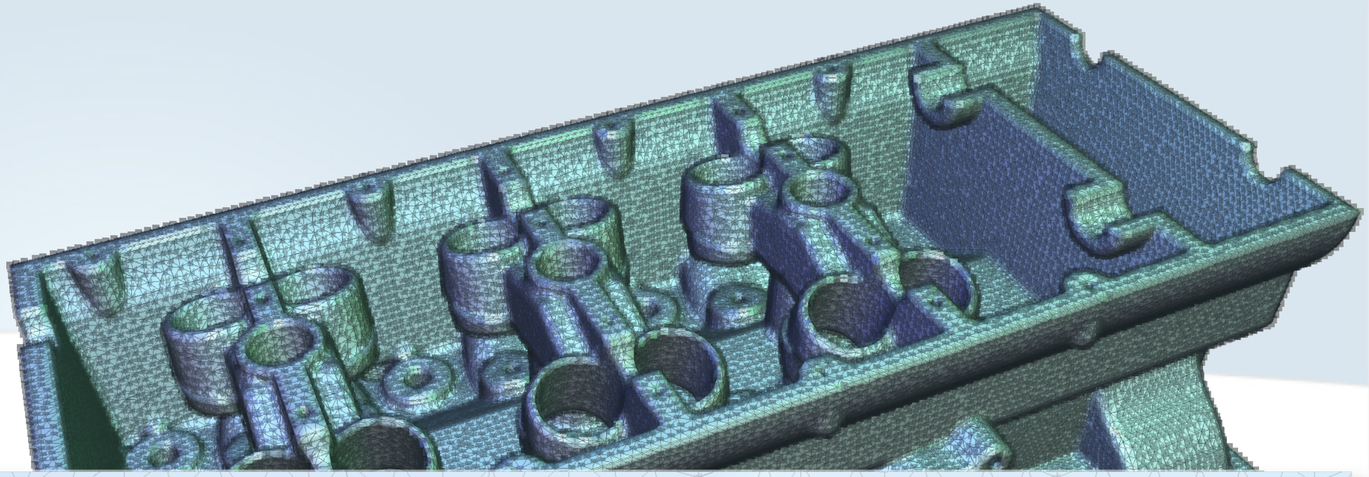


In practice



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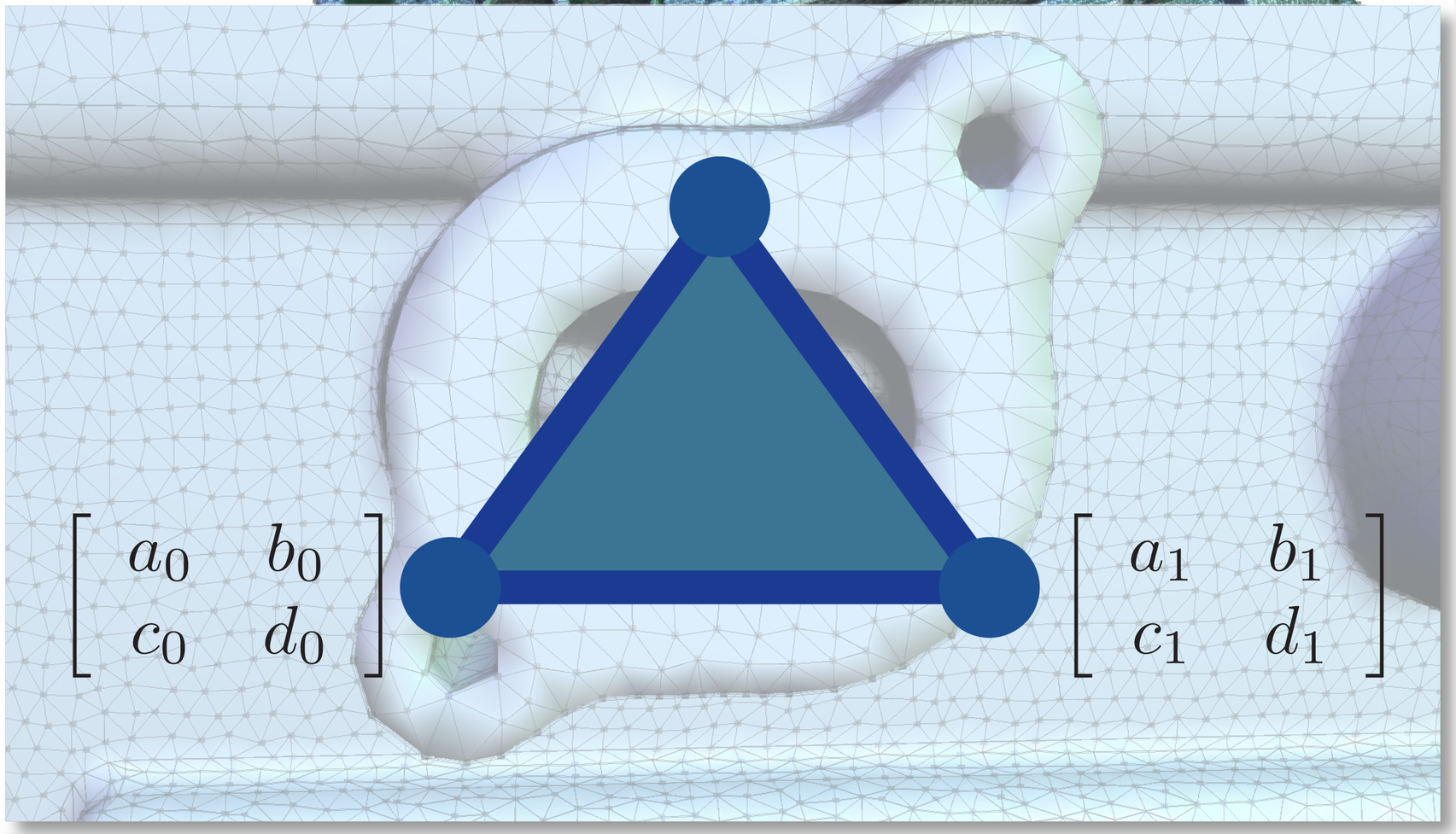

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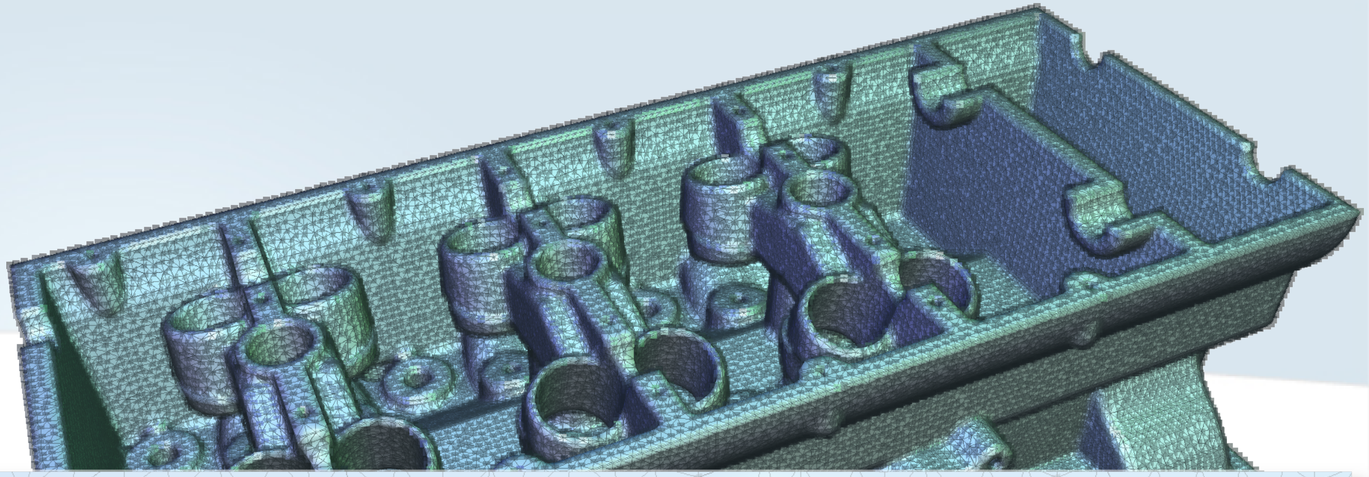

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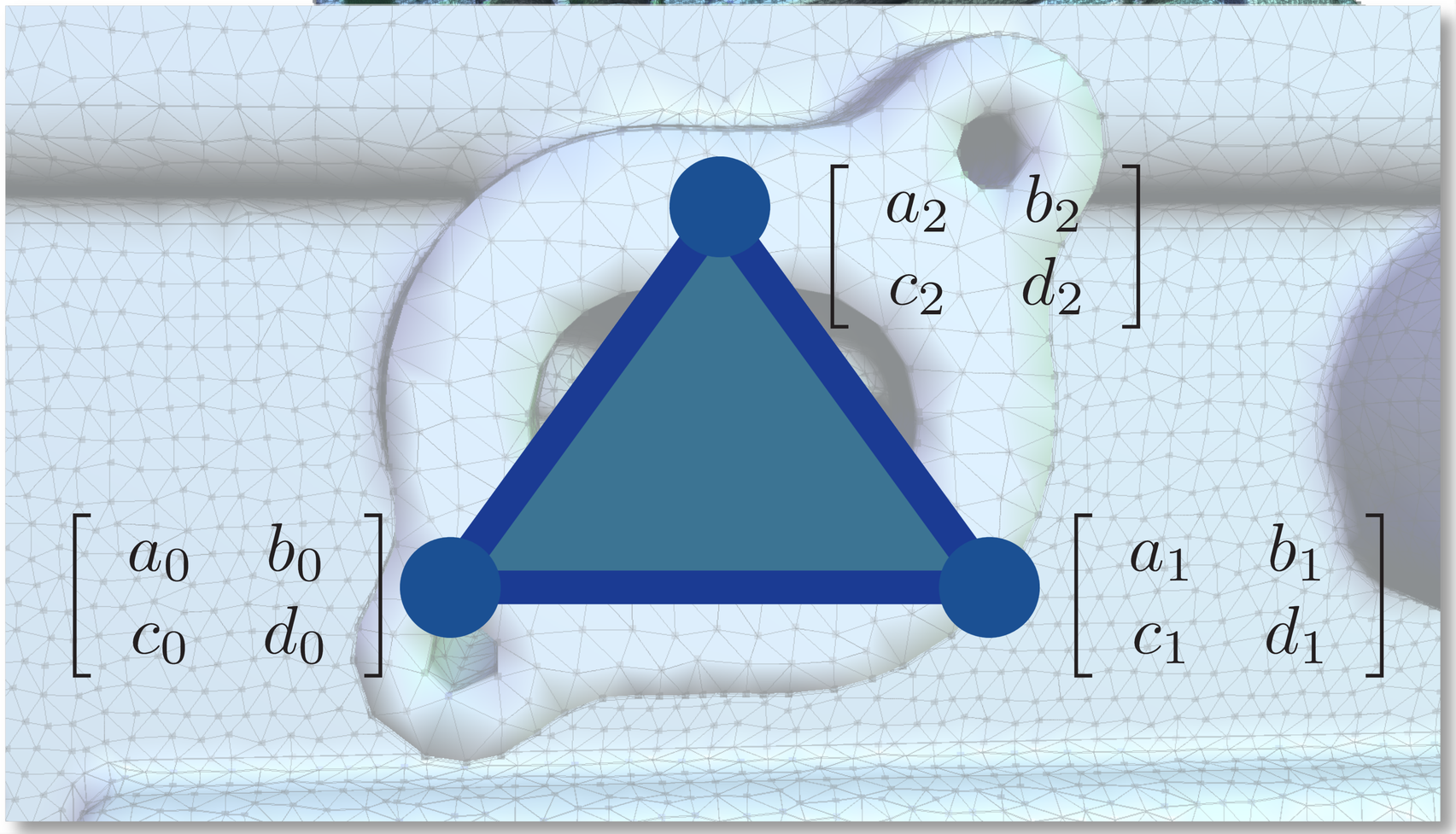

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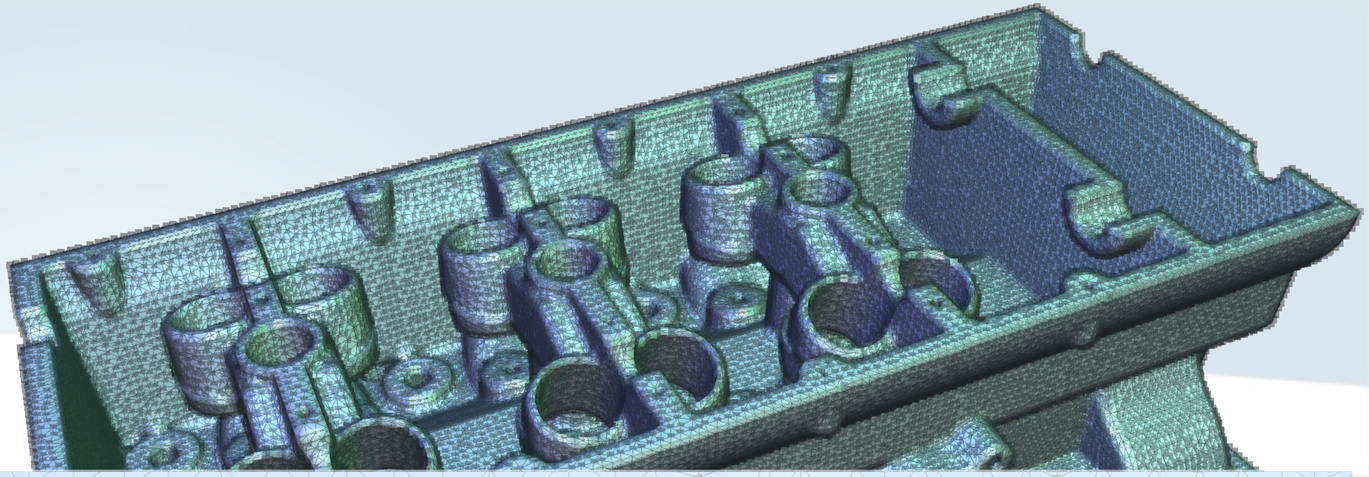

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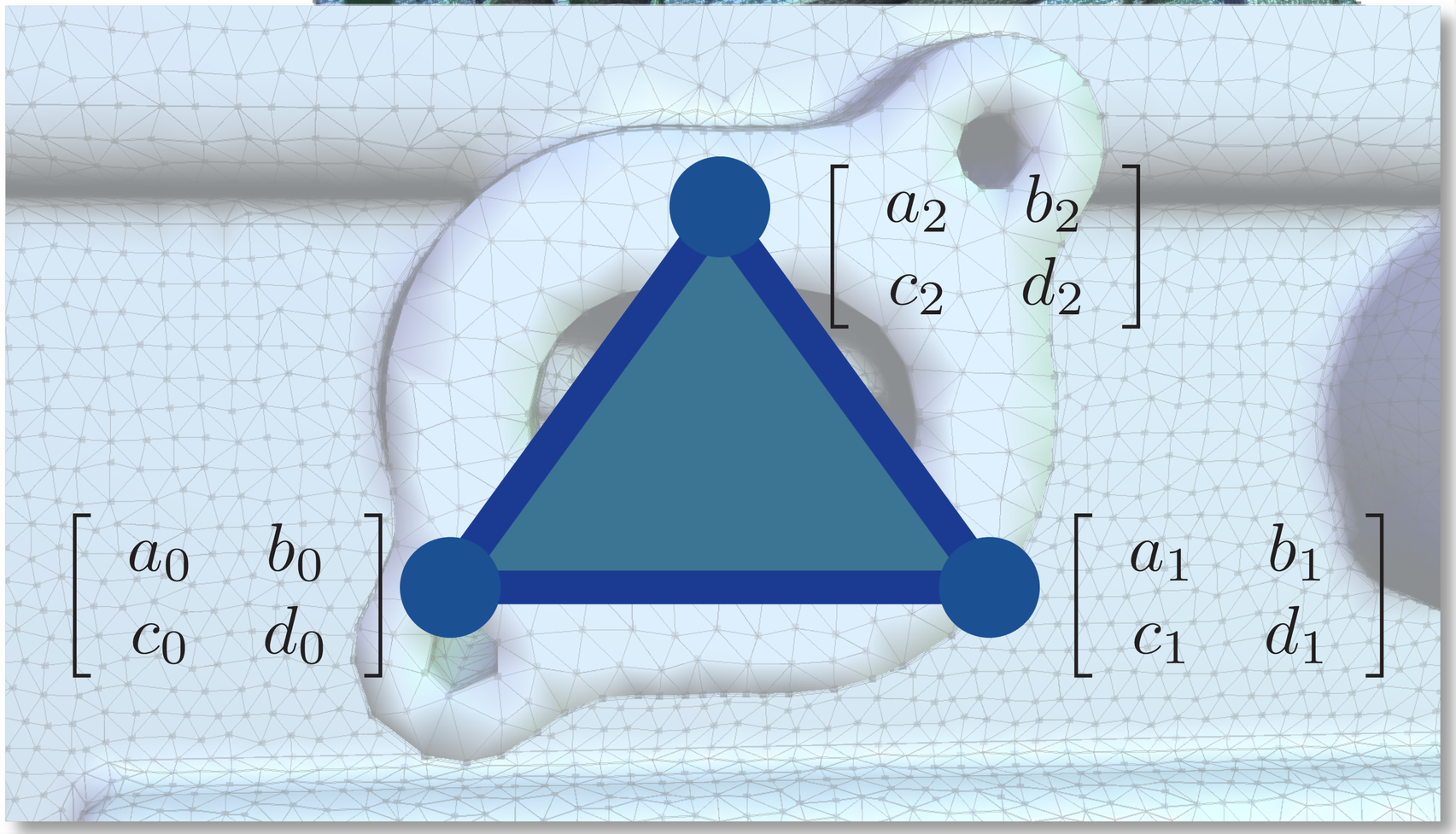

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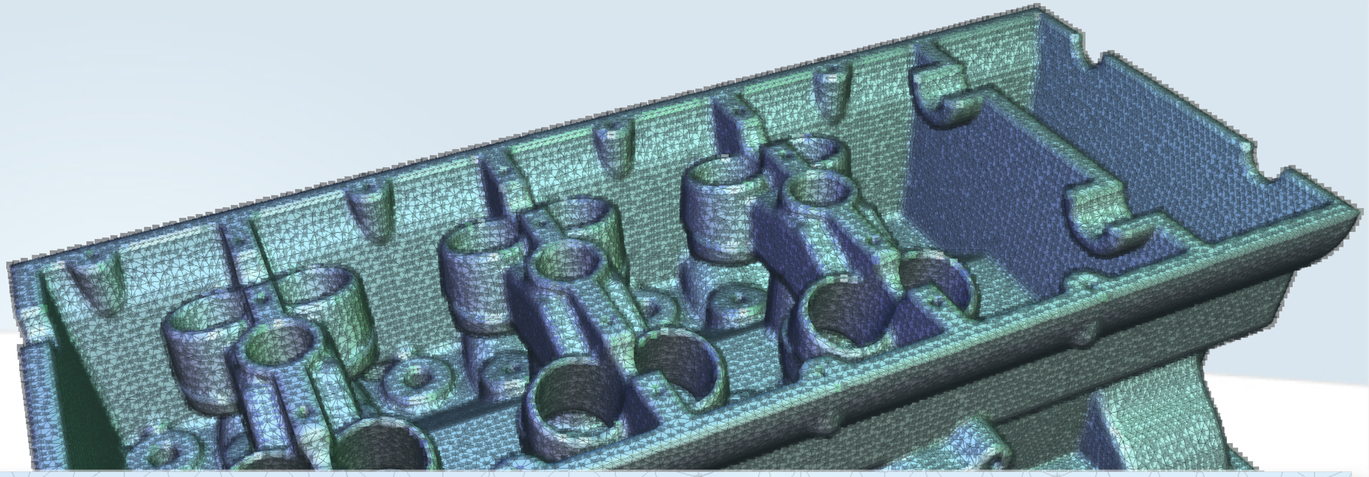

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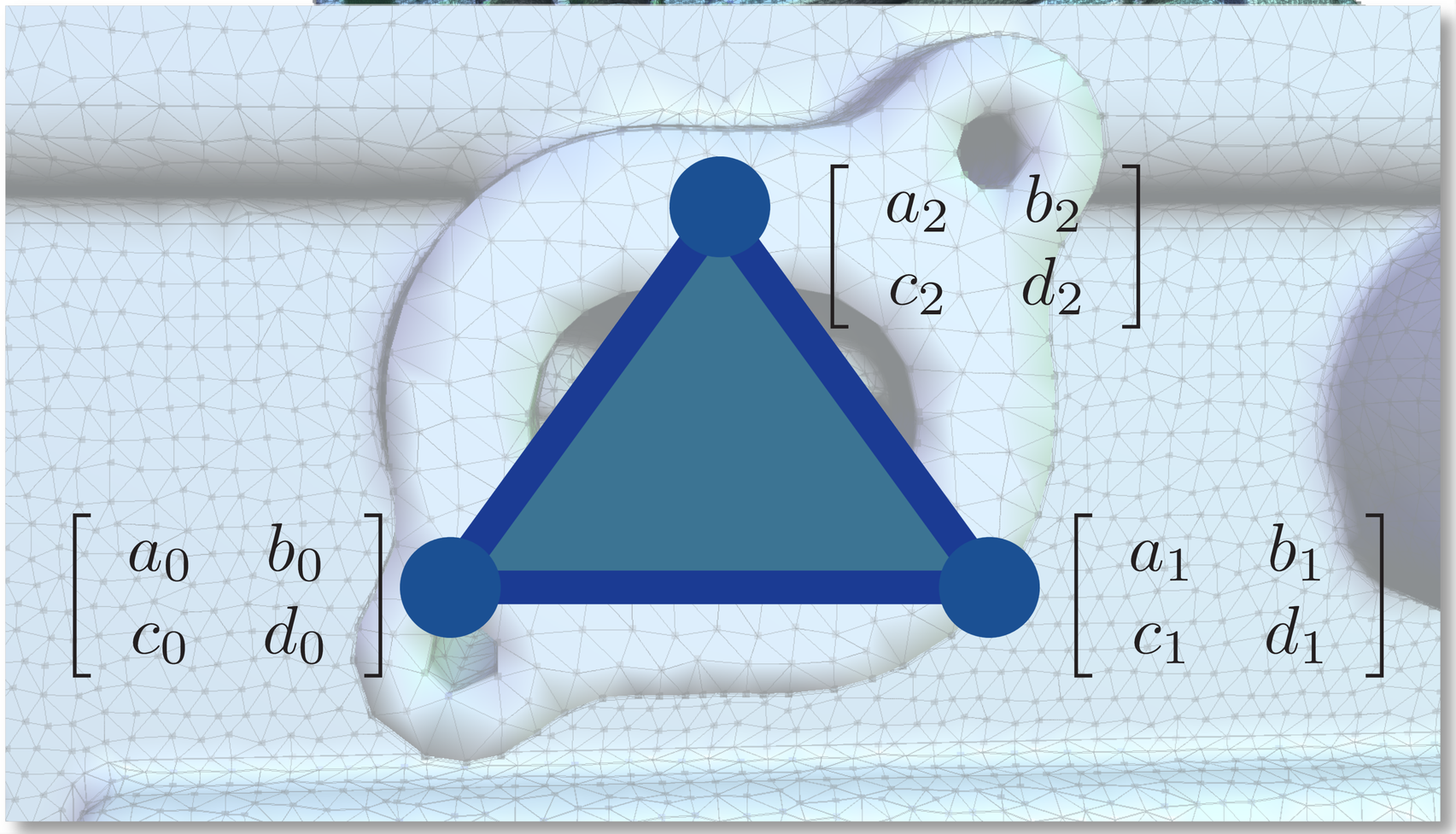

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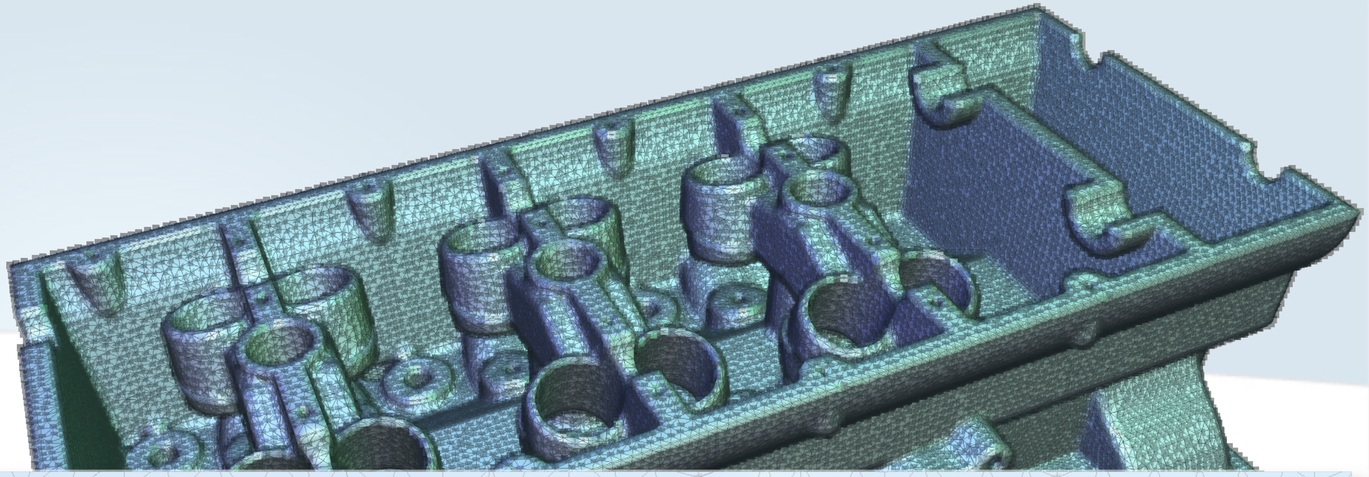

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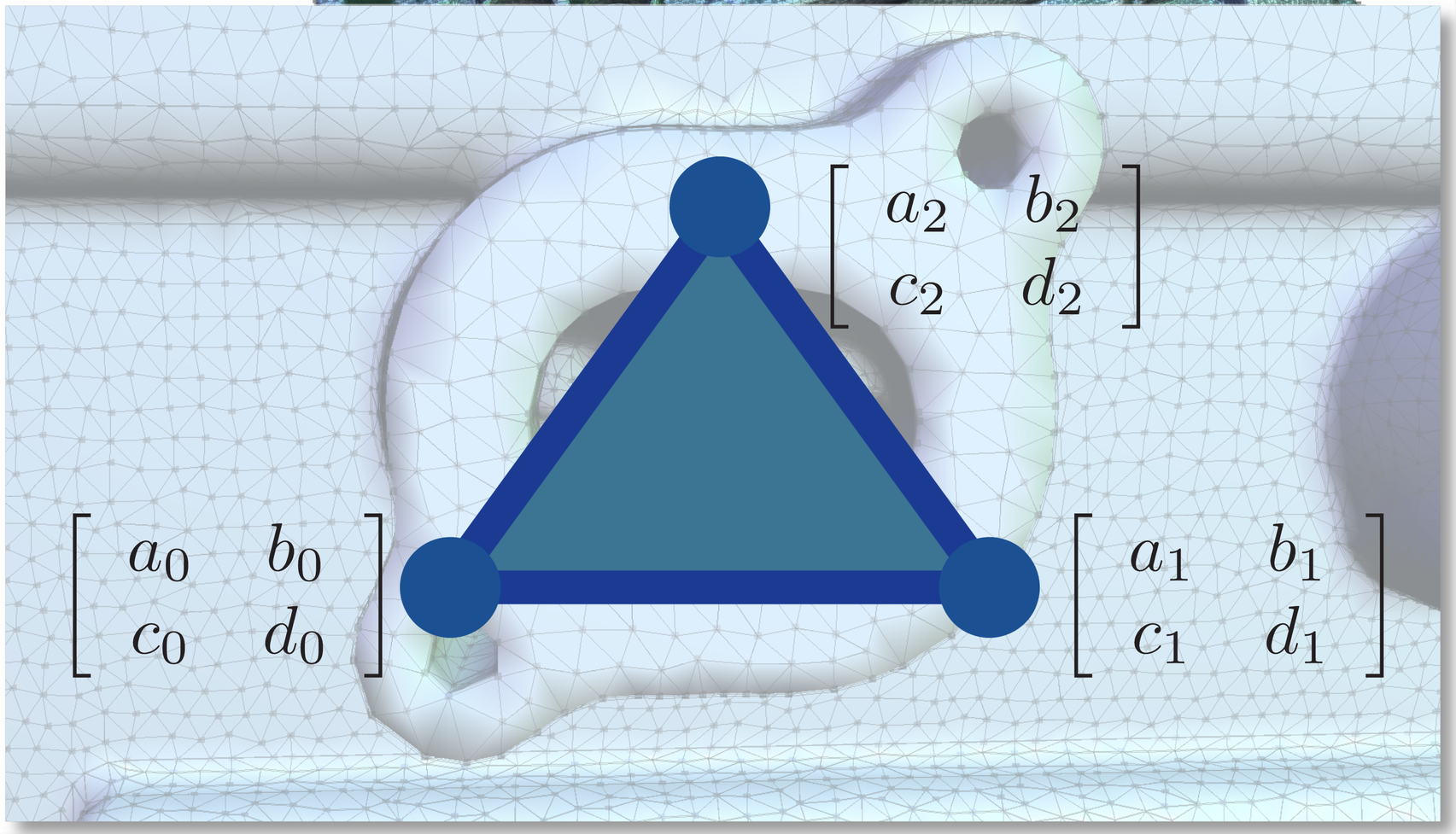

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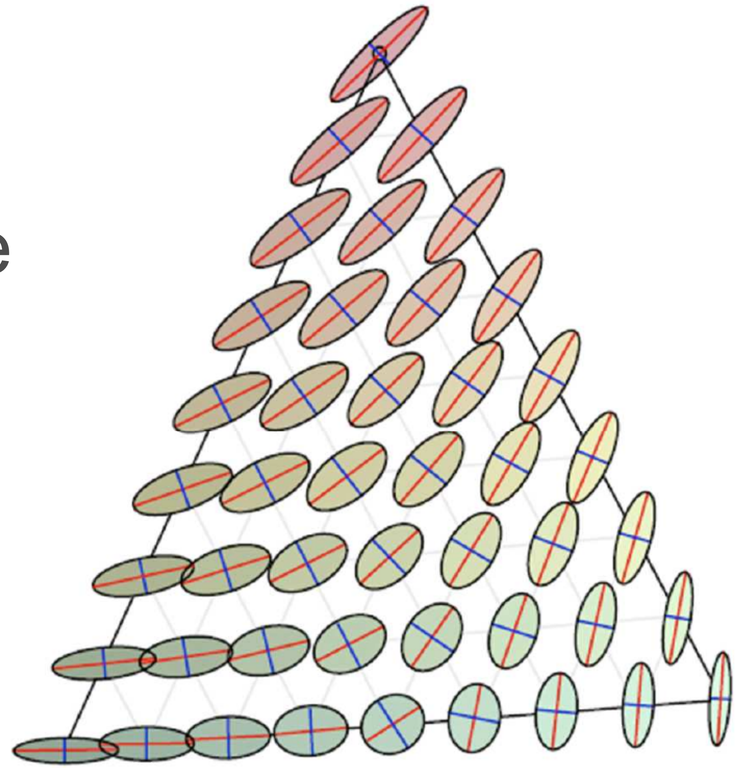

[Hotz]

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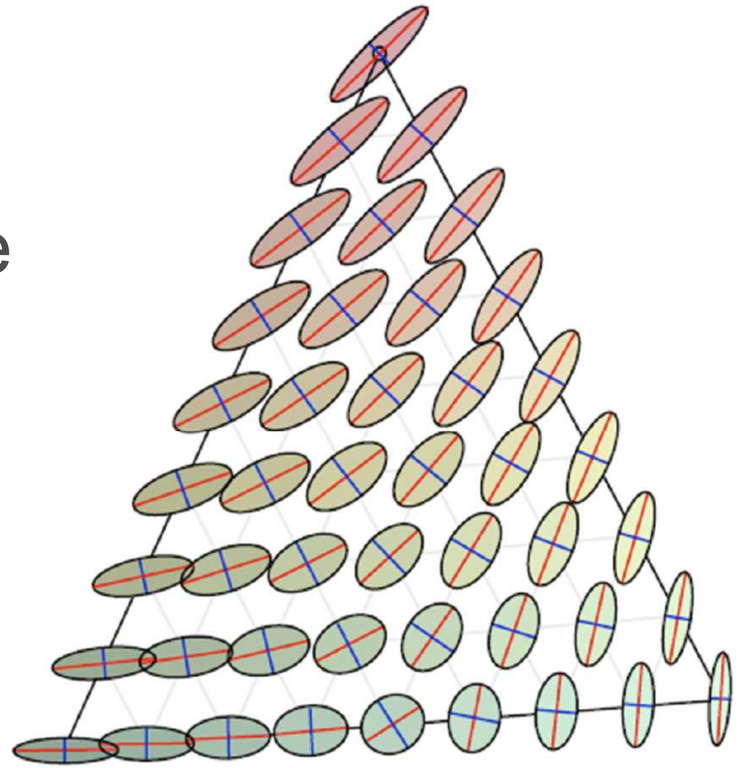

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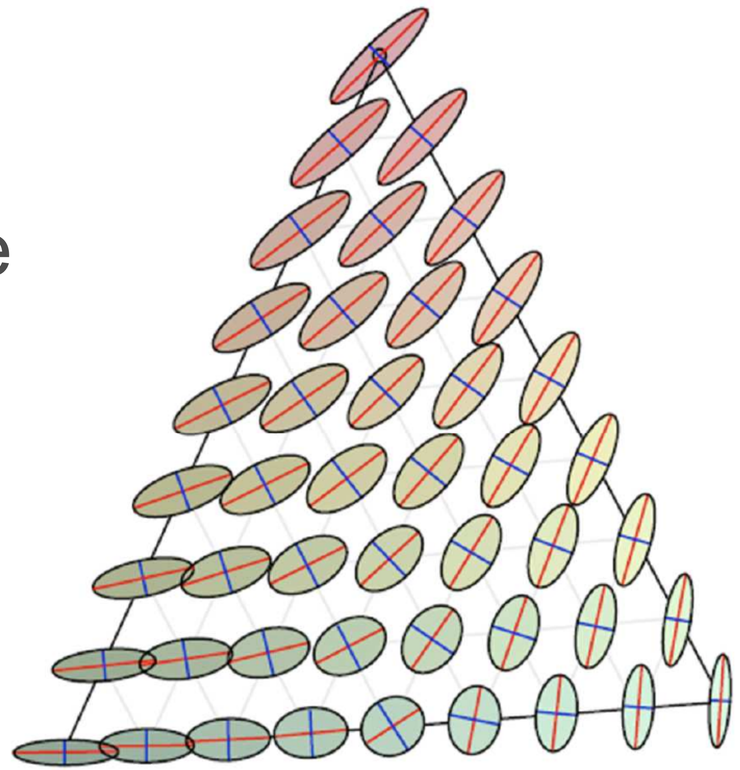

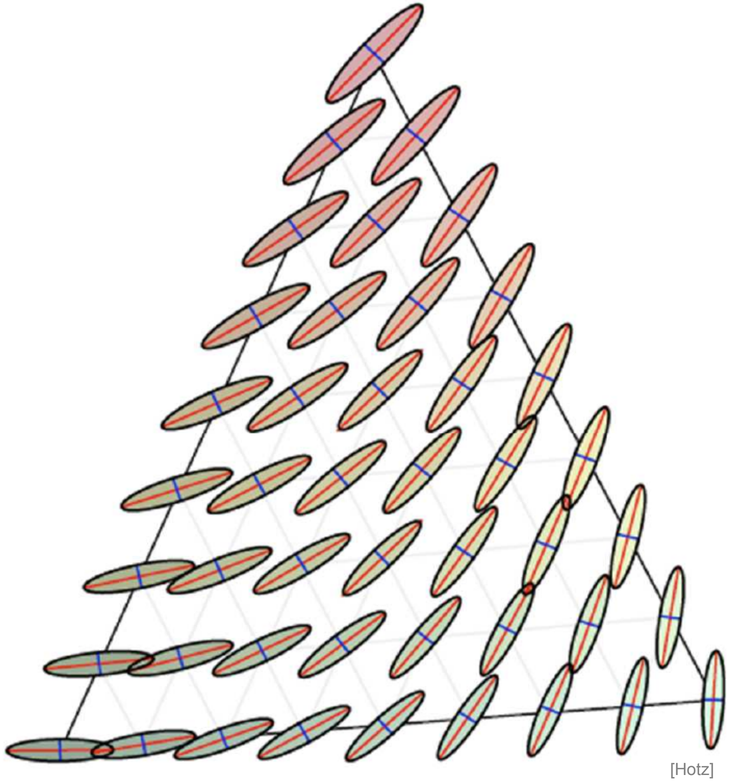

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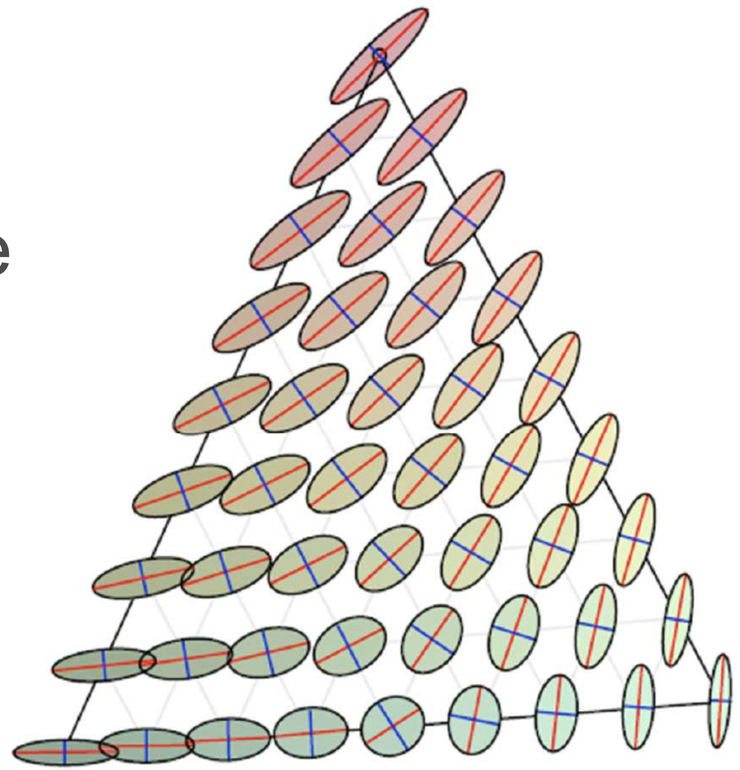

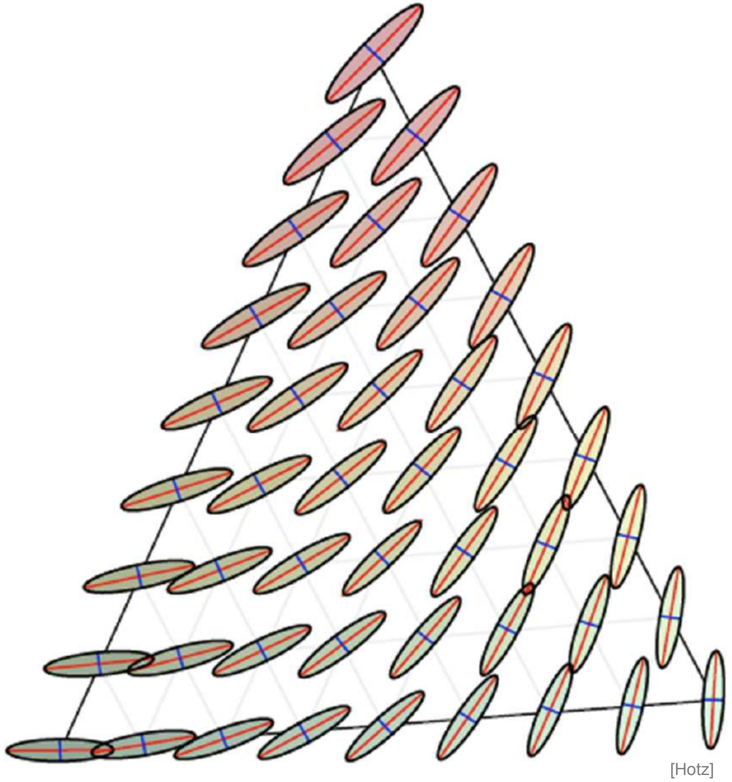

[Hotz]

Glyph packing

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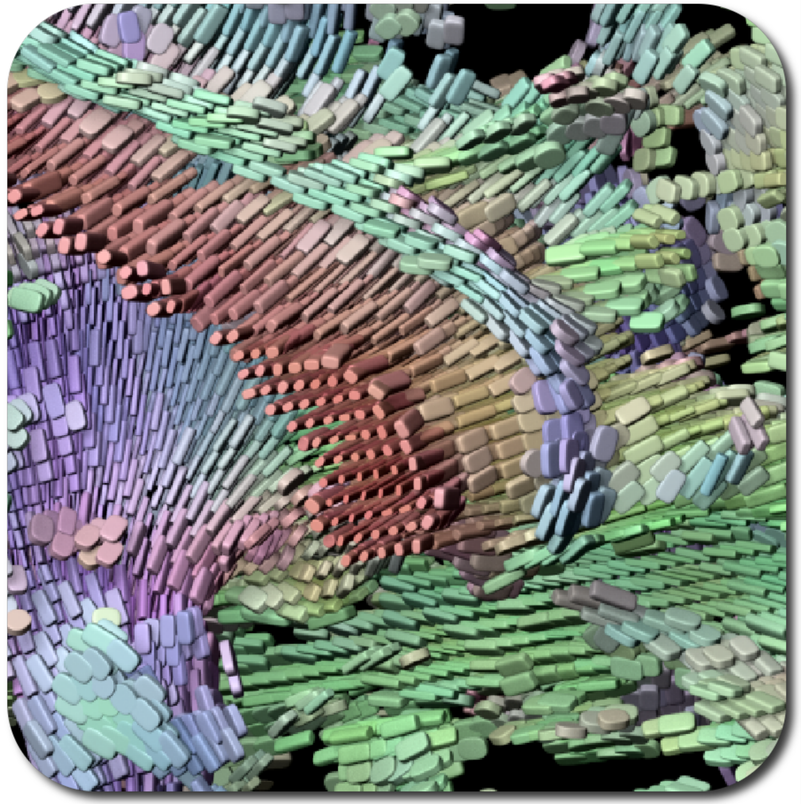

[Kindlmann]

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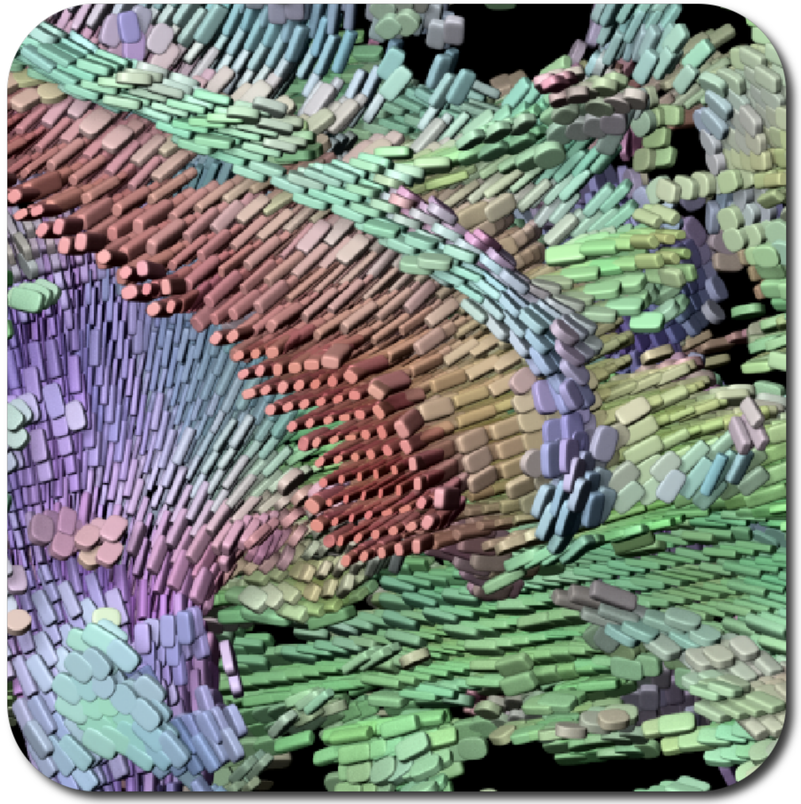

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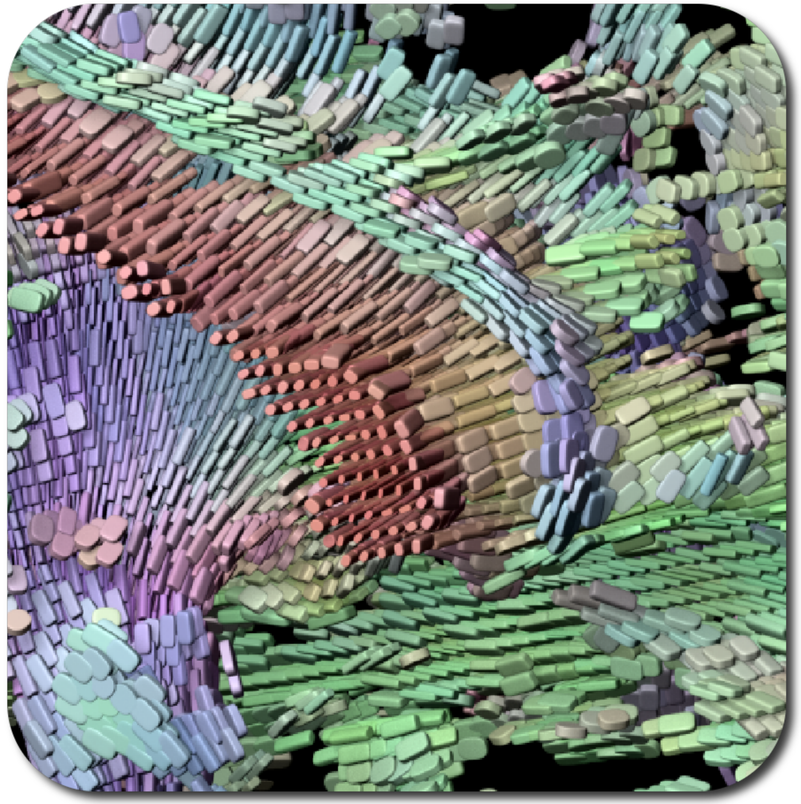

[Kindlmann]

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SVD or PCA

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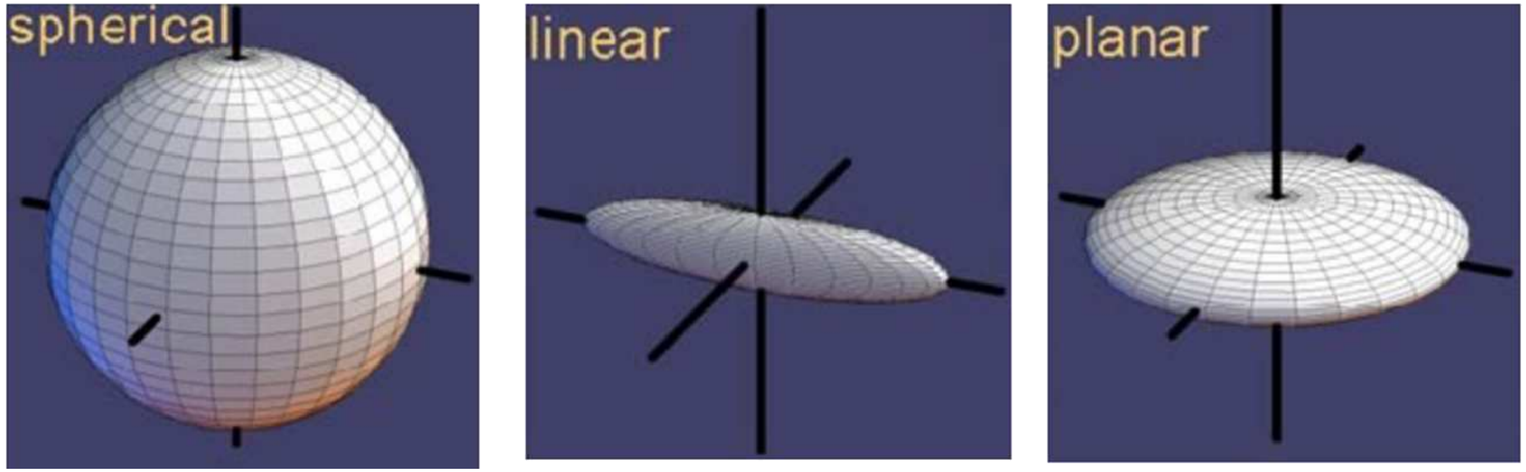
Tensor glyphs

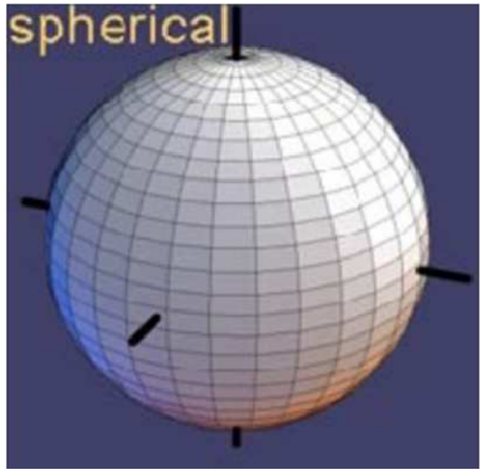
- Ellipsoids

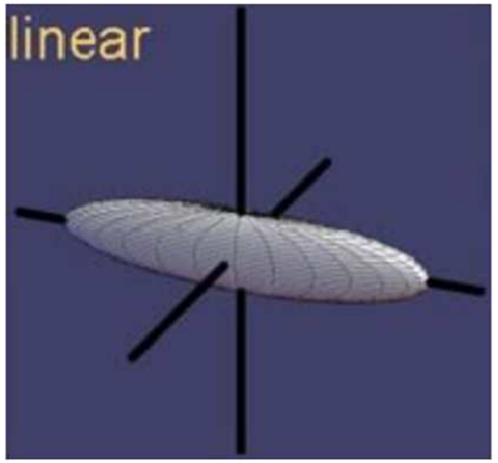


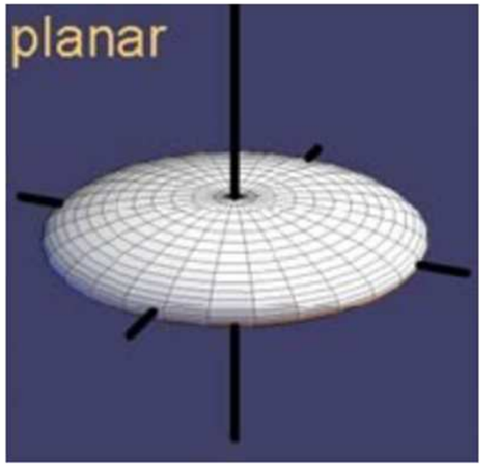
[Kindlmann]

Tensor glyphs

- Ellipsoids
  - Semi-principal axes
    - Eigen vectors







[Kindlmann]

Tensor glyphs

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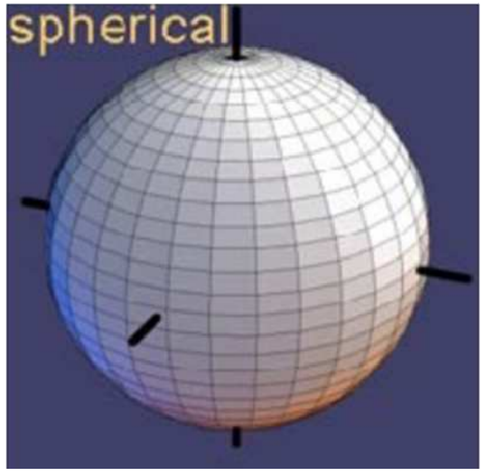
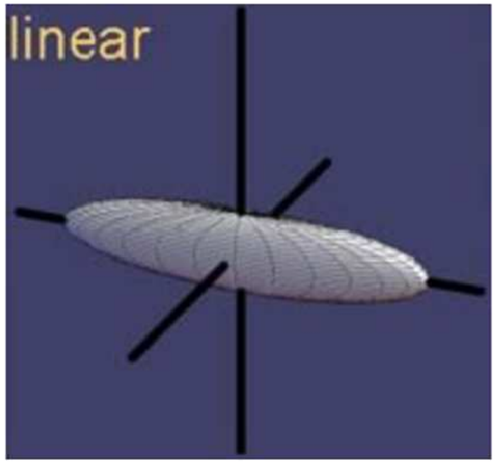
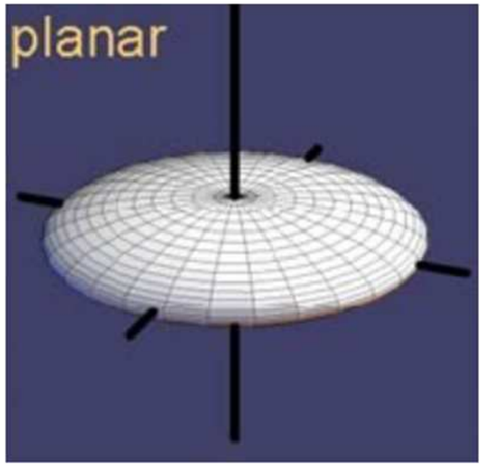
    - Eigen vectors
      - **Direction, not a vector!**







[Kindlmann]

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    - Eigen vectors
      - **Direction, not a vector!**
  - Axis length
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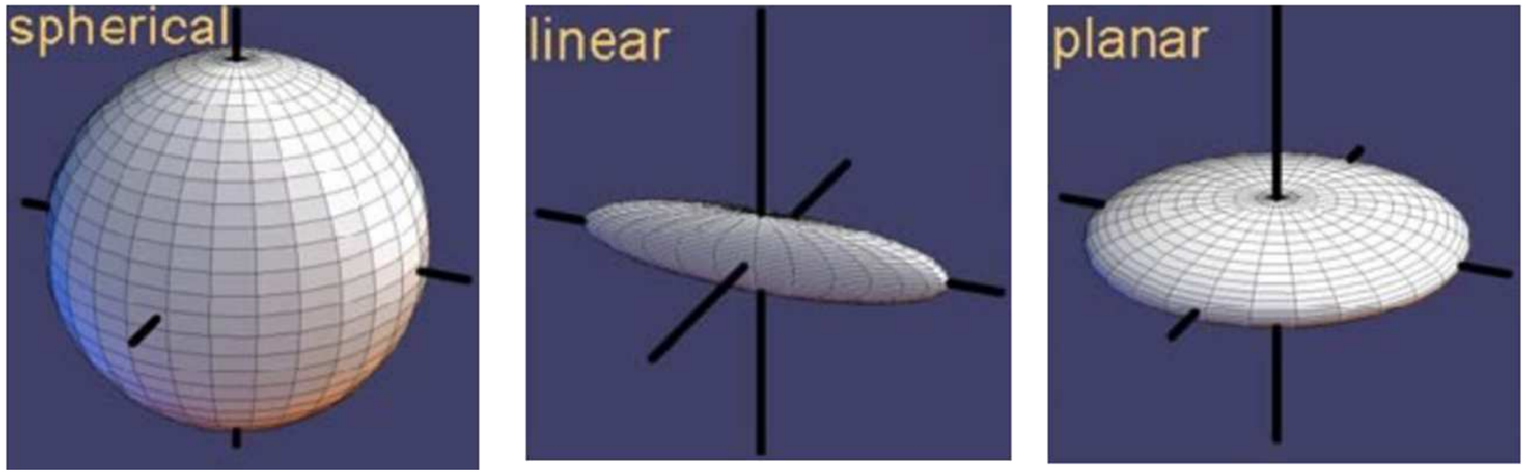

[Kindlmann]

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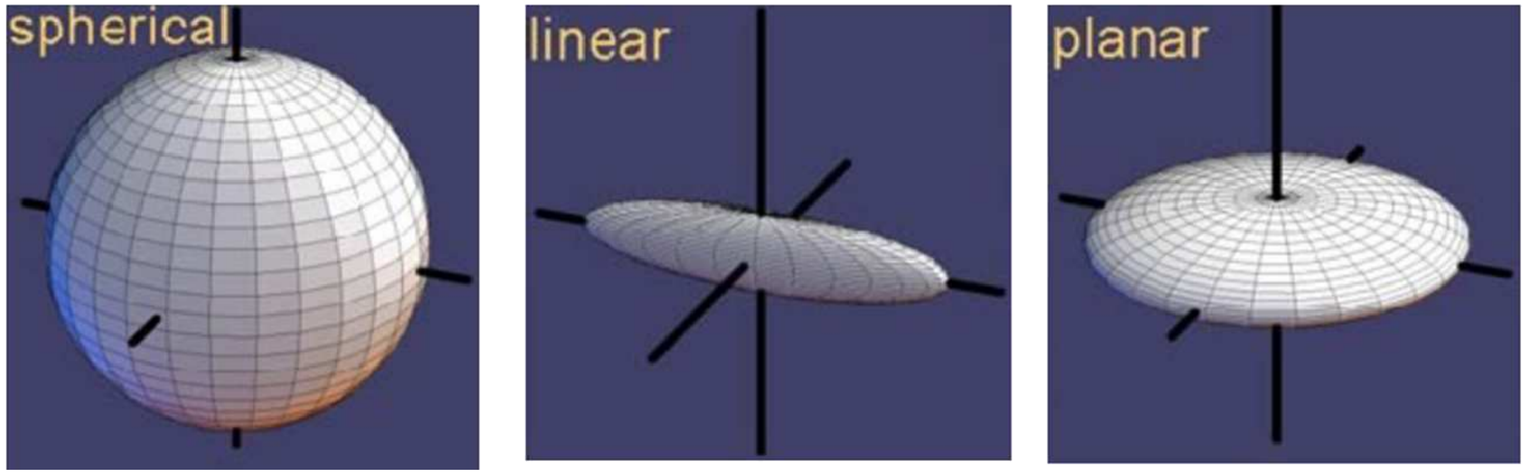
  - Axis length
    - Eigen values
  - Anisotropy information



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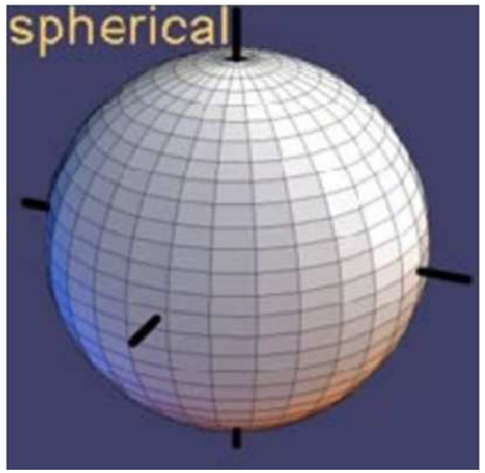
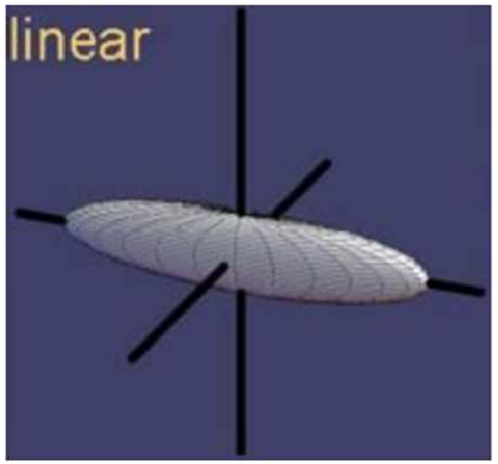
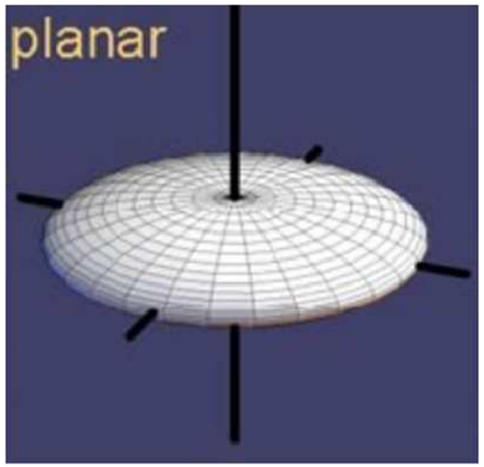
Isotropy



Axial anisotropy



Planar anisotropy

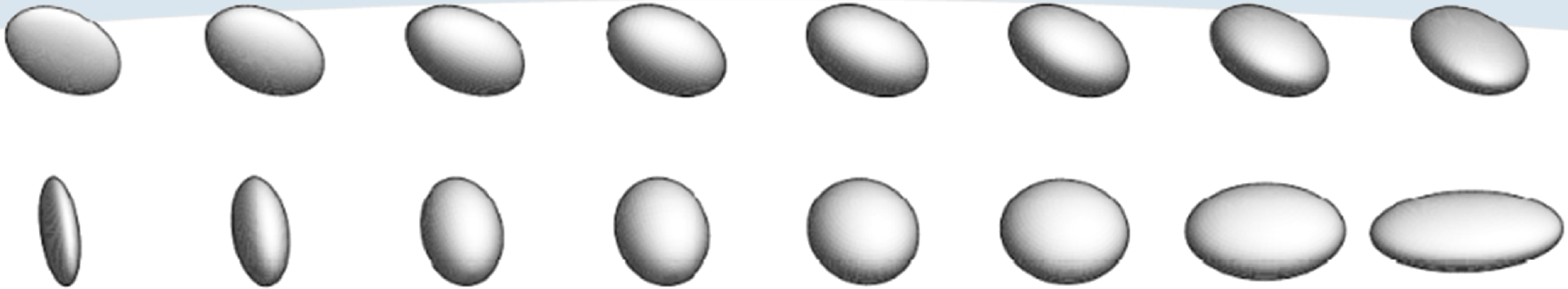


[Kindlmann]

Tensor glyphs

- Ellipsoids in 3D

Tensor glyphs



[Kindlmann]

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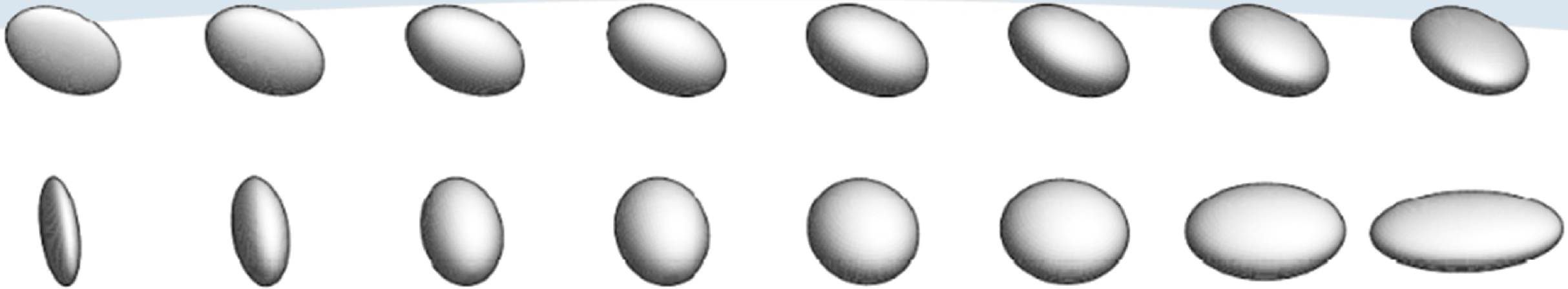
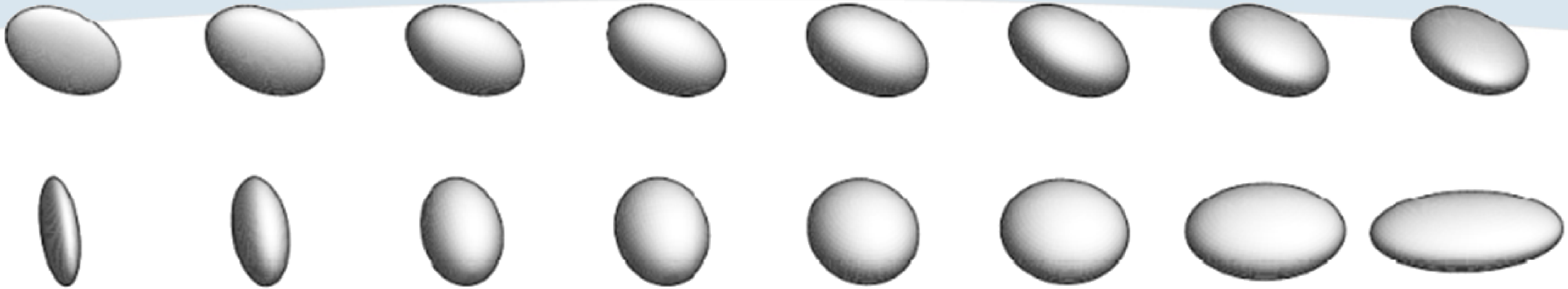


[Kindlmann]

- Ellipsoids in 3D
  - Information loss due to screen projection

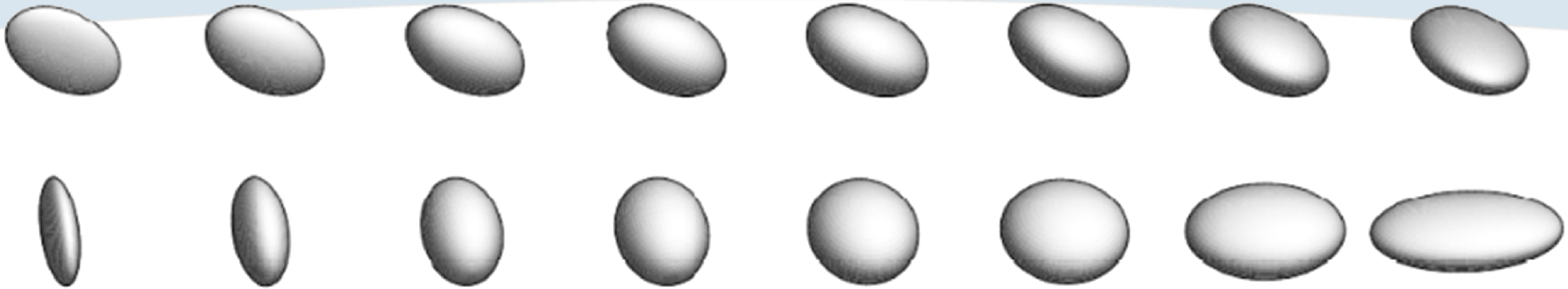
Tensor glyphs



[Kindlmann]

- Ellipsoids in 3D
  - Information loss due to screen projection
  - Continuous shading

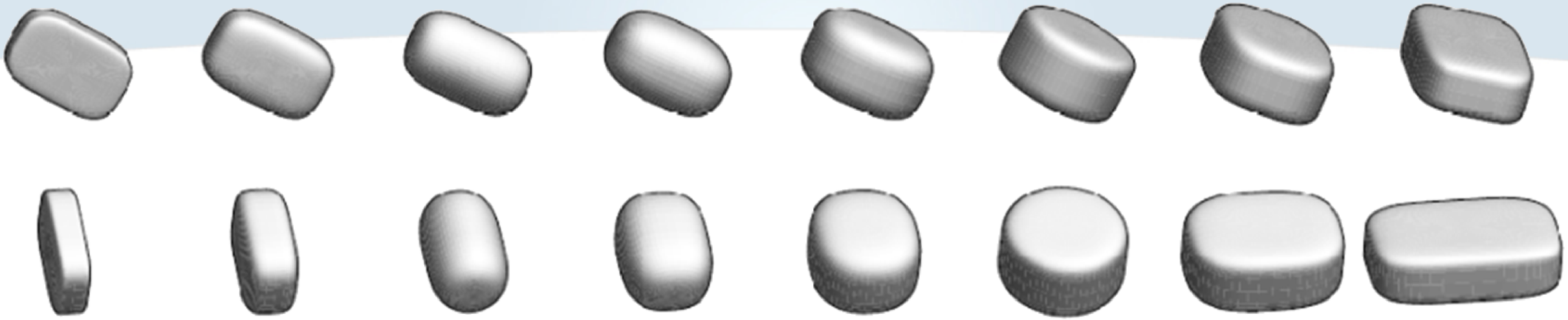
Tensor glyphs



[Kindlmann]

- Ellipsoids in 3D
  - Information loss due to screen projection
  - Continuous shading
    - Hard to evaluate variations across axes

Tensor glyphs

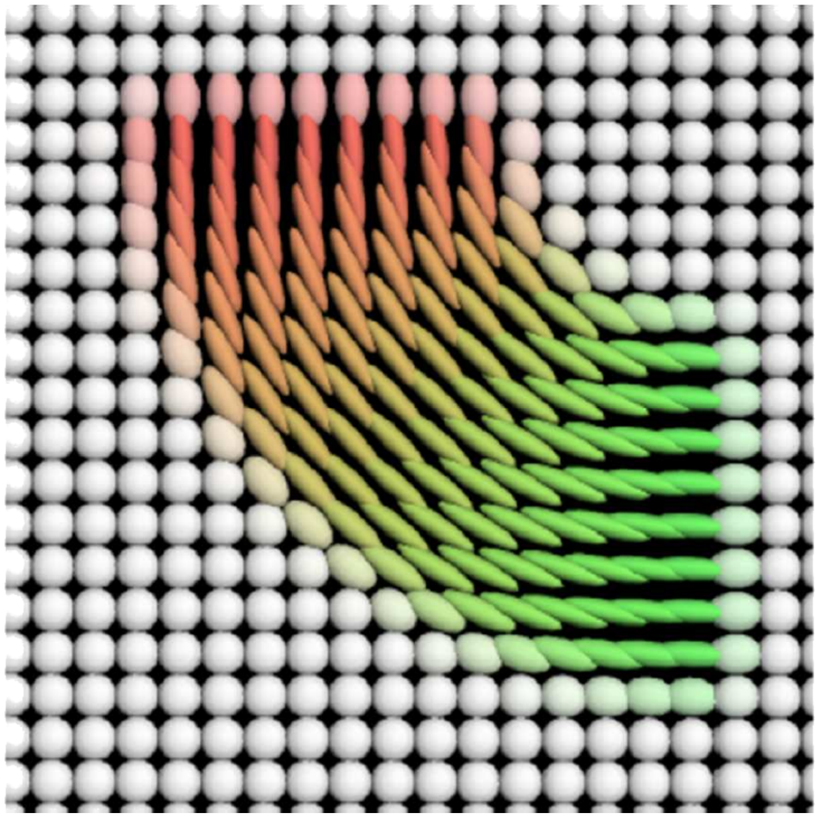


[Kindlmann]

- “Superquadrics”
  - Parallelepiped with smooth edges
  - Exaggerate shading variations across axis

Glyph packing

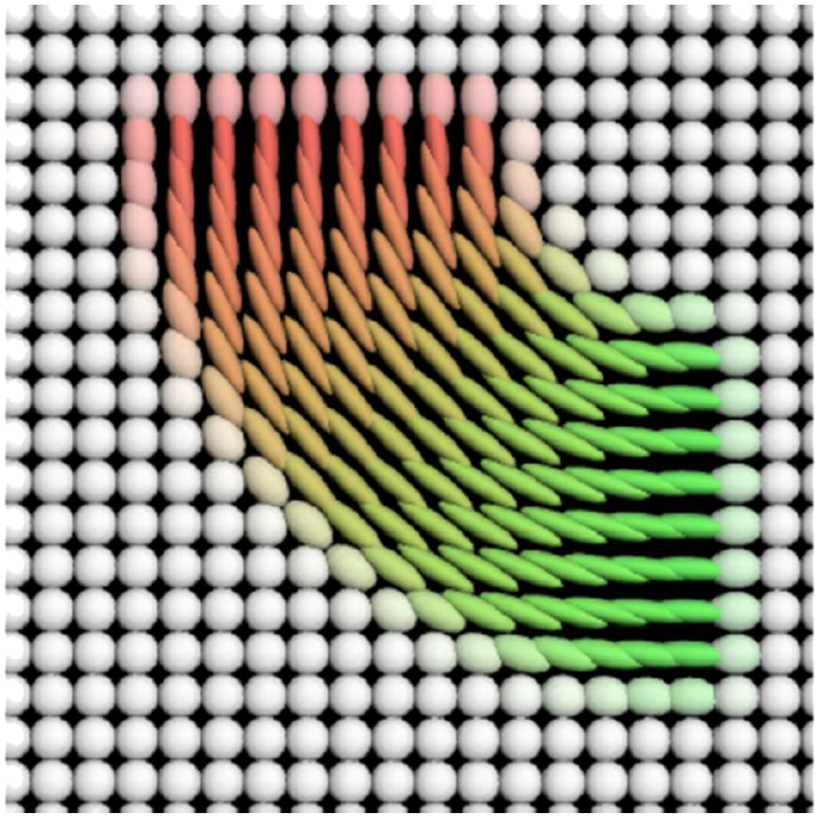
- How to distribute the glyphs



[Kindlmann]

Glyph packing

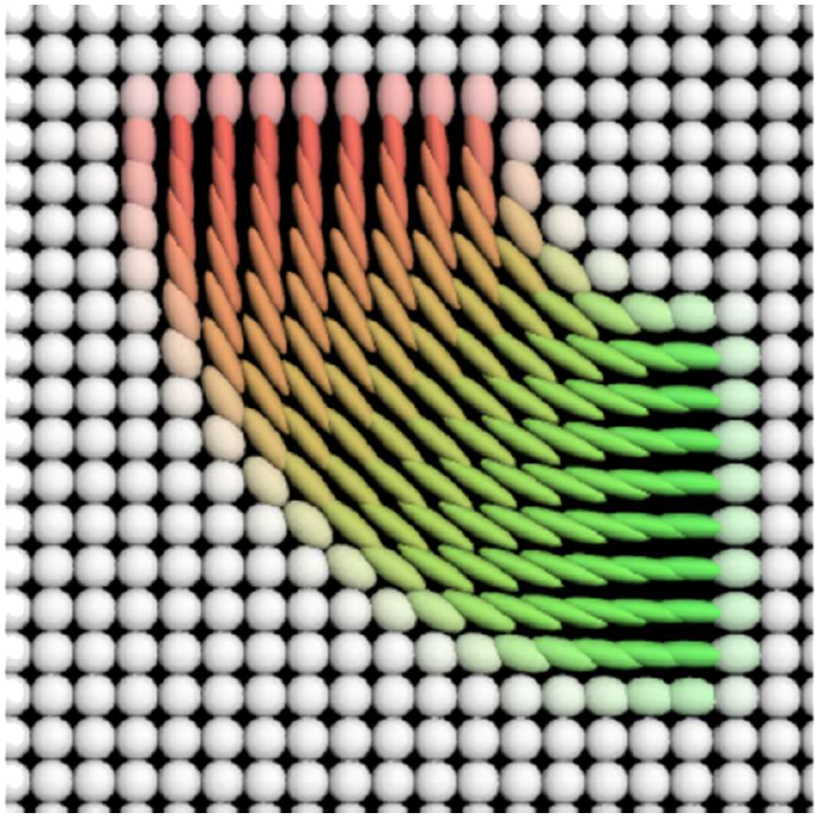
- How to distribute the glyphs
  - To avoid data overload
  - Glyph overlap



[Kindlmann]

Glyph packing

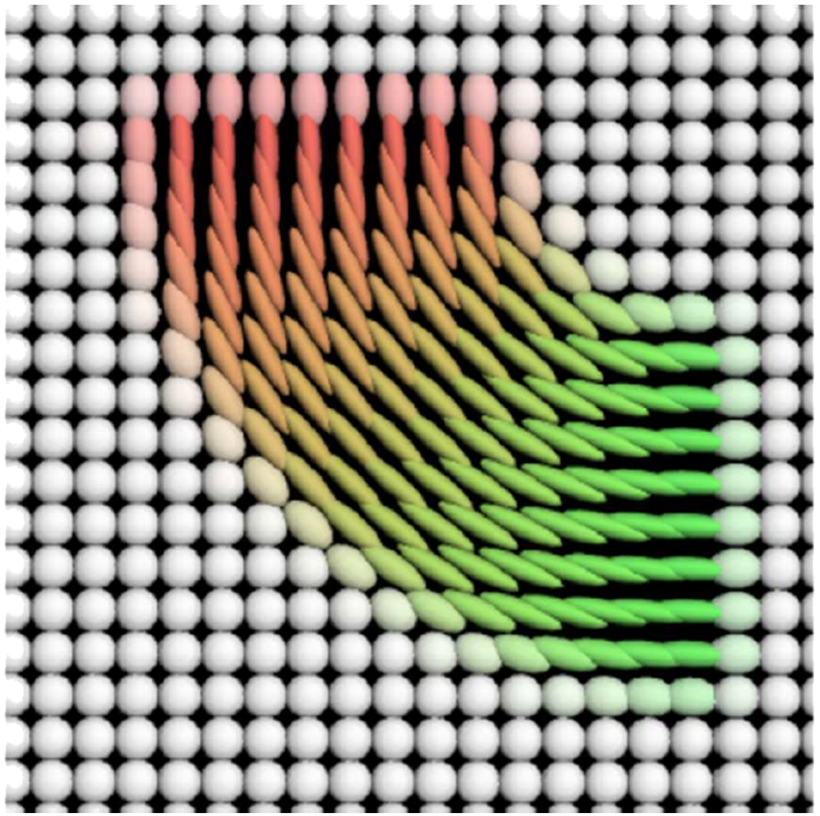
- How to distribute the glyphs
  - To avoid data overload
  - Glyph overlap
- Particle-based energy optimization



[KindImann]

Glyph packing

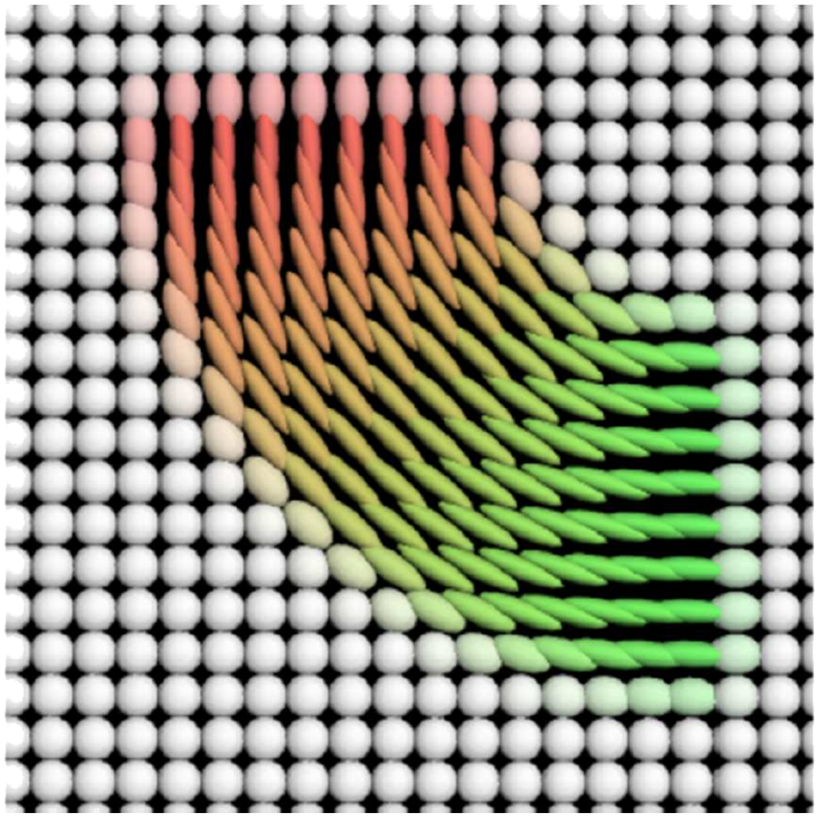
- How to distribute the glyphs
  - To avoid data overload
  - Glyph overlap
- Particle-based energy optimization
  - Given a target number of particles



[Kindlmann]

Glyph packing

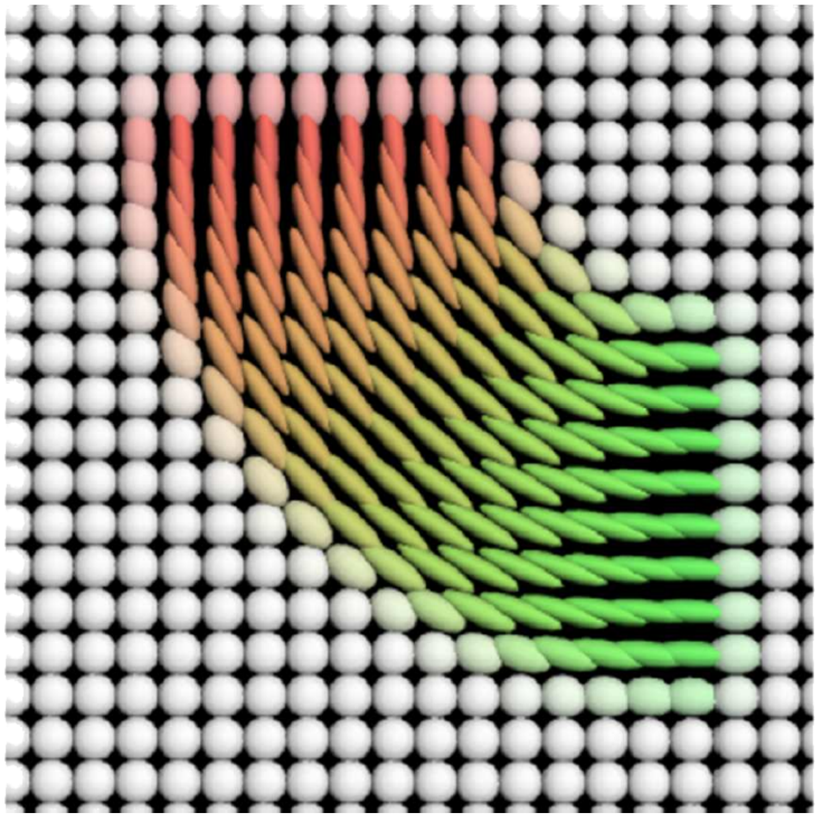
- How to distribute the glyphs
  - To avoid data overload
  - Glyph overlap
- Particle-based energy optimization
  - Given a target number of particles
  - Optimally (globally) scale and place them



[Kindlmann]

Glyph packing

- How to distribute the glyphs
  - To avoid data overload
  - Glyph overlap
- Particle-based energy optimization
  - Given a target number of particles
  - Optimally (globally) scale and place them
    - To minimize overlap

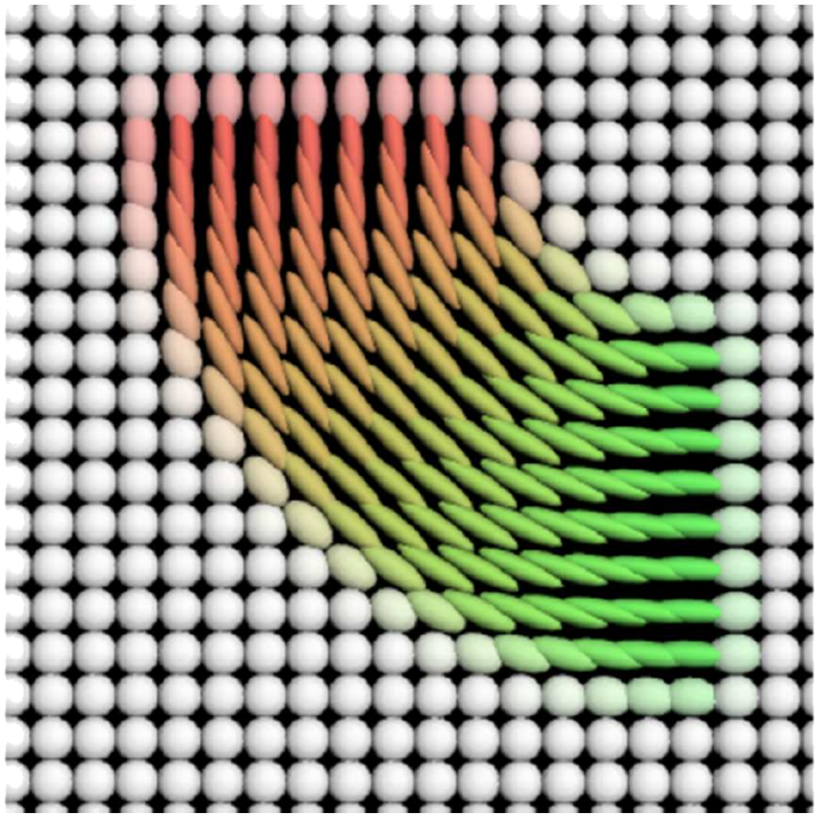


[KindImann]

Glyph packing

- How to distribute the glyphs
  - To avoid data overload
  - Glyph overlap

- Particle-based energy optimization
  - Given a target number of particles
  - Optimally (globally) scale and place them
    - To minimize overlap
  - Maximize the sum of distances between glyphs

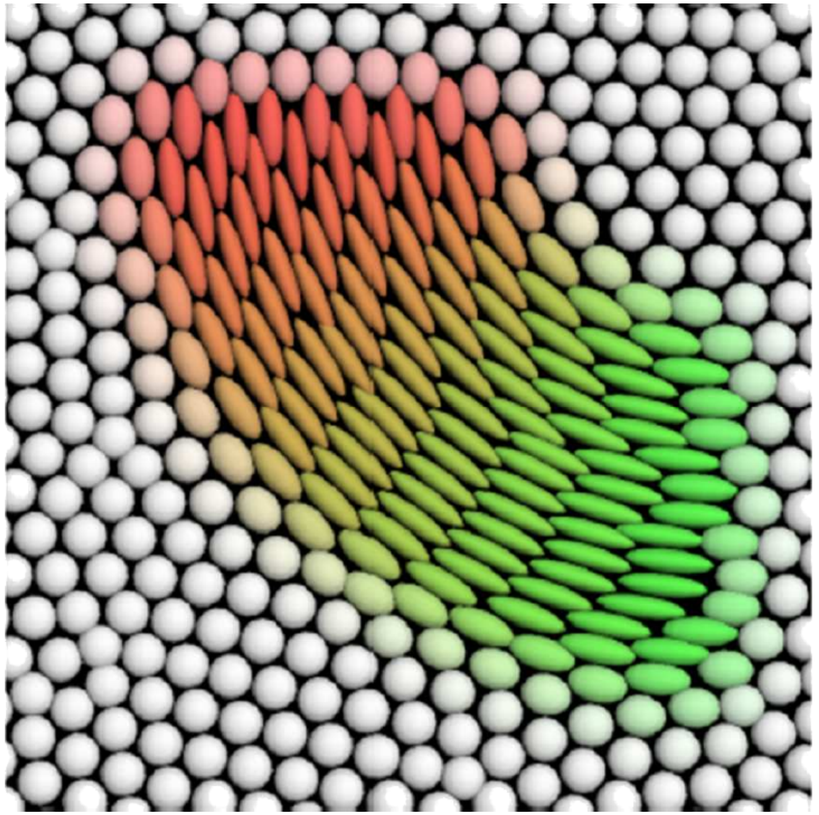


[Kindlmann]

Glyph packing

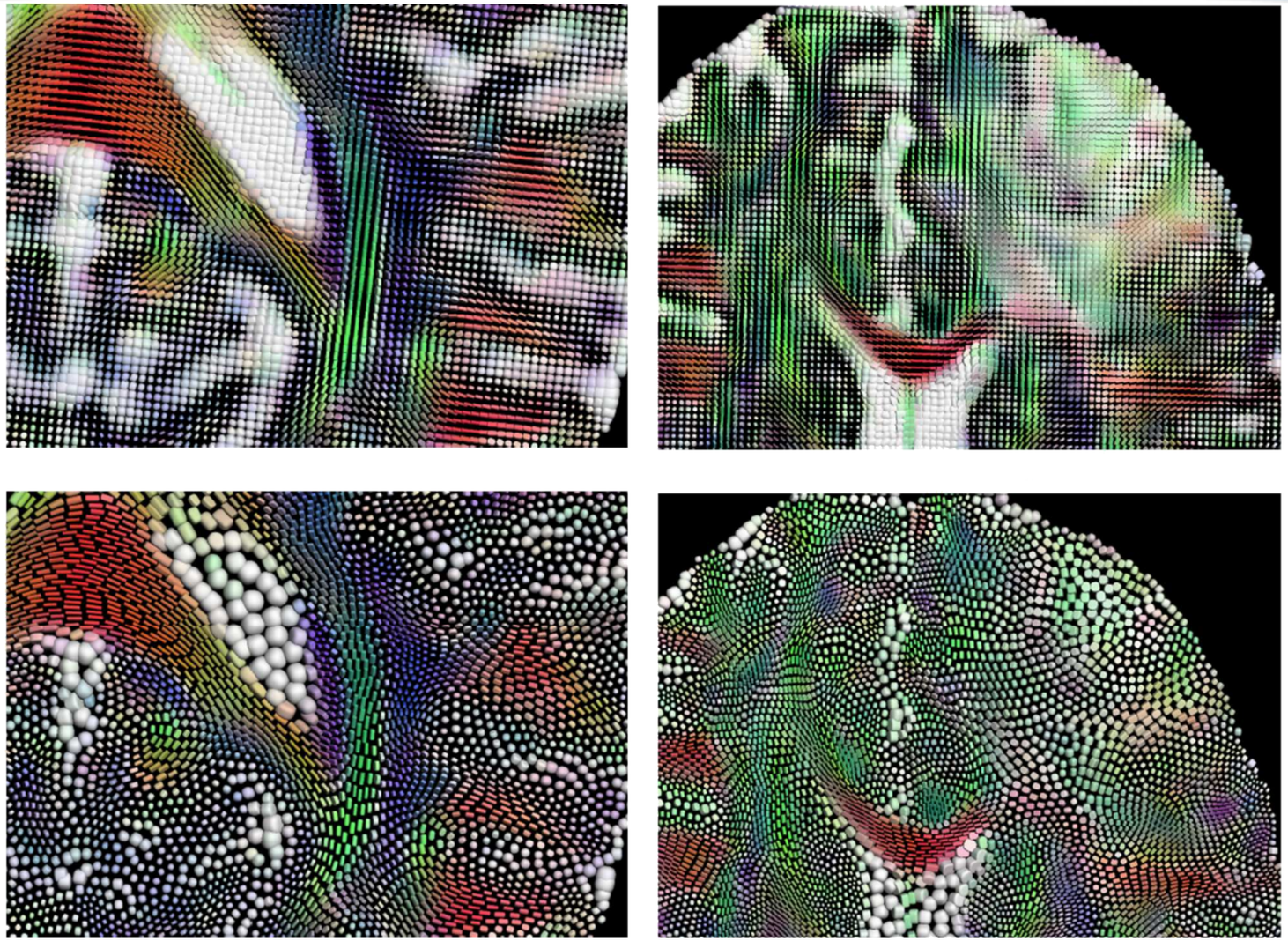
- How to distribute the glyphs
  - To avoid data overload
  - Glyph overlap

- Particle-based energy optimization
  - Given a target number of particles
  - Optimally (globally) scale and place them
    - To minimize overlap
  - Maximize the sum of distances between glyphs



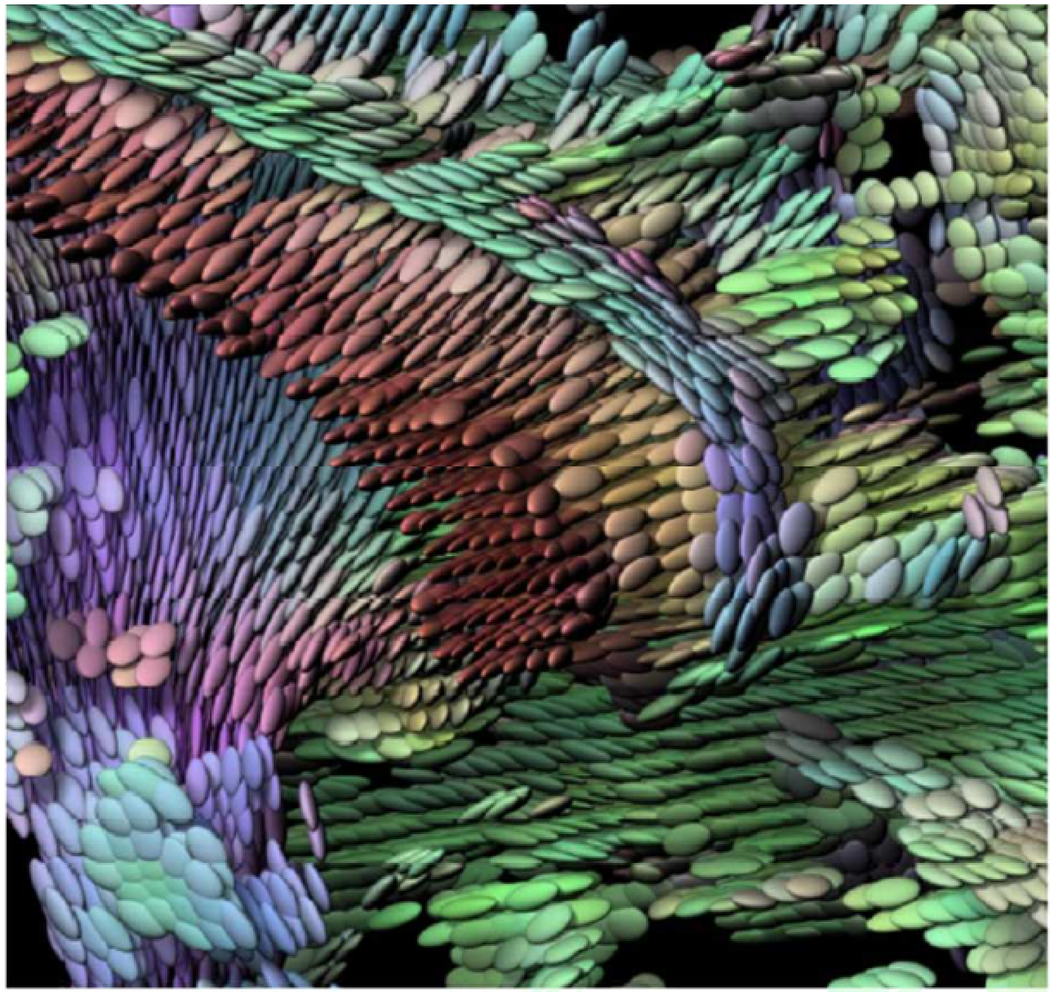
[KindImann]

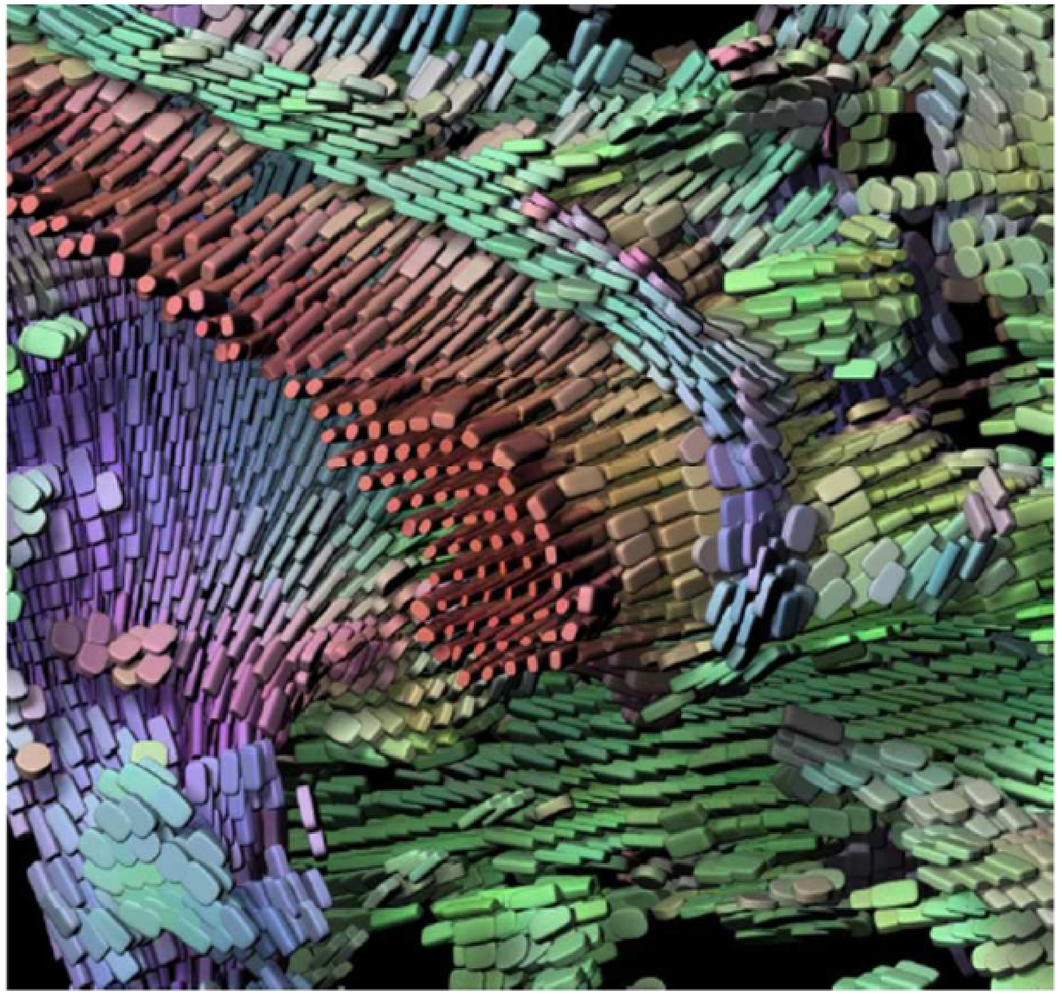
Glyph packing



[Kindlmann]

Putting it all together

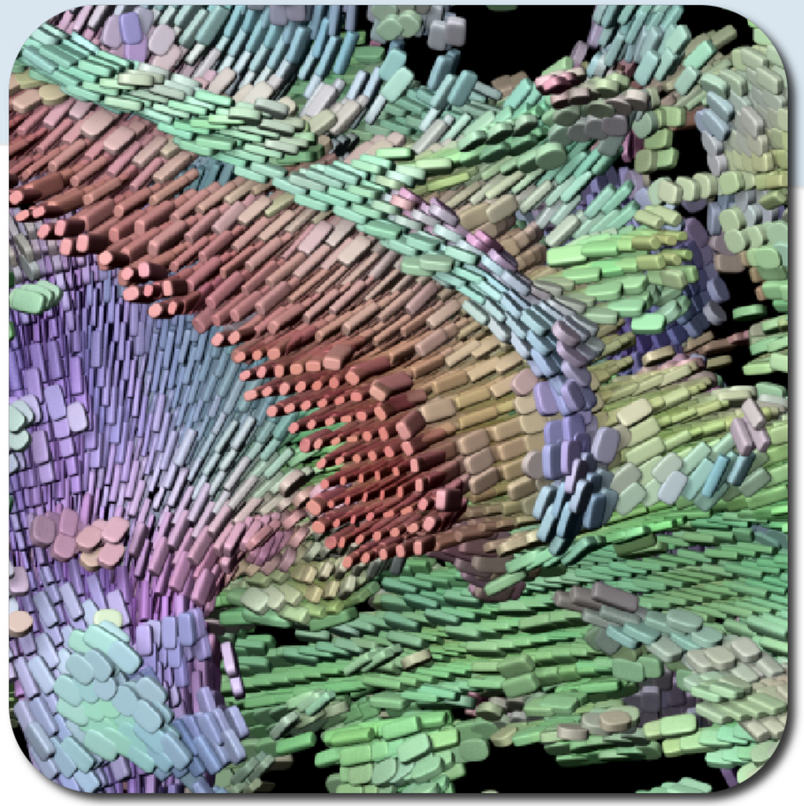




[Kindlmann]

Tensor glyphs

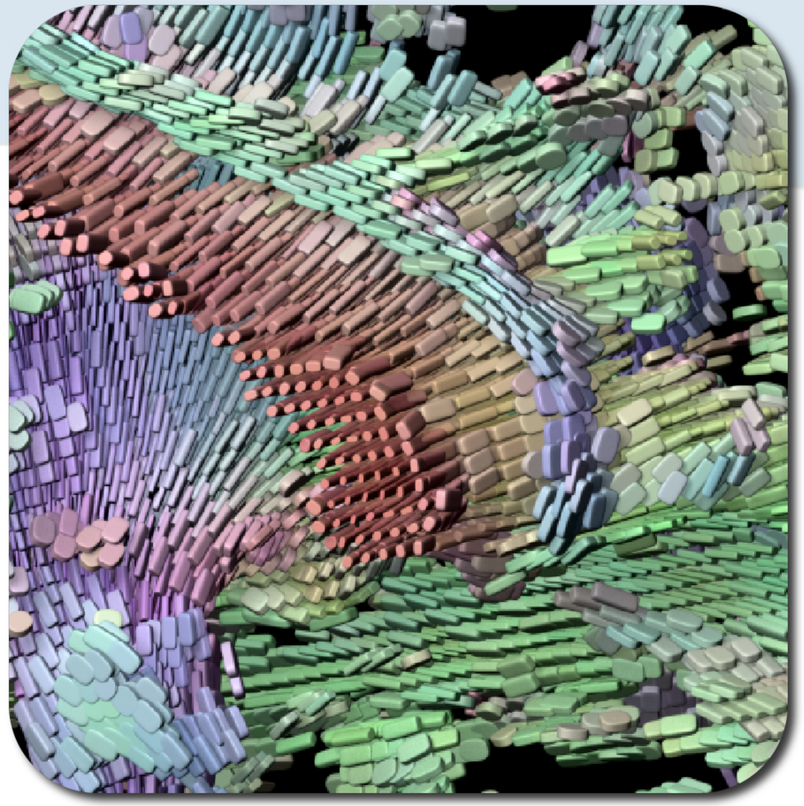
- Provide important insights



[Kindlmann]

Tensor glyphs

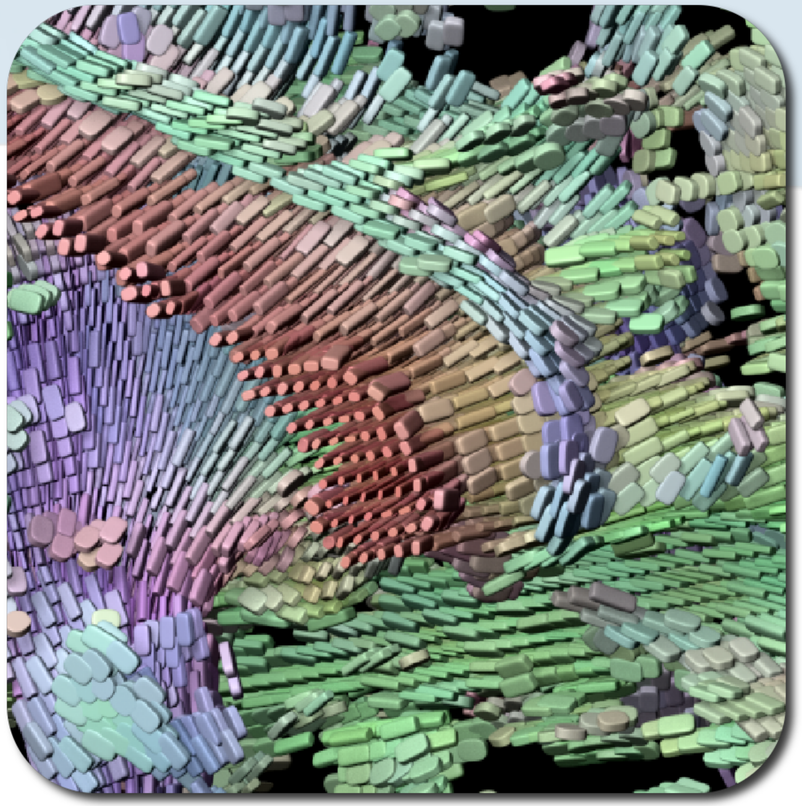
- Provide important insights
  - Direction information



[Kindlmann]

Tensor glyphs

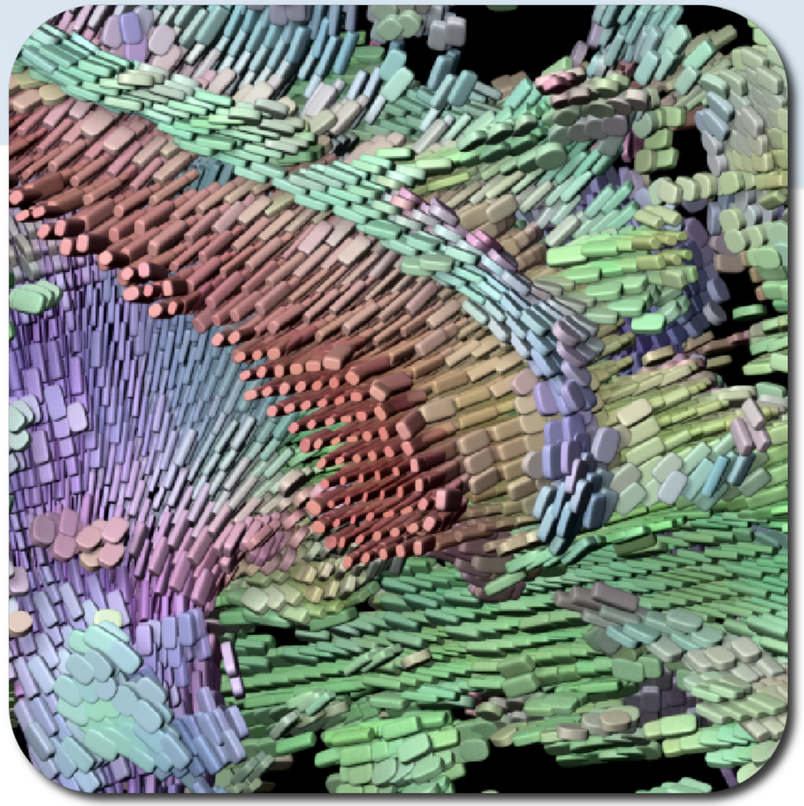
- Provide important insights
  - Direction information
  - Anisotropy information



[Kindlmann]

Tensor glyphs

- Provide important insights
  - Direction information
  - Anisotropy information
- Still
  - Occlusion issues

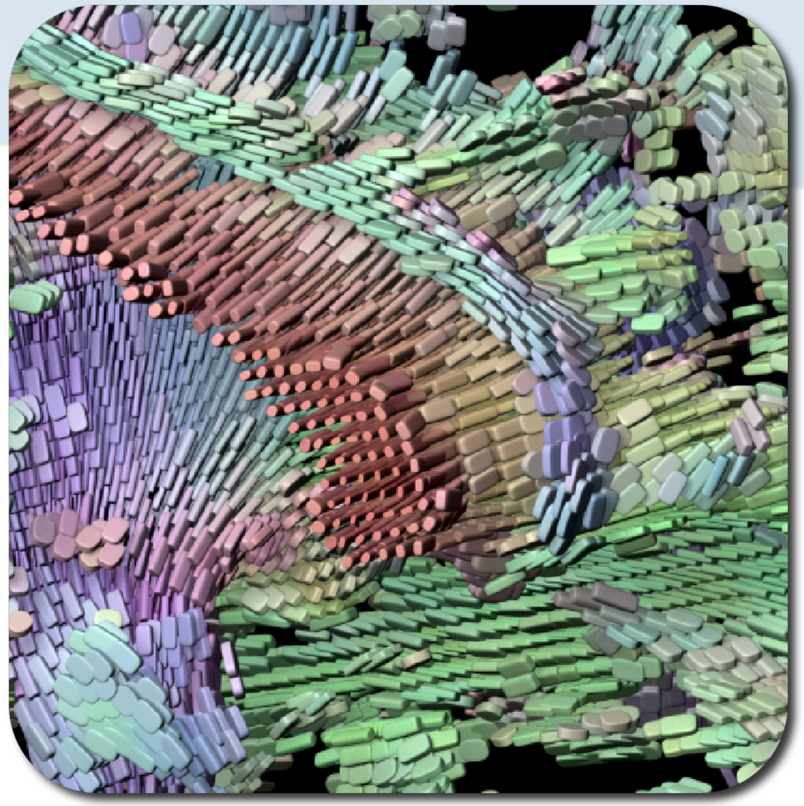


[Kindlmann]

Tensor glyphs

- Provide important insights
  - Direction information
  - Anisotropy information

- Still
  - Occlusion issues
  - No global information

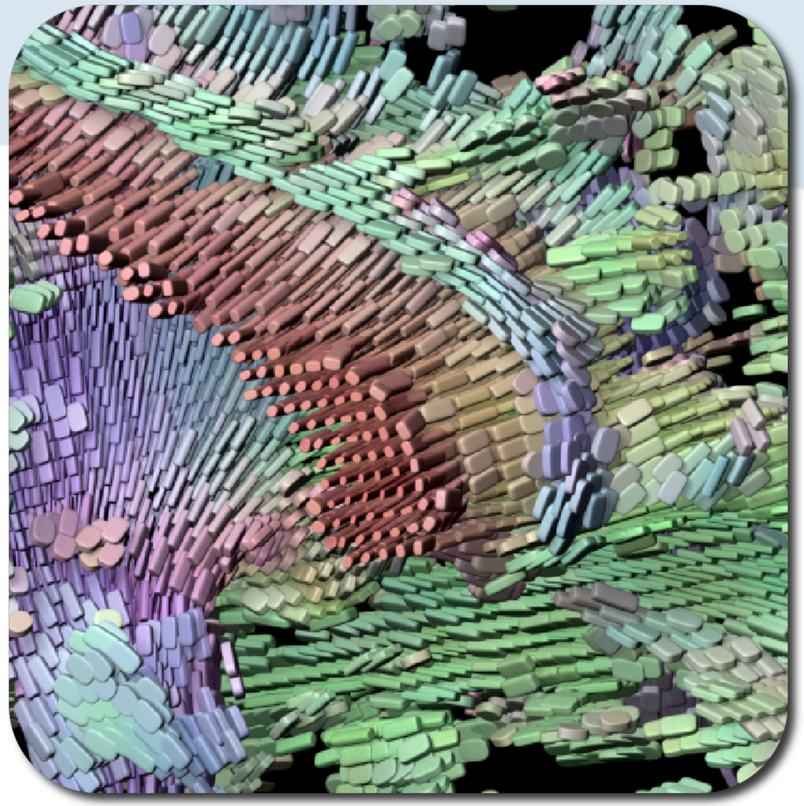


[Kindlmann]

Tensor glyphs

- Provide important insights
  - Direction information
  - Anisotropy information

- Still
  - Occlusion issues
  - No global information
  - Dependent on the number of glyphs
    - Trade-off



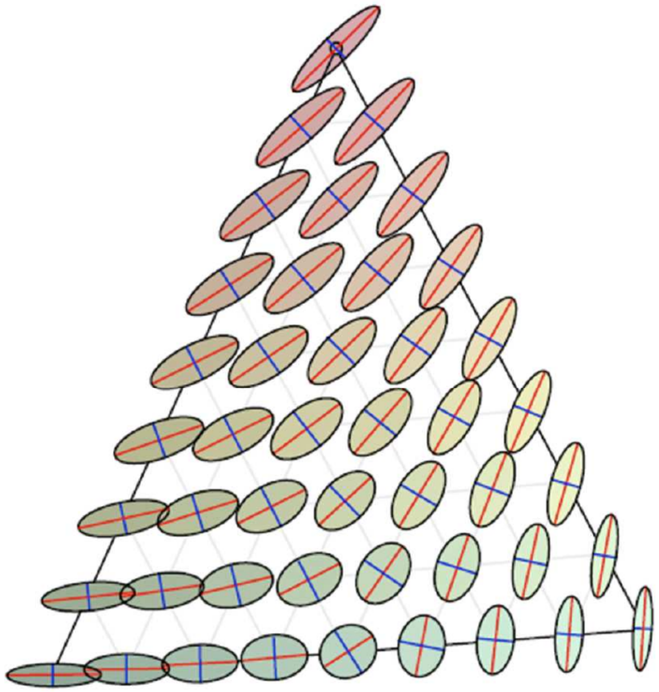
[Kindlmann]

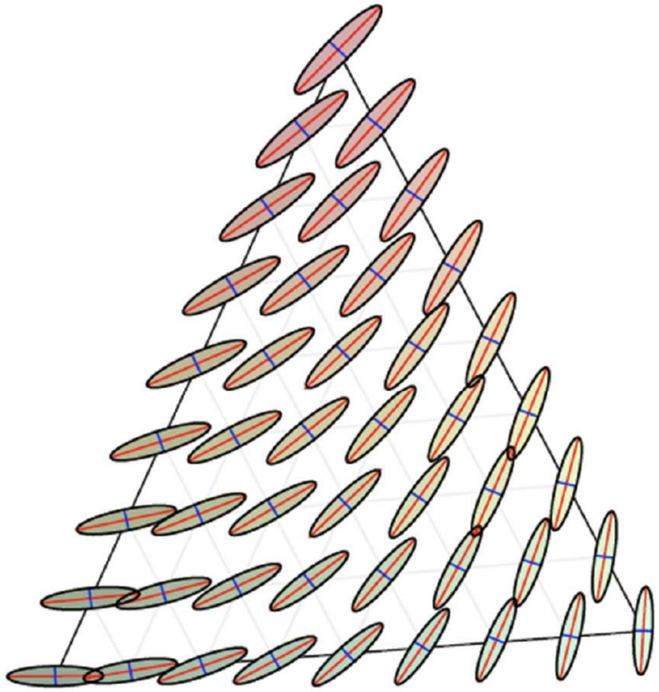
Derived vector fields

- What interesting vector fields could we consider?

Derived vector fields

- What interesting vector fields could we consider?
  - Eigenvectors

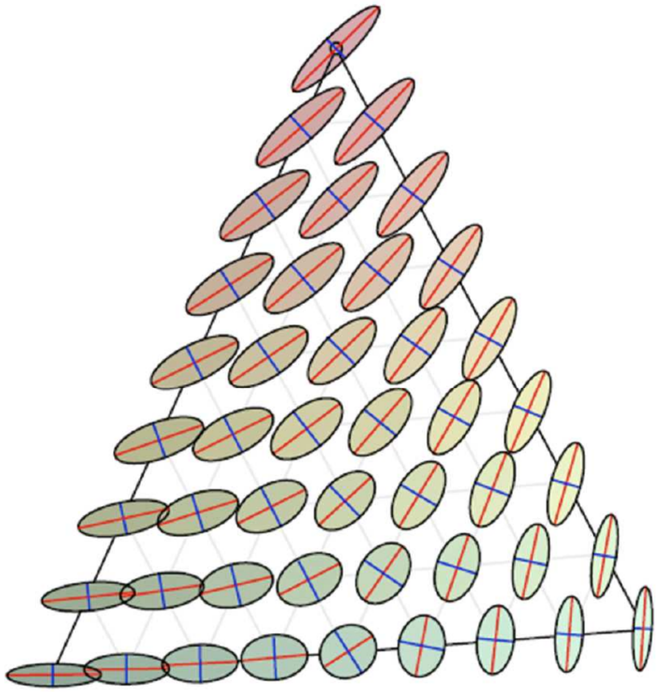


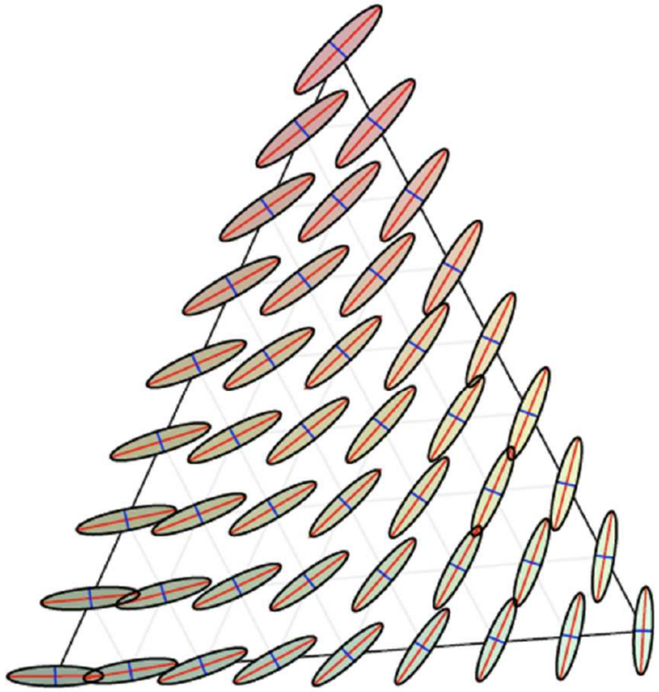


[Hotz]

Derived vector fields

- What interesting vector fields could we consider?
  - Eigenvectors
    - Not a real vector field

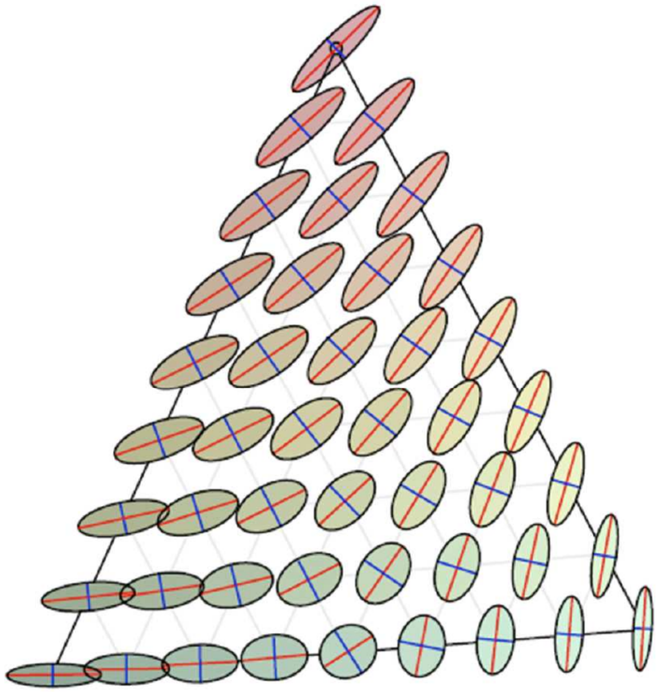


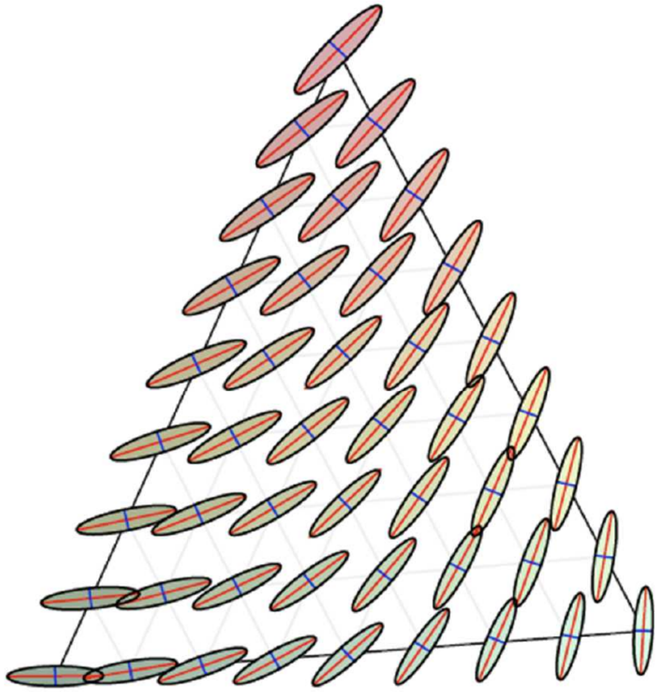


[Hotz]

Derived vector fields

- What interesting vector fields could we consider?
  - Eigenvectors
    - Not a real vector field
      - No magnitude
      - No orientation

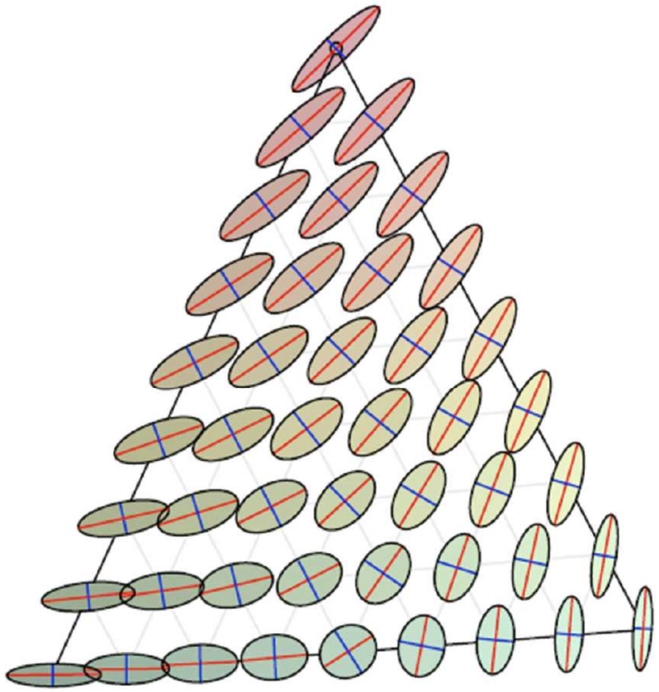


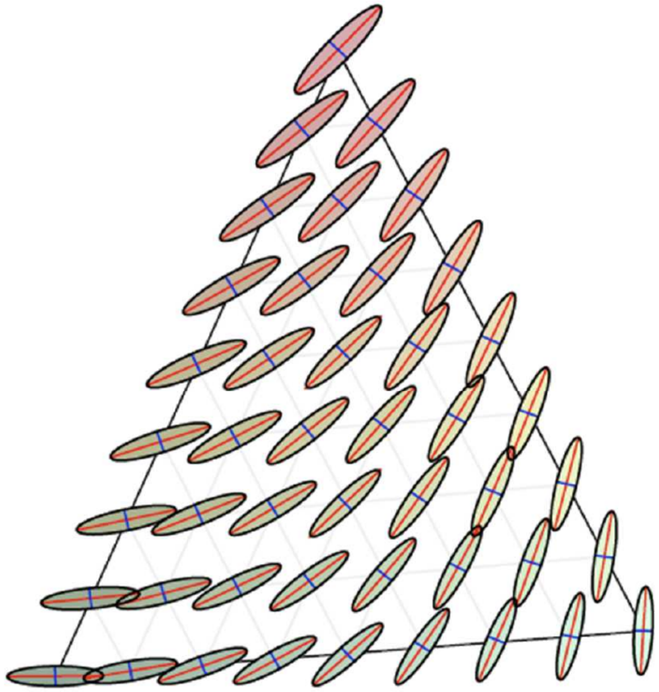


[Hotz]

Derived vector fields

- What interesting vector fields could we consider?
  - Eigenvectors
    - Not a real vector field
      - No magnitude
      - No orientation
  - Magnitude
    - Eigenvalues

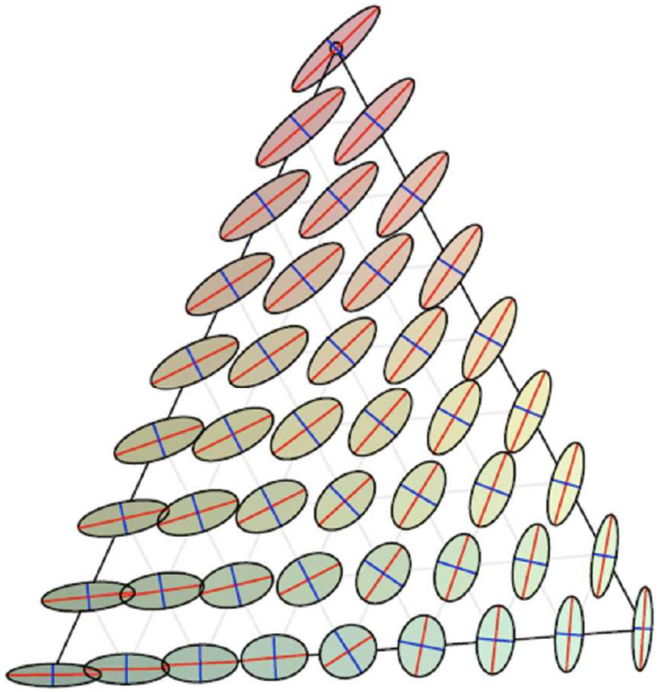


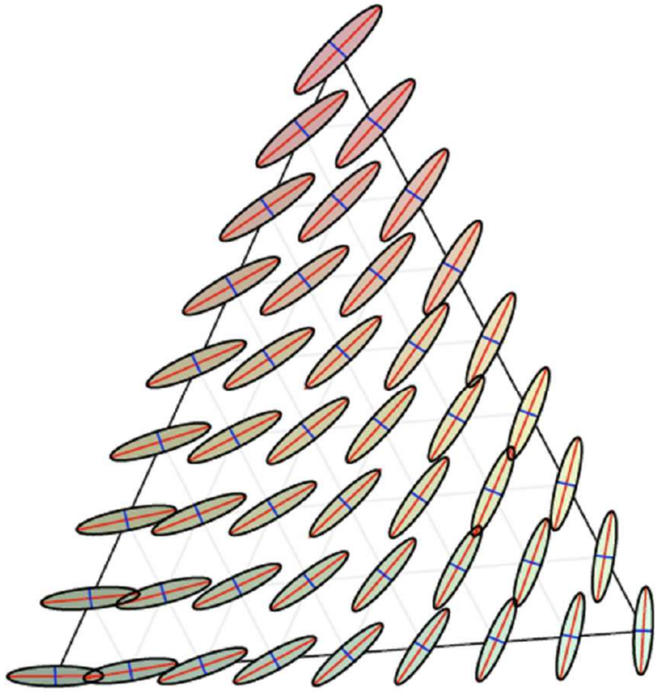


[Hotz]

Derived vector fields

- What interesting vector fields could we consider?
  - Eigenvectors
    - Not a real vector field
      - No magnitude
      - No orientation
    - Magnitude
      - Eigenvalues
  - Ambiguous entity

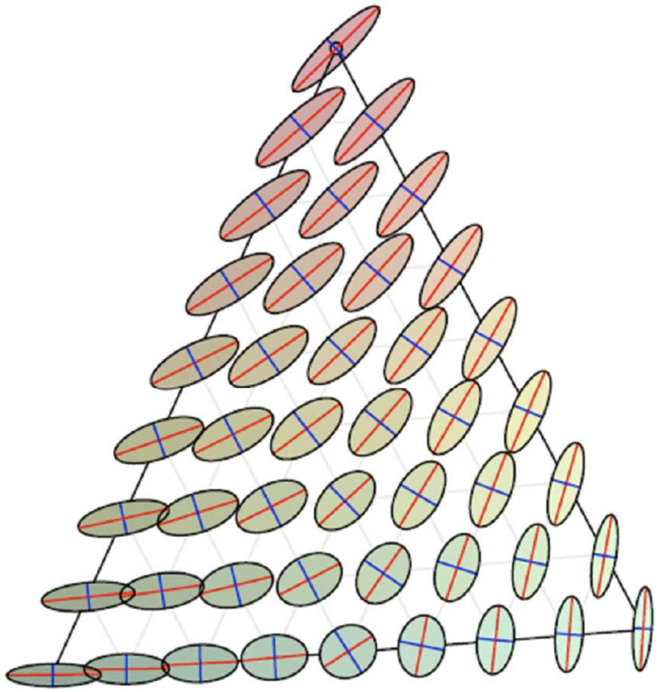


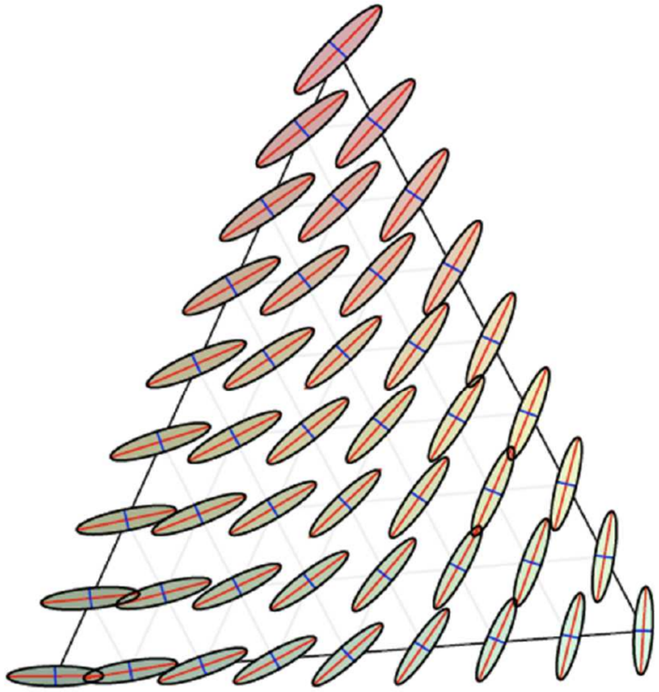


[Hotz]

Derived vector fields

- What interesting vector fields could we consider?
  - Eigenvectors
    - Not a real vector field
      - No magnitude
      - No orientation
  - Magnitude
    - Eigenvalues
  - Ambiguous entity
    - Notion of direction field

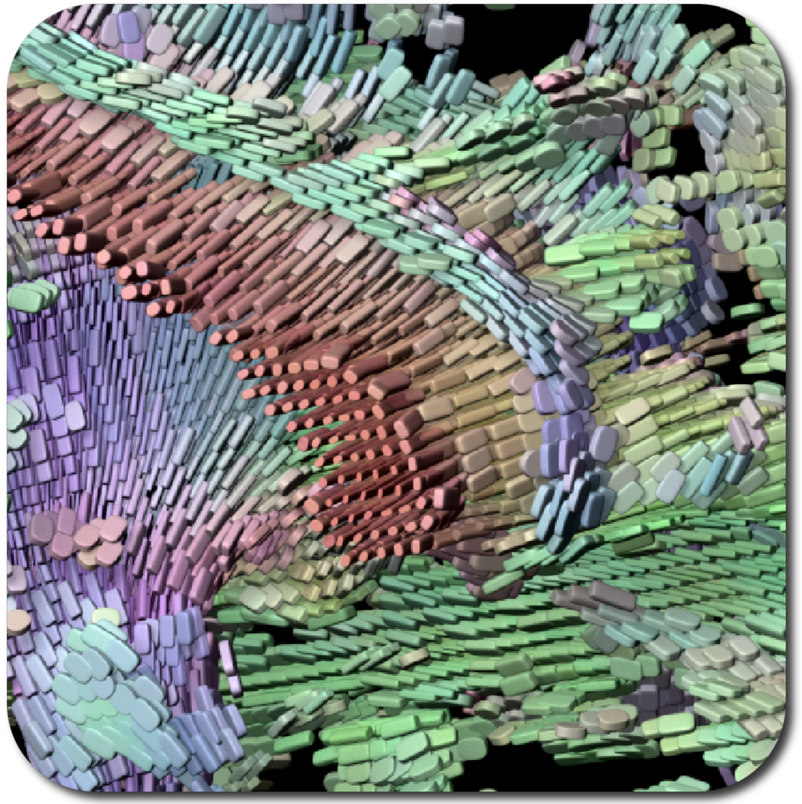




[Hotz]

Direction fields

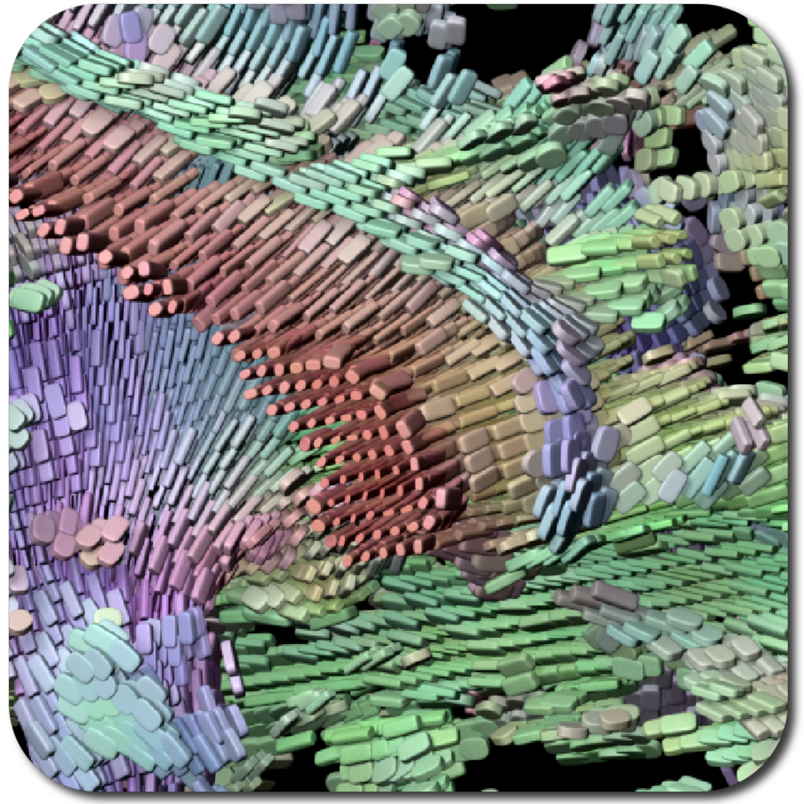
- Set of orthogonal “pseudo” vectors



[Kindlmann]

Direction fields

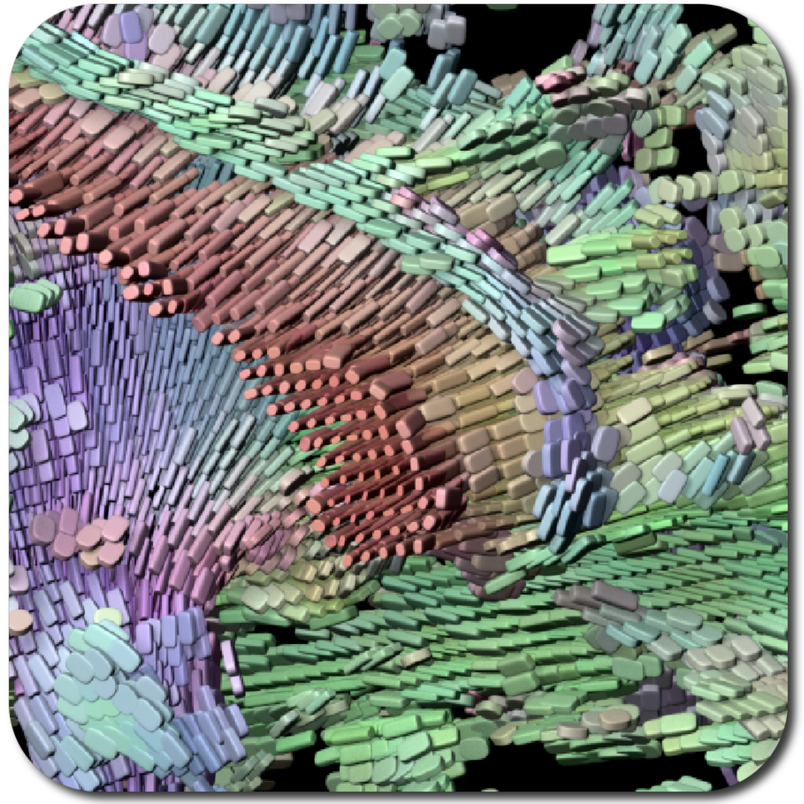
- Set of orthogonal “pseudo” vectors
  - Pulling all the vector field visualization techniques



[Kindlmann]

Direction fields

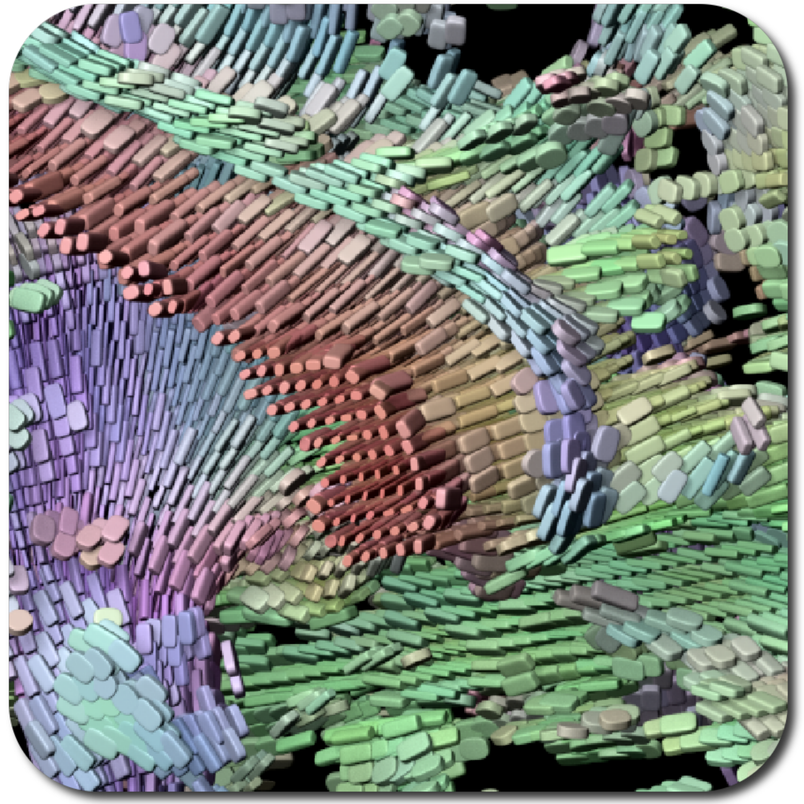
- Set of orthogonal “pseudo” vectors
  - Pulling all the vector field visualization techniques
    - Streamline computation
    - Streamline seeding
    - LIC



[Kindlmann]

Direction fields

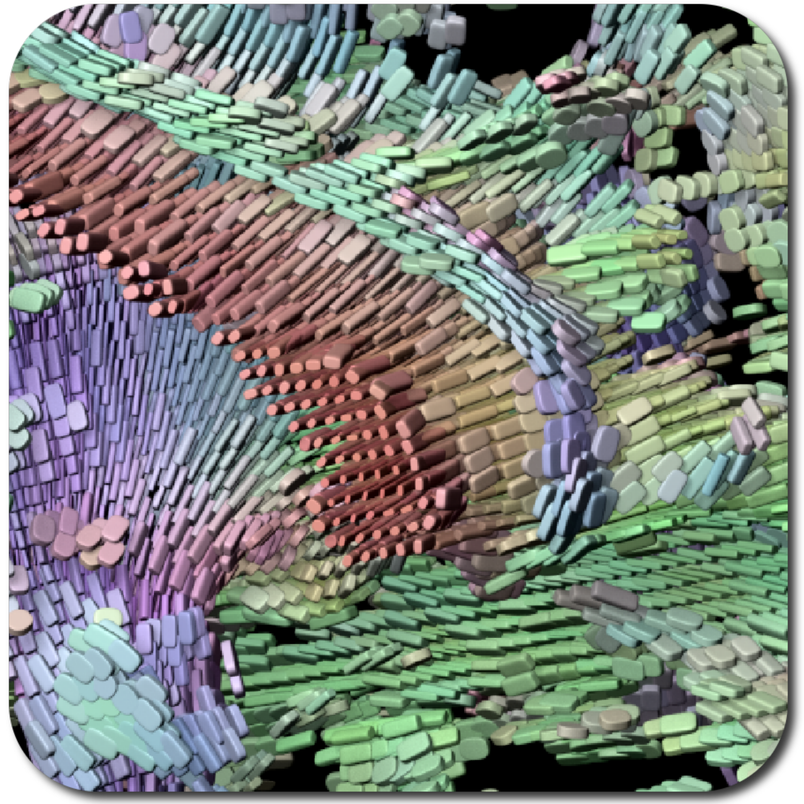
- Set of orthogonal “pseudo” vectors
  - Pulling all the vector field visualization techniques
    - Streamline computation
    - Streamline seeding
    - LIC
  - How to combine the directions?



[Kindlmann]

Direction fields

- Set of orthogonal “pseudo” vectors
  - Pulling all the vector field visualization techniques
    - Streamline computation
    - Streamline seeding
    - LIC
  - How to combine the directions?
    - Get inspiration from …

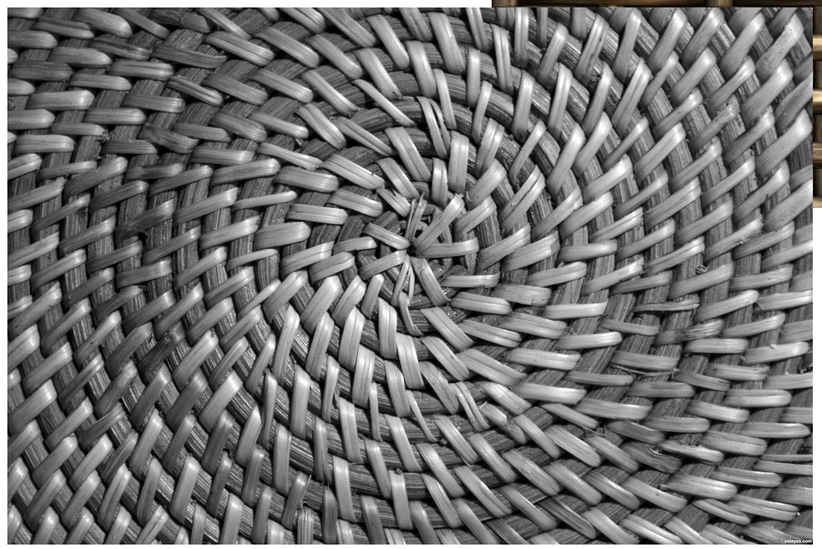


[Kindlmann]

Direction fields

- Set of orthogonal “pseudo” vectors
  - Pulling all the vector field visualization techniques
    - Streamline computation
    - Streamline seeding
    - LIC
  - How to combine the directions?
    - Get inspiration from … craft

[http://www.123rf.com]

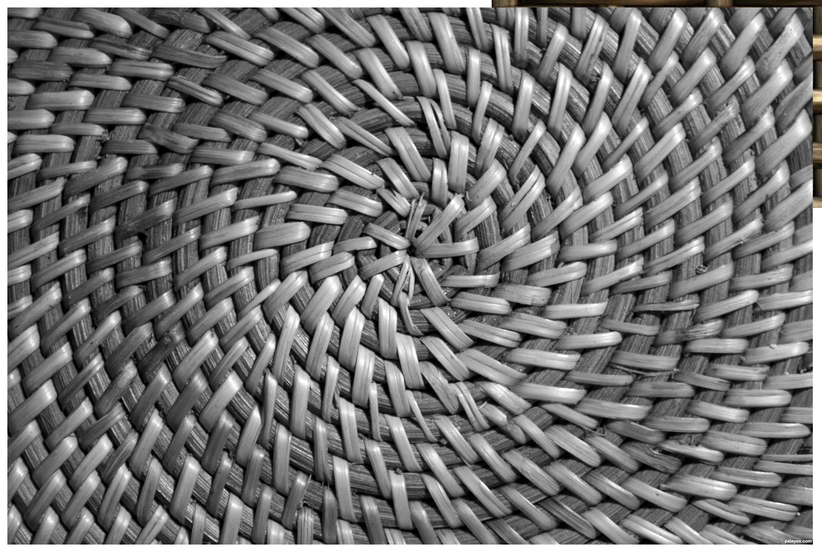


[http://www.pxleyes.com]

Direction fields

- Set of orthogonal “pseudo” vectors
  - Pulling all the vector field visualization techniques
    - Streamline computation
    - Streamline seeding
    - LIC
  - How to combine the directions?
    - Get inspiration from … craft
    - Overlay the directions

[http://www.123rf.com]

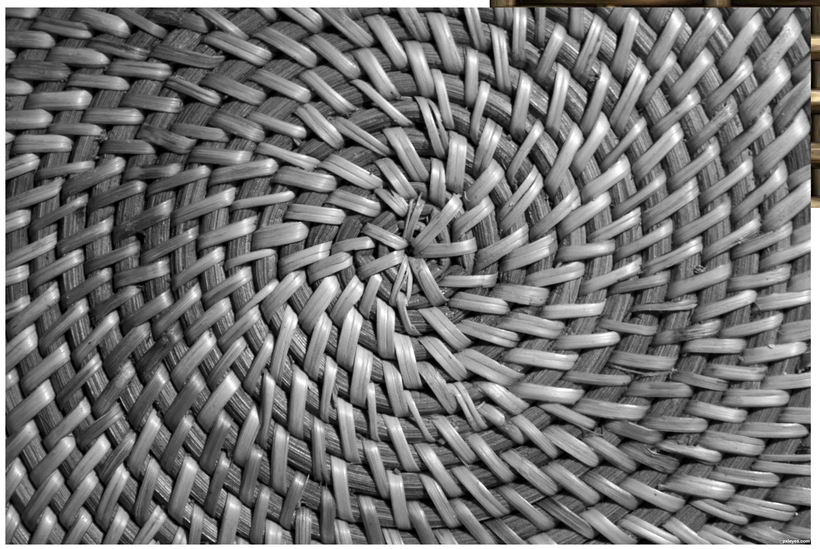


[http://www.pxleyes.com]

Hyper-streamlines

- For each of the d directions

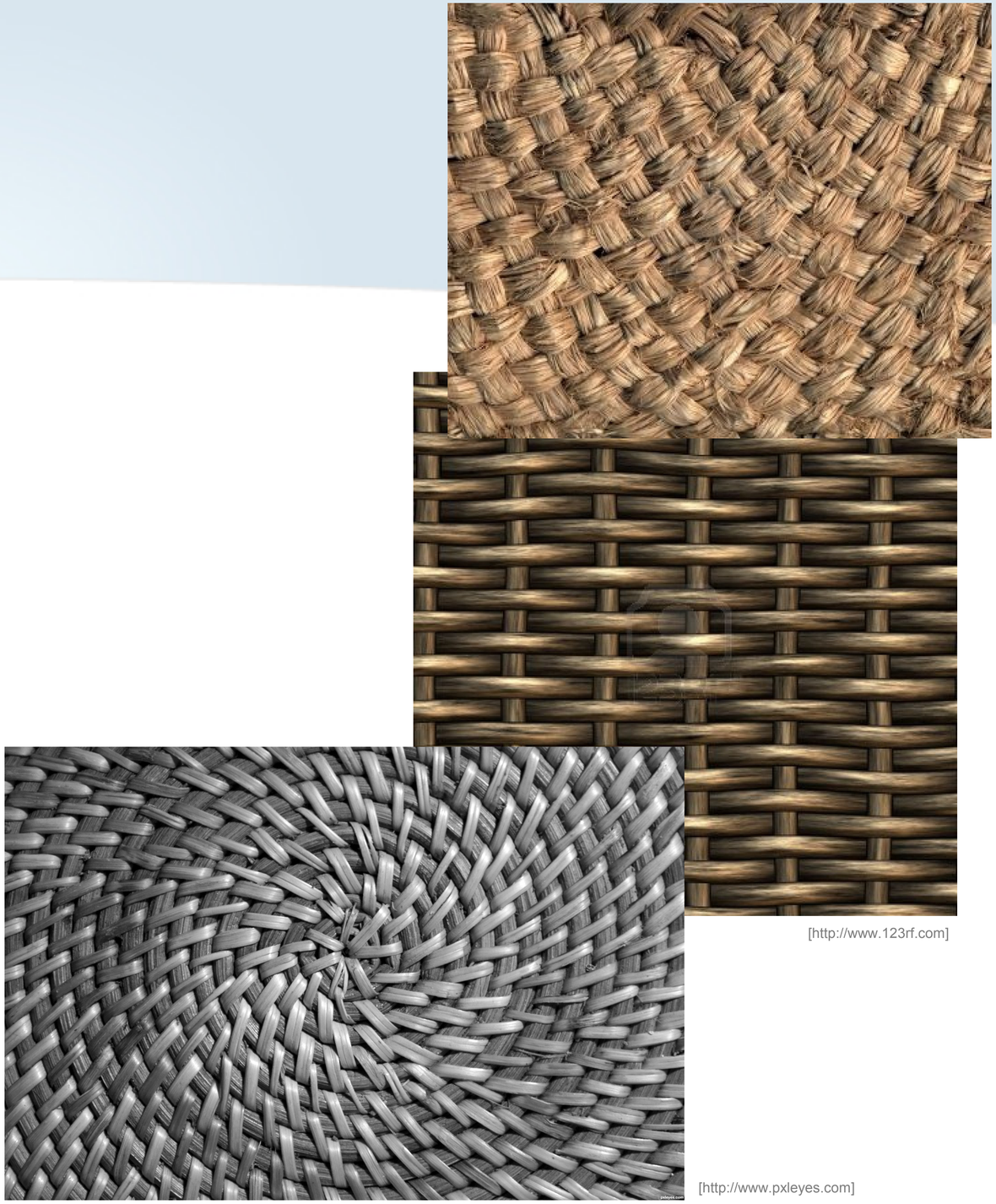
[http://www.123rf.com]



[http://www.pxleyes.com]

Hyper-streamlines

- For each of the d directions
  - Streamline integration

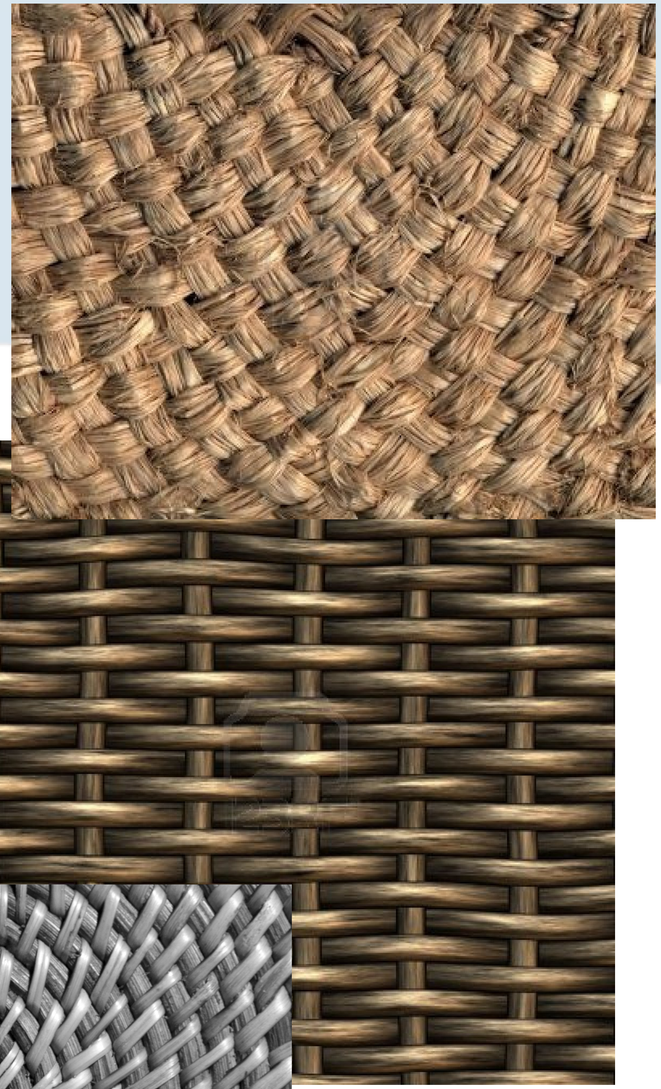


[http://www.123rf.com]

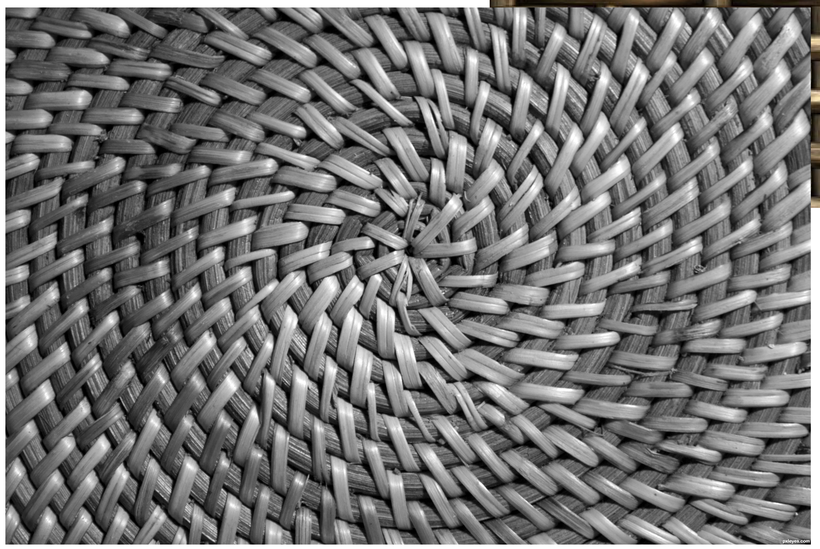
[http://www.pxleyes.com]

Hyper-streamlines

- For each of the d directions
  - Streamline integration
    - We know how to do it for vector fields



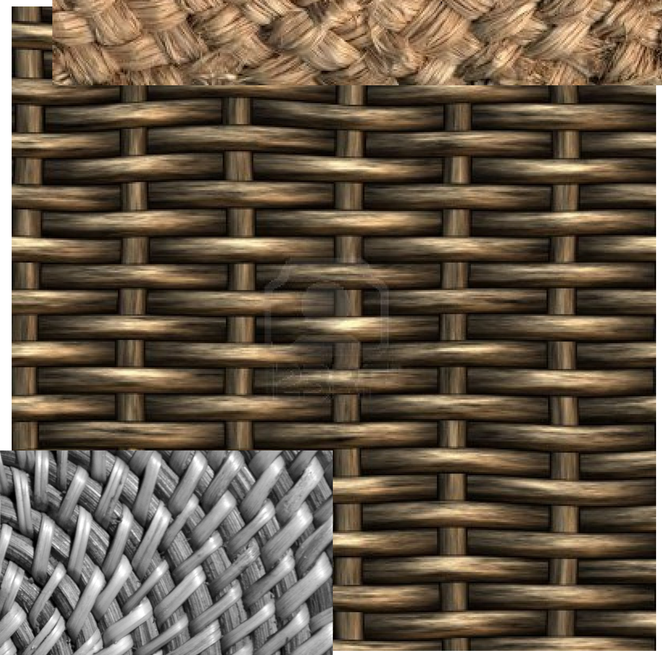
[http://www.123rf.com]



[http://www.pxleyes.com]

Hyper-streamlines

- For each of the d directions
  - Streamline integration
    - We know how to do it for vector fields
  - Problem



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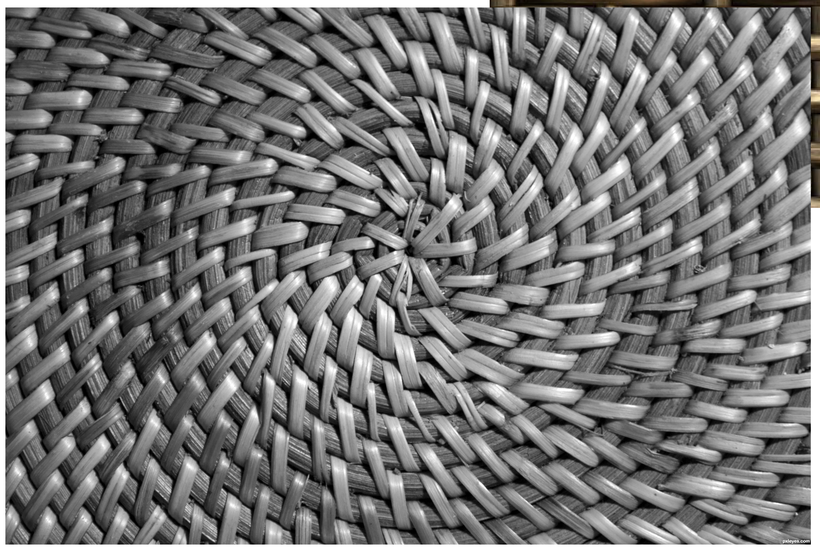
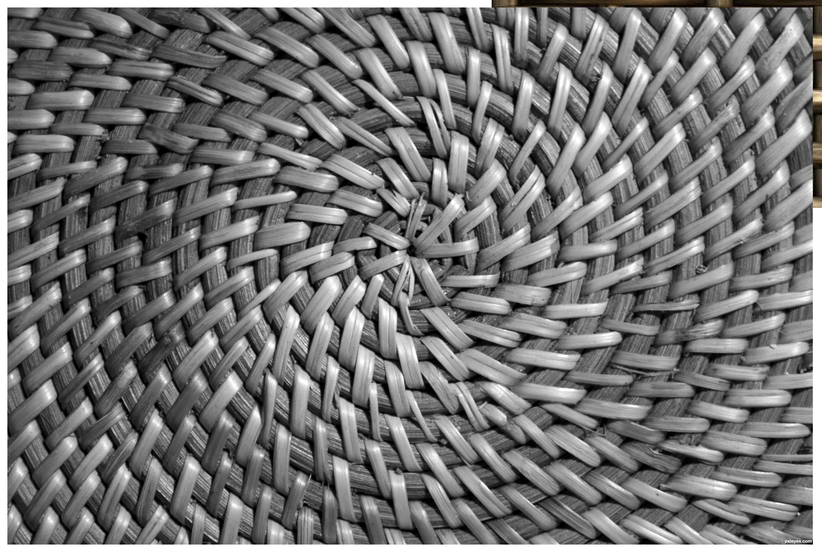


[http://www.pxleyes.com]

Hyper-streamlines

- For each of the d directions
  - Streamline integration
    - We know how to do it for vector fields
  - Problem
    - No orientation

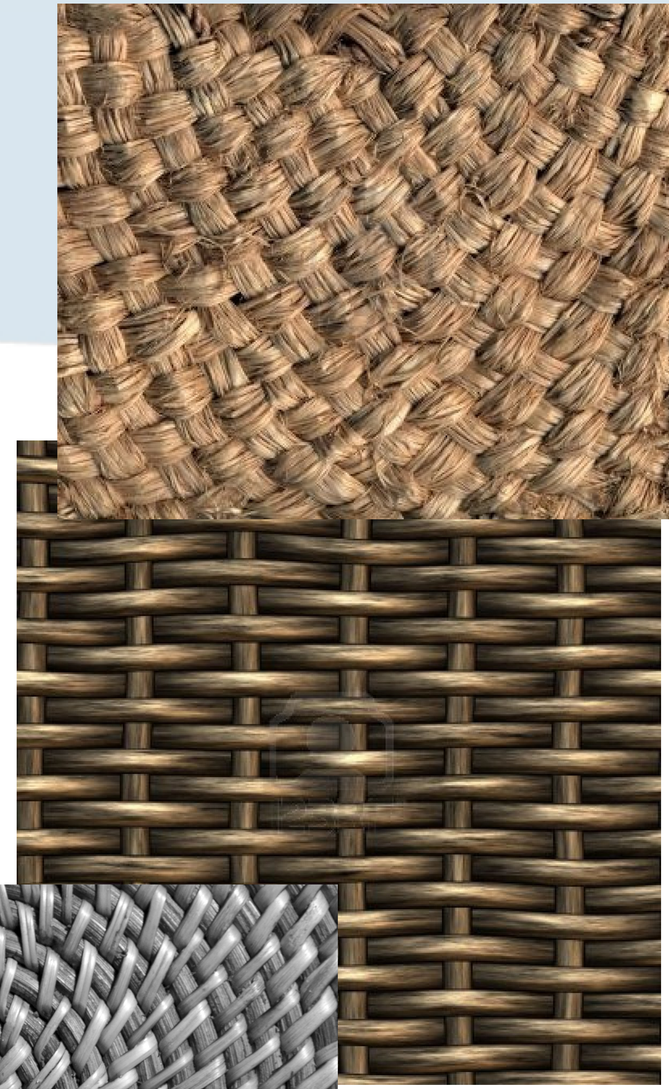
[http://www.123rf.com]



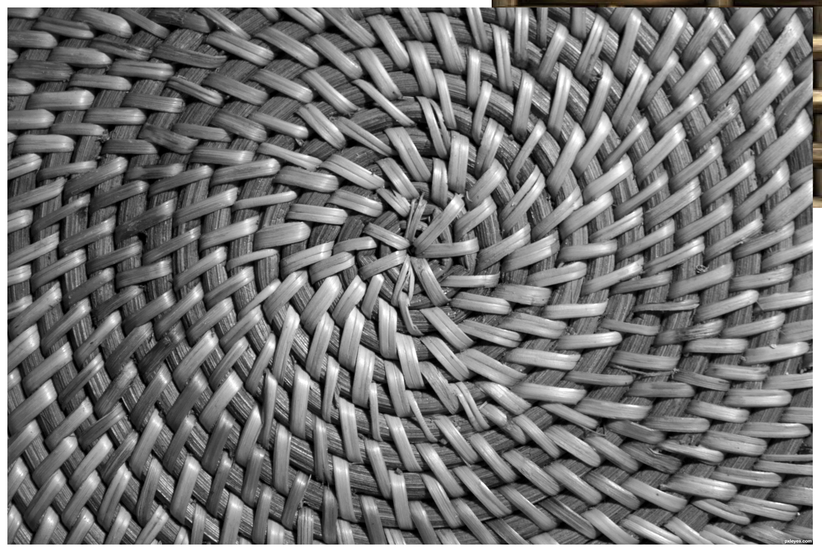
[http://www.pxleyes.com]

Hyper-streamlines

- For each of the d directions
  - Streamline integration
    - We know how to do it for vector fields
  - Problem
    - No orientation
    - No clear matches across cells



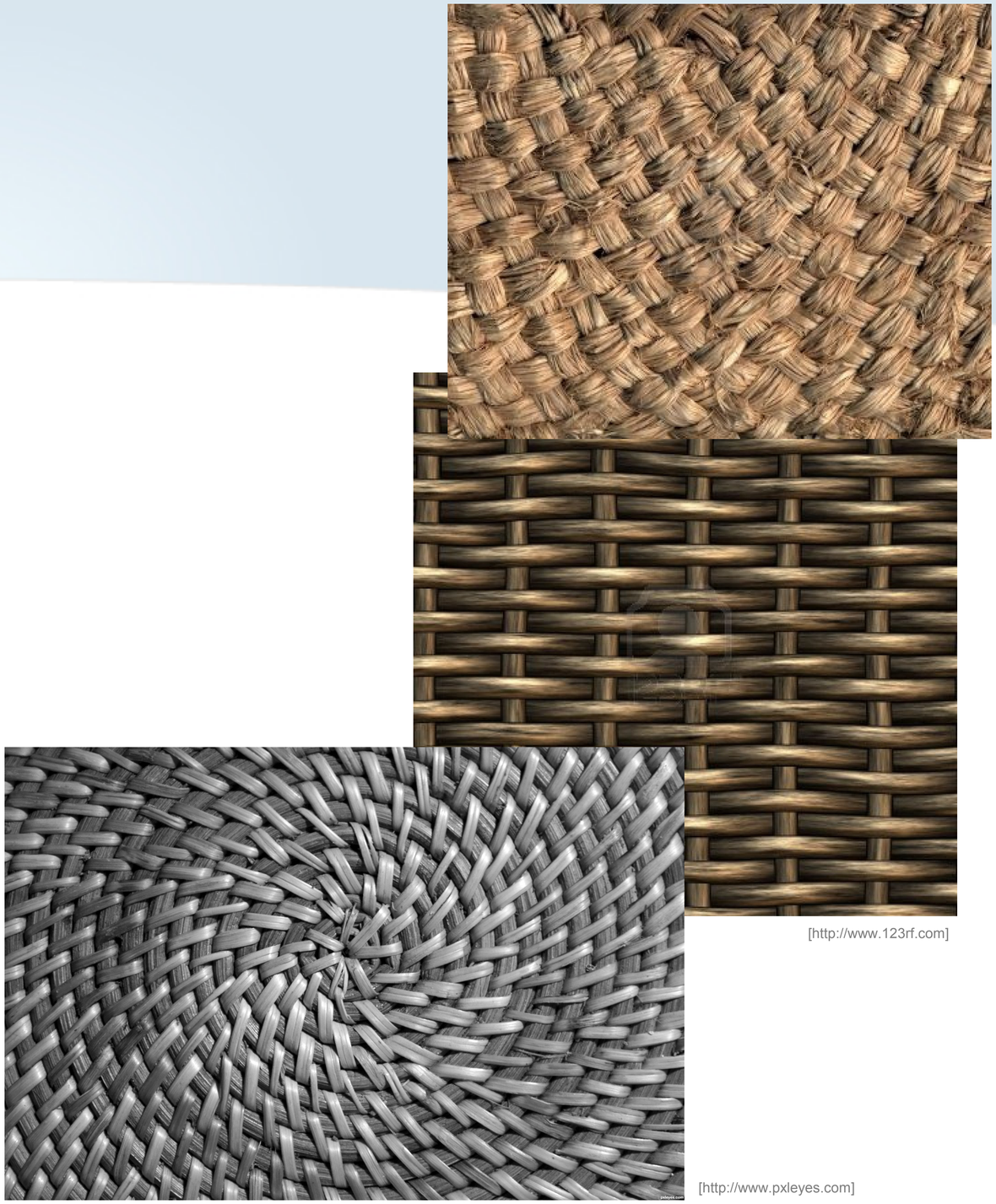
[http://www.123rf.com]



[http://www.pxleyes.com]

Hyper-streamlines

- Other than that...

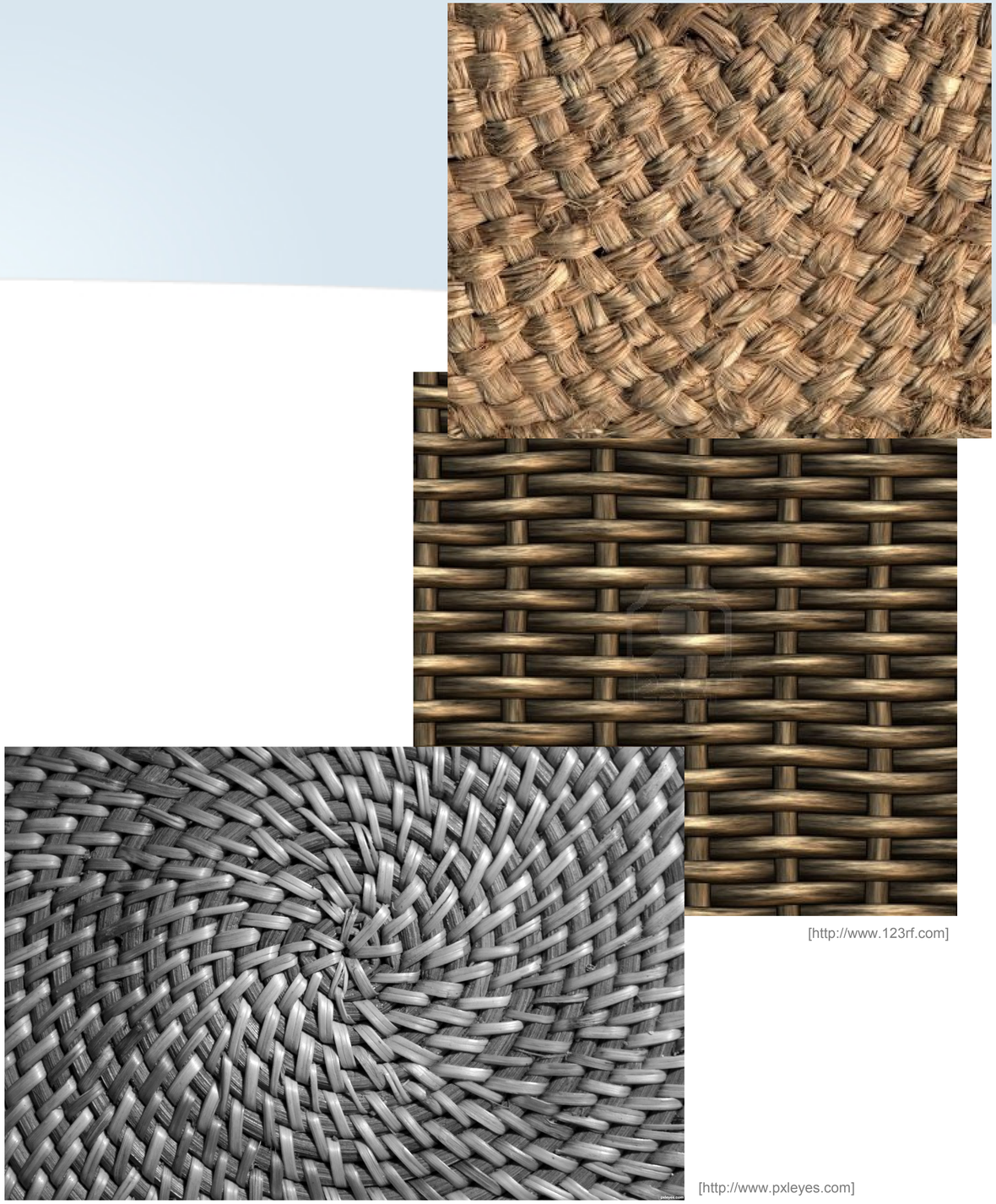


[http://www.123rf.com]

[http://www.pxleyes.com]

Hyper-streamlines

- Other than that...
  - Same integration as for vector fields



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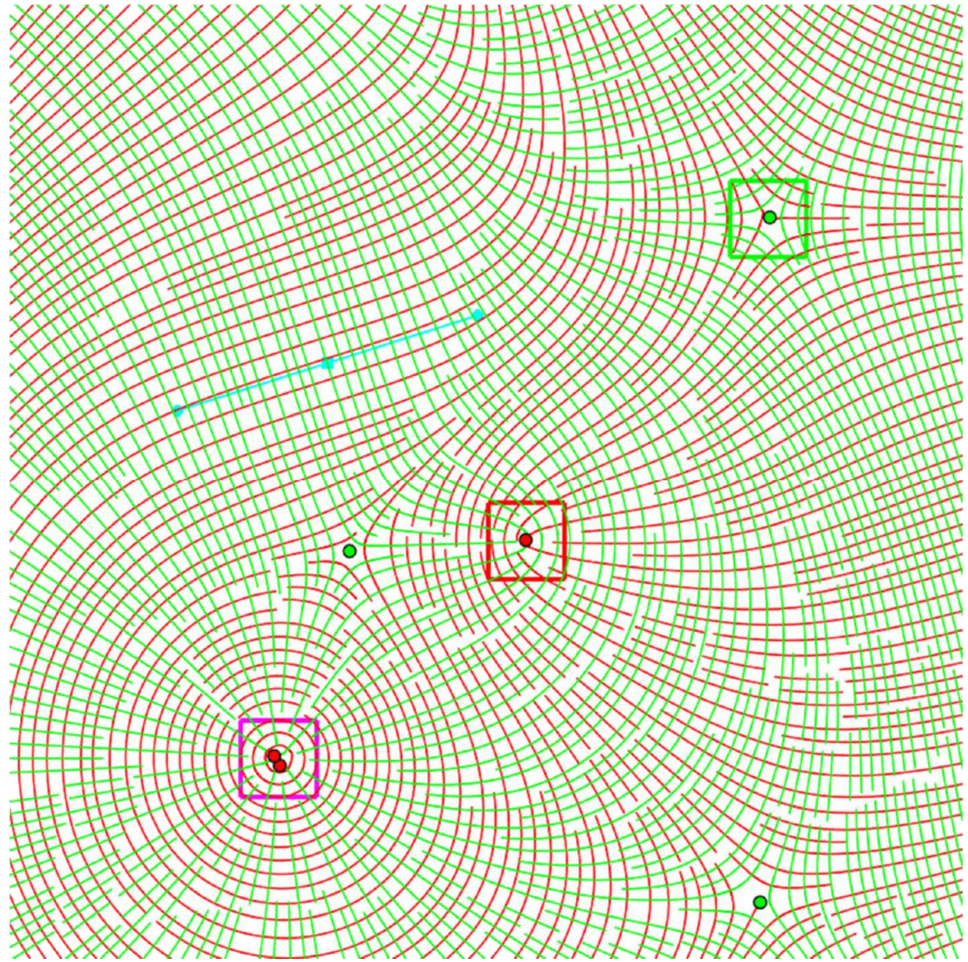
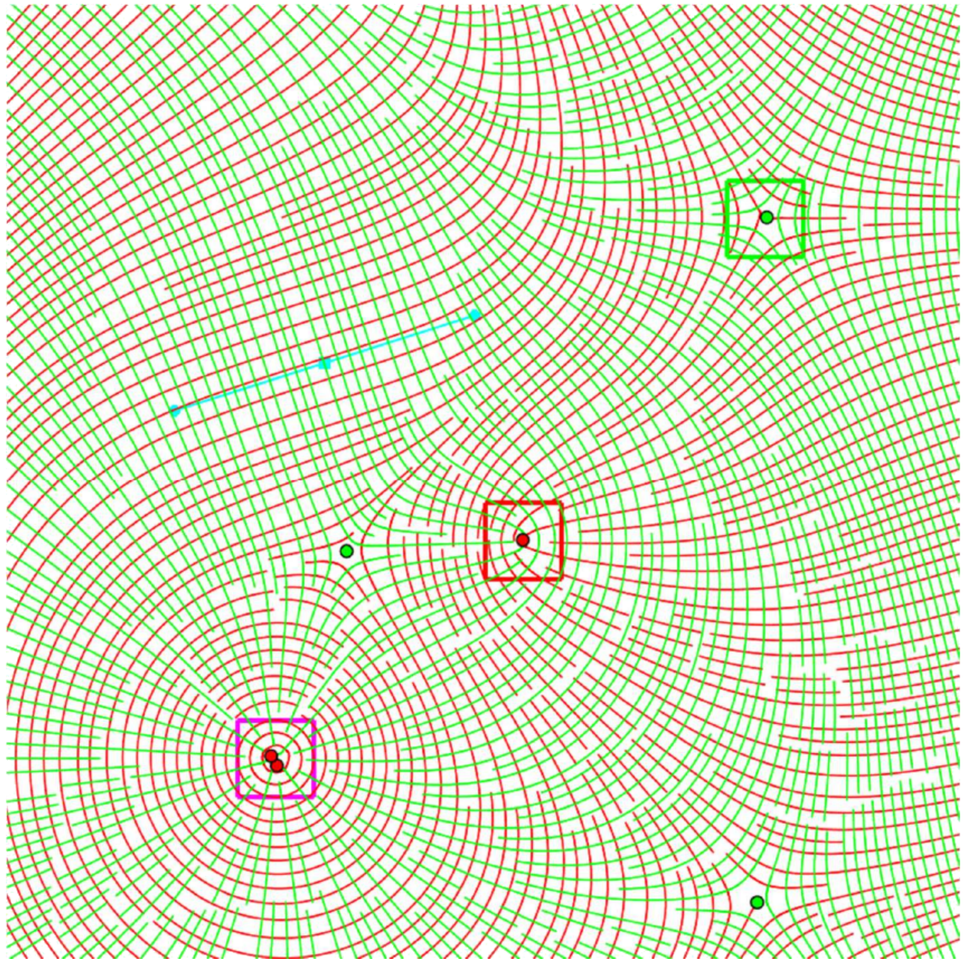


[Chen]

Hyper-streamlines

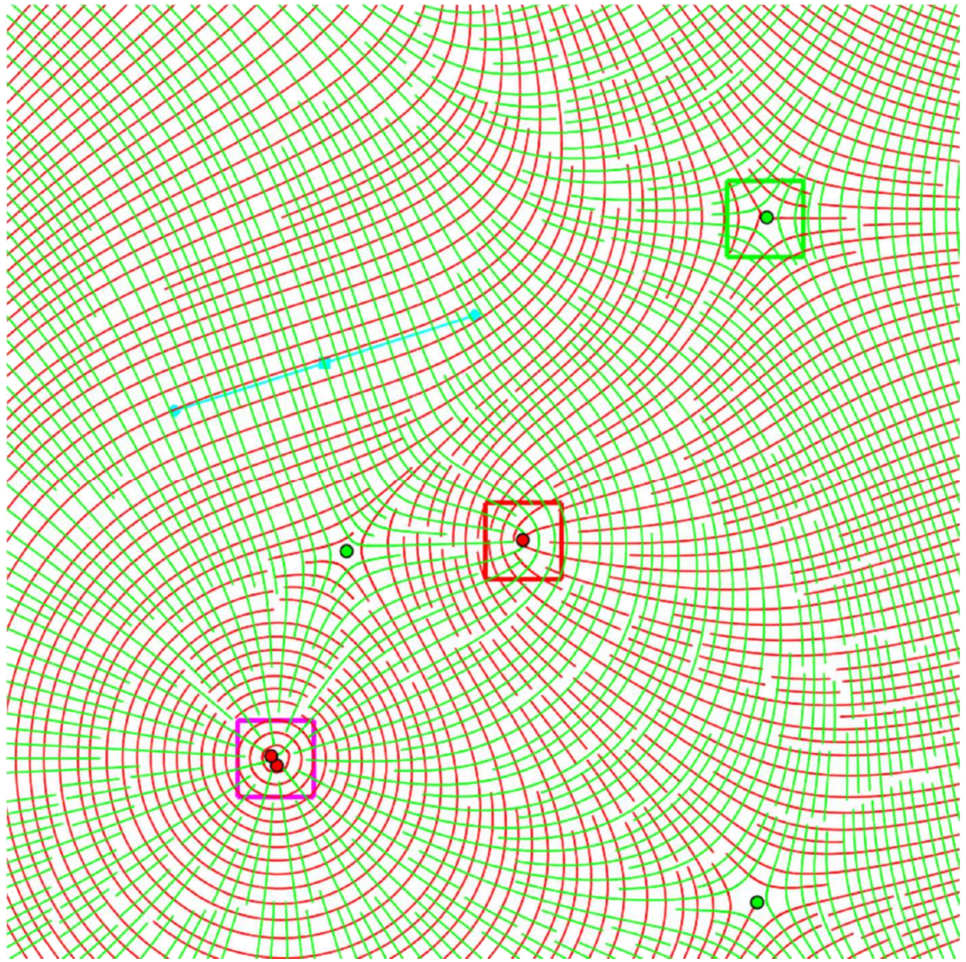
- Other than that...
  - Same integration as for vector fields
- Recap



[Chen]

Hyper-streamlines

- Other than that...
  - Same integration as for vector fields
- Recap
  - Independent computation for minor/major



[Chen]

Hyper-streamlines

- Other than that...
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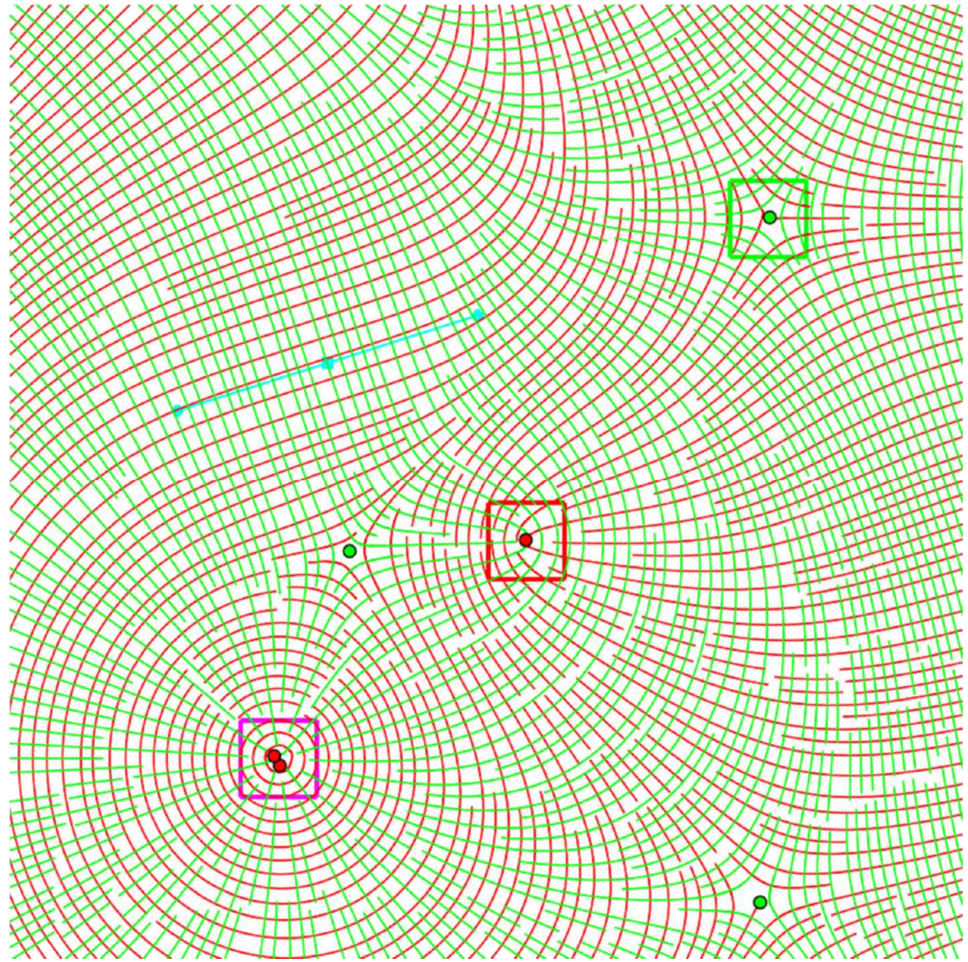
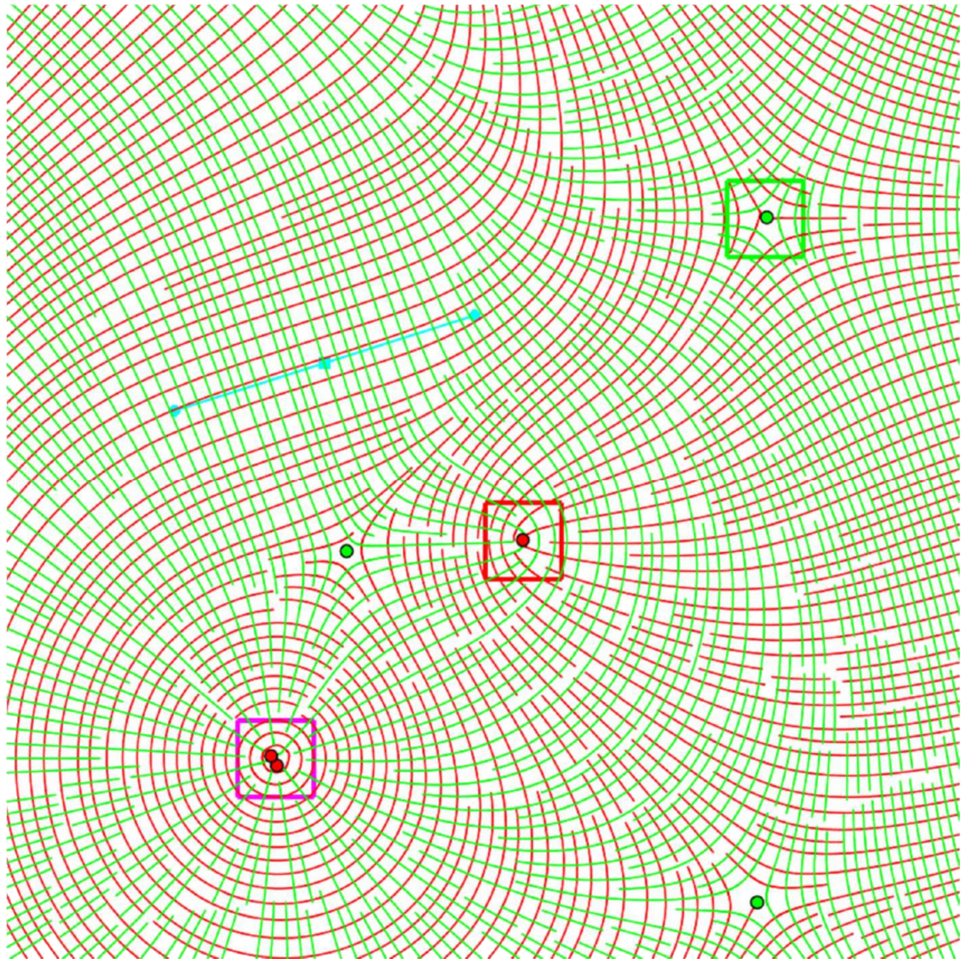
- Recap
  - Independent computation for minor/major
  - Seeding
    - Distance criterion



[Chen]

Hyper-streamlines

- Other than that...
  - Same integration as for vector fields
- Recap
  - Independent computation for minor/major
  - Seeding
    - Distance criterion
  - Integration
    - Euler
    - Runge-Kutta

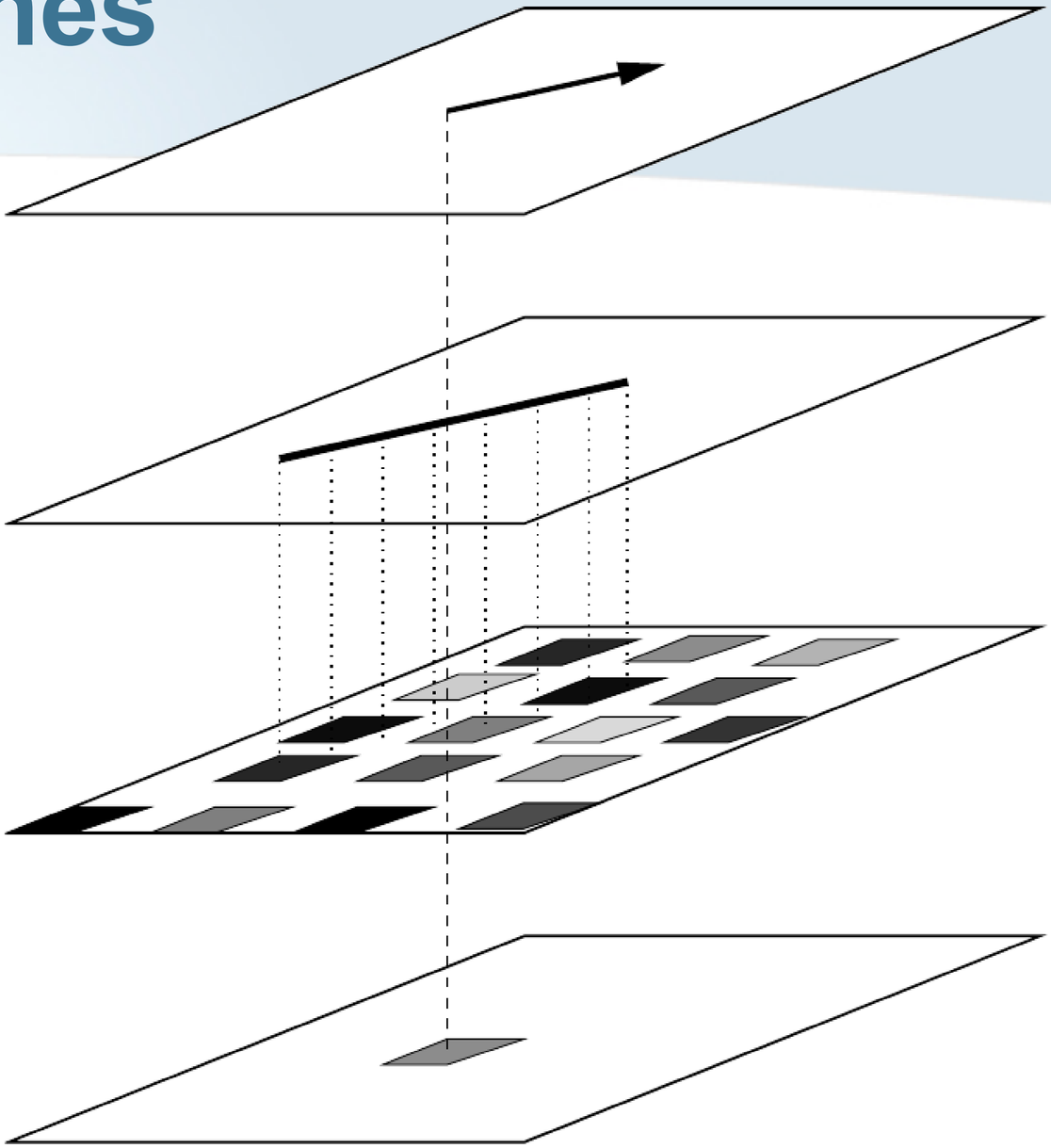


[Chen]

Beyond hyper-streamlines

- Now that we know how to extract hyper-streamlines
  - More global visualization

- Getting inspiration from vector fields
  - Line Integral Convolution



[Helgeland]

Beyond hyper-streamlines

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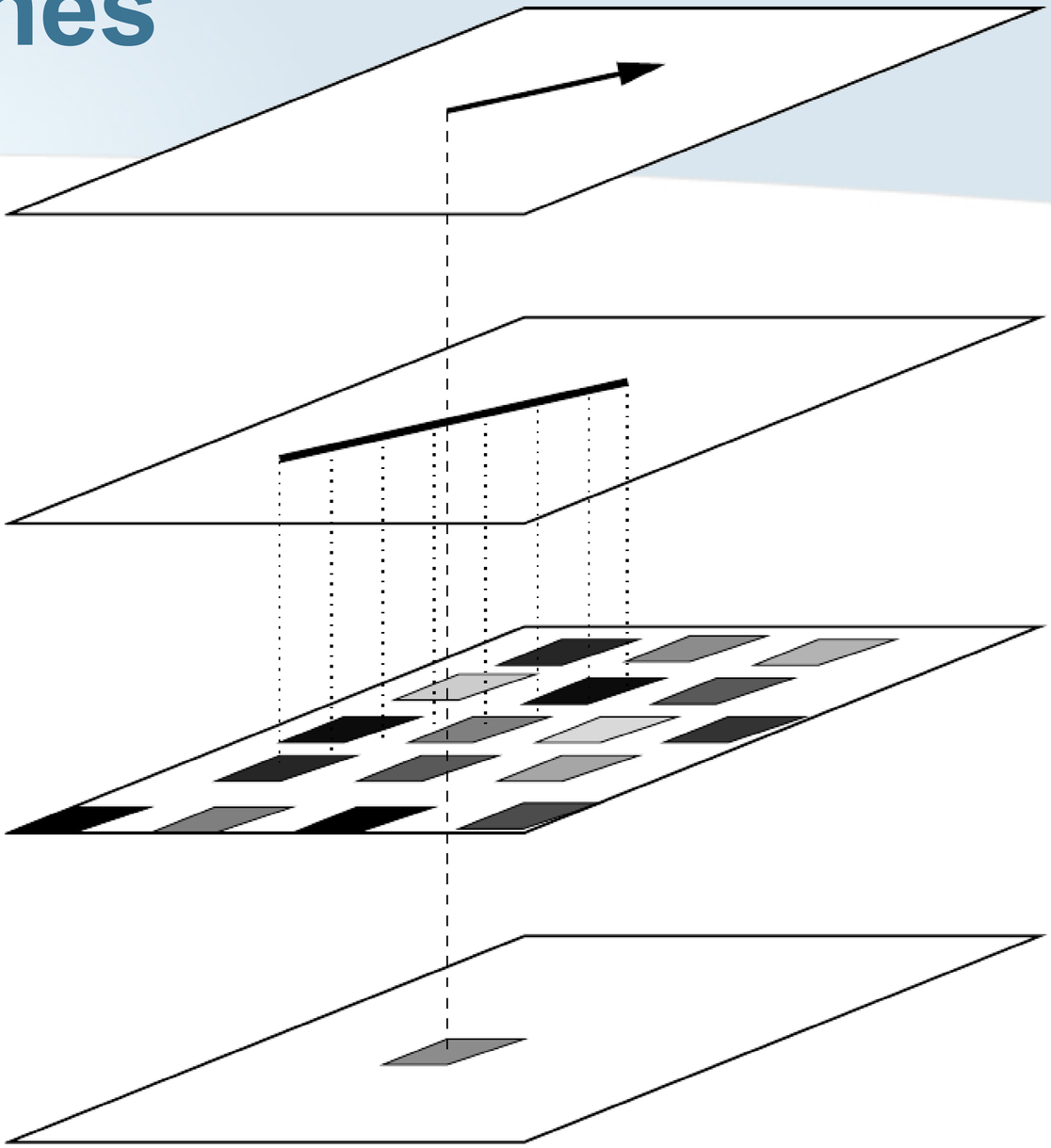
    - d-dimensional convolution kernel



[Helgeland]

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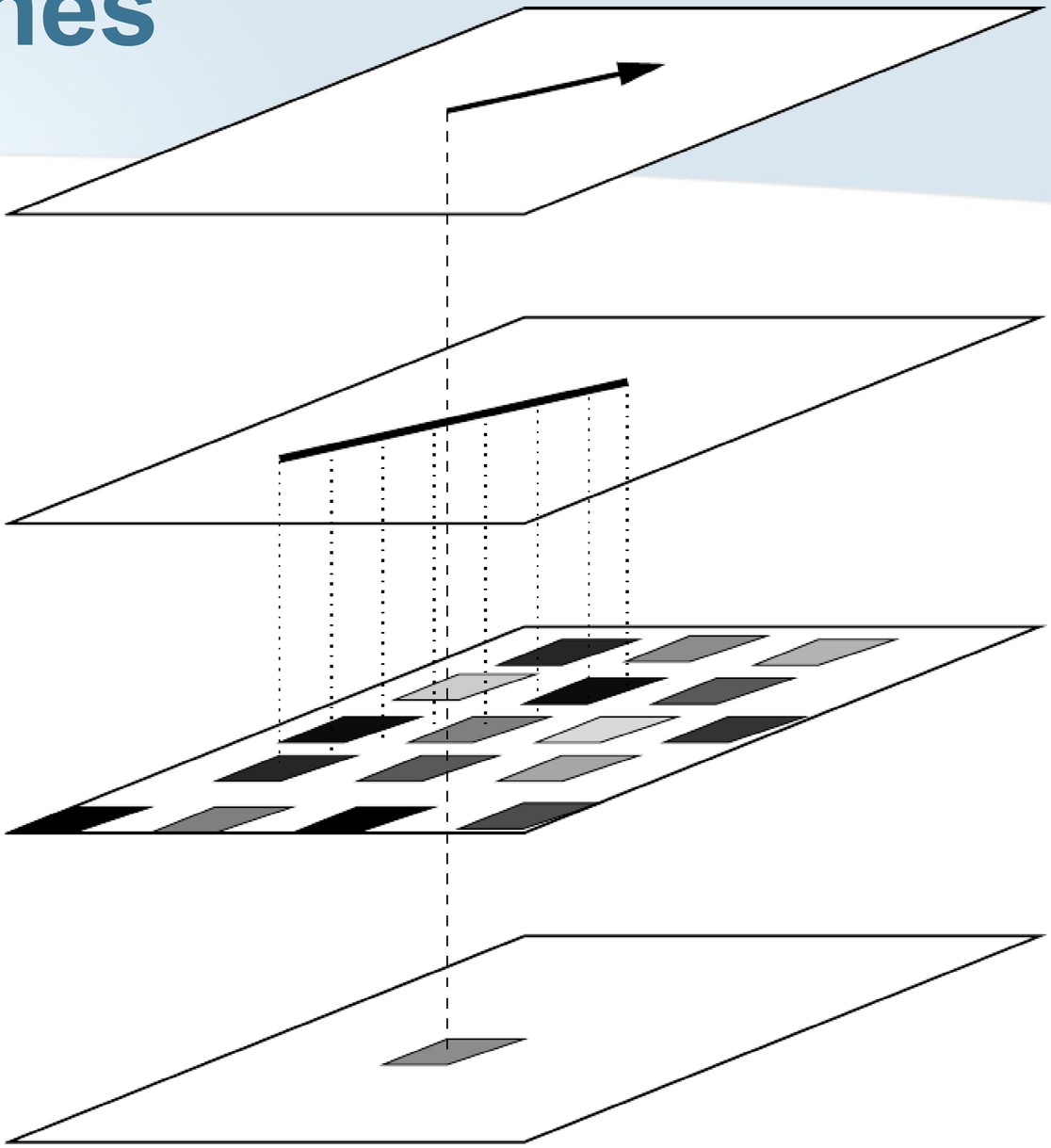
    - d-dimensional convolution kernel
    - Independent computations



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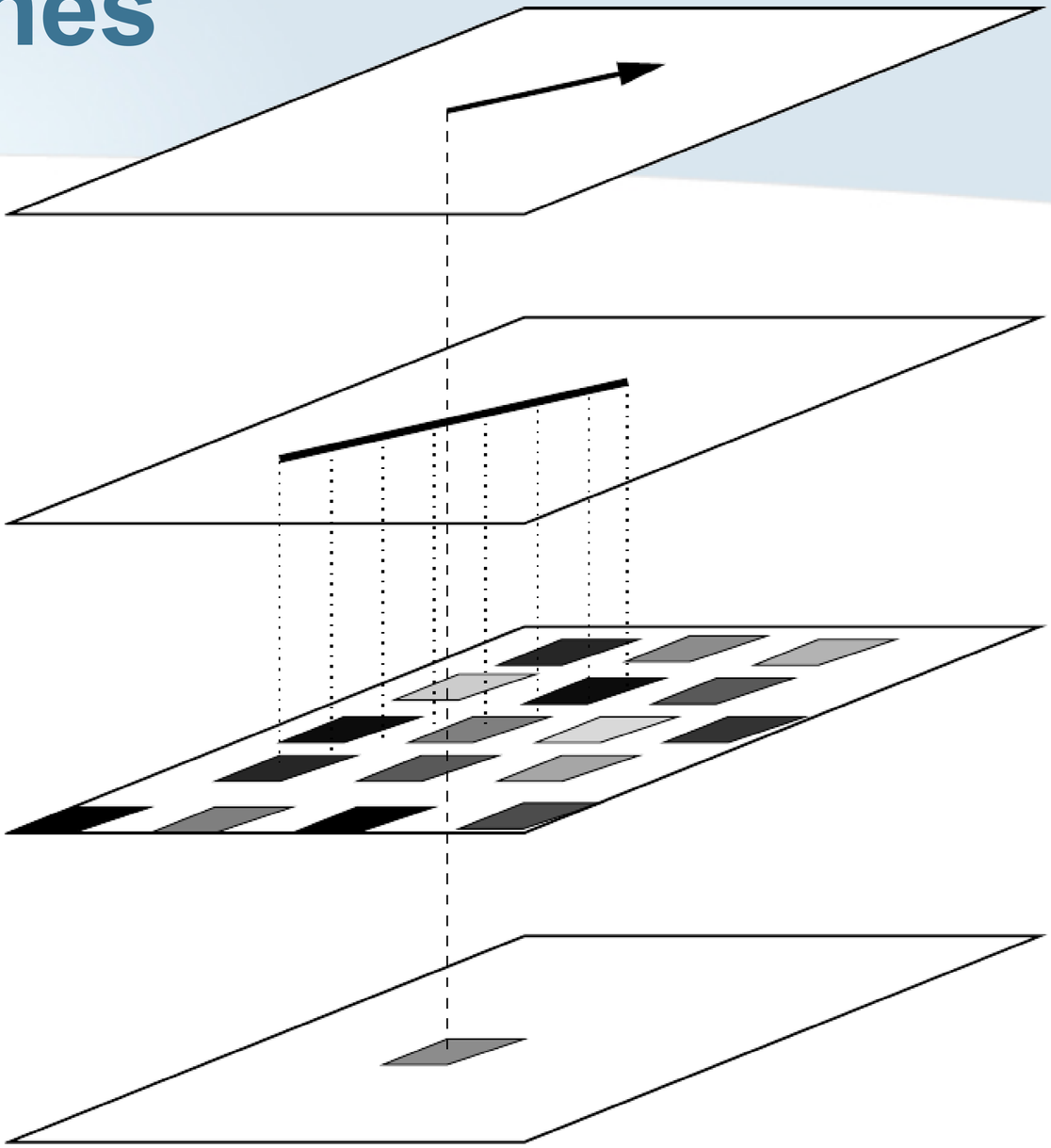

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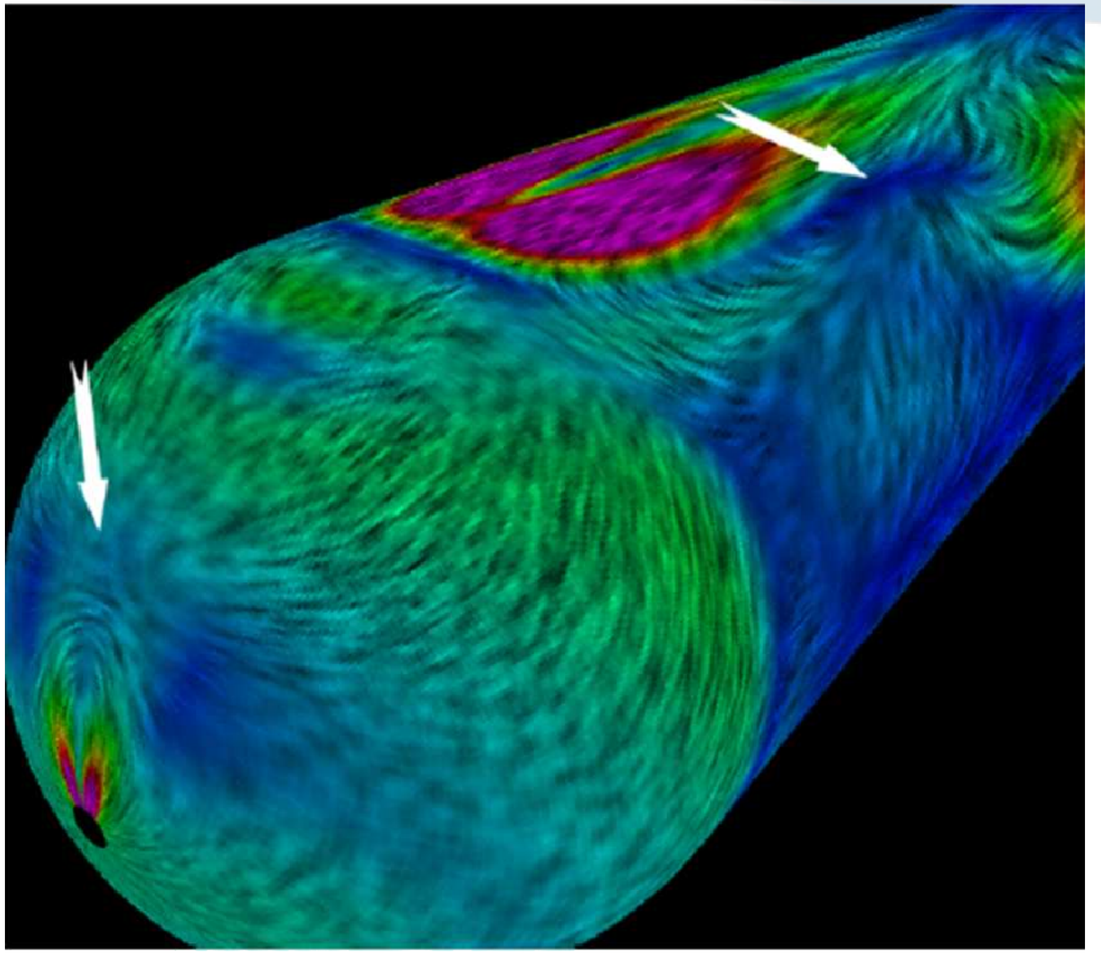

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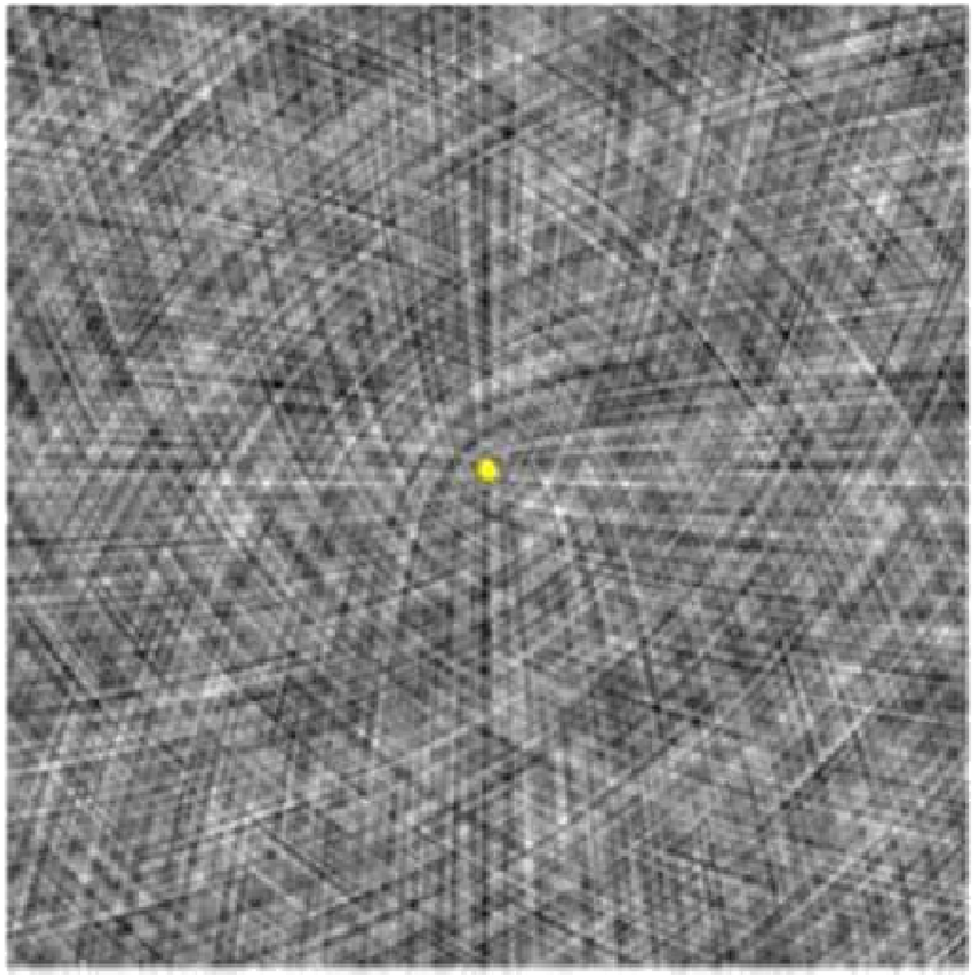

[Zhang]

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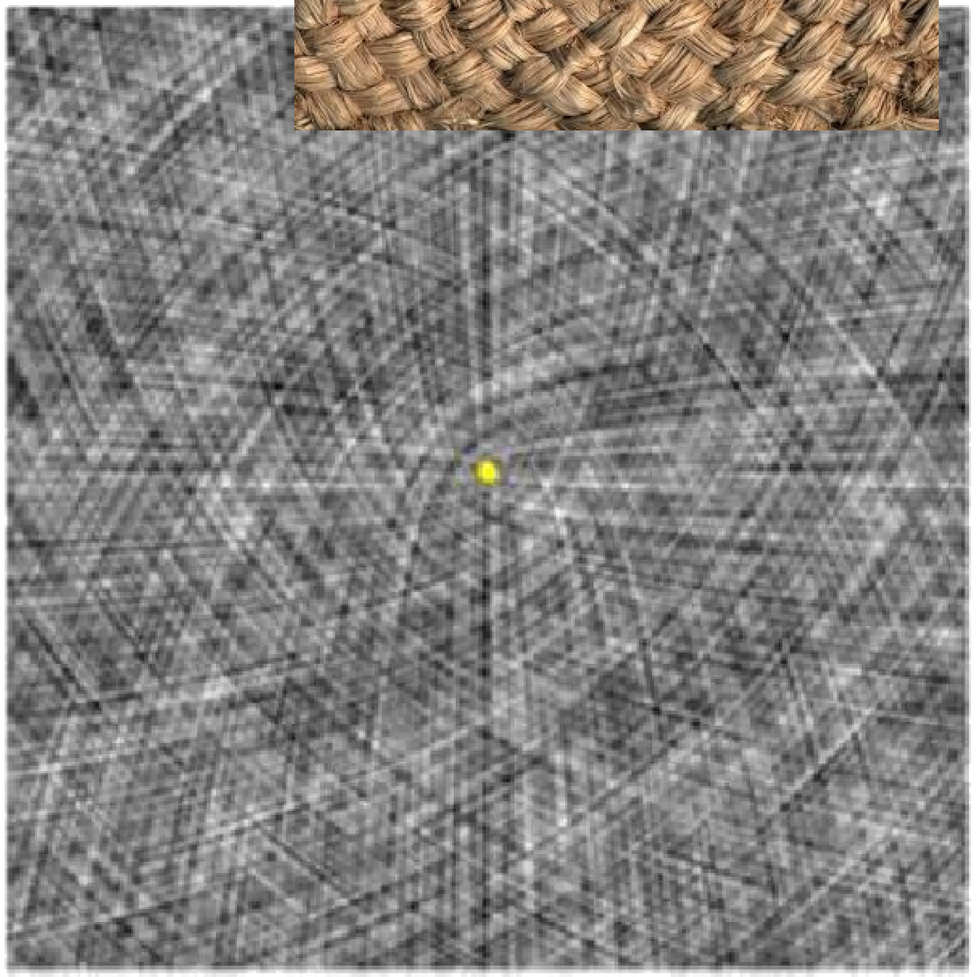
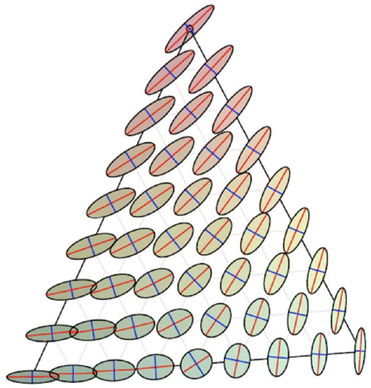
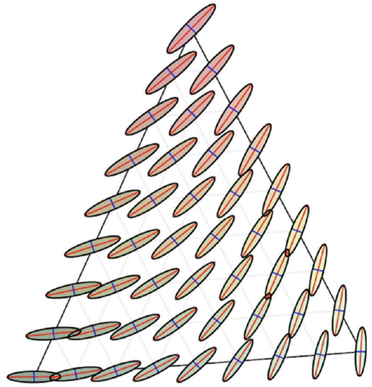
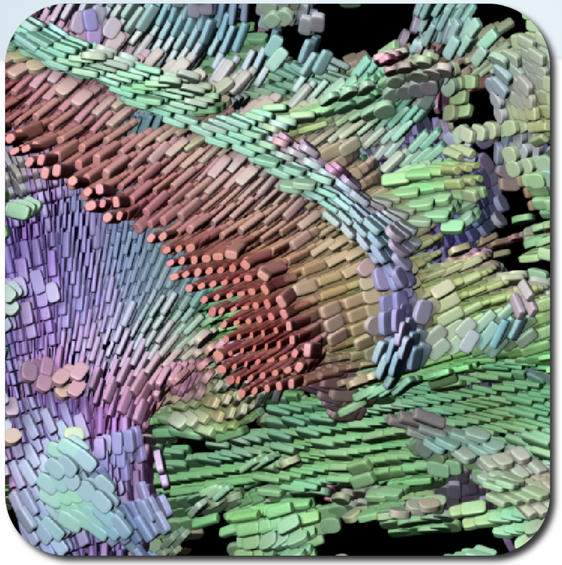


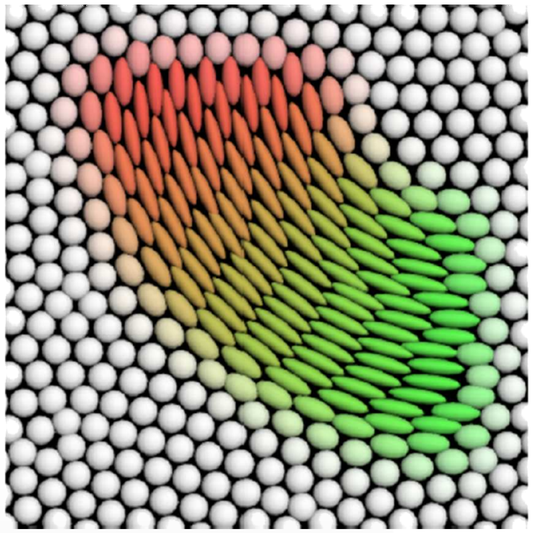
[Zhang]

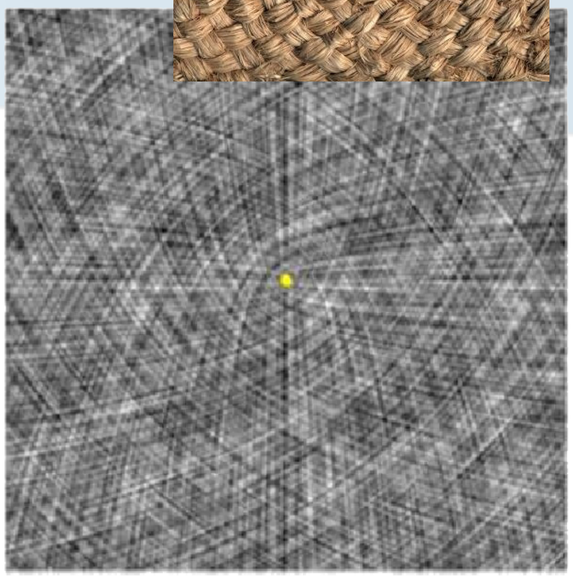
Tensor fields

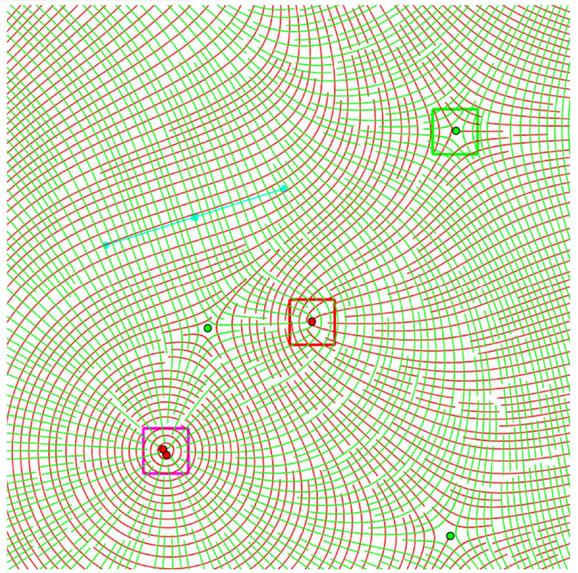


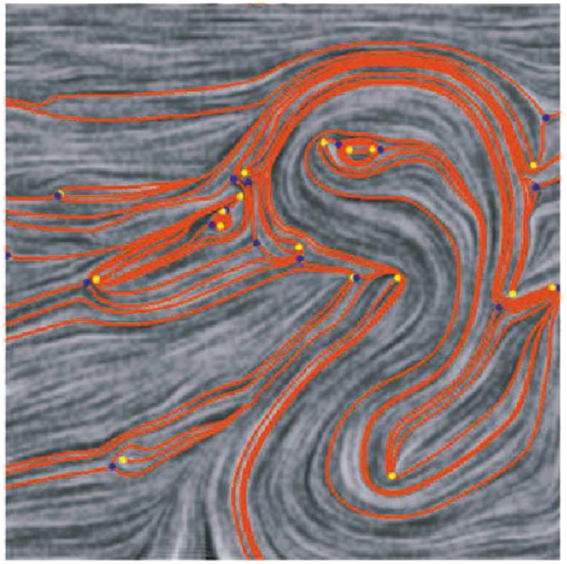










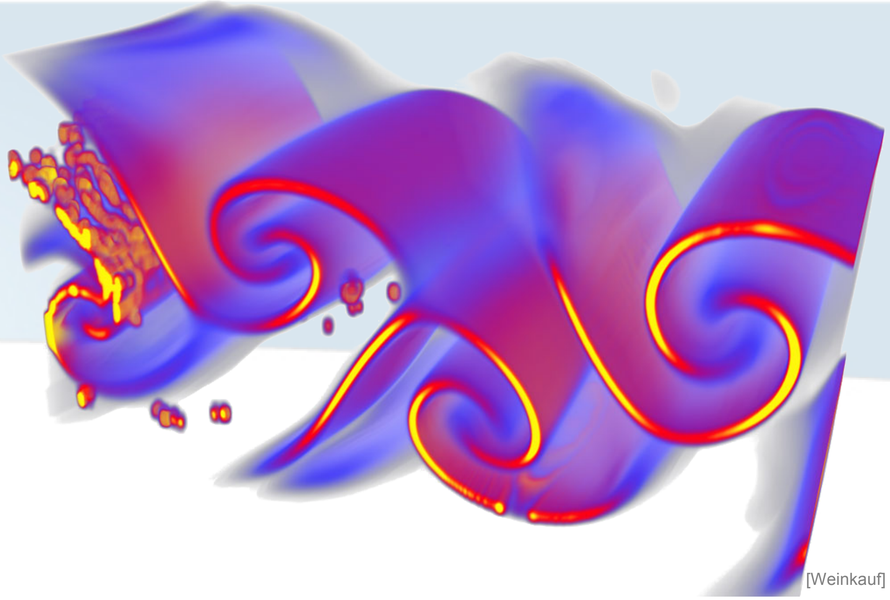


The awful truth

The awful truth

- Simulations usually have a temporal component

The awful truth



[Weinkauf]

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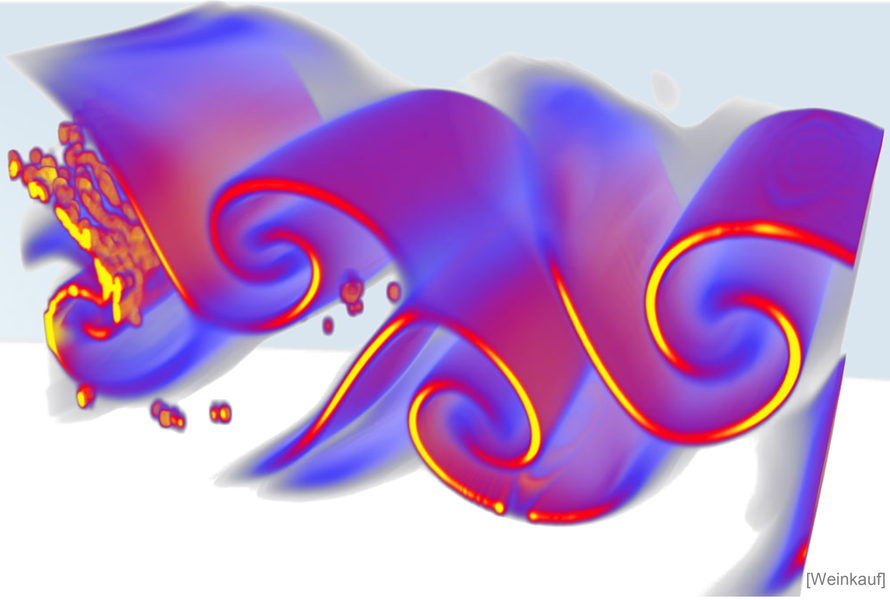

[Weinkauf]

- Simulations usually have a temporal component

- Simulations often come uncertainty evaluation

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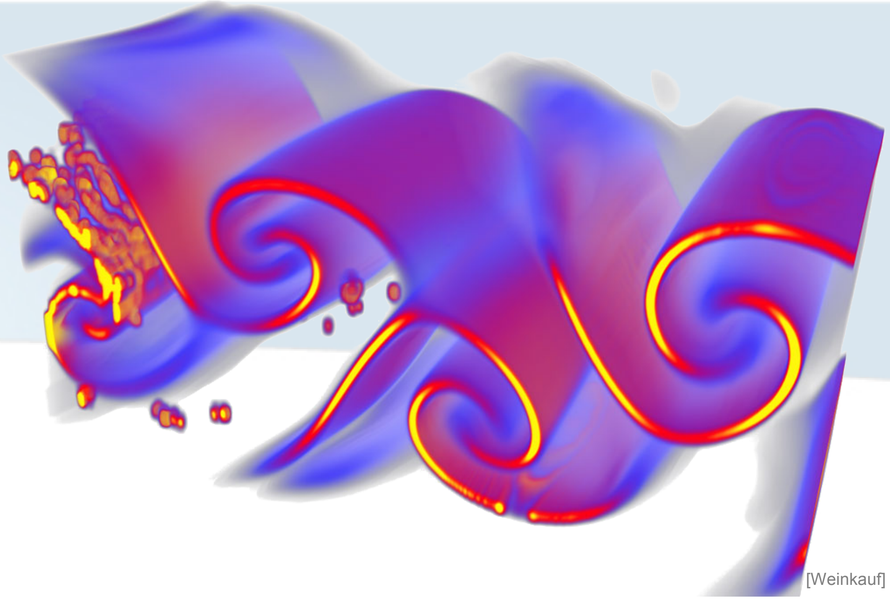

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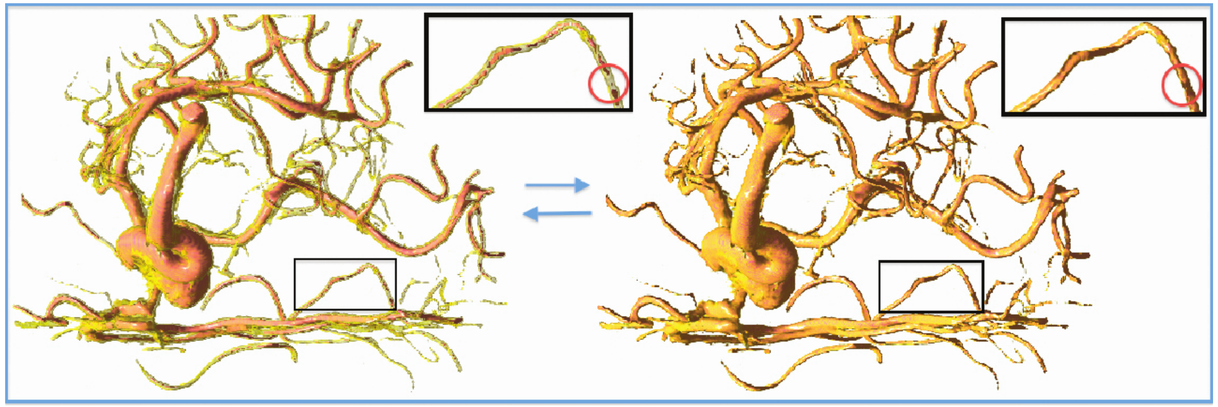

[Fout]

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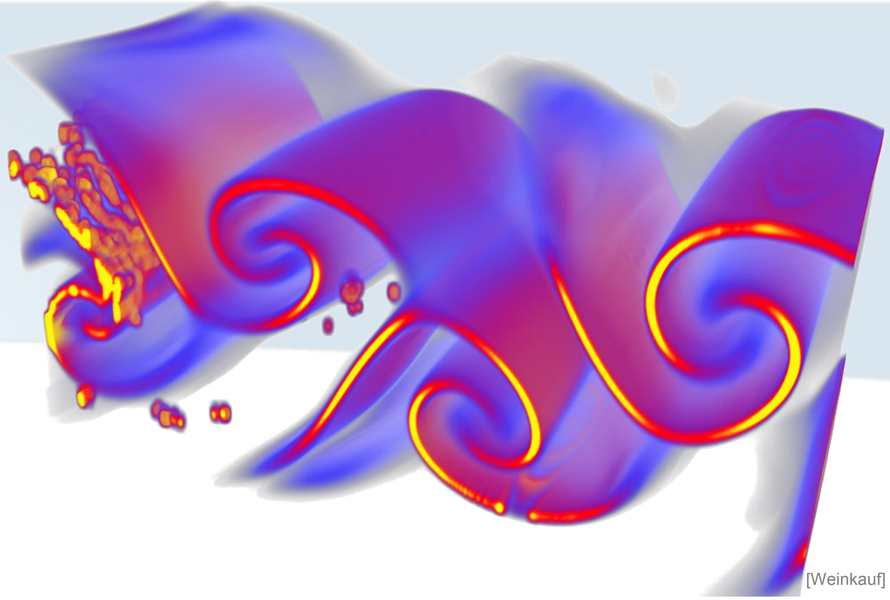

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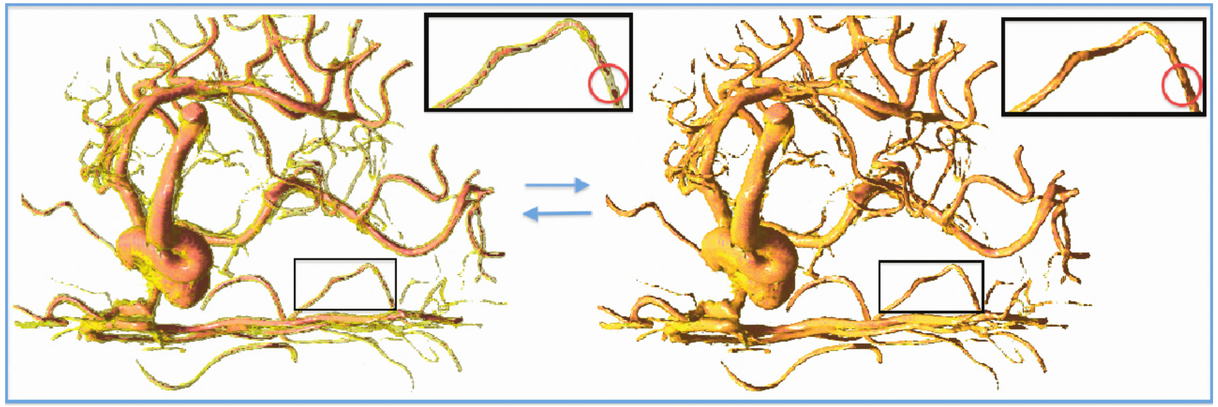
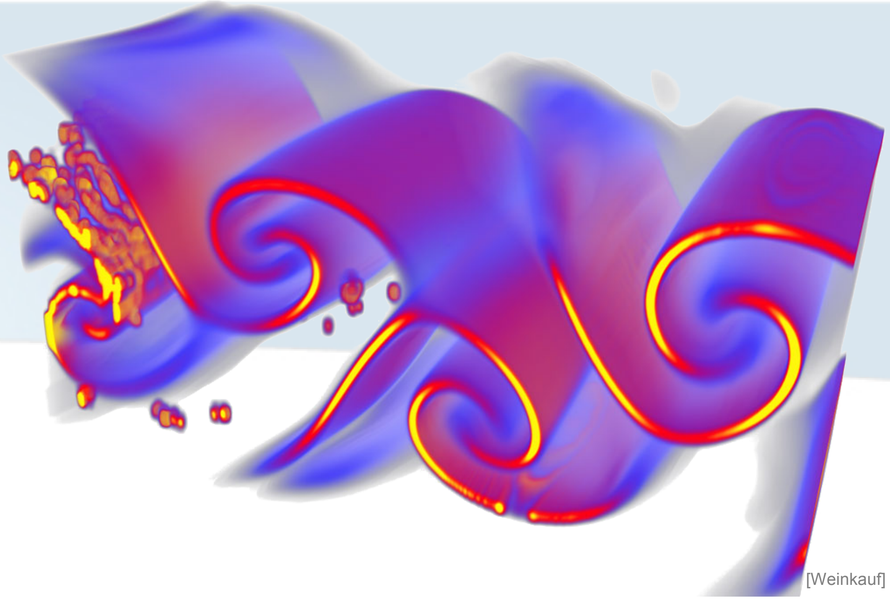


[Fout]

- Simulations often come uncertainty evaluation

- Simulations often yield several fields per data-set

The awful truth



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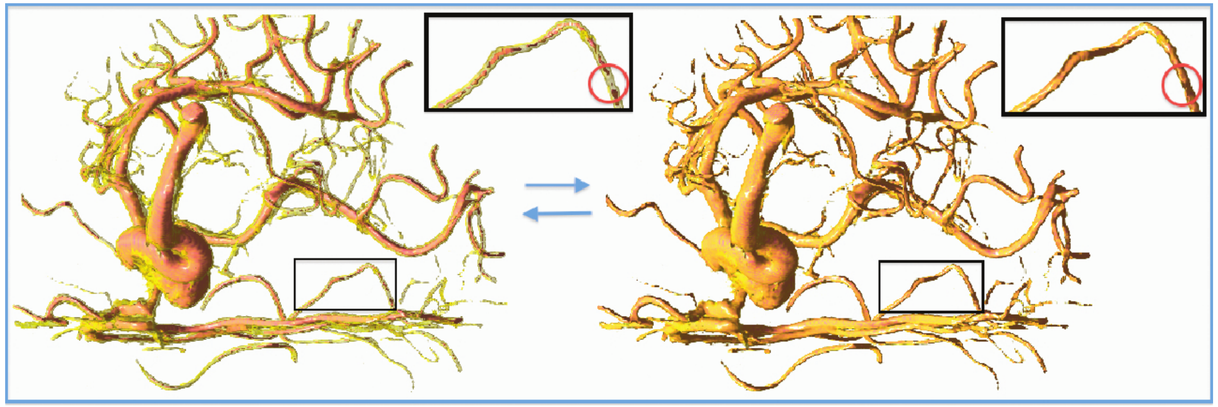

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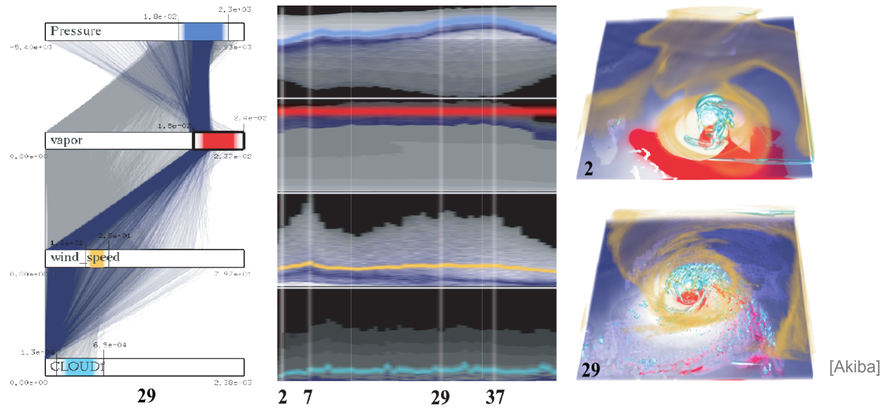

[Akiba]

- Simulations often yield several fields per data-set

References

- tierny@telecom-paristech.fr

- Projects, internships, Ph.D. thesis

- Online class
  - http://www.telecom-paristech.fr/~tierny/visualizationClass.html