The Power Model of Fitts' Law Does Not Encompass the Logarithmic Model

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Fitts' law is a well-known empirical rule of thumb which predicts the average time T it takes people, under time pressure, to reach with some pointer a target of width W located at distance D. Within the classic experimental paradigm settled by Fitts [1], the law is a relation of the form $T = f(D/W)$, where f stands for some strictly increasing function. Two formulations are well-known:

$$T = a + b \cdot \log_2\Big(2\frac{D}{W}\Big) \qquad \text{(Fitts [1])} \qquad (1)$$

$$T = a + b \cdot \Big(\frac{D}{W}\Big)^{1/n} \qquad \text{(Meyer } \textit{et al.} \text{ [2, 4])} \qquad (2)$$

Whether Fitts' law is a logarithmic (1) or a power law (2) has remained unclear so far. The curves look similar over the rather narrow range of D/W that can be actually investigated in the laboratory.

In two widely cited papers [2, 4], Meyer *et al.* have suggested there is no real log vs. power issue about Fitts' law. Arguing that $a + b \cdot (D/W)^{1/n} \to a' + b' \cdot \ln(D/W)$ as the maximum number of submovements $n \to +\infty$, they claimed that the power model of Fitts' law they derived from their substantive theory—the celebrated stochastic optimized submovement theory—encompasses the logarithmic model as a limiting case.

We review the submovement theory [2, 3]: Consider the recursive functional equation predicted by the theory after n submovements, assuming uniformly distributed endpoints:

$$f_n\Big(\frac{D}{W}\Big) = \min_s \left\{ \frac{D/W - 1/2}{s} + \frac{2}{s}\int_{1/2}^{s/2} f_{n-1}(x)\,\mathrm{d}x \right\} \qquad (n > 1) \qquad (3)$$

We derive an easy proof that the solution $T = f_n(D/W)$ is given by the positive root T of the nth order equation

$$2\frac{D}{W} = 1 + 2T + \frac{(2T)^2}{2} + \cdots + \frac{(2T)^n}{n!}. \qquad (4)$$

The resulting model does indeed tend to the logarithmic $T = \ln(2D/W)/2$ as $n \to +\infty$, while for $n = 2$ we do recover the square-root model derived by Meyer *et al.* in [2]. However, our analysis makes it clear why the solution cannot be, even to a rough approximation as n grows large, identified with a power law of the form (2).

Even if one takes (2) for granted, we demonstrate that Meyer *et al.*'s claim is false: there do not even exist sequences a_n, b_n such that the model $a_n + b_n(D/W)^{1/n}$ tends to a logarithmic model as $n \to +\infty$, as was suggested in [4, Fig. 6.13].

Meyer *et al.* [2, 4] have convinced the community of Fitts' law students that their submovement theory leads to a power model that encompasses the logarithmic models. But it appears that (i) their theory does not lead to a genuine power model, and (ii) their supposedly power model does not encompass the logarithmic one. At any rate, awareness that in fact the two classes of candidate mathematical descriptions of Fitts' law are not equivalent should stimulate experimental research.

[1] Fitts, P.M. (1954) *J. Exp. Psychol.* 47, 381–391.

[2] Meyer, D.E., Abrams, R.A., Kornblum, S., Wright, C.E., & Smith, J.E.K. (1988) *Psychol. Rev.* 95, 340–370.

[3] Smith, J.E.K. (1988) in D.R. Brown & J.E.K. Smith (Eds.), *Frontiers of mathematical psychology: Essays in honor of Clyde Coombs* (pp. 193–202). New York: Springer.

[4] Meyer, D.E., Smith, J.E.K., Kornblum, S., Abrams, R.A., & Wright, C.E. (1990) in M. Jeannerod (Ed.), *Attention and performance XIII* (pp. 173–226). Hillsdale, NJ: Erlbaum.