Searching Aligned Groups of Objects with Fuzzy Criteria

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Abstract. The detection of aligned groups of objects is important for satellite image interpretation. This task can be challenging when objects have different sizes. In this paper, we propose a method for extracting aligned objects from a labeled image. In this method we construct a neighborhood graph of the objects of the image, and its dual graph where we incorporate information about the relative direction of the objects, evaluated using fuzzy measures of relative position. The groups of objects satisfying the fuzzy criterion of being locally aligned are extracted from the dual graph. These groups are the candidates for being (globally) aligned. The method was tested on synthetic images, and on objects extracted from real images demonstrating that the method extracts the aligned groups of objects even if the objects have different sizes.

1 Alignment and Related Work

Alignment can be defined as the spatial property possessed by a group of objects arranged in a straight line[1]. Determining the groups of aligned objects is crucial for image interpretation. According to the Gestalt theory, the human perceptual vision system groups objects together using certain rules. Among these rules there is one called continuity of direction which groups together objects in the same direction, and one particular case is the constancy of direction that refers to alignments [5]. An aligned group of objects has the characteristic that it should be seen as a whole, since if its elements are observed in an independent manner then the alignment property is lost. Having to look it as a whole makes alignment detection a difficult task.

Identifying the aligned groups of objects in satellite images is important for several applications. Satellite images provide a huge amount of geographical information, and aligned groups of objects can be seen as a way to reduce this information in a pertinent way. For example in cartography, it is necessary to find groups of aligned buildings for map generalization [12]. Observing if a group of buildings is aligned can give information about the structure of their arrangement, and whether they belong to a urban, rural or residential area [6]. In object detection, complex semantic classes such as parking areas (car parkings,

[1] Definition taken from ThinkMap Visual Thesaurus
http://www.visualthesaurus.com/

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ports, truck parkings or airports) comprise aligned groups of transport vehicles. Therefore, the identification of aligned groups of transport vehicles can be useful for detecting instantiations of these complex classes, and is meaningful for the description of this kind of scenes.

Alignment extraction has been studied in image processing as a low level feature. For instance methods relying on the Hough transform [5] or the Radon transform [7] are used to find groups of points in digital images which fall into a line. Other examples are the identification of aligned segments which have the same orientations as the alignment [5,10,11,8]. However, alignment extraction as a high level feature has been less studied. One example is the work of [4], where an algorithm to detect aligned groups of buildings in maps is presented. In this algorithm buildings with aligned barycenters are extracted, and the quality of the alignments is evaluated based on the criteria of proximity and similarity laws of Gestalt theory. Nevertheless, when the groups are composed of objects of different sizes, it is not possible to detect the alignment by observing just their barycenters (see Fig. 1). Thus, when considering extended objects and not only points the notion of "falling into a line" becomes imprecise. Therefore it is necessary to consider a degree of satisfaction of the relation of alignment.

In this work we propose a novel method to detect alignments of objects that can be applied to objects of different sizes, or to fuzzy objects. In our approach, we use the direction orientation between any two elements of the group to determine their degree of alignment. To measure the orientation between two objects we make use of what we call orientation histogram which is based on the angle histogram introduced by Mijama and Ralescu in [9] (Sec. 2). Our strategy consists in first determining the *locally* aligned groups which are the candidates to form an aligned group of objects. Then we measure the degree of alignment of each candidate group (Sec. 3) and solve conflicts. The results of the method are shown on synthetic and real images in Sec. 5.

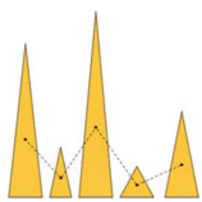


Fig. 1. Problems encountered when the group has objects of different sizes: an aligned group of objects with not aligned barycenters

2 Angle and Orientation Histograms

Angle histograms have proved to be an adequate way for evaluating the directional spatial relation between two objects, since they take into account the shape of the regions [9]. They can be interpreted as a function that captures the directional position between two objects. Let a and b be two objects defined by two regions in the image space $\mathfrak{I}$, that we denote by a and b. The angle histogram from a to b is obtained by computing for each pair of points $p_a \in a$ and $p_b \in b$ the angle between the segment joining them and the horizontal axis, denoted by $\angle(p_a, p_b)$. Angles are organized in a histogram, normalized by the largest frequency:

$$H^a(b)(\theta) = \frac{\sum_{p_a \in a, p_b \in b | \angle(p_a,p_b)=\theta} 1}{\max_{\phi \in [0,2\pi)} \sum_{p_a \in a, p_b \in b | \angle(p_a,p_b)=\phi} 1}. \quad (1)$$

To determine if an object a is in a given direction with respect to an object b (for example "right of"), we can compute the angle histogram $H^a(b)$ and compare it with a template for the relation "right of" by using for instance a conjunctive operator or the compatibility between the computed histogram and the template [9]. Angle histograms are easily extended to fuzzy objects. In addition, they are invariant to simultaneous translation, scaling and rotation of both objects. They are not symmetrical, but they satisfy: $H^a(b)(\theta) = H^b(a)(\theta + \pi)$.

Since we are interested in the orientation of two objects with respect to the horizontal axis, we introduce the notion of orientation histogram, which is simply an angle histogram where the angles are computed modulus π and its support has a length equal to π. For the case where a and b are fuzzy objects with membership function $\mu_a : \mathfrak{I} \to [0,1]$ and $\mu_b : \mathfrak{I} \to [0,1]$, respectively, the orientation histogram is given by:

$$O(a,b)(\theta) = \frac{\sum_{p_a,p_b \in \mathfrak{I} | mod(\angle(p_a,p_b),\pi)=\theta} \mu_a(p_a) \wedge \mu_b(p_b)}{\max_{\phi \in [0,\pi)} \sum_{p_a,p_b \in \mathfrak{I} | mod(\angle(p_a,p_b),\pi)=\phi} \mu_a(p_a) \wedge \mu_b(p_b)}, \quad (2)$$

where $\wedge$ is a t-norm. The orientation histogram is a fuzzy subset of $[0, \pi[$ that represents the orientation between two objects with respect to the horizontal axis, it preserves the same properties as the angle histogram, and in addition it is symmetrical.

To compare if two orientation histograms are similar, it is important to consider the imprecision that is linked to the comparison of two angles that are approximately the same. When a fuzzy morphological dilation [3] is performed on an orientation histogram using a structuring element ν_0, then the high values of the histogram will be propagated to the similar angle values according to ν_0. The structuring element ν_0 is designed such that $\nu_0(\theta - \tilde{\theta})$ represents the degree to which $\tilde{\theta}$ and θ are "approximately" equal (modeled by a trapezoid function in our experiments). Then the similarity degree between two orientation histograms can be given by the maximum height of the intersection of the dilated histograms:

$$sim(O(a,b), O(c,d)) = \max_{\theta \in [0,\pi)} \left[D_{\nu_0}(O(a,b)) \wedge D_{\nu_0}(O(c,d)) \right](\theta), \quad (3)$$

where $\wedge$ is a t norm, and the fuzzy morphological dilation is given by $D_{\nu_0}(\mu)(\theta) = \sup_{\tilde{\theta} \in [0,\pi[} \min(\mu(\tilde{\theta}), \nu_0(\theta - \tilde{\theta}))$ [3].

This degree of similarity can be extended to evaluate the similarity degree between several orientation histograms. Let $\{O(a_i, b_i)\}_{i=0}^{N}$ be a set of orientation histograms. Then the degree of similarity between them is given by:

$$sim\,(O(a_0, b_0), \ldots, O(a_N, b_N)) = \max_{\theta \in [0,\pi[} \bigwedge_{i=0}^{N} D_{\nu_0}(O(a_i, b_i))(\theta). \quad (4)$$

3 Alignment Detection

In this section we propose the definitions of *globally* aligned and *locally* aligned, both relations depend on a neighborhood relation. Let a, b be two objects. We define $N_d(a)$ as the Voronoi neighborhood of a constrained by a distance d, and the binary relation $Neigh(a, b)$ is satisfied if $b \cap N_d(a) \neq \emptyset$.

A group S is said to be *globally* aligned if all its members are connected by the $Neigh$ relation, and if there exists an angle θ such that every member of the group is able to see the other members of the group in a direction θ or $\theta + \pi$ with respect to the horizontal axis. Thus, it is possible to define the degree of *global* alignment as follows:

Definition 1. *Let $S = \{a_0, \ldots, a_N\}$, with $N \geq 3$, be a group of objects in $\Im$, connected by the $Neigh$ relation. The degree of global alignment of S is given by:*

$$\mu_{ALIG}(S) = sim\,(O(a_0, S \setminus \{a_0\}), \ldots, O(a_N, S \setminus \{a_N\}))\,. \quad (5)$$

A group S with $\mu_{ALIG}(S) = \beta$ is called a *globally* aligned group to a degree β.A group $S = \{a_0, \ldots, a_N\}$ is said to be *locally* aligned to a degree β, if for every two pairs of neighboring objects, having one object in common, the orientations between the objects of each pair are similar to a degree β , and also if the group is connected by the neighbor relation. The latter can be summarized by saying that a group S with $|S| \geq 3$ is *locally* aligned to a degree β if it satisfies the following relations:

$R1 : \forall x, y, z\ \ (Neigh(x, y) \wedge Neigh(y, z)) \Rightarrow (sim(O(x, y), O(y, z)) \geq \beta)$

$R2 : \forall a, b\ \exists x_0, \ldots, x_m$ for $m > 1$ such that $x_0 = a, x_m = b$ and $\bigwedge_{i=0}^{m-1} Neigh(x_i, x_{i+1})$

Extracting Locally Aligned Groups of Objects: To extract the locally aligned groups, first we construct a neighborhood graph G_N to obtain the information of which objects are connected via the $Neigh$ relation. In a neighborhood graph $G_N = (V, E)$ the vertices represent the objects of the group, and there is an edge between two vertices if and only if the corresponding objects are neighbors. Notice that only the connected subsets of three vertices x, y and z in G_N which share a common vertex, for example y, satisfy $Neigh(x, y) \wedge Neigh(y, z)$. These connected subsets are called *triplets*. According to $R1$, only the *triplets* $\{x, y, z\}$ for which $sim(O(x, y), O(y, z)) \geq \beta$ can belong to a *locally* aligned group. *Triplets* can be easily identified as the edges of the dual graph, when the dual graph is constructed in the following manner. The dual graph is denoted by $\tilde{G}_N = \{\tilde{V}, \tilde{E}\}$ where each vertex $\tilde{v}_i$ represents an edge in the graph G_N. An edge exists between two vertices $\tilde{v}_i$ and $\tilde{v}_j$ of $\tilde{G}_N$ if the two corresponding edges of the graph G_N have a common vertex. If, additionally, we attribute to each edge (i, j) the similarity degree between the orientation histograms of $\tilde{v}_i$ and $\tilde{v}_j$ that we denote by $\tilde{s}_{ij}$, then it will be possible to verify whether the relation $R1$ holds for its corresponding *triplet*. Figure 2 shows an example of neighborhood graph and its dual graph. Notice that the edges of $\tilde{G}_N$ with a high value represent the *triplets* of objects with a similar orientation histogram. For instance, in the dual graph the edge between the nodes (1 - 2) and (2 - 3) has a similarity value of 1, this edge corresponding to the objects labeled 1, 2 and 3 of Fig. 2(a). In a similar way, edges with a low value represent objects which are not aligned, for example in the dual graph the edge between the nodes (1 - 2) and (6 - 2) has a similarity value of 0.11 and corresponds to the objects labeled 1, 2 and 6, which do not form a *globally* aligned *triplet*.

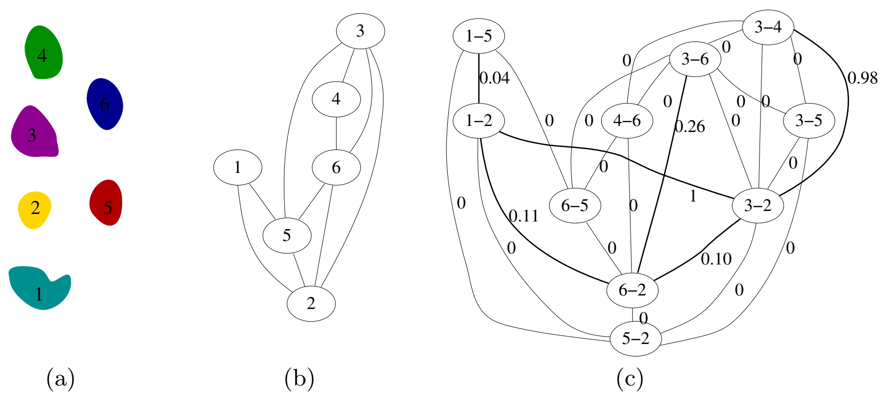


Fig. 2. (a) Labeled image (b) Neighborhood graph (c) Dual graph of (b)

Returning to the conditions expressed by the relations $R1$ and $R2$ of *locally* alignment, the first one states that *triplets* should be *globally* aligned, and the second one that the group should be formed by connected objects according to the $Neigh$ relation. Then a group S satisfies these relations if and only if the subset $\tilde{S} \subseteq \tilde{V}$ which represents the dual of S satisfies the following relations:

$$\tilde{R}1 : \forall \tilde{v}_i, \tilde{v}_j \; Conn(\tilde{v}_i, \tilde{v}_j) \Rightarrow (\tilde{s}_{ij} \geq \beta)$$

$$\tilde{R}2 : \forall \tilde{v}_i, \tilde{v}_j \; \exists \tilde{u}_0, \ldots \tilde{u}_K \text{ for } K > 1 \text{ such that } \tilde{u}_0 = \tilde{v}_i, \tilde{u}_N = \tilde{v}_j \text{ and } \bigwedge_{k=0}^{K-1} Conn(\tilde{u}_0, \tilde{u}_k),$$

where $Conn(\tilde{u}, \tilde{v})$ is true if there exists an edge between $\tilde{u}$ and $\tilde{v}$. Condition $\tilde{R}2$ expresses that $\tilde{S}$ should be connected, since if $\tilde{S}$ is not connected then S is not connected. Therefore, a *locally* aligned group is a subset $S \subseteq V$ for which its dual set $\tilde{S} \subseteq \tilde{V}$ is connected in $\tilde{G}$ and the value of all the edges joining the vertices within $\tilde{S}$ is greater than or equal to β.

To extract the $\tilde{S}_i \subseteq \tilde{V}$ corresponding to the dual sets of the *locally* aligned sets $S_i \subseteq V$, first we extract the connected components $\{C_k\}$ of $\tilde{V}$ which are connected by an edge value greater that β. Then for each C_k we obtain the minimum value of its edges denoted by $cons(C_k)$:

$$cons(C_k) = \min\{\tilde{s}_{ij} | \tilde{v}_i, \tilde{v}_j \in C_k\}$$

If $cons(C_k) < \beta$ then C_k does not satisfy $\tilde{R}1$, thus vertices are removed until $cons(C_k) \geq \beta$. The vertices which are removed are the ones having more conflict with their neighbors in C_k. We say that two connected vertices $\tilde{u_i}$ and $\tilde{v_j}$ are in conflict if $\tilde{s}_{ij}$ is close to zero, that is if the corresponding orientation histograms of both vertices are not similar. We measure the conflict of a vertex $\tilde{v}_t$ with its neighbors in C_k by using what we call the degree of the vertex in C_k given by:

$$deg(\tilde{v}_t) = \frac{\sum_{\tilde{v}_j \in C_k} \tilde{s}_{tj}}{|\{(i,j)|\tilde{v}_j \in C_k\}|}. \qquad (6)$$

It is clear that if $\tilde{v}_t$ is in conflict with several of its connected vertices in C_k then $deg(\tilde{v}_t)$ will be close to 0, and it will be close to 1 if there is no conflict. Then the conflict of a vertex will be given by $1 - deg(\tilde{v}_t)$.

Candidates Evaluation: The *locally* aligned groups $\mathcal{S}$ to a degree β are the possible candidates for being a *globally* aligned group with a degree of satisfaction β. The evaluation is performed by measuring the degree of *global* alignment using Equation (5). Usually the *locally* aligned groups to a degree β are *globally* aligned to a degree β. If the degree of *global* alignment is inferior to β in a group S, then we divide the group by eliminating the vertices in $\tilde{S}$ with the minimum vertex degree (Eq. 6) in $\tilde{S}$, and we repeat this step until there is no conflict in the vertices.

Adding More Elements to the Group: Once the *globally* aligned groups of objects are identified, it is possible to add new objects to the group or fusion two *globally* aligned groups to obtain a larger *globally* aligned group. For each group S_i we perform two morphological directional dilations of the group in the directions θ and $\theta + \pi$, where θ is the orientation of the alignment (the angle which maximizes the conjunction of the orientation histograms $O(a_i, S \setminus \{a_i\})$). These dilations will be denoted by $D_{\nu_\theta}(S_i)$ and $D_{\nu_{\theta+\pi}}(S_i)$. An object a which satisfies the $Neigh$ relation with one of the members of S_i and which is seen by S_i to a degree greater than or equal to β (that is $\mu_{include}(a, D_{\nu_\theta}(S_i) \cup D_{\nu_{\theta+\pi}}(S_i)) \geq \beta$, where $\mu_{include}$ denotes a degree of inclusion [2]) is added to S_i. If a whole group S_j is seen by S_i and one of the elements of S_i is connected to one of the members S_j and both groups have similar orientation, then both groups are fusioned into one.

4 Complexity Analysis

In this section we deal with the cost of the basic operations of the algorithm for extracting *locally* aligned groups and *globally* aligned groups.

First, we consider the complexity of extracting *locally* aligned groups. Consider we have N objects each with at most n_o points. The complexity of the algorithm is $O(N^2)$ since most of steps of the algorithm deal with operations over the graph or its dual. It should be noticed that the step which corresponds to the construction of the orientation histograms has a complexity of $O(N^2 n_o^2)$, since at maximum there are $N(N-1)$ edges on the graph and for each edge an orientation histogram is constructed and the construction of an orientation histogram has a complexity of $O(n_o^2)$.

The complexity of finding a *globally* aligned group from a *locally* aligned group with N_A elements each having at most n_o points lies on the following steps. The first step consists in evaluating the degree of *global* alignment and division of the group in the case where it is not aligned, and this step has a complexity of $O(N_A^2 n_o^2)$. The second step consists in performing the morphological directional dilations of the group in the directions of alignment θ and $\theta + \pi$, and has a complexity of $O(N_I)$ [1], where N_I is the number of points in the image (see [1] for the implementation of the directional morphological dilation using a propagation method). And finally, the complexity of the step of evaluating the degree of inclusion of each object not belonging to the group into the directional dilations of the group is $O((N - N_A)n_o^2)$, where N is the total number of objects. Hence, summing the three steps we obtain that the total complexity is $O(N_A^2 n_o^2 + N_I)$.

5 Results

We applied the method to the objects of the synthetic image of Fig. 3. The method obtains the *locally* aligned group shown in Fig. 3(b) with degree 0.9, and this group is also *globally* aligned with degree 0.85. The group is then extended to add new objects: Fig. 3(c) shows the degree to which each pixel is observed by the group, and finally Fig. 3(d) shows the aligned group after adding the elements. The degree of *global* alignment of the whole group is 0.8. In this example we used objects of different sizes and the method was able to extract the *globally* aligned group. This example highlights the flexibility of the method, since the green and orange objects fall into the line but the orientation between them is different from the one of the *global* alignment.

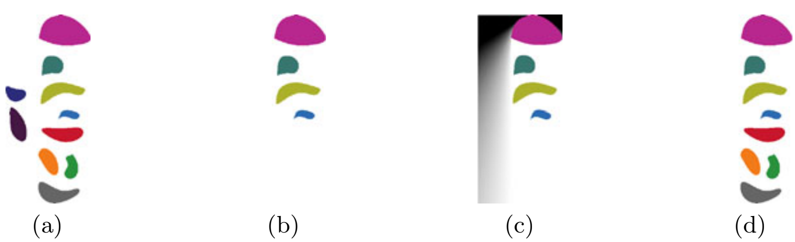
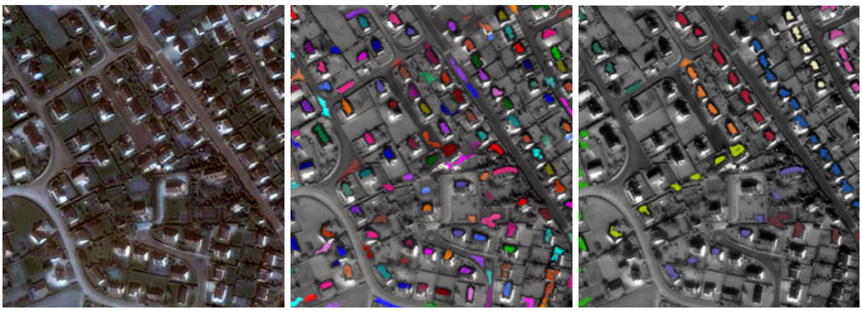


Fig. 3. (a) Labeled image (b) Locally aligned group (c) The region seen by the group of (b) in the direction of the alignment (white = high value of visibility) (d) Group obtained after adding new elements

We also applied the method to the houses extracted from the satellite image of urban area objects in Fig. 4(b). Figure 4(c) shows some of the *globally* aligned subsets of houses obtained. It is not possible to show all the *globally* aligned groups found by the algorithm since there are objects which belong to more than one group. We can observe that the algorithm obtains the most distinctive groups of the image (pink, orange, white, red and blue sets). However, not all the obtained groups are meaningful for the description of the scene (purple and light green sets), since these are subsets which are *globally* aligned but do not give any information about the arrangement of the houses. Finally, note that all the obtained groups satisfy the notion of *global* alignment discussed in Sec. 3.



(a) Original Image (b) Segmented buildings (c) Subsets of aligned buildings

Fig. 4. Some of the *globally* aligned subsets found by the algorithm with a degree greater than 0.9

6 Conclusions

In this work we have introduced the definitions of *globally* and *locally* aligned groups as fuzzy relations, and gave a method to extract them from an image of labeled objects. Both definitions are appropriate to determine alignments of objects of different sizes. The methods and the definitions were tested on objects extracted from real images, giving satisfactory results. In the obtained results, it is possible to notice that not all the obtained groups are meaningful for the interpretation of a scene. Hence it is necessary to combine the obtained alignments with other relations to put the *globally* aligned groups into context, for example find if the *global* or *local* alignments are parallel between them or parallel to a linear structure.

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