

ENS L3 : "Systèmes numériques : de l'algorithme aux circuits"

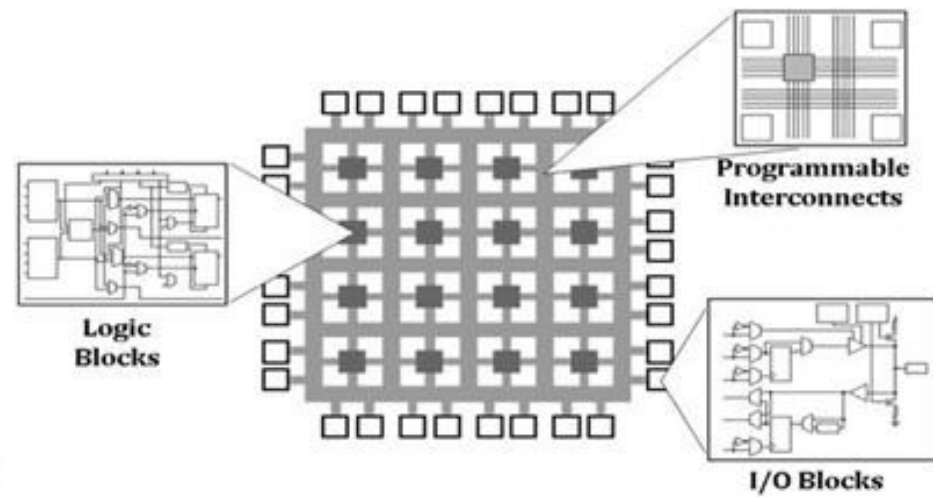
Leçon : langages de description de matériel

Sylvain GUILLEY
6 décembre 2016

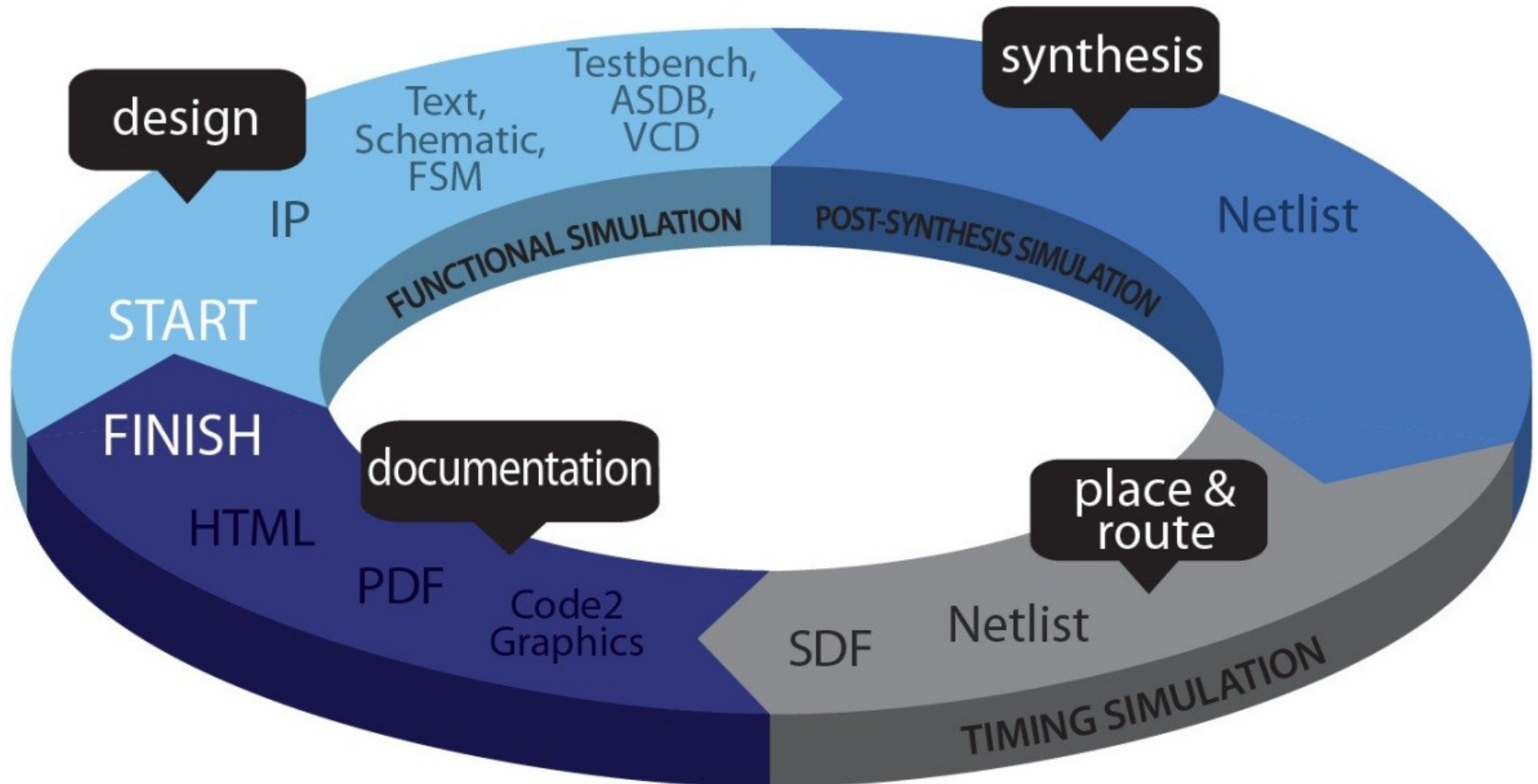
Programme

- EDA (Electronic Design Automation)
- Verilog
- Exemples
 - Elements de base
 - C-element
 - Table de substitution d'AES
 - Pipelines de processeurs
- Synthétisabilité
- Exercices : TD en ligne

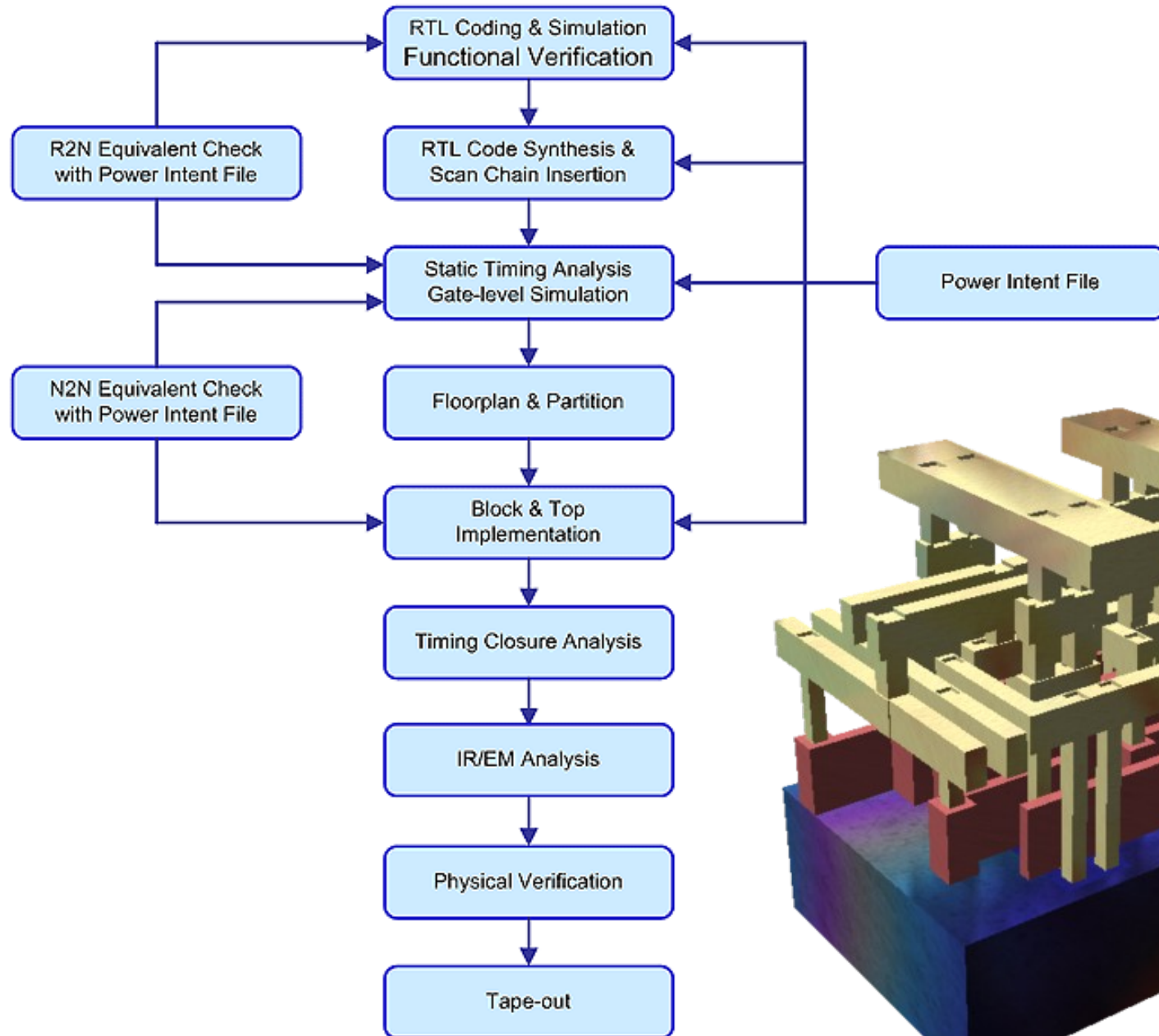
EDA



FPGA Design



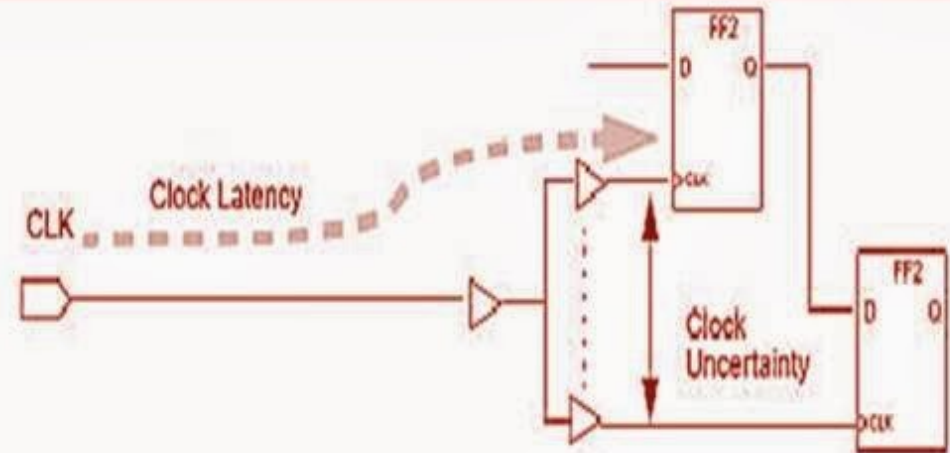
EDA



Commands

set_clock_uncertainty

- ❑ Models clock skew effects on the clock.
- ❑ After CTS real propagated skew is considered.



- ❑ `set_clock_uncertainty [-from from_clock] [-rise_from rise_from_clock] [-fall_from fall_from_clock] [-to to_clock] [-rise_to rise_to_clock] [-fall_to fall_to_clock] [-rise] [-fall] [-setup] [-hold] uncertainty [object_list]`
- ❑ # Specifies the clock uncertainty for clocks or for clock-to-clock transfers.

Examples:

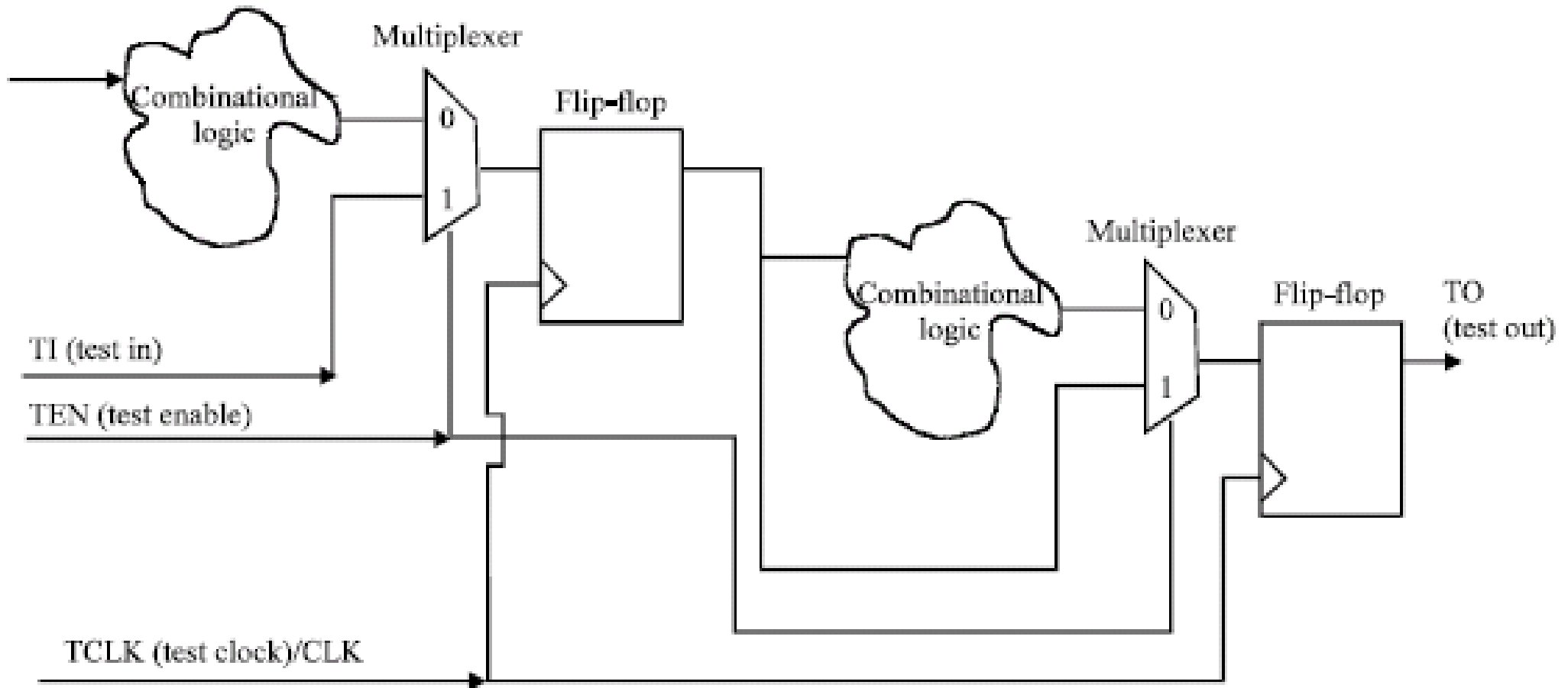
- ❑ `set_clock_uncertainty -setup -rise -fall 0.2 [get_clocks CLK2]`
- ❑ `set_clock_uncertainty -from [get_clocks HSCLK] -to [get_clocks SYSCLK] -hold 0.35`
- ❑ `set_clock_uncertainty -setup 0.475 clock`
- ❑ `set_clock_uncertainty -hold 0.27 clock`

SDC



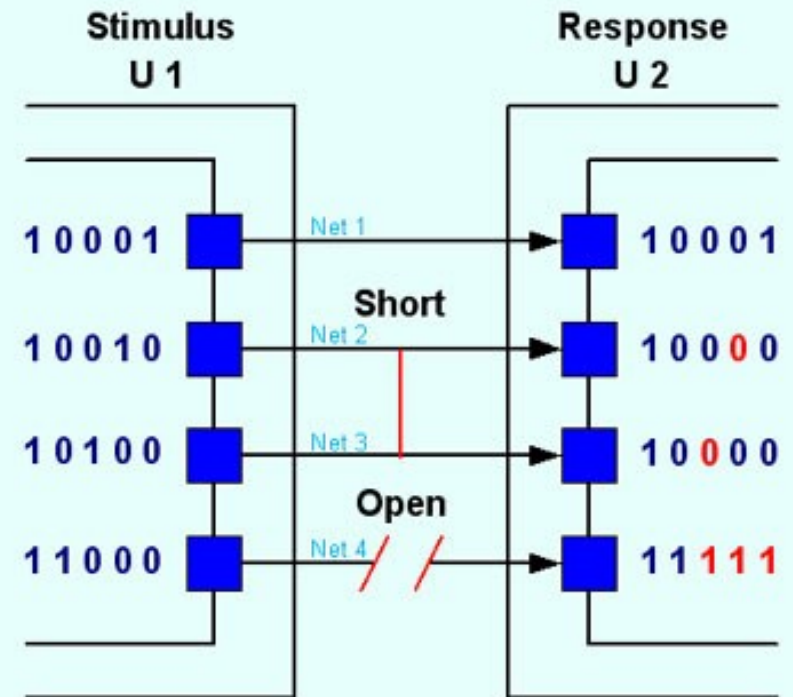
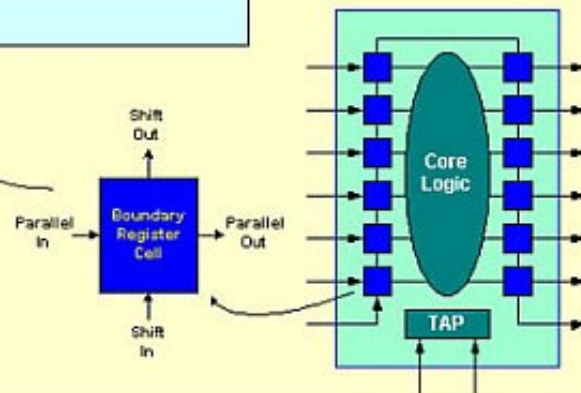
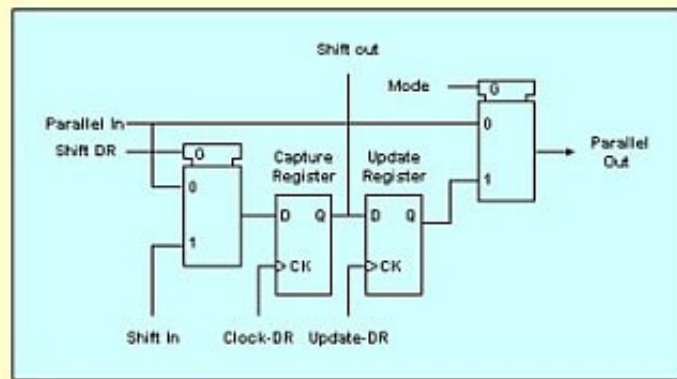
Test des circuits intégrés

Deux modes, en fonction de TEN



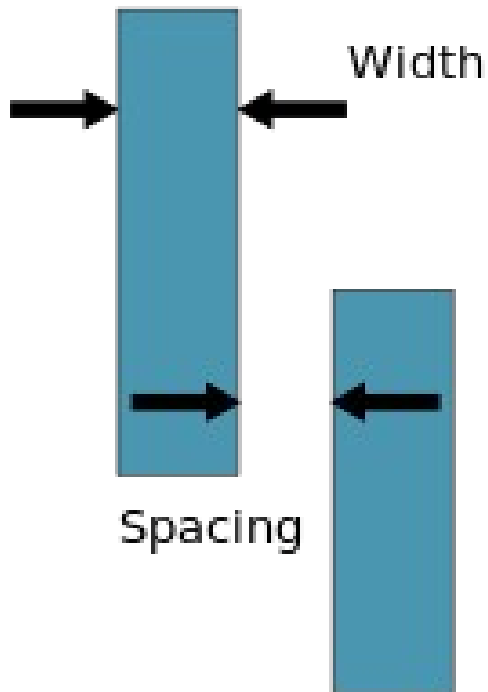
Test des cartes électroniques

Boundary-scan, défini dans le standard IEEE Std.-1149.1

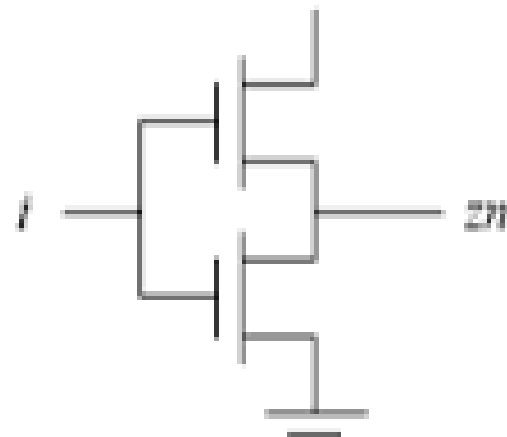


Vérifications finales

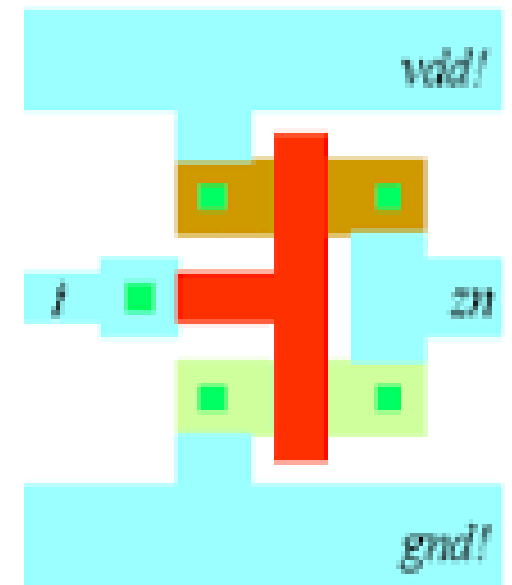
The three basic DRC checks



DRC : Design Rules Check
LVS : Layout Versus Schematic



compare with



Niveaux de description

- Transaction-Level Model
- Register-Transfert Level :
 - Comportemental
 - dataflow
- Netlist (structurel)

Langages

- System-C (IEEE 1666)
 - Testbench
 - Dimensionnement de performances
- Verilog (IEEE 1076), VHDL (IEEE 1364)
 - Simulation + synthèse
 - Netlist

SystemC

```
#include "systemc.h"

SC_MODULE(adder)           // module (class) declaration
{
    sc_in<int> a, b;       // ports
    sc_out<int> sum;

    void do_add()         // process
    {
        sum.write(a.read() + b.read()); //or just sum = a + b
    }

    SC_CTOR(adder)        // constructor
    {
        SC_METHOD(do_add); // register do_add to kernel
        sensitive << a << b; // sensitivity list of do_add
    }
};
```

Syntaxe Verilog

```
// Here is a module definition, called FRED
```

```
module FRED(q, a, b);
```

```
    input a, b;    // These are the inputs
```

```
    output q;     // Make sure your comments are helpful
```

```
    // ..... Guts of module go here .....
```

```
endmodule
```

```
// Here is a module definition with two input busses
```

```
module FRED(q, d, e);
```

```
    input [4:0] d, e;    // Ports d and e are each five bit busses
```

```
    output q;           // q is still one bit wide
```

```
endmodule
```

Syntaxe Verilog

```
// Here are some local nets being defined
module FRED(q, a);
    input a;
    output q;
    wire [2:0] xbus, ybus;    // Two local busses of three bits each
    wire parity_x, parity_y; // Two simple local signals.
    ...

// Here is a module definition with submodule instances.
module FRED(q, a, b);
    input a, b;    // These are the inputs
    output q;     // Make sure your comments are helpful
    wire qbar;    // Local, unported net
    NOR2 mygate(q, a, qbar), myothergate(qbar, b, q);
endmodule
```

Syntaxe Verilog

```
// Here is a module definition with named port mapping.
module FRED(q, a, b);

    input a, b;    // These are the inputs
    output q;     // Make sure your comments are helpful
    wire qbar;    // Local, unported net

    NOR2 mygate(.x1(a), .y(q), .x2(qbar)),
              myothergate(.y(qbar), .x2(q), .x1(b));
endmodule
```

Syntaxe Verilog

```
assign <signal> = <signal-expression> ;
```

For example

```
wire p;  
assign p = (q & r) | (~r & ~s);
```

Il y a 123 mots réservés
dans verilog !

Syntactic sugar :

```
wire p = (q & r) | (~r & ~s);
```

Syntaxe Verilog

```
wire [7:0] bus1, clipbus;  
assign clipbus = (bus1 > 8'd127) ? 8'd127 : bus1;
```

```
wire [23:0] mybus = yourbus & 24'b11110000_11110000_00001111;
```

```
wire [31:0] dbus;  
wire bit7 = dbus[7];  
wire [7:0] lowsevenbits = dbus[7:0];  
wire [31:0] swapbus = dbus[0:31];
```

```
wire [11:0] boo;  
input [3:0] src;  
assign boo[11:8] = src;  
assign boo[7:4] = 4'b0;  
assign boo[3:0] = src + 4'd1;
```

```
assign boo = { src, 4'b0, src + 4'd1 };
```


Syntaxe Verilog

Symbol	Function	Resultant width
~	monadic negate	as input width
—	monadic complement (*)	as input width
!	monadic logic not	unit
*	unsigned binary multiply (*)	sum of arg widths
/	unsigned binary division (*)	difference of arg widths
%	unsigned binary modulus (*)	width of rhs arg
+	unsigned binary addition	maximum of input widths
—	unsigned binary subtraction	maximum of input widths
>>	right shift operator	as left argument
<<	left shift operator	as left argument
==	net/bus comparison	unit
!=	inverted net/bus compare operator	unit
<	bus compare operator	unit
>	bus compare operator	unit
>=	bus compare operator	unit
<=	bus compare operator	unit
&	diadic bitwise and	maximum of both inputs
^	diadic bitwise xor	maximum of both inputs
^^	diadic bitwise xnor (*)	maximum of both inputs
	diadic bitwise or	maximum of both inputs
&&	diadic logical and	unit
	diadic logical or	unit
?:	conditional expression	maximum of data inputs

Symbol	Function
&	and
	or
^	xor
~&	and with final invert
~	or with final invert
^^	xor with final invert

```
wire [9:0] mybus;
wire x = (& (mybus));
```

```
wire yes = p > r;
```

Ternaires :

```
wire [7:0] p, q, r;
wire s0, s1;
```

```
wire [7:0] bus1 = (s0) ? p : q;
wire [7:0] bus2 = (s0) ? p : (s1) ? q : r;
```

Syntaxe Verilog : combinatoire

```
always @(aal_counter) case (aal_counter) // full_case
    0: aal_header <= 8'h00;
    1: aal_header <= 8'h17;
    2: aal_header <= 8'h2D;
    3: aal_header <= 8'h3A;
    default: aal_header <= 8'h74;
endcase
```

Syntaxe Verilog : sequentiel

```
module EXAMPLE(ck, ex);
  input ck;          // This is the clock input
  output ex;

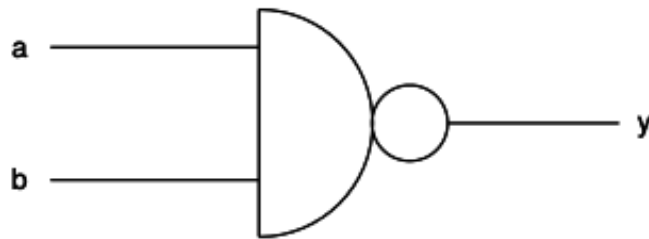
  reg ex, ez;       // ex is both a register and an output
  reg [2:0] q;      // q is a local register for counting.

  always @(posedge ck)
  begin
    if (ez) begin
      if (q == 2) ex <= 1; else ex <= ~ex;
    q <= q + 3'd1;
    end
    ez <= ~ez;
  end
endmodule
```

- **ez** : compteur modulo 2
- **q** : compteur modulo 2^3 , avec enable sur ez
- **ex** : est affecté 2x, mais sous des conditions différentes

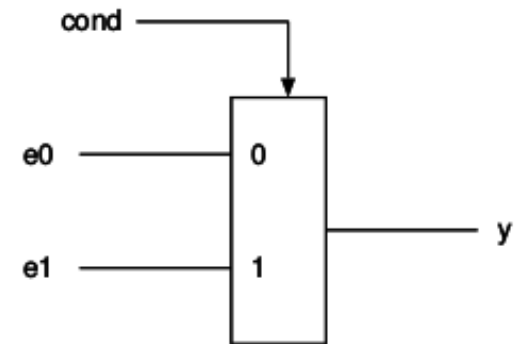
Petits exemples de codes en Verilog

Nand Gate



```
assign y = ~(a & b);
```

2 input Mux

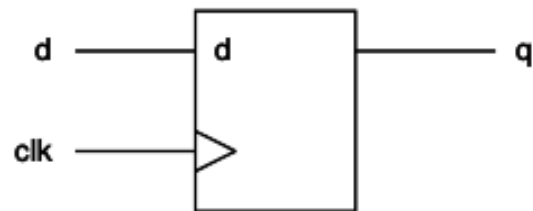


```
if (cond) y = e1;  
else y = e0;
```

```
y = (cond) ? e1 : e0;
```

Petits exemples de codes en Verilog

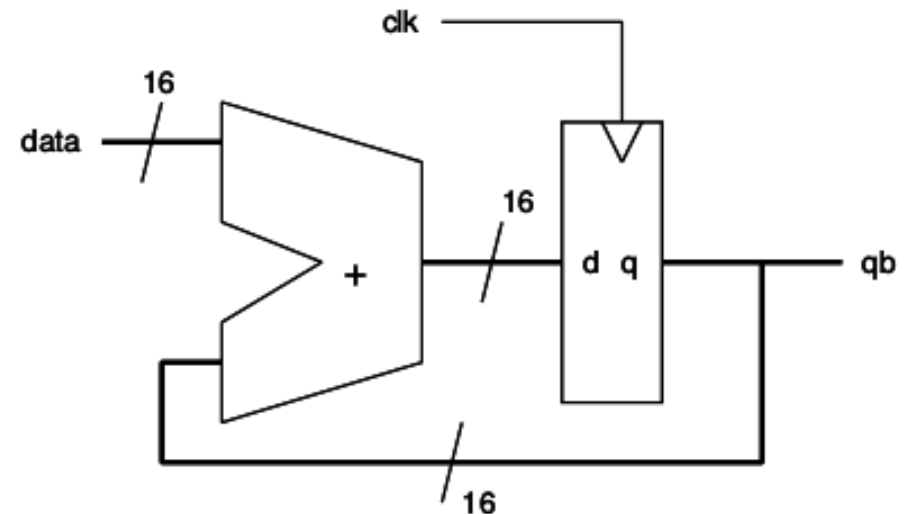
D-type FF



"On the positive edge of the clock
the value on the d input is copied to
the q output"

```
always @(posedge clk) q <= d;
```

Accumulator

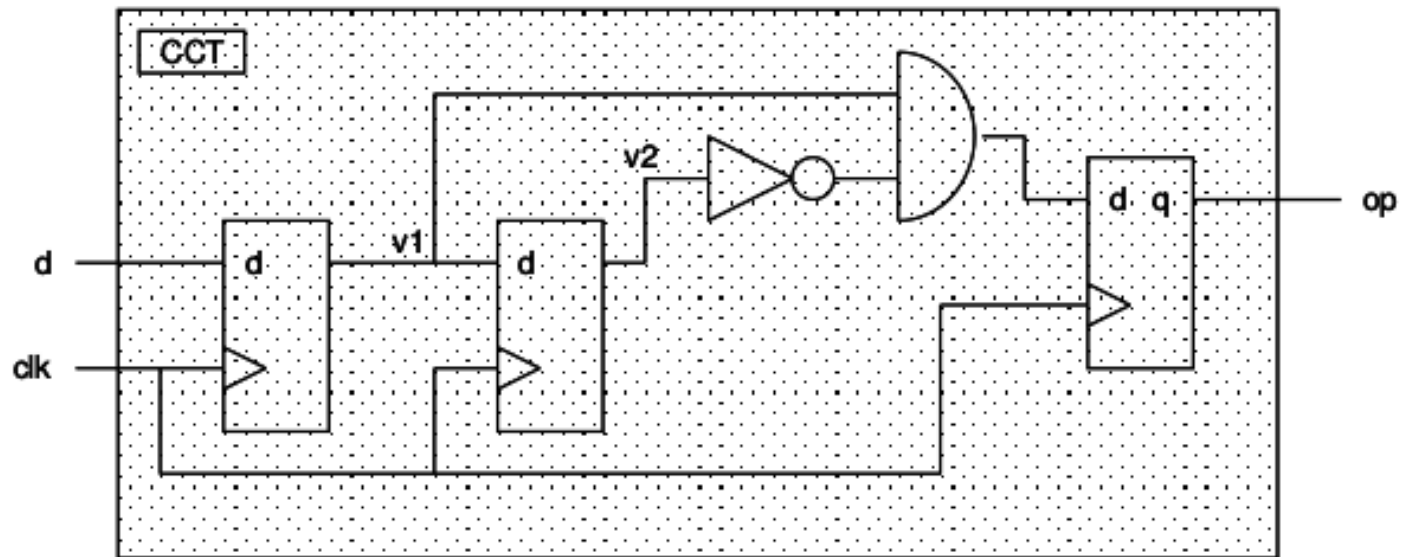


```
wire [15..0] qb;  
reg [15..0] data;  
always @(posedge clk) qb <= qb+data;
```

Petits exemples de codes en Verilog

Little Circuit (pulse generator).

```
module CCT(d, clk, op);  
  
  input d, clk;  
  output op;  
  reg op;  
  reg v1, v2;  
  
  always @(posedge clk)  
    begin  
      v1 <= d;  
      v2 <= v1;  
      op <= v1 & ~v2;  
    end  
  
endmodule
```



Powerful Critical Path Synthesis

- Performs aggressive timing driven re-structuring, mapping and gate-level optimization.
- Logic duplication for reducing the load seen by the critical path.
- Buffer high fan out nets to improve total negative slack.

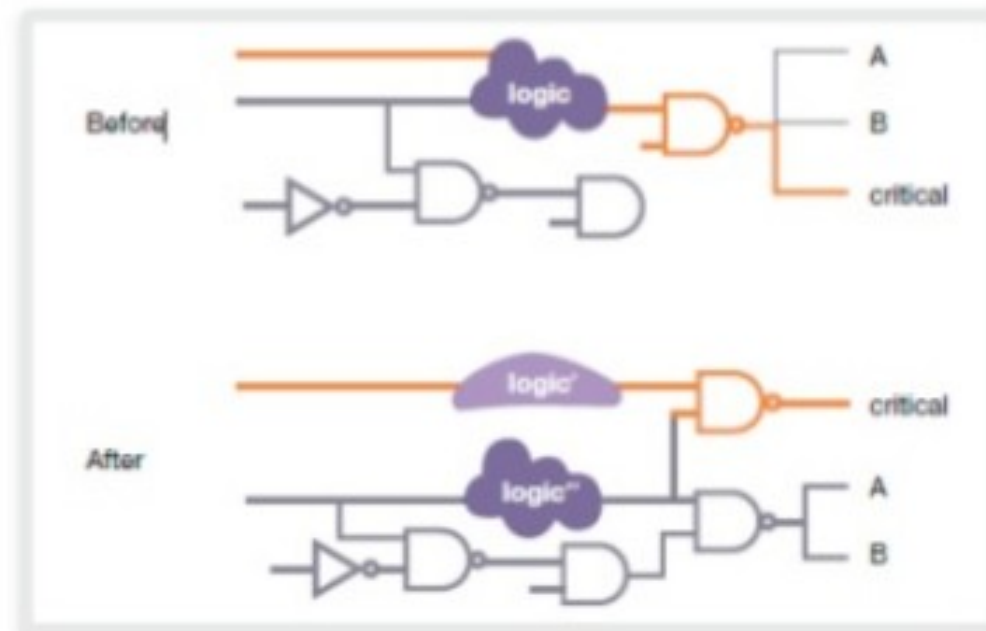
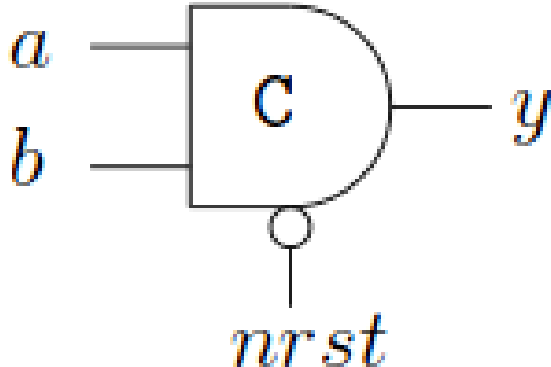


Figure 5. Register duplication [1]

C-Element : comportemental

TAB. 1. Le C-Element à deux entrées accompagné de son comportement décrit en VHDL.

	<pre>if (nrst = 0) then y <= 0; elsif (a = b) then y <= a; end if;</pre>
--	--

C-Element : structures en transistors

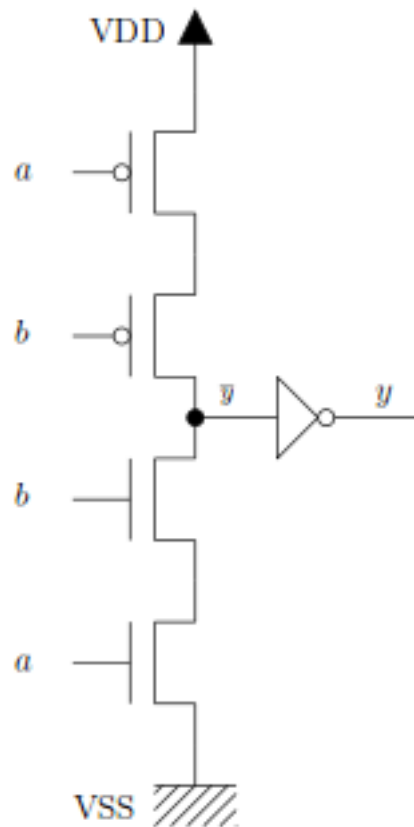


FIG. 1a. C-Element dynamique.

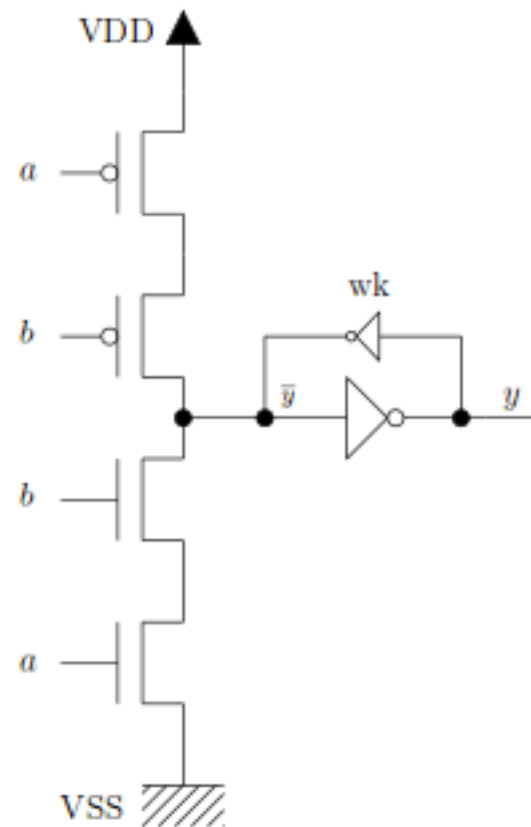


FIG. 1b. C-Element à rebouclage faible.

C-Element : structures en transistors

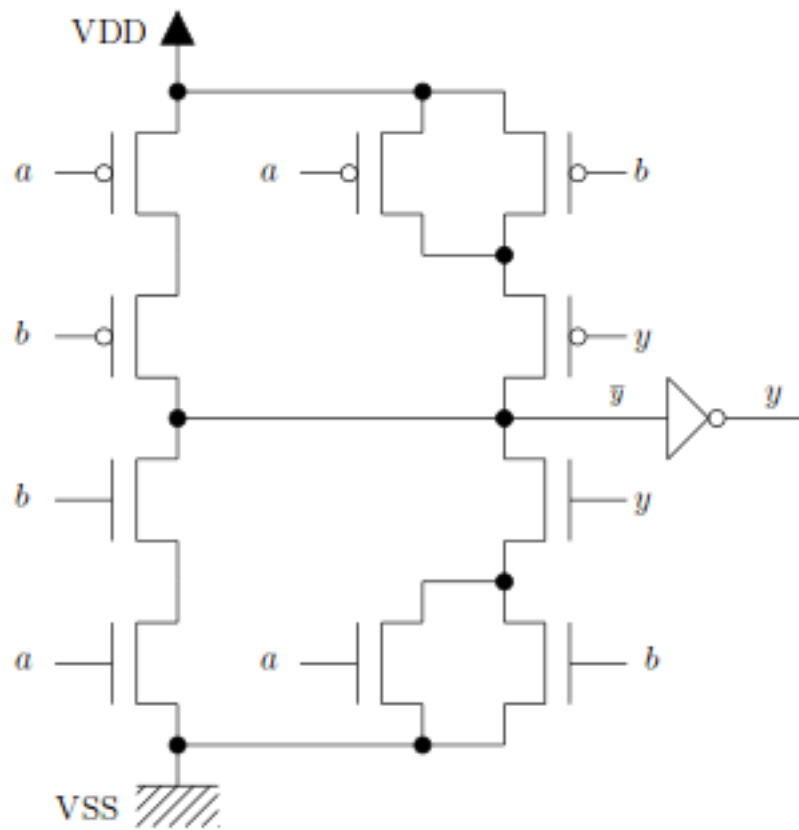


FIG. 1c. C-Element conventionnel.

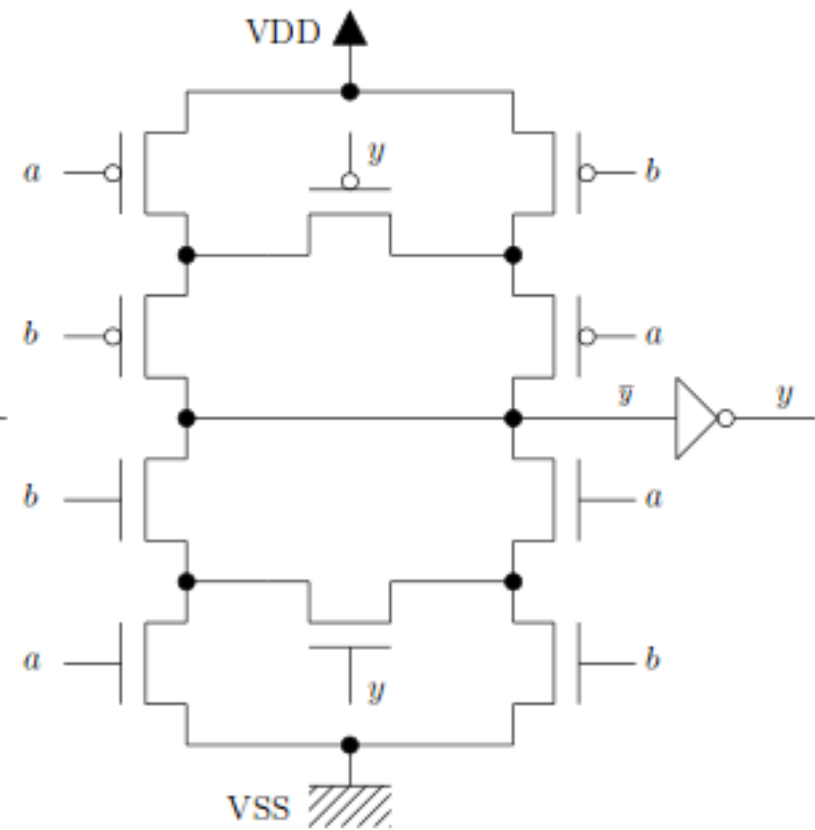


FIG. 1d. C-Element symétrique.

C-Element : structures en transistors

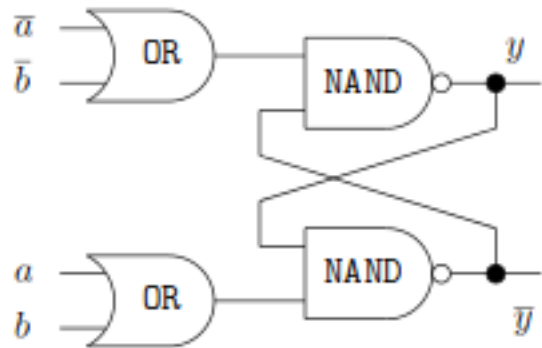


FIG. 2a. Structure en portes du C-Element à bascule RS.

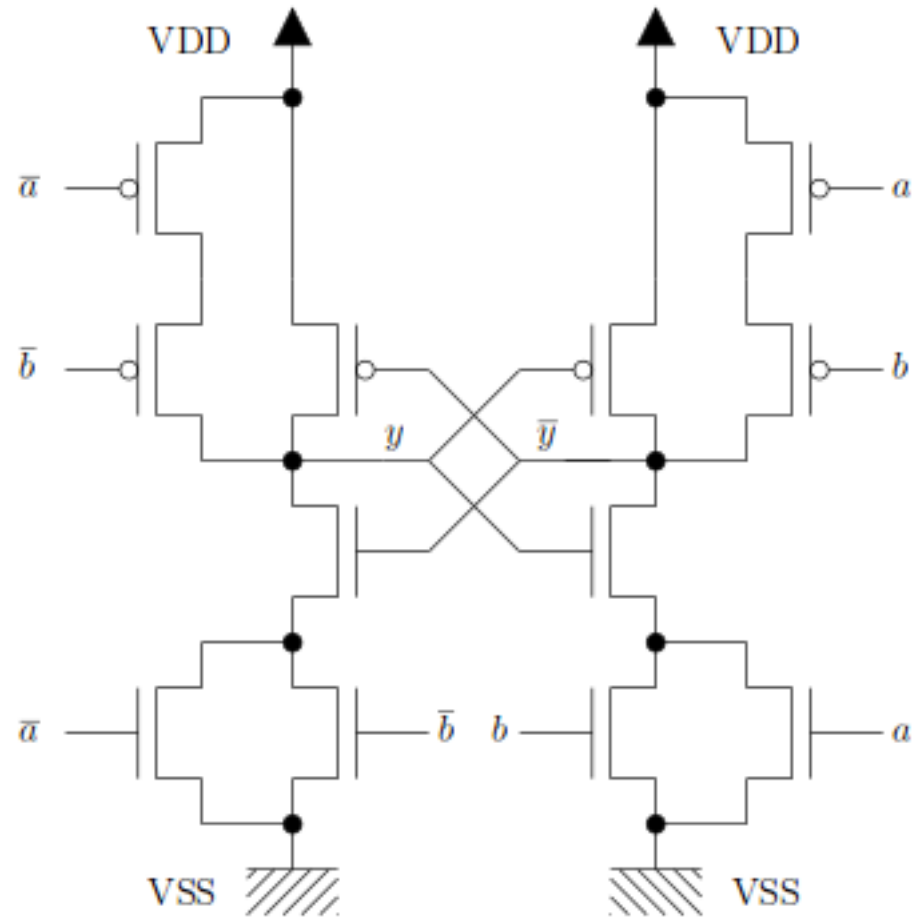
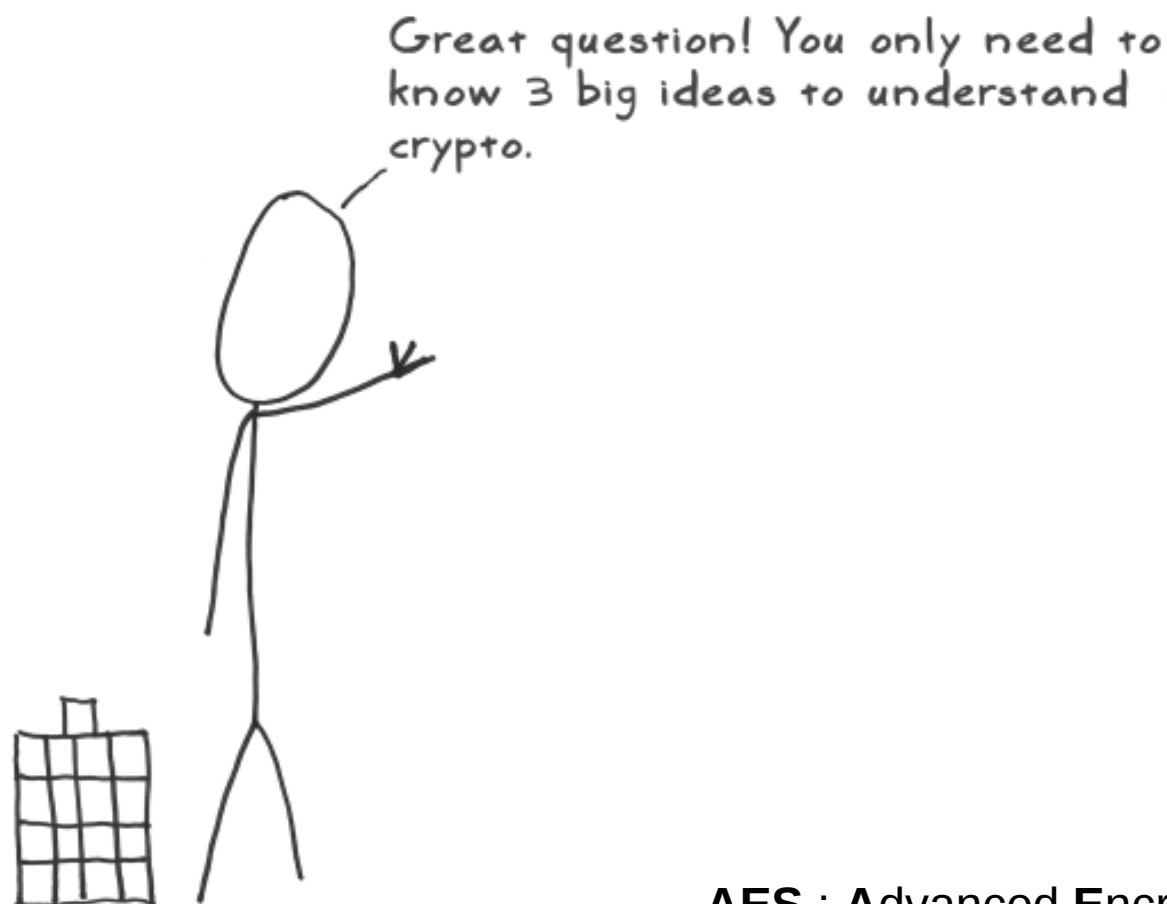


FIG. 2b. Structure en transistors du C-Element à bascule RS de la Fig. 2a.

Sboxes en cryptographie (sym.)



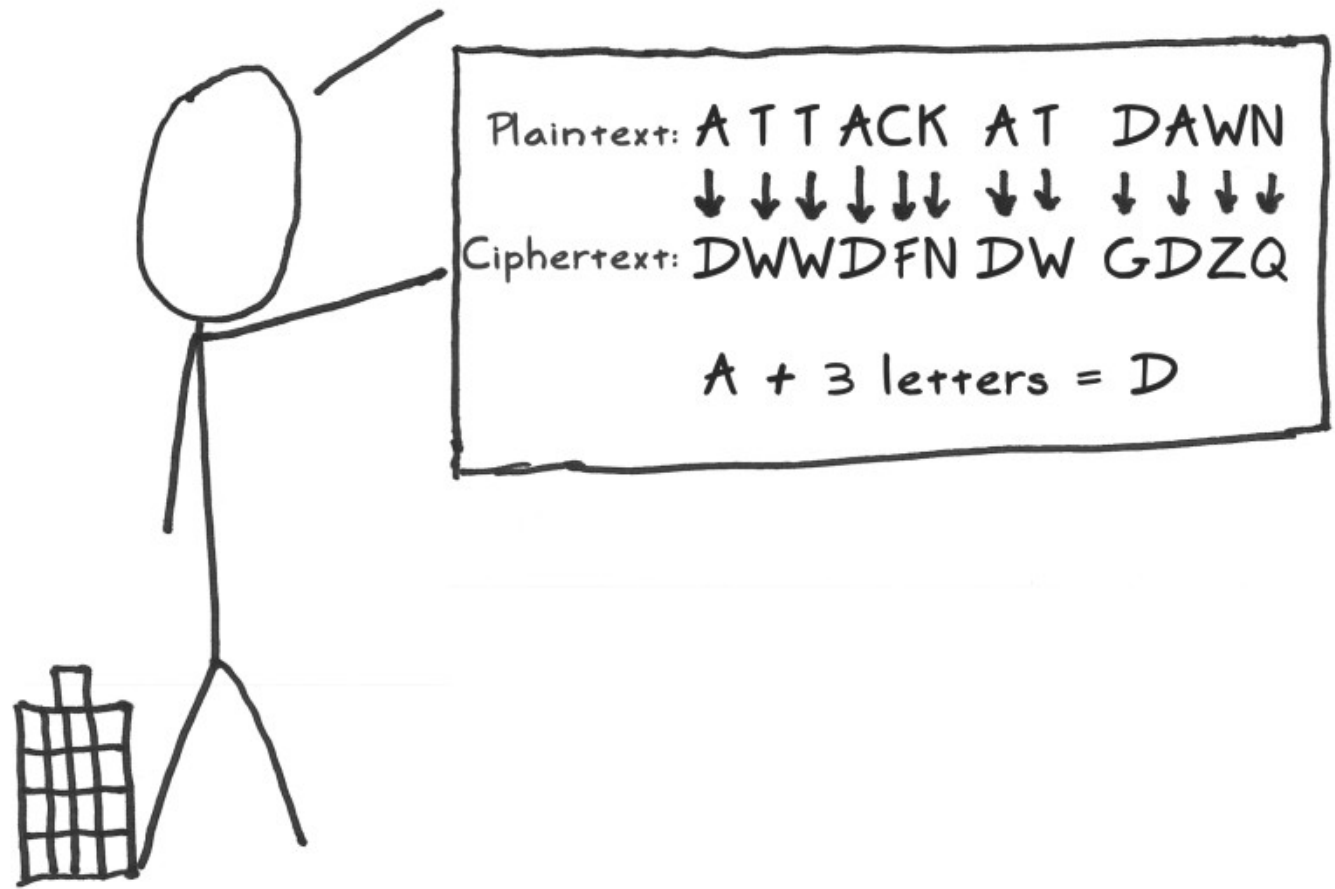
AES : **A**dvanced **E**ncryption **S**tandard

NIST FIPS 197

Included in ISO/IEC 18033-3 standard

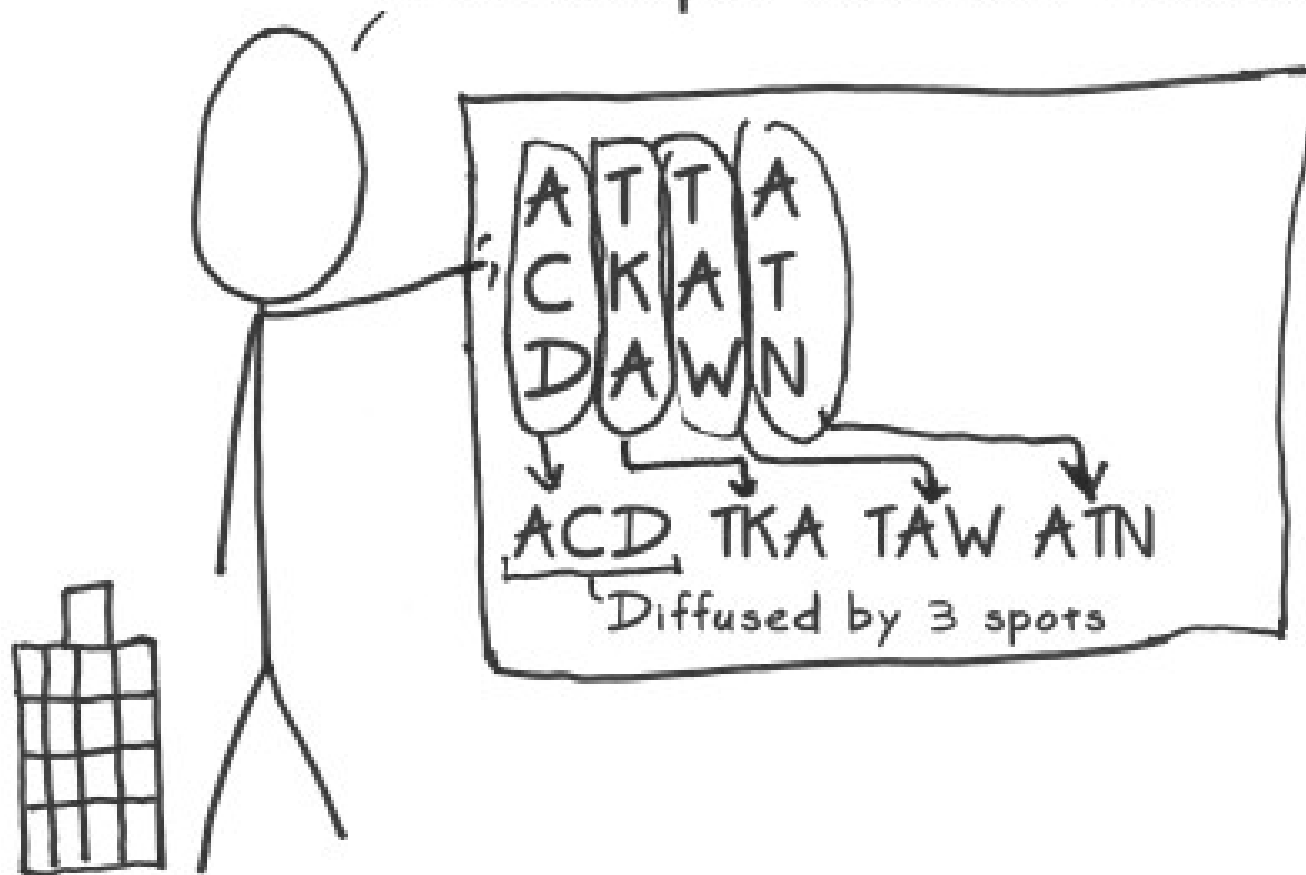
Big Idea #1: Confusion

It's a good idea to obscure the relationship between your real message and your 'encrypted' message. An example of this 'confusion' is the trusty ol' Caesar Cipher:



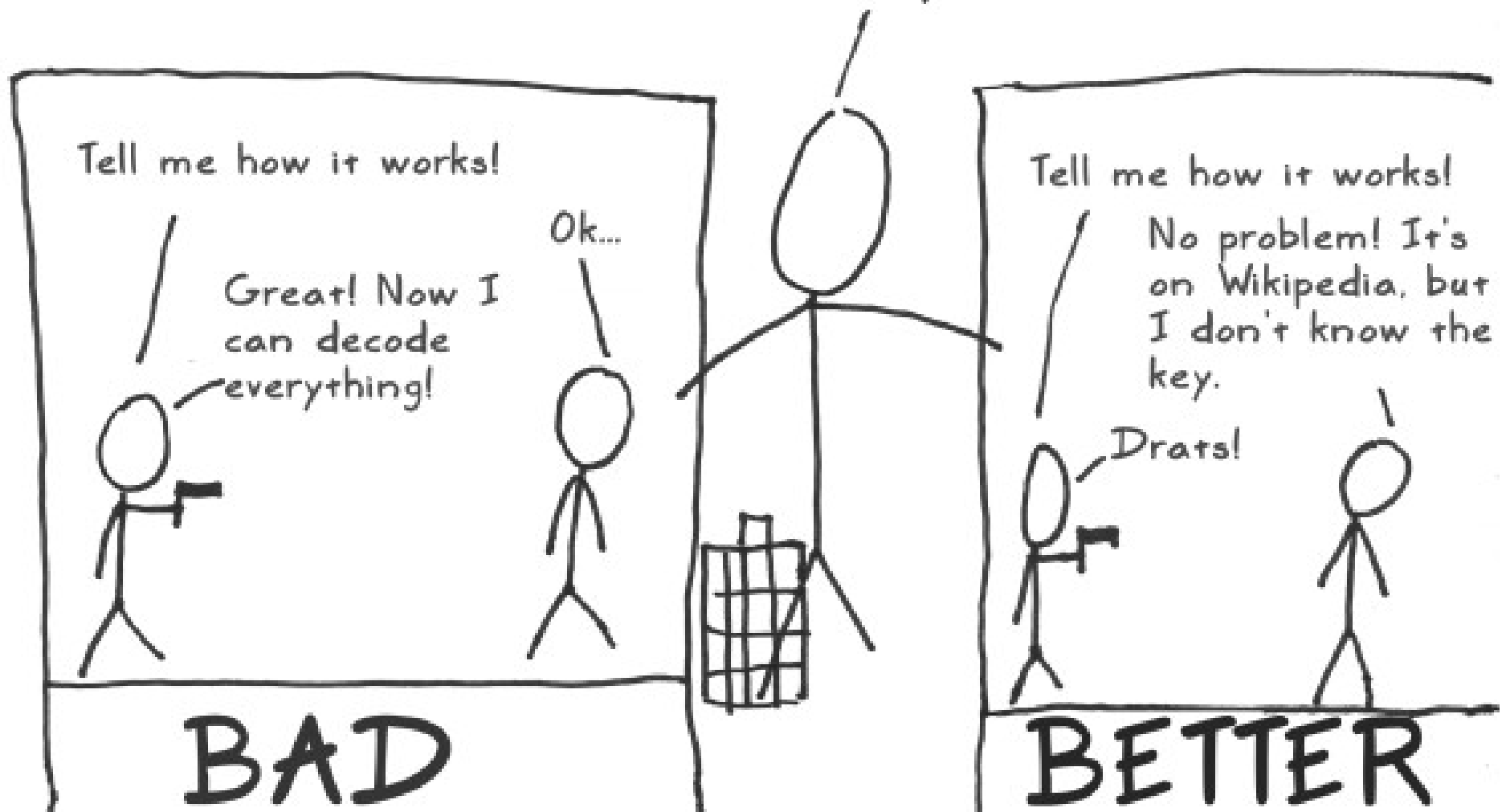
Big Idea #2: Diffusion

It's also a good idea to spread out the message. An example of this "diffusion" is a simple column transposition:



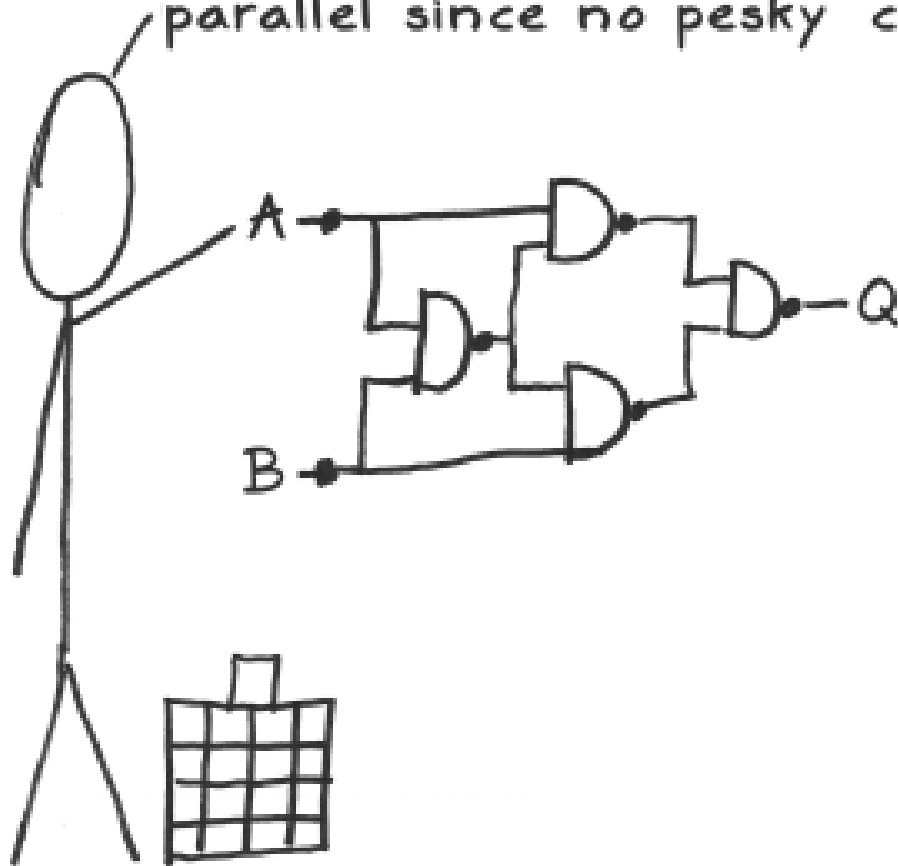
Big Idea #3: Secrecy Only in the Key

After thousands of years, we learned that it's a bad idea to assume that no one knows how your method works. Someone will eventually find that out.



A Tribute to XOR

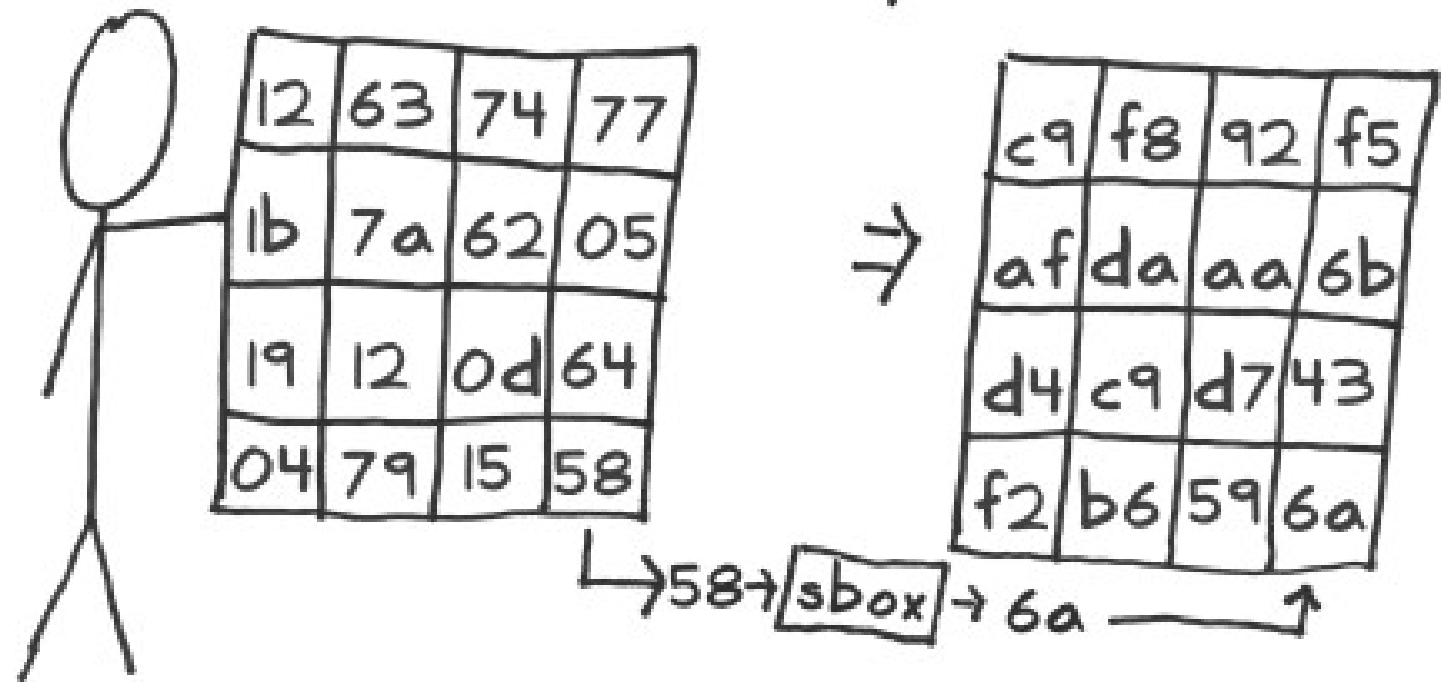
There's a simple reason why I use xor to apply the key and in other spots: it's fast and cheap - a quick bit flipper. It uses minimal hardware and can be done in parallel since no pesky 'carry' bits are needed.



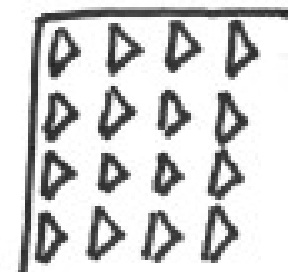
AES ♥ ⊕

Applying Confusion: Substitute Bytes

I use confusion (Big Idea #1) to obscure the relationship of each byte. I put each byte into a substitution box (sbox), which will map it to a different byte:



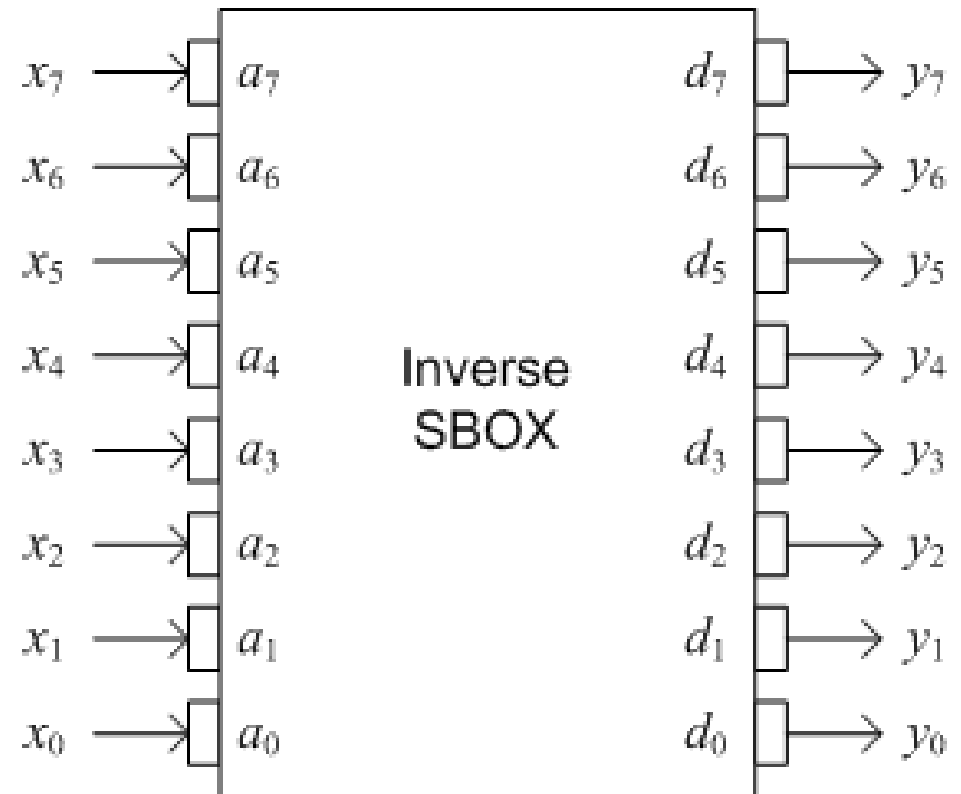
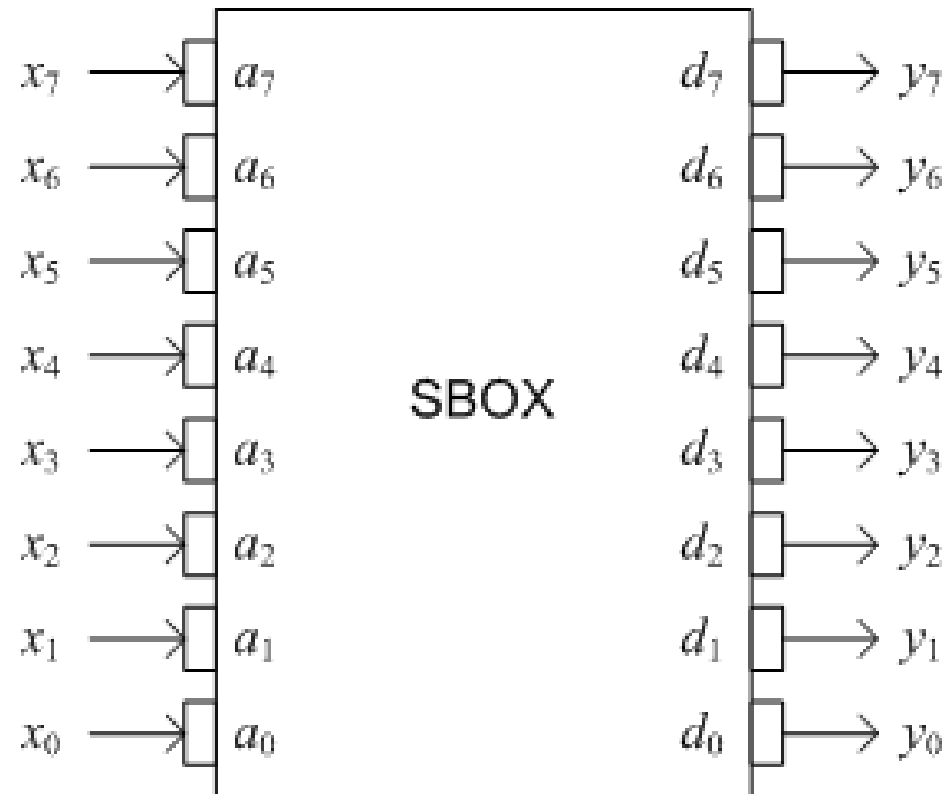
Y Denotes 'confusion'



SubBytes = Substitute Bytes

	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9	xa	xb	xc	xd	xe	xf
0x	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
1x	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
2x	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
3x	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
4x	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
5x	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
6x	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
7x	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
8x	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
9x	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
ax	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
bx	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
cx	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
dx	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
ex	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
fx	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

Hardware view



```

library ieee;
use ieee.std_logic_1164.all;
use ieee.numeric_std.all;
use work.matrix_pkg.all;

entity sbox is
  generic ( sbox_type : string );
  port( clk : in std_ulogic
        ; state_i : in byte
        ; state_o : out byte
        );
end entity;

architecture rtl of sbox is

  type rom_x8 is array(natural range <>) of byte;
  constant rom256x8 : rom_x8 ( 0 to 255 ) :=
    ( x"63", x"7C", x"77", x"7b", x"F2", x"6B", x"6F", x"C5", x"30", x"01", x"67", x"2B", x"FE", x"D7", x"AB", x"76",
      x"CA", x"82", x"C9", x"7D", x"FA", x"59", x"47", x"F0", x"AD", x"D4", x"A2", x"AF", x"9C", x"A4", x"72", x"C0",
      x"B7", x"FD", x"93", x"26", x"36", x"3F", x"F7", x"CC", x"34", x"A5", x"E5", x"F1", x"71", x"D8", x"31", x"15",
      x"04", x"C7", x"23", x"C3", x"18", x"96", x"05", x"9A", x"07", x"12", x"80", x"E2", x"EB", x"27", x"B2", x"75",
      x"09", x"83", x"2C", x"1A", x"1B", x"6E", x"5A", x"A0", x"52", x"3B", x"D6", x"B3", x"29", x"E3", x"2F", x"84",
      x"53", x"D1", x"00", x"ED", x"20", x"FC", x"B1", x"5B", x"6A", x"CB", x"BE", x"39", x"4A", x"4C", x"58", x"CF",
      x"D0", x"EF", x"AA", x"FB", x"43", x"4D", x"33", x"85", x"45", x"F9", x"02", x"7F", x"50", x"3C", x"9F", x"A8",
      x"51", x"A3", x"40", x"8F", x"92", x"9D", x"38", x"F5", x"BC", x"B6", x"DA", x"21", x"10", x"FF", x"F3", x"D2",
      x"CD", x"0C", x"13", x"EC", x"5F", x"97", x"44", x"17", x"C4", x"A7", x"7E", x"3D", x"64", x"5D", x"19", x"73",
      x"60", x"81", x"4F", x"DC", x"22", x"2A", x"90", x"88", x"46", x"EE", x"B8", x"14", x"DE", x"5E", x"0B", x"DB",
      x"E0", x"32", x"3A", x"0A", x"49", x"06", x"24", x"5C", x"C2", x"D3", x"AC", x"62", x"91", x"95", x"E4", x"79",
      x"E7", x"C8", x"37", x"6D", x"8D", x"D5", x"4E", x"A9", x"6C", x"56", x"F4", x"EA", x"65", x"7A", x"AE", x"08",
      x"BA", x"78", x"25", x"2E", x"1C", x"A6", x"B4", x"C6", x"E8", x"DD", x"74", x"1F", x"4B", x"BD", x"8B", x"8A",
      x"70", x"3E", x"B5", x"66", x"48", x"03", x"F6", x"0E", x"61", x"35", x"57", x"B9", x"86", x"C1", x"1D", x"9E",
      x"E1", x"F8", x"98", x"11", x"69", x"D9", x"8E", x"94", x"9B", x"1E", x"87", x"E9", x"CE", x"55", x"28", x"DF",
      x"8C", x"A1", x"89", x"0D", x"BF", x"E6", x"42", x"68", x"41", x"99", x"2D", x"0F", x"B0", x"54", x"BB", x"16");

begin

sbox_ram_gen: if sbox_type = "RAM" generate
  process(clk)
  begin
    if falling_edge(clk) then
      state_o <= rom256x8( to_integer(unsigned(state_i)) );
    end if;
  end process;
end generate;

end architecture;

```

Synthèse logique

```
# Synthesis script.
# Launch with the command:
# >>> rc -gui -files syn.tcl

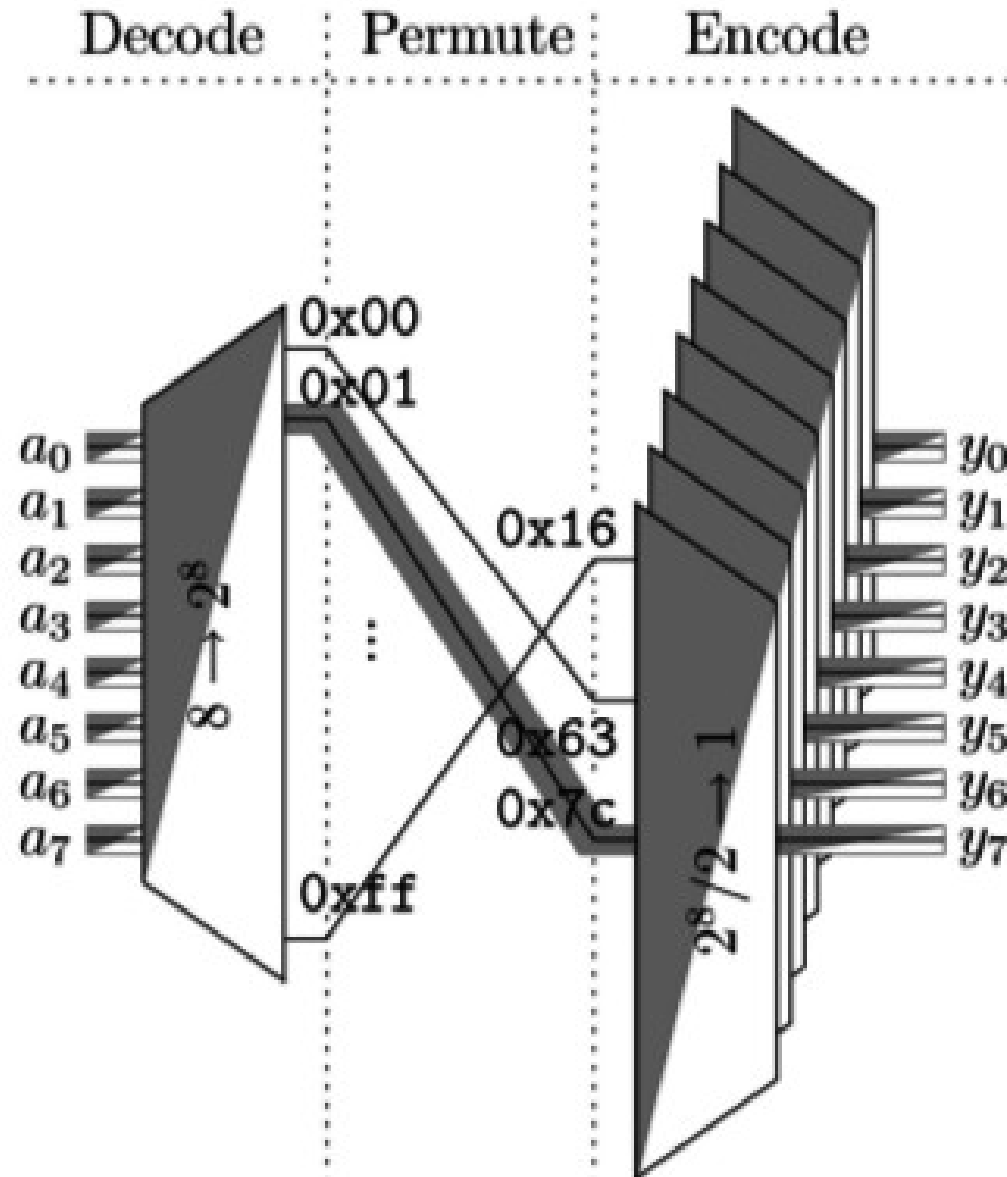
set_attribute library "/comelec/softs/opt/opus_kits/HCMOS9GP_REVG/CORE9GPLL_SNPS_AVT_4.1/SIGNOFF/bc_1.
32V_0C_wc_1.08V_105C/PT_LIB/CORE9GPLL_Worst.lib"
read_hdl -vhdl " ../src/aes/matrix_pkg.vhd                ../src/aes/mixcolumns.vhd                ../src/aes/sbox.v
hd                ../src/aes/shiftrounds.vhd            ../src/aes/state.vhd                    ../src/aes/subbyt
es.vhd            ../src/aes/addroundkey.vhd            ../src/aes/cipher.vhd                  ../src/aes/fsm.vh
d                ../src/aes/keyexpansion.vhd            ../src/aes/aes.vhd                      ../src/aes/aes_25
6_fast.vhd        ../src/sensor/sensor.vhd            ../src/top/top.vhd"

elaborate
synthesize -to_mapped

# Netlist, to be used in encounter
write_hdl -v2001 > top.vm
report power > rpt_power.txt
report area > rpt_area.txt
report memory > rpt_memory.txt
report gates > rpt_gates.txt
report timing > rpt_timing.txt
report timing -lint > rpt_timing.txt

quit
```


Decode – Permute - Encode



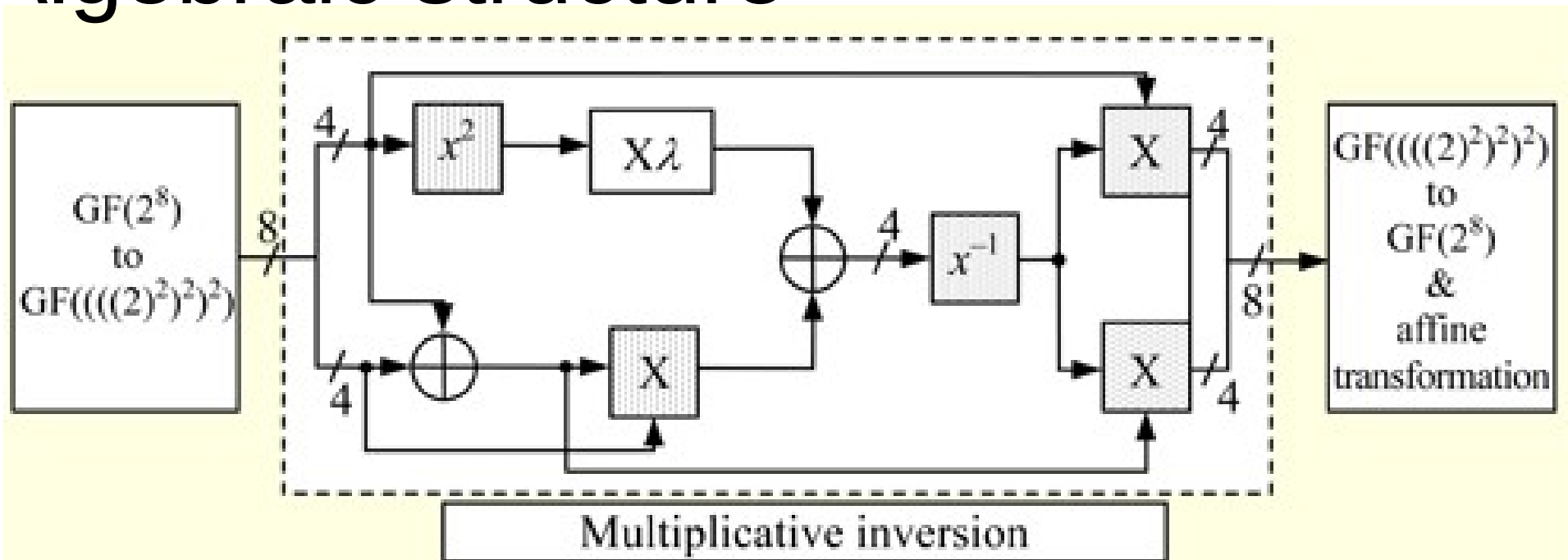
In the polynomial representation, multiplication in $GF(2^8)$ (denoted by \bullet) corresponds with the multiplication of polynomials modulo an **irreducible polynomial** of degree 8. A polynomial is irreducible if its only divisors are one and itself. **For the AES algorithm, this irreducible polynomial is**

$$m(x) = x^8 + x^4 + x^3 + x + 1, \quad (4.1)$$

Sbox : $a \rightarrow b = a^{-1}$, then $b \rightarrow b'$

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Algebraic structure



Conclusion for AES

Architecture \ Critère	Taille	Energie	Temps
Table			x
Decode – Permute – Encode		x	
Algèbre	x		

Pipeline de processeurs

- Example on 6502 processor (8 bit) :
 - download here : <https://github.com/chenxiao07/vhdl-nes/tree/master>
 - read in file `vhdl-nes-master/src/free6502.vhd` lines 765 to 772

```
-- The ALU adder
alu_add <= alu_in1 + alu_add_in2 + alu_add_cin;

-- The ALU itself    This is purely combinatorial logic
process (alu_add, alu_in1, alu_in2, alu_op, c_flag)
begin
  -- default for alu_add inputs
  alu_add_in2 <= alu_in2;
  alu_add_cin <= c_flag;

  case alu_op is
    -- [...]
765    when MC_BIT_AND => alu_out <= alu_in1 and alu_in2;
    when MC_BIT_OR  => alu_out <= alu_in1 or  alu_in2;
    when MC_BIT_XOR => alu_out <= alu_in1 xor alu_in2;
    when MC_BIT_ASL => alu_out <= alu_in1(7 downto 0) & "0";
    when MC_BIT_LSR => alu_out <= alu_in1(0) & "0" & alu_in1(7 downto 1);
    when MC_BIT_ROL => alu_out <= alu_in1(7 downto 0) & c_flag;
772    when MC_BIT_ROR => alu_out <= alu_in1(0) & c_flag & alu_in1(7 downto 1);
    when others     => alu_out <= alu_in1;
  end case;
end process;
```

Pipeline de processeurs

Example on LEON processor (32 bit) :

see `gaisler/leon3v3/iu3.vhd`

```
procedure logic_op(r : registers; aluin1, aluin2, mey : word;
    ymsb : std_ulogic; logicres, y : out word) is
variable logicout : word;
begin
    case r.e.aluop is
    when EXE_AND    => logicout := aluin1 and aluin2;
    when EXE_ANDN  => logicout := aluin1 and not aluin2;
    when EXE_OR    => logicout := aluin1 or aluin2;
    when EXE_ORN   => logicout := aluin1 or not aluin2;
    when EXE_XOR   => logicout := aluin1 xor aluin2;
    when EXE_XNOR  => logicout := aluin1 xor not aluin2;
    when EXE_DIV   =>
        if DIVEN then logicout := aluin2;
        else logicout := (others => '-'); end if;
    when others => logicout := (others => '-');
    end case;
    if (r.e.ctrl.wy and r.e.mulstep) = '1' then
        y := ymsb & r.m.y(31 downto 1);
    elsif r.e.ctrl.wy = '1' then y := logicout;
    elsif r.m.ctrl.wy = '1' then y := mey;
    elsif MACPIPE and (r.x.mac = '1') then y := mulo.result(63 downto 32);
    elsif r.x.ctrl.wy = '1' then y := r.x.y;
    else y := r.w.s.y; end if;
    logicres := logicout;
end;
```

Synthétisabilité

Unsupported Verilog Language Constructs

Synthesis tools are not so intelligent. They try to infer hardware from HDL description.

Most synthesis tools does not support the following Verilog constructs:

- Unsupported definitions and declarations

- primitive definition
- time declaration
- event declaration
- triand, trior, tri1, tri0, and triregnet types
- Ranges and arrays for integers

- Unsupported statements

- defparam statement
- initial statement
- repeat statement
- delay control
- event control
- wait statement
- fork-join statements
- deassign statement
- force statement
- release statement
- Assignment statement with a variable used as a bit-select on the left side of the equal sign

- Unsupported operators

- Case equality and inequality operators (=== and !==)
- Division and modulus operators for variables

- Unsupported gate-level constructs

- nmos, pmos, cmos, rmos, rmos, rmos, pullup, pulldown, tranif0, tranif1, rtran, rtrainf0, and rtrainf1gate types

- Unsupported miscellaneous constructs

- Hierarchical names within a module

Assistance à la synthèse

- VHDL

- attribute keep of clock_signal_name:
signal is "true";

- Verilog

- // synthesis attribute keep of
clock_signal_name is true;

- Comme des scripts :

- set_dont_touch [get_cells
I_cdnuser_cts/S_CKsunderM_DLY_1]

Fibonacci

- $U_0 = 0$
- $U_1 = 1$
- $U_2 = U_1 + U_0$
- $U_3 = U_2 + U_1$
- ...

THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®

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(Greetings from [The On-Line Encyclopedia of Integer Sequences!](#))

Search: **seq:0,1,1,2,3,5,8,13**

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Sort: [relevance](#) | [references](#) | [number](#) | [modified](#) | [created](#) Format: [long](#) | [short](#) | [data](#)

[A000045](#)

Fibonacci numbers: $F(n) = F(n-1) + F(n-2)$ with $F(0) = 0$ and $F(1) = 1$.

+20
3752

(Formerly M0692 N0256)

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 0,4

COMMENTS

Also sometimes called Lamé's sequence.

$F(n+2)$ = number of binary sequences of length n that have no consecutive 0's.

$F(n+2)$ = number of subsets of $\{1,2,\dots,n\}$ that contain no consecutive integers.

$F(n+1)$ = number of tilings of a $2 \times n$ rectangle by 2×1 dominoes.

$F(n+1)$ = number of matchings (i.e., Hosoya index) in a path graph on n vertices:

$F(5)=5$ because the matchings of the path graph on the vertices A, B, C, D are the empty set, $\{AB\}$, $\{BC\}$, $\{CD\}$ and $\{AB, CD\}$. - [Emeric Deutsch](#), Jun 18 2001

$F(n)$ = number of compositions of $n+1$ with no part equal to 1. [Cayley, Grimaldi]

Positive terms are the solutions to $z = 2*x*y^4 + (x^2)*y^3 - 2*(x^3)*y^2 - y^5 - (x^4)*y + 2*y$ for $x,y \geq 0$ (Ribenoim, page 193). When $x=F(n)$, $y=F(n+1)$ and $z>0$ then $z=F(n+1)$.

Fibonacci en SystemVerilog

```
module fibonacci(clk,
                reset_n,
                U);

    input logic    clk;
    input logic    reset_n;
    output logic [7:0] U;

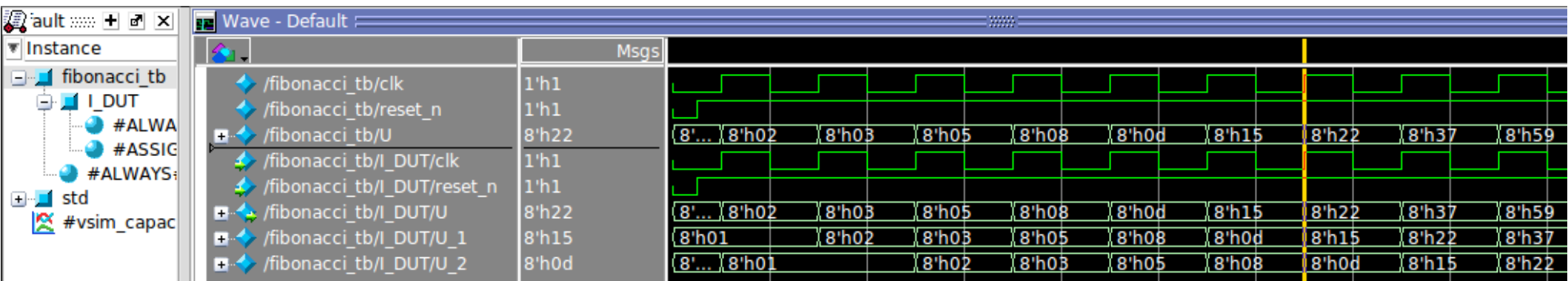
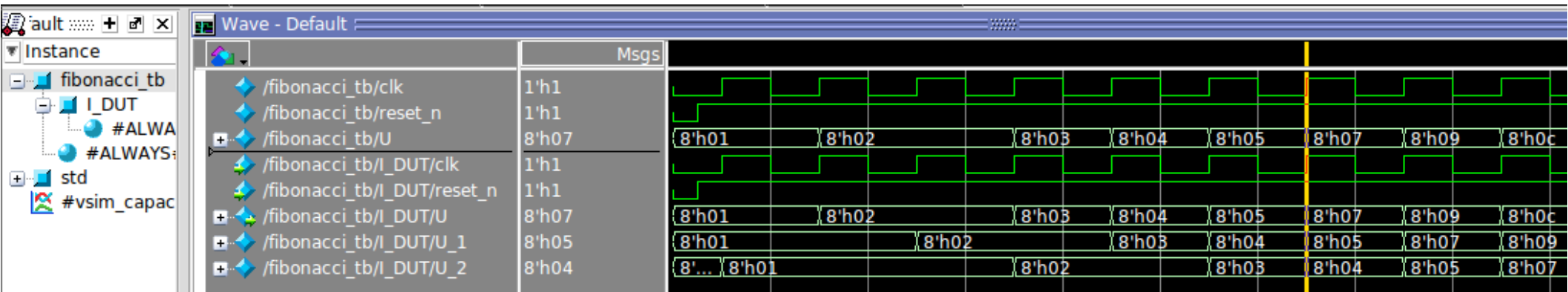
    // Un-1 (= Un au cycle précédent)
    logic [7:0]    U_1;

    // Un-2 (= Un-1 au cycle précédent)
    logic [7:0]    U_2;

    always @(posedge clk or negedge reset_n)
        if(reset_n==0)
            begin
                U    <= 1;
                U_1 <= 1;
                U_2 <= 0;
            end
        else
            begin
                // Un = Un-1 + Un-2
                U    <= U_1 + U_2;
                // Un-1 = Un retardé d'un cycle
                U_1 <= U;
                // Un-2 = Un-1 retardé d'un cycle
                U_2 <= U_1;
            end
    endmodule
```

Simulation

- `vlog fibonacci.sv fibonacci_tb.sv`
- `vsim fibonacci_tb`



Exercice d'arithmétique

```
#include <stdint.h>

uint32_t guess( uint32_t x, uint32_t y )
{
    return ( x & y ) + (( x ^ y )>>1);
}
```

Exercice d'arithmétique

```
#include <stdint.h>

uint32_t guess( uint32_t x, uint32_t y )
{
    return ( x & y ) + (( x ^ y )>>1);
}
```

Lemmas :

- $(x+y) = (x | y) + (x \& y)$
- $(x \oplus y) = (x | y) - (x \& y)$

Exercice d'arithmétique

```
#include <stdint.h>

uint32_t guess( uint32_t x, uint32_t y )
{
    return ( x & y ) + (( x ^ y )>>1);
}
```

Lemmas :

Otherwise, consider

$(a \gg 1) + (b \gg 1) + ((a \% 2 + b \% 2) \gg 1)$

- $(x+y) = (x | y) + (x \& y)$
- $(x \oplus y) = (x | y) - (x \& y)$

TD (Travaux Dirigés)

- Exercices :
 - <http://perso.telecom-paristech.fr/~guilley/ENS/20161206/TP/>
- Entraînement de type MOOC :
 - <http://hdl.telecom-paristech.fr/>