

# PERFORMANCE ANALYSIS OF BLIND CARRIER FREQUENCY OFFSET AND SYMBOL TIMING DELAY ESTIMATORS IN FLAT-FADING CHANNELS

Y. Wang<sup>1</sup>, E. Serpedin<sup>1</sup>, P. Ciblat<sup>2</sup>, and P. Loubaton<sup>3</sup>

<sup>1</sup> Dept. of Electrical Engineering, Texas A&M University, College Station, TX 77843, USA

<sup>2</sup> Université Catholique de Louvain, Laboratoire “Téledétection et Télécommunications”, Belgium

<sup>3</sup> Université de Marne-la-Vallée, Laboratoire “Systèmes de Communication”, Marne-la-Vallée, France

## ABSTRACT

By exploiting the received signal’s second-order cyclostationary statistics, blind algorithms have been recently proposed for joint estimation of the frequency offset and the symbol timing delay of a linearly modulated waveform transmitted through a flat-fading channel. The goal of this paper is to establish and analyze the asymptotic (large sample) performance of the Gini-Giannakis [4] and Ghogho-Swami-Durrani [3] estimators as a function of the pulse shape bandwidth and the oversampling factor. It is shown that the performance of these estimators improves as the pulse shape bandwidth increases, and the best performance is obtained by selecting small values for the oversampling factor.

## 1. INTRODUCTION

In mobile radio channels the loss of synchronization often occurs [5], and re-acquisition must be performed in a fast and reliable way without sacrificing bandwidth for periodic re-training. Therefore, developing optimal blind synchronization architectures appears as an important problem. Recently, blind carrier frequency offset and symbol timing delay estimators that exploit the second-order cyclostationary (CS) statistics, introduced by oversampling the received waveform, have been proposed in [3], [4], [7].

The goal of this paper is to analyze and design criteria for improving the performance of the existing blind carrier frequency offset and symbol timing delay estimators [3], [4] with respect to (w.r.t.) the pulse shape bandwidth and the oversampling factor. The theoretical asymptotic performance of the Gini-Giannakis (GG) [4] and Ghogho-Swami-Durrani (GSD) [3] estimators is established, and it is shown that significant performance improvement can be obtained by selecting small values for the oversampling factor ( $P = 2, 3$ ) and pulse shapes with larger bandwidths. Both experimental and theoretical results show that larger oversampling factors ( $P = 4, \dots, 8$ ) are not justifiable from a computational and performance analysis viewpoint. By dividing the receiver samples into several disjoint subsets and by exploiting separately the resulting subsets, it is shown that the performance of the GG estimator can be improved by averaging the corresponding estimates that are obtained from each subset.

## 2. MODELING ASSUMPTIONS

Consider the baseband representation of a linearly modulated signal transmitted through a flat-fading channel. The receiver output can be expressed as<sup>1</sup>:

$$x_c(t) = \mu_c(t)e^{j2\pi f_e t} \sum_l w(l)h_c(t - \epsilon T - lT) + v_c(t), \quad (1)$$

<sup>1</sup>We use the subscript  $c$  to denote continuous-time signals.

where  $\mu_c(t)$  is the fading-induced noise,  $w(l)$ ’s are zero-mean unit variance independent and identically distributed (i.i.d.) symbols,  $h_c(t)$  denotes the convolution of the transmitter’s signaling pulse and the receive filter,  $v_c(t)$  is the additive noise,  $T$  is the symbol period,  $f_e$  and  $\epsilon$  stand for the frequency offset and the timing delay, respectively, which are the parameters to be estimated by exploiting the second-order CS-statistics of the received waveform.

By oversampling the received signal  $x_c(t)$  (see eq. (1)) with the sampling period  $T_s := T/P$  ( $P \geq 2$ ), the following discrete-time model is obtained:

$$x(n) = \mu(n)e^{j2\pi f_e Tn/P} \sum_l w(l)h(n - lP) + v(n), \quad (2)$$

with  $x(n) := x_c(nT_s)$ ,  $\mu(n) := \mu_c(nT_s)$ ,  $v(n) := v_c(nT_s)$ , and  $h(n) := h_c(nT_s - \epsilon T)$ . In order to simplify the derivation of the asymptotic performance of estimators [3], [4], we assume the following:

(AS1)  $w(n)$  is a zero-mean i.i.d. sequence with  $\sigma_w^2 = 1$ .

(AS2)  $\mu(n)$  is a constant fading-induced noise with unit energy.

(AS3)  $v(n)$  is a zero-mean white process independent of  $w(n)$ , with variance  $\sigma_v^2$ .

(AS4) the combined filter  $h_c(t)$  is a raised cosine pulse of bandwidth  $[-(1 + \rho)/2T, (1 + \rho)/2T]$ , where the parameter  $\rho$  represents the rolloff factor ( $0 \leq \rho < 1$ ) [6, Ch. 9].

(AS5) frequency offset  $f_e$  is small enough so that the mismatch of the receive filter due to  $f_e$  can be neglected [4]. Generally,  $f_e T < 0.05$  is assumed.

## 3. BLIND CARRIER FREQUENCY OFFSET AND TIMING DELAY ESTIMATORS

In this paper, the time-varying correlation of  $x(n)$  is defined as  $c_{2x}(n; \tau) := E\{x^*(n)x(n + \tau)\}$ , where  $\tau$  is an integer lag. Straightforward calculations show that  $c_{2x}(n; \tau) = c_{2x}(n + P; \tau)$ ,  $\forall n, \tau$ . Being periodic,  $c_{2x}(n; \tau)$  admits a Fourier Series (FS) expansion whose FS-coefficients, termed cyclic correlations, are given by the following expression for  $P \geq 3$  [3], [4]:

$$C_{2x}(k; \tau) = \frac{\sigma_w^2}{P} e^{j2\pi f_e T\tau/P} e^{-j2\pi k\epsilon} G_2(k; \tau) e^{j\pi k\tau/P} \cdot e^{-j2\pi k t_0/T} + \sigma_v^2 \delta(\tau) \delta(k), \quad (3)$$

where  $G_2(k; \tau) := (P/T) \int_{-P/2T}^{P/2T} H_c(F - k/2T) H_c(F + k/2T) \exp(j2\pi F\tau T/P) dF$ ,  $H_c(F)$  denotes the Fourier Transform (FT) of  $h_c(t)$ , and  $t_0$  is a known time delay chosen to ensure causality of  $h_c(t)$ .

In practice, the cyclic correlations  $C_{2x}(k; \tau)$  have to be estimated from a finite number of samples  $N$ . The standard sample estimate of  $C_{2x}$  is given by (see e.g., [2] and [4]):

$$\hat{C}_{2x}(k; \tau) = \frac{1}{N} \sum_{n=0}^{N-\tau-1} x^*(n)x(n+\tau)e^{-j2\pi kn/P}, \quad \tau \geq 0. \quad (4)$$

The GG estimator determines the frequency offset  $f_e$  and the timing delay  $\epsilon$  from the following eqs. [4, eqs. (10), (11)]:

$$\hat{f}_e = \frac{P}{4\pi T\tau} \arg\{\hat{C}_{2x}(1; \tau)\hat{C}_{2x}(-1; \tau)\}, \quad (5)$$

$$\hat{\epsilon} = -\frac{1}{2\pi} \arg\{\hat{C}_{2x}(1; \tau)e^{-\frac{j2\pi f_e T\tau}{P}}e^{-\frac{j\pi\tau}{P}}e^{\frac{j2\pi t_0}{T}}\}. \quad (6)$$

As described in [3], the accuracy of the estimators in (5) and (6) does not vary significantly with  $\tau$ . In this paper, we choose  $\tau = 1$  for the GG estimator. One can see that in this case, the frequency offset estimators corresponding to GSD [3, eq. (7)] and GG [4, eq. (10)] algorithms coincide. However, the timing delay estimator corresponding to the GSD algorithm [3, eq. (8)] is different from the GG estimator (6) and is given by the following relation:

$$\hat{\epsilon} = -\frac{1}{2\pi} \arg\{\hat{C}_{2x}(1; 0)e^{\frac{j2\pi t_0}{T}}\}. \quad (7)$$

#### 4. PERFORMANCE ANALYSIS

The estimators of  $f_e$  and  $\epsilon$  are asymptotically unbiased and consistent [3], [4]. In this section, we will establish the asymptotic variances of  $\hat{f}_e$  and  $\hat{\epsilon}$ , which are defined as  $\lim_{N \rightarrow \infty} NE\{(\hat{f}_e - f_e)^2\}$  and  $\lim_{N \rightarrow \infty} NE\{(\hat{\epsilon} - \epsilon)^2\}$ , respectively. Because of lack of space, only the results for  $P \geq 4$  are presented here. If we define the normalized unconjugated/conjugated asymptotic variances of the cyclic correlations by means of the following relations [2]:

$$\begin{aligned} \left[ \mathbf{\Gamma}^{(k,m)} \right]_{u,v} &:= \lim_{N \rightarrow \infty} NE \left\{ \left( \hat{C}_{2x}(k, u) - C_{2x}(k, u) \right) \right. \\ &\quad \cdot \left. \left( \hat{C}_{2x}(m, v) - C_{2x}(m, v) \right)^* \right\}, \\ \left[ \tilde{\mathbf{\Gamma}}^{(k,m)} \right]_{u,v} &:= \lim_{N \rightarrow \infty} NE \left\{ \left( \hat{C}_{2x}(k, u) - C_{2x}(k, u) \right) \right. \\ &\quad \cdot \left. \left( \hat{C}_{2x}(m, v) - C_{2x}(m, v) \right) \right\}, \end{aligned}$$

where  $k, m = \pm 1$ , then the following proposition, which is an extension of the result presented in [2], can be established:

**Proposition 1.** *The asymptotic variances of the cyclic correlations are given by:*

$$\begin{aligned} \left[ \mathbf{\Gamma}^{(-1,-1)} \right]_{u,v} &= \sum_{\tau} C_{2x}(0; \tau + u - v)C_{2x}^*(0; \tau)e^{j2\pi\tau/P} \\ &\quad + \kappa PC_{2x}(-1; u)C_{2x}^*(-1; v), \\ \left[ \mathbf{\Gamma}^{(1,1)} \right]_{u,v} &= \sum_{\tau} C_{2x}(0; \tau + u - v)C_{2x}^*(0; \tau)e^{-j2\pi\tau/P} \\ &\quad + \kappa PC_{2x}(1; u)C_{2x}^*(1; v), \\ \left[ \mathbf{\Gamma}^{(1,-1)} \right]_{u,v} &= \sum_{\tau} C_{2x}(1; \tau + u - v)C_{2x}^*(-1; \tau)e^{j2\pi(v-\tau)/P} \\ &\quad + \kappa PC_{2x}(1; u)C_{2x}^*(-1; v), \\ \left[ \mathbf{\Gamma}^{(-1,1)} \right]_{u,v} &= \left[ \mathbf{\Gamma}^{*(1,-1)} \right]_{v,u}, \end{aligned}$$

where  $\kappa$  stands for the kurtosis of symbol sequence  $w(n)$ .

Since  $C_{2x}(k; \tau) = e^{j2\pi k\tau/P}C_{2x}^*(-k; -\tau)$  [1], it follows that:

$$\left[ \tilde{\mathbf{\Gamma}}^{(k,m)} \right]_{u,v} = e^{j2\pi mv/P} \left[ \mathbf{\Gamma}^{(k,-m)} \right]_{u,-v}.$$

#### 4.1. GG Estimator

By exploiting Proposition 1 and the eqs. (5) and (6), the asymptotic variances of  $\hat{f}_e$  and  $\hat{\epsilon}$  can be obtained and are given by:

**Proposition 2.** *The asymptotic variance of the frequency offset estimator (5) is given by:*

$$\lim_{N \rightarrow \infty} NE\{(\hat{f}_e - f_e)^2\} = \zeta_1^2 \cdot \left( \frac{\mathcal{V}_{11}}{\alpha_1^2} + \frac{\mathcal{V}_{12}}{\beta_1^2} - \frac{2\mathcal{V}_{13}}{\alpha_1\beta_1} \right),$$

where<sup>2</sup>:

$$\zeta_1 := \frac{P}{4\pi T} \cdot \frac{\tan(4\pi T f_e/P)}{1 + \tan^2(4\pi T f_e/P)},$$

$$\alpha_1 := C_{2x}(1; 1)C_{2x}(-1; 1) - C_{2x}^*(1; 1)C_{2x}^*(-1; 1),$$

$$\beta_1 := C_{2x}(1; 1)C_{2x}(-1; 1) + C_{2x}^*(1; 1)C_{2x}^*(-1; 1),$$

$$\begin{aligned} \mathcal{V}_{11} &= 2\text{re} \left( C_{2x}^2(1; 1)\tilde{\mathbf{\Gamma}}_{1,1}^{(-1,-1)} \right) + 2\text{re} \left( C_{2x}^2(-1; 1)\tilde{\mathbf{\Gamma}}_{1,1}^{(1,1)} \right) \\ &\quad + 4\text{re} \left( C_{2x}(1; 1)C_{2x}(-1; 1)\tilde{\mathbf{\Gamma}}_{1,1}^{(-1,1)} \right) \\ &\quad - 4\text{re} \left( C_{2x}(1; 1)C_{2x}^*(-1; 1)\mathbf{\Gamma}_{1,1}^{(-1,1)} \right) \\ &\quad - 2C_{2x}(1; 1)C_{2x}^*(1; 1)\mathbf{\Gamma}_{1,1}^{(-1,-1)} \\ &\quad - 2C_{2x}(-1; 1)C_{2x}^*(-1; 1)\mathbf{\Gamma}_{1,1}^{(1,1)}, \end{aligned}$$

$$\begin{aligned} \mathcal{V}_{12} &= 2\text{re} \left( C_{2x}^2(1; 1)\tilde{\mathbf{\Gamma}}_{1,1}^{(-1,-1)} \right) + 2\text{re} \left( C_{2x}^2(-1; 1)\tilde{\mathbf{\Gamma}}_{1,1}^{(1,1)} \right) \\ &\quad + 4\text{re} \left( C_{2x}(1; 1)C_{2x}(-1; 1)\tilde{\mathbf{\Gamma}}_{1,1}^{(-1,1)} \right) \\ &\quad + 4\text{re} \left( C_{2x}(1; 1)C_{2x}^*(-1; 1)\mathbf{\Gamma}_{1,1}^{(-1,1)} \right) \\ &\quad + 2C_{2x}(1; 1)C_{2x}^*(1; 1)\mathbf{\Gamma}_{1,1}^{(-1,-1)} \\ &\quad + 2C_{2x}(-1; 1)C_{2x}^*(-1; 1)\mathbf{\Gamma}_{1,1}^{(1,1)}, \end{aligned}$$

$$\begin{aligned} \mathcal{V}_{13} &= j2 \left\{ \text{im} \left( C_{2x}^2(1; 1)\tilde{\mathbf{\Gamma}}_{1,1}^{(-1,-1)} \right) + \text{im} \left( C_{2x}^2(-1; 1)\tilde{\mathbf{\Gamma}}_{1,1}^{(1,1)} \right) \right. \\ &\quad \left. + 2\text{im} \left( C_{2x}(1; 1)C_{2x}(-1; 1)\tilde{\mathbf{\Gamma}}_{1,1}^{(-1,1)} \right) \right\}. \end{aligned}$$

**Proposition 3.** *The asymptotic variance of the timing delay estimator (6) is given by:*

$$\lim_{N \rightarrow \infty} NE\{(\hat{\epsilon} - \epsilon)^2\} = \zeta_2^2 \left( \frac{\mathcal{V}_{21}}{\alpha_2^2} + \frac{\mathcal{V}_{22}}{\beta_2^2} - \frac{2\mathcal{V}_{23}}{\alpha_2\beta_2} \right),$$

where:

$$\zeta_2 := \frac{1}{2\pi} \cdot \frac{\tan(2\pi\epsilon)}{1 + \tan^2(2\pi\epsilon)}, \quad \psi_1 := e^{j\pi(2t_0/T-1)/P},$$

$$\begin{aligned} \alpha_2 &:= \psi_1 C_{2x}(1; 1)e^{-j2\pi f_e T/P} - \psi_1^* C_{2x}^*(1; 1)e^{j2\pi f_e T/P}, \\ \beta_2 &:= \psi_1 C_{2x}(1; 1)e^{-j2\pi f_e T/P} + \psi_1^* C_{2x}^*(1; 1)e^{j2\pi f_e T/P}, \end{aligned}$$

<sup>2</sup>“re” and “im” stand for the real and imaginary part, respectively.

$$\begin{aligned}
\mathcal{V}_{21} &= 2\text{re}\left(\psi_1^2 e^{-j4\pi f_e T/P} \tilde{\Gamma}_{1,1}^{(1,1)}\right) - 2\Gamma_{1,1}^{(1,1)} \\
&\quad + 4\text{re}\left(\psi_1^2 e^{-j2\pi f_e T/P} \lambda_2 C_{2x}(1;1)\right) \\
&\quad - 4\text{re}\left(e^{-j2\pi f_e T/P} \lambda_1 C_{2x}^*(1;1)\right) \\
&\quad - 4\text{im}\left(-\frac{j2\pi T}{P} e^{-j2\pi f_e T/P} \psi_1 C_{2x}(1;1)\right)^2 \cdot \text{avar}(f_e), \\
\mathcal{V}_{22} &= 2\text{re}\left(\psi_1^2 e^{-j4\pi f_e T/P} \tilde{\Gamma}_{1,1}^{(1,1)}\right) + 2\Gamma_{1,1}^{(1,1)} \\
&\quad + 4\text{re}\left(\psi_1^2 e^{-j2\pi f_e T/P} \lambda_2 C_{2x}(1;1)\right) \\
&\quad + 4\text{re}\left(e^{-j2\pi f_e T/P} \lambda_1 C_{2x}^*(1;1)\right) \\
&\quad + 4\text{re}\left(-\frac{j2\pi T}{P} e^{-j2\pi f_e T/P} \psi_1 C_{2x}(1;1)\right)^2 \cdot \text{avar}(f_e), \\
\mathcal{V}_{23} &= j2\text{im}\left(\left(-\frac{j2\pi T}{P} e^{-j2\pi f_e T/P} \psi_1 C_{2x}(1;1)\right)^2\right) \cdot \text{avar}(f_e) \\
&\quad + j2\text{im}\left(\psi_1^2 e^{-j4\pi f_e T/P} \tilde{\Gamma}_{1,1}^{(1,1)}\right) \\
&\quad + j4\text{im}\left(\psi_1^2 e^{-j2\pi f_e T/P} \lambda_2 C_{2x}(1;1)\right), \\
\lambda &:= \left(\frac{1}{\alpha_1} - \frac{1}{\beta_1}\right) \left[C_{2x}(1;1) \tilde{\Gamma}_{1,1}^{(1,-1)} + C_{2x}(-1;1) \tilde{\Gamma}_{1,1}^{(1,1)}\right] \\
&\quad - \left(\frac{1}{\alpha_1} + \frac{1}{\beta_1}\right) \left[C_{2x}^*(1;1) \Gamma_{1,1}^{(1,-1)} + C_{2x}^*(-1;1) \Gamma_{1,1}^{(1,1)}\right], \\
\lambda_1 &:= \lambda \zeta_1 \frac{j2\pi T}{P} e^{j2\pi f_e T/P}, \quad \lambda_2 := -\lambda \zeta_1 \frac{j2\pi T}{P} e^{-j2\pi f_e T/P},
\end{aligned}$$

and  $\text{avar}(f_e)$  denotes the asymptotic variance of  $\hat{f}_e$ .

## 4.2. GSD Estimator

When compared with the GG algorithm (6), the symbol timing delay estimator corresponding to the GSD algorithm is obtained from the eq. (6) by fixing  $\tau = 0$ . Note that such a choice of  $\tau$  decouples the symbol timing delay estimator (6) from the frequency offset estimator (5) in the sense that the estimation of  $\epsilon$  does not require an initial estimate of  $f_e$  [3]. The following result holds:

**Proposition 4.** *The asymptotic variance of the timing delay estimator (7) is given by:*

$$\lim_{N \rightarrow \infty} NE\{(\hat{\epsilon} - \epsilon)^2\} = \zeta_2^2 \left( \frac{\mathcal{V}_{31}}{\alpha_3^2} + \frac{\mathcal{V}_{32}}{\beta_3^2} - \frac{2\mathcal{V}_{33}}{\alpha_3 \beta_3} \right),$$

with:

$$\begin{aligned}
\mathcal{V}_{31} &= 2\text{re}(\psi_2^2 \tilde{\Gamma}_{0,0}^{(1,1)}) - 2\Gamma_{0,0}^{(1,1)}, \quad \psi_2 := e^{j2\pi t_0/T} \\
\mathcal{V}_{32} &= 2\text{re}(\psi_2^2 \tilde{\Gamma}_{0,0}^{(1,1)}) + 2\Gamma_{0,0}^{(1,1)}, \quad \mathcal{V}_{33} = j2\text{im}(\psi_2^2 \tilde{\Gamma}_{0,0}^{(1,1)}), \\
\alpha_3 &:= \psi_2 C_{2x}(1;0) - \psi_2^* C_{2x}^*(1;0) = j2\text{im}(\psi_2 C_{2x}(1;0)), \\
\beta_3 &:= \psi_2 C_{2x}(1;0) + \psi_2^* C_{2x}^*(1;0) = 2\text{re}(\psi_2 C_{2x}(1;0)).
\end{aligned}$$

## 5. SIMULATION EXPERIMENTS

In this section, the experimental results and theoretical asymptotic bounds are compared. The experimental results are obtained by performing a number of 400 Monte Carlo trials assuming that the

transmitted symbols are i.i.d. QPSK symbols. The transmit and receive filters are square-root raised cosine filters and the additive noise is generated as Gaussian white noise. To render the discrete-time noise uncorrelated, a front end filter with two-sided bandwidth  $P/T$  is used [3]. All the simulations are performed with  $f_e T = 0.011$  and  $\epsilon T = 0.37$ .

1) *Performance w.r.t. the oversampling rate  $P$ :* By changing the oversampling rate  $P$ , we compare the MSEs of GG and GSD estimators with their theoretical asymptotic variances. The number of symbols  $N$  is set to 400 and the rolloff factor of the filter is  $\rho = 0.5$ . SNR is fixed at 20 dB. The results are depicted in Figures 1–2. Both frequency offset and timing delay estimators show that increasing the oversampling rate will impair the performance. This is due to the fact that for larger  $P$ , less cyclic correlation information is obtained. Moreover, although more samples are collected as  $P$  increases, their correlation increases too, which is known to increase the estimators' variance [4].

2) *Performance w.r.t. the filter bandwidth:* By varying the rolloff factor  $\rho$ , we can obtain different bandwidths for the combined filter  $h_c(t)$ . Larger values of  $\rho$  correspond to wider bandwidths. Consider the parameters  $P = 8$ ,  $N = 400$  and SNR = 20 dB. From Figures 3–4, one can see that a smaller  $\rho$  causes a poorer performance. This is due to the small values of the second-order cyclic correlations. In fact, since  $x(n)$  is given by the equation (2), it is well known that under assumption (AS2), the cyclic spectrum of  $x(n)$ , which is defined as the FT of  $C_{2x}(k; \tau)$  w.r.t.  $\tau$  [4], can be expressed for  $k \neq 0$  as (cf. [8]):

$$S_{2x}(k; f) = \frac{1}{P} H(f - f_e T_s) H^*(f - f_e T_s - k/P) e^{-j2\pi k \epsilon}, \quad (8)$$

where  $H(f)$  is the discrete-time FT of  $h(\cdot)$ . Based on (8) and since  $h_c(t)$  is bandlimited, it follows that as the bandwidth decreases, the supports of the functions  $f \rightarrow H(f - f_e T_s)$  and  $f \rightarrow H^*(f - f_e T_s - k/P)$  become more and more disjoint, which leads to less cyclic correlation information. Also, it turns out that the timing delay estimator corresponding to the GG method performs slightly better than the GSD estimator.

3) *Averaging improves the performance of the GG estimator:* Assume that the received waveform and a time delayed replica of the received waveform are both oversampled with the oversampling factor  $P = 2$ . In addition, assume that the GG estimator is applied separately on the two resulting sets of samples and that the resulting frequency and timing delay estimates are averaged. Surprisingly, Figures 5–6 reveal the fact that this new estimator improves significantly the performance of the GG estimator even in the presence of a very small rolloff factor ( $\rho = 0.1$ ,  $N = 400$ ).

## 6. CONCLUSIONS

In this paper, we have analyzed the asymptotic performance of the blind carrier frequency offset and timing delay estimators [3], [4], which rely on the second-order cyclostationary statistics generated by oversampling the output of the receive filter. We have derived the asymptotic variance expressions of  $\hat{f}_e$  and  $\hat{\epsilon}$  and shown that a smaller oversampling rate ( $P = 2, 3$ ) and a wider pulse shape bandwidth ( $\rho \in [0.6, 0.9]$ ) can improve the estimation accuracy as well as reduce the computational complexity of the estimators.

Due to space constraints, we have only illustrated the performance analysis for circular input sequences (QPSK) and oversampling rates  $P \geq 4$ . The analysis of the more general estimators that consider arbitrary oversampling factors  $P$  and arbitrary input constellations, in the presence of time-varying fading effects, together

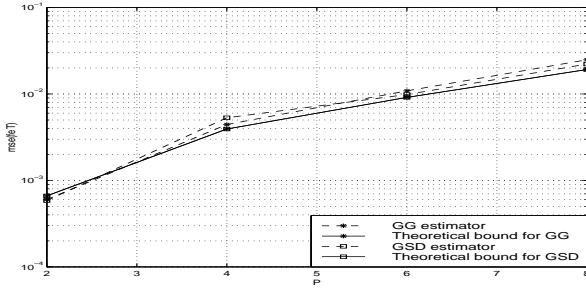


Fig. 1. MSE of  $f_e T$  versus oversampling rate  $P$

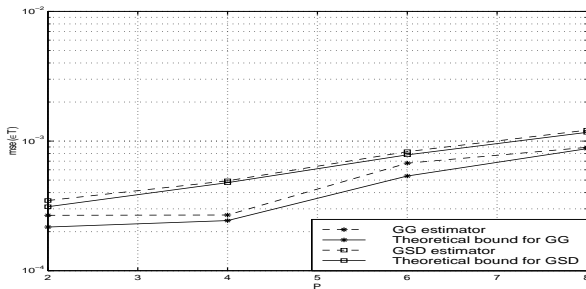


Fig. 2. MSE of  $\epsilon T$  versus oversampling rate  $P$

with a rigorous performance analysis of the proposed estimator, which relies on the averaging of the estimates obtained by applying the GG estimator on different subsets of samples, have already been implemented and are to be reported in a future paper.

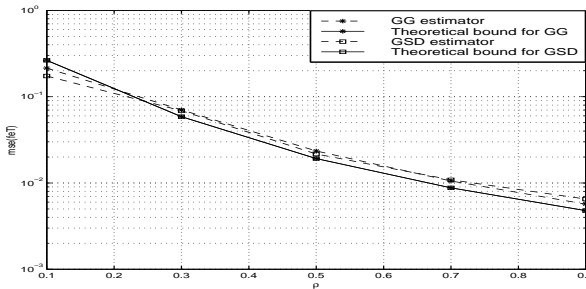


Fig. 3. MSE of  $f_e T$  versus rolloff factor  $\rho$

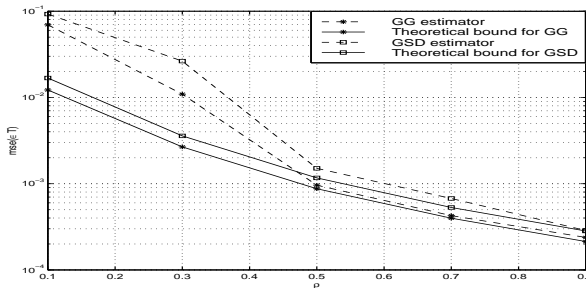


Fig. 4. MSE of  $\epsilon T$  versus rolloff factor  $\rho$

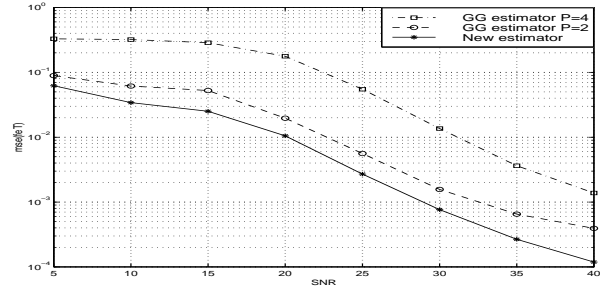


Fig. 5. MSE of  $f_e T$  versus SNR (dB)

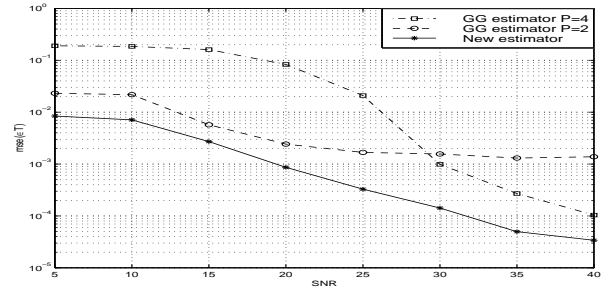


Fig. 6. MSE of  $\epsilon T$  versus SNR (dB)

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