Structure segmentation and recognition in images guided by structural constraint propagation

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Abstract. In some application domains, such as medical imaging, the objects that compose the scene are known as well as some of their properties and their spatial arrangement. We can take advantage of this knowledge to perform the segmentation and recognition of structures in medical images. We propose here to formalize this problem as a constraint network and we perform the segmentation and recognition by iterative domain reductions, the domains being sets of regions. For computational purposes we represent the domains by their upper and lower bounds and we iteratively reduce the domains by updating their bounds. We show some preliminary results on normal and pathological brain images.

1 INTRODUCTION

Image segmentation and recognition is a key problem in scene interpretation. In some application domains, such as medical imaging, the objects that compose the scene are known as well as some of their properties and their spatial arrangement. This knowledge may be properly encoded as a symbolic graph. Two main approaches can then be derived. The first one consists in matching this graph representation with image regions obtained from a preliminary segmentation (e.g. [5]). Since it is usually difficult to segment the image into semantically meaningful entities, this type of approach often relies on an over-segmentation, which makes the matching more complex (no isomorphism can be expected). The second type of approach uses the graph as a guide in a sequential process. In [4], the structures are sequentially segmented using a deformable model, which is constrained to fulfill some spatial relations with previously segmented structures. However the result is highly dependent on the segmentation order and the segmentation of one structure cannot benefit from partial information available about not already segmented structures.

In this paper, we propose a new method to overcome these limitations. The idea is to express the problem as a constraint propagation process, exploiting the capability of constraint networks to solve combinatorial problems [18]. The propagation can be performed either by adding or simplifying constraints or by reducing the domains of variables. In the scope of qualitative spatial reasoning, the first option has been investigated in particular to solve satisfiability problems, for instance with RCC-8 relations [16] or qualitative relative positions [10]. We propose here to investigate the second option, i.e. the iterative reduction of the variable domains.

We first recall in Section 2 some definitions on structural representations. Section 3 is the core of the paper. We define the constraint network, domains, domain bounds, the structural constraints and the contracting operators for several types of structural knowledge. In Section 4 we present a propagation process and a decision process based on minimal surface extraction. In Section 5 some preliminary segmentation and recognition results are presented on brain magnetic resonance images (MRI).

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2 PRELIMINARIES

Structural Knowledge Representation – The structural arrangement of anatomical structures is known and almost stable, even in the presence of a pathology. This knowledge, supposed to be consistent, can be appropriately encoded by an hypergraph [7] where the vertices correspond to spatial objects and the edges (between one or several nodes) may represent either:

- known properties of objects, such as the connectivity, a priori range of volumes,
- relative positions between structures,
- appearance properties, such as homogeneity or contrast. Such characteristics depend on the imaging modality (MRI in our example).

Since such knowledge is usually expressed in linguistic terms (in anatomical textbooks for instance [19]), fuzzy sets constitute an appealing framework for its formal modeling: to represent spatial relations, to account for different types of imprecision, related to the imperfections of the image, and to the intrinsic vagueness of some relations [1]. Membership functions defining these fuzzy sets can be learned from a data base of examples.

Fuzzy Sets [6] – Let X be a bounded subset of $\mathbb{Z}^n$. A fuzzy set on X will be denoted by its membership function $\mu : X \rightarrow [0,1]$. We denote α-cuts by μ_α and by $\mathcal{F}$ the set of fuzzy sets defined on X. $(\mathcal{F}, \leq)$ is a complete lattice for the usual order on fuzzy sets. The supremum $\vee$ and infimum $\wedge$ are the $\max$ and $\min$ respectively. The smallest element is denoted by $0_\mathcal{F}$ and the largest element by $1_\mathcal{F}$. We denote the fuzzy complementation by $c(\mu)(x) = 1 - \mu(x)$, the Lukasiewicz t-norm by $\top(x,y) = \max(0, x+y-1)$ and t-conorm by $\perp(x,y) = \min(1, x+y)$, for x, y in $[0,1]$.

3 STRUCTURAL RECOGNITION PROBLEM AS A CONSTRAINT NETWORK

3.1 Structural segmentation and recognition problem

Let $I : X \rightarrow \mathbb{R}^+$ be a grey level image. We want to extract a set of N structures $\chi = \{O_i | i \in [1..N]\}$ present in that image. Each of these variables O_i is represented as a fuzzy subset $\mu_i \in \mathcal{F}$ of X and takes values in a domain $D_i \subseteq \mathcal{F}$. The set of domains associated with χ is denoted by $\mathcal{D}$. This recognition problem is constrained by the prior knowledge described in Section 2. Let us assume for instance that the knowledge base contains the relation "A is to the right of B". The recognition amounts to find two fuzzy sets μ_1 and μ_2 satisfying the binary constraint $C^{dir}_{A,B}(\mu_1, \mu_2) = 1$. The formal expression of these constraints is described in Section 3.3 for several types of relations. We will denote by $\mathcal{C}$ the set of constraints.

Our segmentation and recognition problem can thus be associated with a constraint network $\langle \chi, \mathcal{D}, \mathcal{C} \rangle$. A solution $\{\mu_i | \mu_i \in D_i, i \in [1..N]\}$ of our problem has to fulfill all constraints. Ideally this problem would have a unique solution. However it is generally under-constrained and different solutions are possible. Through contracting operators we will simplify our problem to obtain domains as close as possible to the set of solutions. In the following we always assume that the problem is satisfiable.

3.2 Domain definition

The definition above involves the representation and the manipulation of domains which are subsets of $\mathcal{F}$. In practice, membership values are discretized, and if k is the cardinality of the current discretization of $[0, 1]$ and n the cardinality of X, the cardinality of $\mathcal{F}$ is then k^n (10^{131072} for the 2D examples presented in Section 5). Handling such a set is generally not computationally tractable and we have to consider a simplified version of it. In [15], the authors represent this subset by its Minimum Bounded Rectangle (MBR) (i.e. the smallest rectangle in 2D that includes all elements of the domain). This very compact representation is nevertheless not able to capture the geometry of objects and provides a poor representation (consider for instance a diagonal line) that will limit the efficiency of the constraint propagation process.

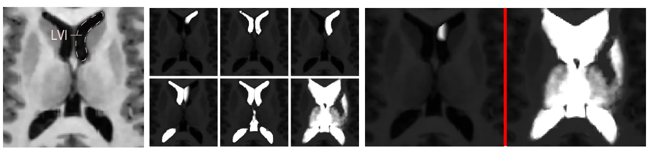
Considering the lattice structure of $\mathcal{F}$, we propose here to define the domain bounds as the supremum and infimum of fuzzy sets over the domain. Let $D_A \subseteq \mathcal{F}$ be the domain associated with an object A. We define the upper bound $\overline{A}$ of D_A as: $\overline{A} = \vee\{\nu \in D_A\}$. It can also be interpreted as an over-estimation of μ_A. The lower bound $\underline{A}$ is defined as: $\underline{A} = \wedge\{\nu \in D_A\}$ and is an under-estimation of μ_A. We can notice that $\forall \nu \in D_A, \underline{A} \leq \nu \leq \overline{A}$.

For instance a tiny domain for the left lateral ventricle LVl (delineated in Figure 1(a)) is defined as the six fuzzy sets in (b). Note that the third one is μ_{LVl}. The lower and upper bounds $(\underline{LVl}, \overline{LVl})$ of this domain are presented in (c).

Based on these notations, we represent the domain associated with a structure A by its bounds:

$$(\underline{A}, \overline{A}) = \{\nu \in \mathcal{F} | \underline{A} \leq \nu \leq \overline{A}\}.$$

Note that if $\underline{A} \nleq \overline{A}$, the domain $(\underline{A}, \overline{A})$ is empty and the problem is unsatisfiable. Let $(\underline{A}^1, \overline{A}^1)$ and $(\underline{A}^2, \overline{A}^2)$ be two non empty domains for the structure A. We consider the following partial order: $(\underline{A}^1, \overline{A}^1) \preccurlyeq (\underline{A}^2, \overline{A}^2)$ if $\forall x \in X$, $\underline{A}^1(x) \geq \underline{A}^2(x)$ and $\overline{A}^1(x) \leq \overline{A}^2(x)$. The associated supremum and infimum operators are respectively defined as: $(\underline{A}^1, \overline{A}^1) \vee (\underline{A}^2, \overline{A}^2) = (\underline{A}^1 \wedge \underline{A}^2, \overline{A}^1 \vee \overline{A}^2)$ and $(\underline{A}^1, \overline{A}^1) \wedge (\underline{A}^2, \overline{A}^2) = (\underline{A}^1 \vee \underline{A}^2, \overline{A}^1 \wedge \overline{A}^2)$.



(a) (b) (c)

Figure 1. A cropped axial view of a brain MRI. (a) Contour of left lateral ventricle (LVl). (b) A domain for LVl that contains six fuzzy sets. (c) Lower bound $\underline{LVl}$ and upper bound $\overline{LVl}$.

3.3 Contracting operators

3.3.1 General issues

The constraints involved in the knowledge data base are expressed as symbolic relations. Each constraint is defined as a function $C : \mathcal{F}^k \rightarrow \{0, 1\}$ if k objects are involved in the relation. As detailed below, it will be expressed in terms of fuzzy sets representing the objects and the spatial or appearance relations. Due to the size of the domains, contracting operators that exhaustively browse the domains (to achieve arc consistency for instance) cannot by applied. We thus define weaker contraction operators that compute new domain bounds from the initial domain bounds. A contracting operator is written as: $\dfrac{\langle \psi; \mathcal{D}; \mathcal{C} \rangle}{\langle \psi; \mathcal{D}'; \mathcal{C} \rangle}$ where ψ is the set of variables involved in the set of constraints $\mathcal{C}$, $\mathcal{D}$ and $\mathcal{D}'$ are the associated domains represented by their bounds, with $\mathcal{D}' \preccurlyeq \mathcal{D}$. Notice that the contracting operators will generally not achieve arc consistency nor 2B consistency [9]. Indeed the domain may contain two values that fulfill all constraints but their supremum or infimum does not necessarily.

3.3.2 Directional relative position

In [1] a method to characterize the directional relative position between objects using mathematical morphology was proposed. Suppose for instance that the caudate nucleus CNl (delineated in Figure 2(a)) is located on the right of the left ventricle LVl (delineated by dashed line). The relation "on the right" can be characterized by a structuring element ν. The fuzzy dilation $\delta_\nu(\mu_{LVl})$ of μ_{LVl} by ν (displayed in (b)) defines a fuzzy set that corresponds to the points on the right of LVl. We consider that such a relation from an object A to an object B is satisfied if it is for all points of B, and we also impose that B is included in the complement of A. The associated constraint can be defined as:

$$C^{dir}_{A,B}(\mu_1, \mu_2) = \begin{cases} 1 & \text{if } \mu_2 \leq \top(\delta_\nu(\mu_1), c(\mu_1)), \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that the objects A and B are respectively defined over the domains $(\underline{A}, \overline{A})$ and $(\underline{B}, \overline{B})$. The elements μ of $(\underline{B}, \overline{B})$ that satisfy $C^{dir}_{A,B}$ according to the current domain of A are such that: $\exists \zeta \in (\underline{A}, \overline{A}), \mu \leq \top(\delta_\nu(\zeta), c(\zeta))$, hence $\mu \leq \top(\delta_\nu(\overline{A}), c(\underline{A}))$, since the dilation and $\top$ are increasing and the complementation is decreasing. The contracting operator associated with the constraint $C^{dir}_{A,B}$ is derived from this inequality.

DIRECTION CONTRACTING OPERATOR:

$$\frac{\langle A, B; (\underline{A}, \overline{A}), (\underline{B}, \overline{B}); C^{dir}_{A,B} \rangle}{\langle A, B; (\underline{A}, \overline{A}), (\underline{B}, \overline{B} \wedge \top(\delta_\nu(\overline{A}), c(\underline{A}))); C^{dir}_{A,B} \rangle}$$

Considering the same example, Figure 2 shows the upper bound $\overline{LVl}$ (c) and $\overline{CDl}$ (d) of the domains of LVl and CDl (the lower bound is here the empty set). The dilation $\delta_\nu(\overline{LVl})$ is displayed in (e) and we can see in (f) the updated upper bound $\overline{CDl}$. The definition of the initial bounds will be addressed in Section 4.

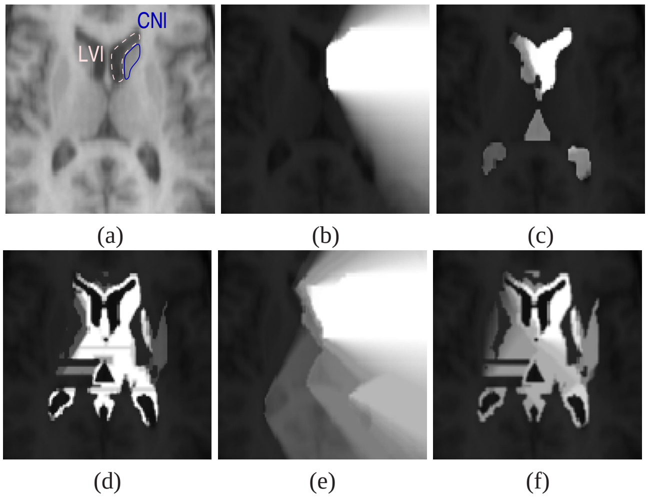


Figure 2. A cropped axial view of a brain MRI. (a) Contours of the left lateral ventricle (LVl) and the left caudate nucleus (CNl). (b) Fuzzy set that represents the points on the right of LVl. (c) $\overline{LVl}$. (d) $\overline{CNl}$. (e) On the right of $\overline{LVl}$. (f) $\overline{CNl}$ updated.

3.3.3 Distances

Distances from fuzzy objects may be computed using mathematical morphology [1]. Let us assume that we have some knowledge about the distance between two objects A and B, that can be modeled as a fuzzy interval. The region of space satisfying such a relation to a reference object μ_1 is defined as the set difference between two dilations, using two structuring elements ν_1 and ν_2 defined in the spatial domain and derived from the fuzzy interval: $\top(c(\delta_{\nu_1}(\mu_1)), \delta_{\nu_2}(\mu_1))$. Two fuzzy sets μ_1 and μ_2 satisfy the distance constraint between A and B if:

$$C^{dist}_{A,B}(\mu_1,\mu_2) = \begin{cases} 1 & \text{if } \mu_2 \leq \top(c(\delta_{\nu_1}(\mu_1)), \delta_{\nu_2}(\mu_1)), \\ 0 & \text{otherwise.} \end{cases}$$

DISTANCE CONTRACTING OPERATOR:

$$\frac{\langle A, B; (\underline{A}, \overline{A}), (\underline{B}, \overline{B}); C^{dist}_{A,B}\rangle}{\langle A, B; (\underline{A}, \overline{A}), (\underline{B}, \overline{B} \wedge \top(c(\delta_{\nu_1}(\underline{A})), \delta_{\nu_2}(\overline{A}))); C^{dist}_{A,B}\rangle}$$

3.3.4 Inclusion

Consider now two objects A and B with A included in B. The associated constraint can be expressed as:

$$C^{in}_{A,B}(\mu_1,\mu_2) = \begin{cases} 1 & \text{if } \mu_1 \leq \mu_2, \\ 0 & \text{otherwise.} \end{cases}$$

INCLUSION CONTRACTING OPERATOR:

$$\frac{\langle A, B; (\underline{A}, \overline{A}), (\underline{B}, \overline{B}); C^{in}_{A,B}\rangle}{\langle A, B; (\underline{A}, \overline{A} \wedge \overline{B}), (\underline{B} \vee \underline{A}, \overline{B}); C^{in}_{A,B}\rangle}$$

The inclusion prior can be extended to a partition prior, for instance if an object A can be decomposed into subparts $\{B_i\}$.

3.3.5 Connectivity

If A is a connected object, its domain can be restricted to connected fuzzy sets (definitions of fuzzy connectivity can be found in [17, 14]). We denote by $\mathcal{H}$, $\mathcal{H} \subseteq \mathcal{F}$, the set of connected fuzzy sets.

$$C^{conn}_{A}(\mu_1) = \begin{cases} 1 & \text{if } \mu_1 \in \mathcal{H}, \\ 0 & \text{otherwise.} \end{cases}$$

A new upper bound can be obtained as: $\xi^1_{\underline{A}}(\overline{A}) = \bigvee\{\nu \in \mathcal{H} | \underline{A} \leq \nu \leq \overline{A}\}$. However it can be shown that this filter is not robust (a small error on $\underline{A}$ may cause a large error on the result). As discussed in [14], we prefer the following formulation:
$\xi^2_{\underline{A}}(\overline{A}) = \bigvee\{\nu \in \mathcal{H} \mid \nu \leq \overline{A}$ and $\max_{x\in X} \nu(x) \leq \mu_{\leq}(\underline{A}, \nu)\}$, where $\mu_{\leq}$ stands for the Lukasiewicz implicator, i.e. $\mu_{\leq}(\underline{A}, \nu) = \min_{x\in X} \min(1, 1 - \underline{A}(x) + \nu(x))$.

CONNECTIVITY CONTRACTING OPERATOR:

$$\frac{\langle A; (\underline{A}, \overline{A}); C^{conn}_{A}\rangle}{\langle A; (\underline{A}, \xi^2_{\underline{A}}(\overline{A})); C^{conn}_{A}\rangle}$$

3.3.6 Volume

A volume prior is represented as a membership function $\mu_{V_{\min}} : \mathbb{R}^+ \to [0, 1]$. The constraint is formulated as (see [14] for details):

$$C^{vol}_{A}(\mu) = \begin{cases} 1 & \text{if } \max_{x\in X} \mu(x) \\ & \quad \leq \max_{v\in\mathbb{R}^+} \min(\mu_V(\mu)(v), \mu_{V_{\min}}(v)), \\ 0 & \text{otherwise,} \end{cases}$$

where $\mu_V(\mu)(v) = \sup\{\alpha, |\mu_\alpha| \geq v\}$, $|\mu_\alpha|$ denoting the cardinality (i.e. the volume) of the α-cut μ_α.

The reduction of the domain to the fuzzy sets that satisfy this prior, will generally not change the bounds. However if we also suppose that the object is connected, the upper bound can be filtered according to: $\xi_{\mu_{V_{\min}}}(\overline{A}) = \bigvee\{\nu \in \mathcal{H} | \nu \leq \overline{A}$ and $C^{vol}_{A}(\nu) = 1\}$.

VOLUME AND CONNECTIVITY CONTRACTING OPERATOR:

$$\frac{\langle A; (\underline{A}, \overline{A}); C^{conn}_{A} \wedge C^{vol}_{A}\rangle}{\langle A; (\underline{A}, \xi_{\mu_{V_{\min}}}(\overline{A})); C^{conn}_{A} \wedge C^{vol}_{A}\rangle}$$

3.3.7 Adjacency

A degree of adjacency between A and B can be defined as [1]: $\mu_{adj}(\mu_A, \mu_B) = \sup_{x,y\in X} \min(\mu_A(x), \mu_B(y), n(x, y))$ where $n(x, y)$ stands for a connectivity degree between two points x and y of X. We define the following constraint:

$$C^{adj}_{A,B}(\mu_1,\mu_2) = \begin{cases} 1 & \text{if } \min(\max_{x\in X} \mu_1(x), \\ & \quad \max_{x\in X} \mu_2(x)) = \mu_{adj}(\mu_1,\mu_2), \\ 0 & \text{otherwise.} \end{cases}$$

As in the volume case, a domain reduction by an adjacency constraint does not affect its bounds. Therefore, we also consider the adjacency jointly with a connectivity prior, and define the following filter: $\xi^{adj}_{\overline{A}}(\overline{B}) = \bigvee\{\nu \in \mathcal{H} | \nu \leq \overline{B}$ and $C^{adj}_{A,B}(\overline{A}, \nu) = 1\}$.

ADJACENCY AND CONNECTIVITY CONTRACTING OPERATOR:

$$\frac{\langle A, B; (\underline{A}, \overline{A}), (\underline{B}, \overline{B}); C^{adj}_{A,B} \wedge C^{conn}_{A} \wedge C^{conn}_{B}\rangle}{\langle A, B; (\underline{A}, \xi^{adj}_{\overline{B}}(\overline{A})), (\underline{B}, \xi^{adj}_{\overline{A}}(\overline{B})); C^{adj}_{A,B} \wedge C^{conn}_{A} \wedge C^{conn}_{B}\rangle}$$

3.3.8 Contrast

The following constraint will play a key role in the propagation process, since it will be computed from image data. We suppose here that the contrast between the structures is roughly known and stable, which is the case in MRI (the lateral ventricles are for instance hypointense compared with the white matter on T1 weighted MRI). We first define the grey level membership function associated with a spatial object as: $\mu^I_A(v) = \sup_{x\in X, I(x)=v}\mu_A(x)$, where I is the intensity function and v a grey level value (conversely a spatial membership function μ can be obtained from a grey level one μ^I as $\mu(x) = \mu^I \circ I(x)$).

We rely on the definition of Michelson for the contrast [12]: $c = \frac{v_1 - v_2}{v_1 + v_2}$, where v_1 and v_2 are two grey levels. According to the extension principle [20], we obtain the following membership function for the contrast between two fuzzy objects A and B, with grey level membership functions μ^I_A and μ^I_B:
$\mu^c_{A,B}(c) = \sup_{(v_1,v_2)\in\mathbb{R}^{+2}, c=\frac{v_1-v_2}{v_1+v_2}} \min(\mu^I_A(v_1), \mu^I_B(v_2))$.
Conversely, if we consider a contrast prior $\mu^c_{A,B}$, we can obtain the set of grey levels that satisfy this contrast prior from object A as: $\mu^I(v) = \sup_{(v_1,v_2)\in\mathbb{R}^{+2}, v=v_1*v_2}\min(\mu^I_A(v_1), \mu^{k^{-1}}_{A,B}(v_2))$ with $\mu^{k^{-1}}_{A,B}(v) = \sup_{c\in[-1,1], v=\frac{1-c}{1+c}}\mu^c_{A,B}(c)$ and from object B as: $\mu^I(v) = \sup_{(v_1,v_2)\in\mathbb{R}^{+2}, v=v_1*v_2}\min(\mu^I_B(v_1), \mu^k_{A,B}(v_2))$ with $\mu^k_{A,B}(v) = \sup_{c\in[-1,1], v=\frac{1+c}{1-c}}\mu^c_{A,B}(c)$.

$$C^{cont}_{A,B}(\mu_1,\mu_2) = \begin{cases} 1 & \text{if } \forall v\in\mathbb{R}^+, \\ & \quad \mu^I_1(v) \le \sup_{\substack{(v_1,v_2)\in\mathbf{R}^{+2}\\ v=v_1*v_2}}\min(\mu^I_2(v_1), \mu^k_{A,B}(v_2)) \\ & \quad \text{and } \forall v\in\mathbb{R}^+, \\ & \quad \mu^I_2(v) \le \sup_{\substack{(v_1,v_2)\in\mathbf{R}^{+2}\\ v=v_1*v_2}}\min(\mu^I_1(v_1), \mu^{k^{-1}}_{A,B}(v_2)) \\ 0 & \text{otherwise.}\end{cases}$$

CONTRAST CONTRACTING OPERATOR:

$$\frac{\langle A,B;(\underline{A},\overline{A}),(\underline{B},\overline{B});C^{cont}_{A,B}\rangle}{\begin{array}{c}\langle A,B;(\underline{A},\overline{A}\wedge(\sup_{\substack{(v_1,v_2)\in\mathbf{R}^{+2}\\ v=v_1*v_2}}\min(\mu^I_{\overline{B}}(v_1),\mu^k_{A,B}(v_2))\circ I)),\\ (\underline{B},\overline{B}\wedge(\sup_{\substack{(v_1,v_2)\in\mathbf{R}^{+2}\\ v=v_1*v_2}}\min(\mu^I_{\overline{A}}(v_1),\mu^{k^{-1}}_{A,B}(v_2))\circ I));C^{cont}_{A,B}\rangle\end{array}}$$

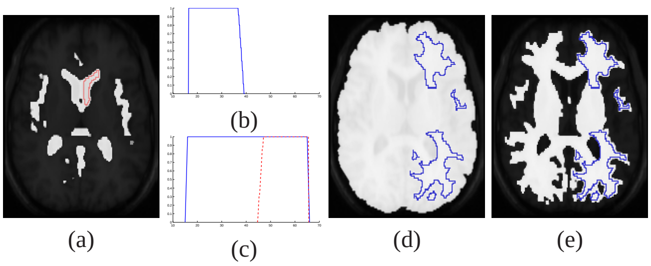


Figure 3. (a) $\overline{LVl}$. (b) $\mu^I_{\overline{LVl}}$. (c) Original $\mu^I_{\overline{WMl}}$ (plain), updated one (dashed). $\overline{WMl}$ before (d) and after (e) application of the contrast contracting operator.

This is illustrated in Figure 3. Suppose for instance that the fuzzy set displayed in (a) is the upper bound $\overline{LVl}$ of the domain of the left lateral ventricle. The associated grey level membership function $\mu^I_{\overline{LVl}}$ is shown in (b). An upper bound $\overline{WMl}$ for the left white matter structures is displayed in (d) (the contour of WMl is also shown) and $\mu^I_{\overline{WMl}}$ in (c). The application of the contrast contracting operator restricts $\mu^I_{\overline{WMl}}$ to the membership in (c) (dashed), which corresponds to the updated $\overline{WMl}$ (e).

4 CONSTRAINT PROPAGATION

We describe here a simple propagation algorithm to perform the segmentation and recognition of a set of structures χ. First we initialize the domains of these structures to $(0_{\mathcal{F}}, 1_{\mathcal{F}})$ and we restrict the set of constraints to those that involve only variables in χ. The constraints are then sequentially applied to reduce the variable domains, i.e. to reduce the upper bound and increase the lower one.

The constraints could be applied sequentially without any ordering. However in most cases the constraint computation would be useless and time consuming. Different factors may influence the benefit of the computation of a constraint. Among them we consider the amount of change (since the last computation) of the bounds of the variables involved in the constraint[3] and the computation cost of the constraint CC (function of the complexity of each involved operation such as dilations). We define a priority P for each constraint, initialized to $P(0) = \frac{card(X)}{CC}$. At each step of the propagation process the highest priority constraint is selected and the associated contracting operator is computed. The priority of the constraint is then set to 0. The application of this contracting operator may induce changes on the domain of its variables. When this occurs, the priority P of a constraint that depends on one of the variables is updated as follows:

$$P(i+1) = P(i) + \frac{\sum_{x\in X}(\overline{A}^1(x) - \overline{A}^2(x)) + (\underline{A}^2(x) - \underline{A}^1(x))}{CC},$$

were $(\underline{A}^1, \overline{A}^1)$ and $(\underline{A}^2, \overline{A}^2)$ are respectively the domains before and after a change on the variable and $P(i)$ is the priority value at step i. The process stops when the priority of all constraints is equal to 0.

[3] In $AC-3$ algorithm [11], the list of constraints to update would correspond to those with a non-zero amount of change.

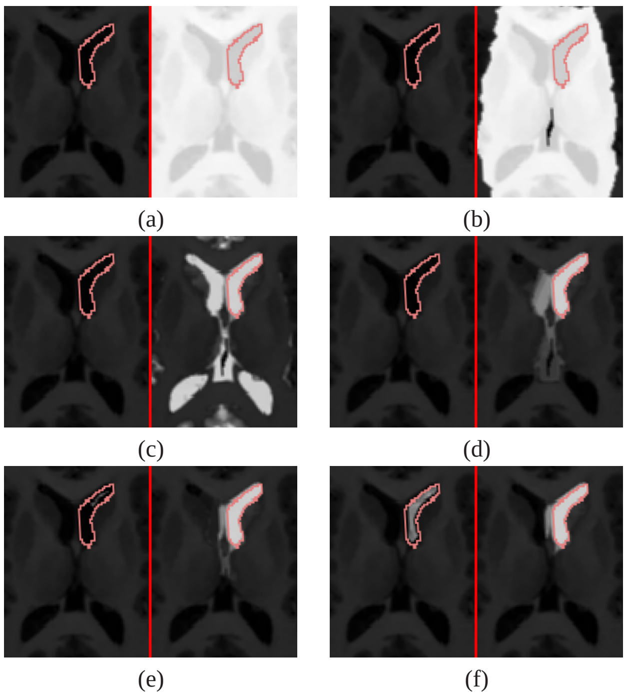


Figure 4. $\underline{LVl}$ (left) and $\overline{LVl}$ (right) at step 0 (a), 500 (b), 1000 (c), 2500 (d), 10000 (e) and 20000 (f) of the propagation process. The target object LVl is delineated.

Ideally the upper and lower bounds of the different domains will converge to the same fuzzy set. However this will generally not occur and there remains some indecision at least on object boundaries. Even if the propagation significantly reduces the search space, it is still time consuming to apply a backtracking algorithm to extract an optimal solution according to some cost function. Therefore we propose to refine the segmentation of each structure by using the method proposed in [13], based on minimal surface optimization [3]. The segmentation problem consists in finding the closed curve that minimizes a metric based on the obtained bounds. This can efficiently be solved using a graph-cuts based method [2] for instance.

5 PRELIMINARY RESULTS ON NORMAL AND PATHOLOGICAL BRAIN

We illustrate here some preliminary results on 2D brain MRI. Our knowledge base contains about 3000 relations involving 34 variables that correspond to visible structures on MRI. If we consider the left caudate nucleus, it is for instance strictly on the right of the left lateral ventricle, fairly on the left of the putamen, much brighter than the lateral ventricle, darker than the white matter and somewhat darker than the putamen.

We now describe the recognition process for a few structures of the 2D brain MRI presented in Figure 5(a). We suppose that the brain was previously extracted. The associated domain is defined as a singleton. Its lower and upper bounds are thus equal. We initialized all other domains to $(0_{\mathcal{F}}, 1_{\mathcal{F}})$. The propagation is then performed, completing in about 5 hours on a 3.0 GHz Pentium 4 CPU. We show in Figure 4 the upper and lower bounds of the left lateral ventricle at different steps of the propagation process. The prior information provides a good discrimination with other structures and the upper and lower bounds are close to the solution at the end of the propagation. The extraction of a crisp segmentation can then easily be performed using the method in [13]. We show in Figure 5(b) the segmentation results for the internal structures.

We show also a result on a case affected by a brain tumor in Figure 5 (c-d). The tumor induces various degrees of deformation and may also involve structural modifications. The case presented here is affected by a cortical tumor which was previously extracted [8]. We modify the knowledge base, just to include that the tumor is a subpart of the brain. We do not modify the other relations. The segmentation results for internal structures is shown in Figure 5(d). We can observe that the result remains correct, despite the shape modification induced on some structures by the tumor.

6 CONCLUSION

We have proposed in this paper a new formulation of the segmentation and recognition task in the case of a known structural arrangement as the resolution of a constraint network. Preliminary results were shown on 2D MRI brain. They illustrate that the constraint propagation is very efficient in providing domain bounds close to the object, thus considerably reducing the search space. Future work aims at improving the efficiency of the propagation process to make it applicable in 3D cases. A deeper study for pathological cases will also be performed, in particular to account for strong structural changes on the internal structures potentially induced by subcortical tumors.

ACKNOWLEDGEMENTS

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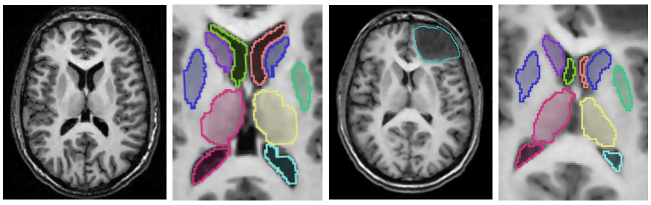

(a) (b) (c) (d)

Figure 5. (a) 2D T1 weighted brain MRI. (b) Cropped view of segmentation results for the internal structures. (c) 2D MRI of a brain affected by a tumor. (d) Segmentation results for internal structures.

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