Abductive reasoning for image interpretation based on spatial concrete domains and description logics

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Résumé

L'interprétation d'images a pour objectif non seulement de détecter et reconnaître des objets dans une scène mais aussi de fournir une description sémantique tenant compte des informations contextuelles dans toute la scène. Le problème de l'interprétation d'images peut être formalisé comme un problème de raisonnement abductif, c'est-à-dire chercher la meilleure explication en utilisant une base de connaissances. Dans ce travail, nous présentons une nouvelle approche utilisant une méthode par tableau pour la génération et la sélection d'explications possibles de l'image donnée lorsque les connaissances, exprimées en logique de description, comportent des concepts décrivant les objets mais aussi les relations spatiales entre ces objets. La meilleure explication est sélectionnée en exploitant les domaines concrets pour évaluer le degré de satisfaction des relations spatiales entre les objets.

Mots Clef

Interprétation d'images, abduction, logiques de description, tableau sémantique, relations spatiales, représentations floues, domaines concrets.

Abstract

Image interpretation aims not only at detecting and recognizing objects in a scene but also at deriving a semantic description considering contextual information in the whole scene. Image interpretation can be formalized as an abductive reasoning problem, i.e. an inference to the best explanation using a background knowledge. In this work, we present a framework using a tableau method for generating and selecting potential explanations of the given image when the background knowledge is encoded in description logics, and includes concepts describing objects and their spatial relations. The best explanation is selected according to a minimality criterion based on the satisfaction degree of spatial relations between the objects, computed in concrete domains.

Keywords

Image interpretation, abduction, description logics, semantic tableau, spatial relations, fuzzy representations, concrete domains.

1 Introduction

As advanced as AI has become, it still remains a big challenge for computers to accomplish complex understanding tasks as humans do, one of which is how to accurately associate perceptual data with appropriate concepts. This relation between visual percepts and high level linguistic expressions is called *semantic gap* [20]. In this work, beyond a single object recognition based on low level features such as color and shape, we focus on a complex description which relies on contextual information like spatial relations between objects as well as prior knowledge on the application domain. We then formalize the interpretation task as an abductive reasoning problem.

Abductive reasoning (abduction) is a backward chaining inference from the observation to the *best* explanations considering the expert knowledge of the domain. Interpretation was considered as abduction for natural language understanding in [11]. Afterwards abduction was applied for a robot system [19], scene understanding problems [17, 18] and image interpretation [1]. A digital image is a numerical representation which does not represent explicitly semantic information. Prior knowledge is intensively used by experts who interpret visually an image. It should then also be used by machines to associate semantics with the image. As illustrated in [1, 15, 17], high level semantics extraction from an image benefits from prior knowledge, such as sport or anatomical knowledge. Description Logics (DLs) are a family of formal knowledge representation formalisms [2] for structural prior knowledge, and was thus used for abduction in [1, 5, 8, 17].

Our aim is to extract high level semantic information from a given image and translate it at a linguistic level. Concretely, we are interested in the interpretation of cerebral images with tumors. High level information consists of descriptions of pathologies as well as of brain structures and spatial relations among them. For instance, according to different levels of anatomical prior knowledge on brain pathology, two possible hypotheses (explanations) could be given for Figure 1:

- an abnormal structure is present in the brain,
- a peripheral non-enhanced tumor is present in the right hemisphere[1].

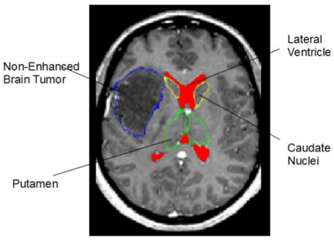


Figure 1: A slice of a pathological brain volume (magnetic resonance imaging (MRI)), where some structures are annotated.

The terminological knowledge of pathological anatomy is formalized in Description Logics and the observation of the scene is represented as instances of the terminologies. The reasoning process is based on a tableau method [2] in order to generate a set of consistent hypotheses, and the *best* explanation is selected based on preference criteria.
Semantic and cardinality minimality are two most common criteria in Description Logics [3, 5, 8]. The ranking of explanations is performed in weighted abduction by assigning a cost value to literals in the logic formalism, and in probabilistic abduction by modeling the prior distribution of concepts. In image interpretation, the localization and the spatial relations between the objects are reliable information to derive a description of the spatial entities in a complex scene [4]. Concrete domains are useful to link real world data and abstract reasoning. In [10, 14], Description Logics with concrete domains were proposed for qualitative spatial reasoning, as an efficient way to connect conceptual terminologies to spatial entities and spatial relations in the image domain. In this work, we propose a new preference criterion based on spatial relations to select the *best* explanation in our context. We evaluate the satisfaction degree of qualitative spatial descriptions of a generated hypothesis in the concrete domain (image), hence this evaluation ranking can be used to select the best explanation among the candidates. In particular, the qualitative spatial relations involved in the knowledge representation are encoded in a fuzzy set framework and computed in the image domain. This fuzzy set framework bridges the gap between the concepts and real world data, useful for spatial reasoning. In addition, this allows us to derive a quantitative estimation of the degree of satisfaction of the relations involved in a high level conceptual hypothesis, and thus an evaluation of the hypothesis itself leading to the ranking mentioned above. For example, in the context of medical imaging, obtaining a description of an abnormal structure in a brain requires to derive a hypothesis from the evidence in the image and from anatomical knowledge. The preferred explanation of the observation is then selected based on spatial restrictions such as localization and spatial relations with respect to other structures.
We first present the whole framework of abduction for image interpretation and the related notions in Section 2. We then detail the logical inference process to derive preferred explanations in Section 3. The computation process for estimating the degree of satisfaction of qualitative spatial relations is presented in Section 4. In Section 5 an illustrative example is explained, and we end with a conclusion and future directions in Section 6.

[1] We use the classical "left is right" convention for display. The "right" structure is on the left side in Figure 1 (i.e. on the right side of the brain).

2 Image interpretation as an abduction problem

2.1 Image interpretation framework

Figure 2 shows the major components of our framework in this work. The main components encompass an input image to be interpreted, a prior knowledge base of the application domain and the reasoning service for the purpose of image interpretation. The input image is first translated into symbolic representations in terms of logical formulas. This preprocessing step can be performed either manually or using segmentation and recognition methods as in [9, 16]. The image space is also the concrete domain in which spatial knowledge will be represented. The

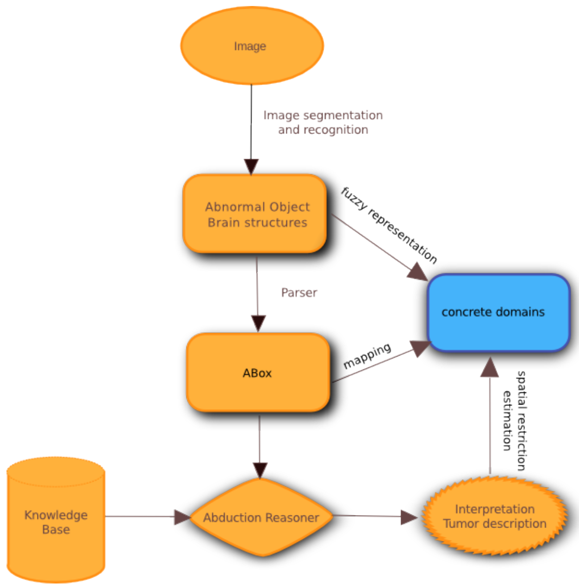


Figure 2: A general diagram of the image interpretation framework.

segmented structure is denoted by an individual associated with a spatial region in the concrete domain. Hypotheses are formulated with the help of the reasoning process taking both the observation (here the image or segmented structures) and the background knowledge into account. The preferred hypothesis depends on the satisfaction of spatial relations in the concrete domain.

2.2 Knowledge representation

In order to represent qualitative spatial relations as well as the inverse and transitive properties useful to reason on such relations, we consider $\mathcal{ALCHI}_{\mathcal{R}^+}$ including inverse roles, symmetric roles and transitive role axioms [12] in this paper. This language supports the transitive and inverse role properties such as $r \equiv s^-$ (inverse role) and $r \circ r \sqsubseteq r$ (transitive role axiom). Accordingly, spatial relations are represented by roles and the spatial properties can be modeled via role axioms. The terminological knowledge is represented using a set of general concepts inclusion (GCIs), in the form $C \sqsubseteq D$ where C and D are two concepts built from a pair of disjoint finite sets N_c (atomic concepts) and N_r (atomic roles), and constructors including concept negation $\neg$, concept conjunction $\sqcap$, concept disjunction $\sqcup$, existential restriction $\exists r.C$ and universal restriction $\forall r.C$.

The semantics is given by an interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where $\Delta^{\mathcal{I}}$ is a non-empty set which indicates the entire "world" of the application domain, and $\cdot^{\mathcal{I}}$ is an interpretation function which maps concepts and individual symbols to $\Delta^{\mathcal{I}}$ and roles to $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$.

The knowledge base used in our framework is a triplet $\mathcal{K} = \{\mathcal{T}, \mathcal{R}, \mathcal{A}\}$, where terminologies (TBox, denoted by $\mathcal{T}$) describe basic axioms of the background knowledge, role axioms (RBox, denoted by $\mathcal{R}$) consist of role properties, and assertions (ABox, denoted by $\mathcal{A}$) involve the facts in the observation (such as information extracted from an image).

An example of a knowledge base referring to brain anatomy is as follows, where LVl and LVr denote left and right lateral ventricles, and CNl and CNr left and right caudate nuclei.

$$
\begin{aligned}
TBox = \{BrainTumor &\sqsubseteq BrainDisease\\
LVl &\sqsubseteq BrainStructure\\
LVr &\sqsubseteq BrainStructure\\
CNl &\sqsubseteq BrainStructure\\
CNr &\sqsubseteq BrainStructure\\
SmallDeformingTumor &\sqsubseteq BrainTumor \sqcap \exists hasEnhancement.NonEnhanced\}
\end{aligned}
$$

$$
\begin{aligned}
RBox = \{rightOf &\equiv leftOf^-\\
above &\equiv below^-\\
closeTo &\equiv closeTo^-\\
farFrom &\equiv farFrom^-\\
isPartOf \circ isPartOf &\sqsubseteq isPartOf\\
hasPart \circ hasPart &\sqsubseteq hasPart\\
isPartOf &\equiv hasPart^-\}
\end{aligned}
$$

$$
\begin{aligned}
ABox = \{&a : BrainTumor\\
&b : NonEnhanced\\
&\langle a, b\rangle : hasEnhancement\}
\end{aligned}
$$

2.3 Concept abduction

In this paper, we consider the image interpretation as a concept abduction problem to derive a high level description of the observed tumor. The objects in the observed image are recognized and represented as individuals in the ABox. An ABox abduction problem can be seen as a set of concept abduction problems with respect to each individual. In this work, we focus on the high level description of the obseved tumor. The concept to be explained is constructed on the basis of the individual describing the detected tumor and contextual information in the ABox.

Definition 1 (Concept Abduction)**.** *Let $\mathcal{L}$ be a DL, $\mathcal{K} = \{\mathcal{T}, \mathcal{R}, \mathcal{A}\}$ be a knowledge base in $\mathcal{L}$, and C, D two concepts in $\mathcal{L}$, supposed to be satisfiable with respect to $\mathcal{K}$. The concept abduction problem $\mathcal{P}$ in DL is expressed as follows: given an observation concept $\mathcal{O}$, a satisfiable concept $\mathcal{H}$ with respect to $\mathcal{K}$ is an explanation if $\mathcal{K} \models \mathcal{H} \sqsubseteq \mathcal{O}$ and $\mathcal{H} \not\models \mathcal{O}$.*

The observation concept in concept abduction is a most specific concept of the individual to be explained.

Definition 2 (**Most specific concept** [1])**.** *Given a TBox $\mathcal{T}$ and an associated interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ in a DL $\mathcal{L}$, let $X \subseteq \Delta^{\mathcal{I}}$ be a subset of the interpretation space and E a defined concept of $\mathcal{L}$. The concept E is defined as the most specific concept of X with respect to $\mathcal{I}$ if:*

- *$X \subseteq E^{\mathcal{I}}$,*
- *for every defined concept $F \in \mathcal{L}$ with $X \subseteq F^{\mathcal{I}}$, we have $\mathcal{T} \models E \sqsubseteq F$.*

Taking the ABox in the previous subsection as an example, the most specific concept of $a^{\mathcal{I}}$ is:
$BrainTumor \sqcap \exists hasEnhancement.NonEnhanced$

Definition 3 (Subconcept [12])**.** *The set $sub(D)$ of the subconcepts of a concept D contains all the concepts occurring in D, and is defined recursively as:*

$$
\begin{aligned}
sub(A) &= \{A\} \text{ for concept names } A \in N_C\\
sub(C \sqcap E) &= \{C \sqcap E\} \cup sub(C) \cup sub(E)\\
sub(C \sqcup E) &= \{C \sqcup E\} \cup sub(C) \cup sub(E)\\
sub(\exists r.C) &= \{\exists r.C\} \cup sub(C)\\
sub(\forall r.C) &= \{\forall r.C\} \cup sub(C)
\end{aligned}
$$

For example,

$$
\begin{aligned}
sub(\exists leftOf.CNl \sqcap \exists closeTo.CNl) = \{&\exists leftOf.CNl \sqcap \exists closeTo.CNl,\\
&\exists leftOf.CNl,\\
&\exists closeTo.CNl,\\
&CNl\}
\end{aligned}
$$

2.4 Fuzzy concrete domain

Definition 4 (*Fuzzy concrete domain*)**.** *A concrete domain is a pair $\mathcal{D} = \{\Delta_\mathcal{D}, \Phi_\mathcal{D}\}$, where $\Delta_\mathcal{D}$ is a subset of the image domain S (S being $\mathbb{Z}^3$ for a 3D image), and $\Phi_\mathcal{D}$ is a set of functions containing:*

- *a mapping function f associating an individual in the ABox with a fuzzy region of $\Delta_\mathcal{D}$;*
- *an evaluation function e^t_r assigning to a pair of individuals (r, t) a satisfaction degree of a spatial relation, where r is the reference object and t is the target object.*

For example, a region in a MR image is recognized to be the right lateral ventricle. Then, an individual $c : LVr$ is initialized in the ABox. The region $f(c)$ is associated with c by the mapping function. To estimate a satisfaction degree of a relationship $rightOf$ of the target object $a : BrainTumor$ with respect to $c : LVr$, $e^a_c(rightOf)$ is computed by evaluating the satisfaction degree of the proposition "the region $f(a)$ is to the $right\ of$ the region $f(c)$" in the image domain.

2.5 Spatial restriction criterion

Minimality criteria are required to select the *best* hypothesis among the candidate ones. Semantic minimality is one of the most common minimality criteria. Other minimality criteria are discussed in [3] in the context of the DL $\mathcal{EL}$.

Definition 5 (Semantic minimality)**.** *For an abduction problem $\mathcal{P} = \langle \mathcal{T}, \mathcal{H}, \mathcal{O} \rangle$, and $\{\mathcal{H}_1, \ldots, \mathcal{H}_n\}$ a set of potential hypotheses, $\mathcal{H}_i$ is a $\sqsubseteq -minimal$ explanation if there does not exist an explanation $\mathcal{H}_j$ for $\mathcal{P}$ such that $\mathcal{H}_j \sqsubseteq \mathcal{H}_i$.*

The most general hypothesis is selected based on the semantic minimality with respect to the hierarchy of the knowledge base. However, this selection criterion does not reflect the spatial information in the image. Therefore, we propose a preference criterion for hypotheses ranking based on spatial relations restrictions in concrete domains (image domain). The spatial restriction criterion is a quantitative estimation of a concept C involving spatial descriptions $\exists sr.D$ or $\forall sr.D$ where sr is a spatial relation role such as $leftOf$, $closeTo$. The satisfaction degree SD is computed as follows:

- given an instance $a : C$, $E = \exists sr.D$ and $E \in Sub(C)$, and a set of instances I, where $i \in I$ is an instance of a concept D in the ABox ($i : D$), $SD(E) = \sup_{i \in I}(e^a_i(sr))$;
- given an instance $a : C$, $E = \forall sr.D$ and $E \in Sub(C)$, and a set of instances I, where $i \in I$ is an instance of a concept D in the ABox ($i : D$), $SD(E) = \inf_{i \in I}(e^a_i(sr))$;
- for a conjunction of spatial descriptions $C = \sqcap_{j \in J} C_j$, $SD(C) = \inf_{j \in J}(SD(C_j))$;
- for a disjunction of spatial descriptions $C = \sqcup_{j \in J} C_j$, $SD(C) = \sup_{j \in J}(SD(C_j))$.

3 Abductive reasoning using semantic tableau method

In this section, we present the concept abduction method using the semantic tableau method, inspired from [5]. The *best* explanation is then selected based on the spatial restriction criterion. As all observed objects in the ABox can be formulated by the most specific concept, our problem is modeled as a concept abduction.
We present several auxiliary definitions that will be used later.

Definition 6 (Negation normal form)**.** *Negation normal form (NNF) is a concept expression such that the negation constructor appears only before atomic concepts. The rules of transformation are described as follows:*

- $\neg(\neg C) \equiv C$,
- $\neg(C \sqcup D) \equiv \neg C \sqcap \neg D$,
- $\neg(C \sqcap D) \equiv \neg C \sqcup \neg D$,
- $\neg(\exists r.C) \equiv \forall r.\neg C$,
- $\neg(\forall r.C) \equiv \exists r.\neg C$

For example, the negation normal form of the concept $\neg(BrainStructure \sqcap \exists leftOf.CNl)$ is $\neg BrainStructure \sqcup \forall leftOf.\neg CNl$.

Definition 7 (Conjunctive normal form [6])**.** *Conjunctive normal form (CNF) is a concept expression where complex concepts are replaced by the conjunction of their superconcept taking TBox axioms into account. For a concept C and a TBox $\mathcal{T} = \{C \sqsubseteq D\}$, $CNF(C, \mathcal{T}) = C \sqcap D$.*

For example, the conjunctive normal form of the concept $SmallDeformingTumor$ with respect to the TBox described in Section 2 is $SmallDeformingTumor \sqcap BrainTumor \sqcap \exists hasEnhancement.NonEnhanced$.

Definition 8 (Internalized concept [2])**.** *Let $\mathcal{T}$ be a TBox and a set of axioms formulated as $C_i \sqsubseteq D_i$. The internalized concept of the TBox is defined as follows:*

$$C_\mathcal{T} \equiv \sqcap_{(C_i \sqsubseteq D_i \in \mathcal{T})}(\neg C_i \sqcup D_i)$$

For example, the internalized concept of the axiom $LVl \sqsubseteq BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl$ is $\neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)$.
We reformulate the subsumption in terms of satisfiability: the concept $H \sqcap \neg O$ is not satisfiable with respect to $\mathcal{T}$, where H is a hypothesis, O is an observation, $\mathcal{T}$ is a TBox. This problem can be reduced by testing the satisfiability of a concept $H \sqcap \neg O \sqcap C_\mathcal{T}$, where $C_\mathcal{T}$ is the internalized concept of $\mathcal{T}$. The tableau algorithm is an efficient decision

procedure for the concept satisfiability problem [2]. This method tries to construct a model of a concept C with respect to the given terminological knowledge. All the concepts are required to be expressed in negation normal form. A concept H that causes unsatisfiability of $H \sqcap \neg O \sqcap C_{\mathcal{T}}$ is a potential hypothesis, i.e. the tableau built from this concept is closed. We follow this strategy and propose an extension of the work by Colucci *et al.* in [5].

Definition 9 (A tableau for $\mathcal{ALCHI}_{\mathcal{R}^+}$). *Let D be an $\mathcal{ALCHI}_{\mathcal{R}^+}$ concept in NNF and let R_D be the set of roles in $\mathcal{ALCHI}_{\mathcal{R}^+}$. A tableau T for D is defined as a triplet $(\mathbf{S}, \mathcal{L}, \mathcal{E})$, where $\mathbf{S}$ is a set of interpretation elements; $\mathcal{L}$ relates each interpretation element to a set of concepts occurring in D ($\mathcal{L} : \mathbf{S} \to \mathbb{P}(sub(D))$ [2]); $\mathcal{E}$ relates each pair of interpretation elements to a set of roles in R_D ($\mathcal{E} : \mathbf{S} \times \mathbf{S} \to \mathbb{P}(R_D)$).*
Let x and y be two interpretation elements in $\mathbf{S}$ ($x, y \in \mathbf{S}$), C, E be two concepts occurring in D and $r \in R_D$. The tableau to check the satisfiability of D is constructed as a tree structure where each node corresponds to an element of interpretation $x \in \Delta^{\mathcal{I}}$. The node is labeled with a set of concepts $\mathcal{L}(x)$. The edge between the nodes x and y is labeled with corresponding roles $r \in \mathcal{E}(\langle x, y \rangle)$. The following rules are applied for the construction:

1. *if $C \in \mathcal{L}(x)$, then $\neg C \notin \mathcal{L}(x)$;*
2. *if $C \sqcap E \in \mathcal{L}(x)$, then $C \in \mathcal{L}(x)$ and $E \in \mathcal{L}(x)$;*
3. *if $C \sqcup E \in \mathcal{L}(x)$, then $C \in \mathcal{L}(x)$ or $E \in \mathcal{L}(x)$;*
4. *if $\exists r.C \in \mathcal{L}(x)$, then there exists some $y \in \mathbf{S}$ such that $r \in \mathcal{E}(\langle x, y \rangle)$ and $C \in \mathcal{L}(y)$;*
5. *if $\forall r.C \in \mathcal{L}(x)$, then for all $y \in \mathbf{S}$ such that $r \in \mathcal{E}(\langle x, y \rangle)$, $C \in \mathcal{L}(y)$;*
6. *if $\forall r.C \in \mathcal{L}(x)$ and r is a transitive role, then for all $y \in \mathbf{S}$ such that $r \in \mathcal{E}(\langle x, y \rangle)$, $\forall r.C \in \mathcal{L}(y)$.*
7. *$r \in \mathcal{E}(\langle x, y \rangle)$ iff $r^- \in \mathcal{E}(\langle y, x \rangle)$.*
8. *if $r \in \mathcal{E}(\langle x, y \rangle)$ and $r \sqsubseteq v$ (or $r^- \sqsubseteq v^-$) then $v \in \mathcal{E}(\langle x, y \rangle)$.*

Definition 10. *(Clash) A branch contains a clash (i.e. the branch is closed), when $\{C, \neg C\} \subseteq \mathcal{L}(x)$ for a node x and a concept C.*

$$\mathcal{L}(x) = \{C,\ \neg C\}$$
$$|$$
$$\boxtimes$$

A branch is said to be complete when there exists a clash in some node x or none of the rules mentioned above can be applied in the tableau. For a given concept D, D is $satisfiable$ if all the branches in the tableau are complete and at least one branch is open, otherwise D is $unsatisfiable$. By applying expansion rules, the construction process of the tableau is performed until no more rule can be applied or a clash occurs. The hypotheses are generated by constructing a conjunctive concept H including at least the complement of one concept in each open branch.
In the selection process, the *best* explanation is selected among a set of consistent hypotheses. In our example, we prefer a high level description of the pathology, which is most specific and has a maximal satisfaction degree of the spatial relations involved in the concept description. Suppose that $\mathcal{H}_1, \cdots, \mathcal{H}_n$ are n most specific candidates. Every $\mathcal{H}$ can be written in CNF with respect to $\mathcal{T}$. An estimation is then performed in the concrete domain (image domain) and $SD(CNF(\mathcal{H}_i))$ can be computed. We will choose the one with maximal value of the satisfaction degree as the *best* explanation. The evaluation of the spatial criterion will be detailed in the next section.

4 Fuzzy representations of spatial relations

In this work, we consider the spatial relations as constraints to evaluate the satisfaction of a hypothesis. The estimation is performed in the concrete domain (image space) using mathematical morphology operators [4] in a fuzzy set framework. The computation aims at answering the question "to which degree a spatial relation is satisfied between two objects".
Let S denote the image space, typically $\mathbb{Z}^3$ for 3D images. A fuzzy set in S is defined by a membership function $\mu : S \to [0, 1]$ where for $x \in S$, $\mu(x)$ represents the degree to which x belongs to the fuzzy set. For a crisp set, membership degrees take only values 0 and 1. When using spatial representations of spatial relations, $\mu(x)$ will represent the degree to which a spatial relation is satisfied at x with respect to a reference object.

Inclusion Since roles such as $is\ part\ of$ and $has\ part$ are often involved in structural descriptions, inclusion is an important spatial relation. In the crisp case, the inclusion satisfaction degree of "X is a part of Y" often takes values 0 ($X \not\subseteq Y$) or 1 ($X \subseteq Y$). It can also be defined as a number in $[0, 1]$, e.g. $\frac{\sharp(X \cap Y)}{\sharp(X)}$, where $\sharp()$ computes the volume of the object. In the fuzzy case, the satisfaction degree of an inclusion relation of a fuzzy object u in a fuzzy object v is defined using fuzzy set operations [7]: $\inf_{x \in S} I(u(x), v(x))$, where I is a fuzzy implication. The extension of the volume based definition can be formalized as: $\frac{\sum_{x \in S} \min(u(x), v(x))}{\sum_{x \in S} u(x)}$.

Directions Qualitative directional relationships, such as *left of*, *in front of*, *above*, are frequently used in expert knowledge representations (particularly in brain anatomical knowledge [4, 13]) to describe a relative location with respect to a reference object. Such relationships are imprecise even for crisp objects. Fuzzy representations are suitable to model these imperfections in the image domain,

[2] $\mathbb{P}(sub(D))$ is the power set of $sub(D)$.

and can be defined using fuzzy dilation [4]. For instance for "right of", the reference object is dilated using a fuzzy structuring element modeling the semantics of "right of", defining the region of space to the right of it. Then the inclusion of the target object in this fuzzy region is evaluated.

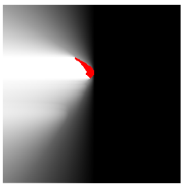


Figure 3: Region to the "right of" the right caudate nucleus (red region).

Distances Distances give metric information in the spatial domain. Qualitative representations of relations such as *close to* and *far from* are also frequently used in anatomical knowledge. Each qualitative distance value can be represented as a fuzzy set on the real line as illustrated in Figure 4. For example, the relation $far\ from$ is an increasing function that maps a distance value to a satisfaction degree in $[0, 1]$. This function can then be used to define the region of space *far from* the reference object, e.g. the fuzzy region of *far from* with respect to the right lateral ventricle, as shown in Figure 5a, where the brightness of pixels represents the degree of *far from* with respect to the reference object. The degree to which another object is close to the reference object can then be computed. This satisfaction degree can also be computed as $\mu_{far}(d)$, where d is the distance between the two objects (minimum, average or Hausdorff distance), as shown in Figure 5b and μ_{far} the fuzzy set defining "far" on the real line as in Figure 4. In our experiments we use this second approach.

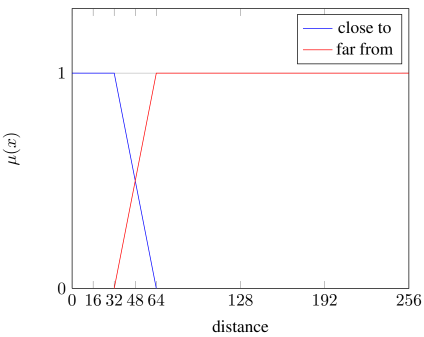
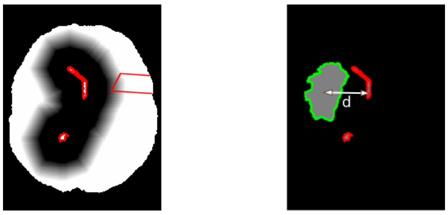


Figure 4: Membership functions of qualitative distance values.

5 An illustrative example

In this section, we give an example to show how the proposed image interpretation framework can be used for a high level semantic description extraction in a pathological brain image context. The following TBox describes a pathological anatomical knowledge and the ABox represents the parsing results of the structures detected using image processing tools [9, 16]. We take the same RBox as in Section 2.2 for the illustration.



(a) Fuzzy region "far from" the right lateral ventricle (the object with red contour).

(b) Distance measurement between two objects (minimum, average or Hausdorff distance can be used).

Figure 5: Two methods of distance computation.

$$
\begin{aligned}
TBox = \{Hemisphere &\sqsubseteq \exists isPartOf.Brain\\
BrainStructure &\sqsubseteq \exists isPartOf.Brain\\
BrainDisease &\sqsubseteq \exists isPartOf.Brain \sqcap \neg BrainStructure\\
BrainTumor &\sqsubseteq BrainDisease\\
LVl &\sqsubseteq BrainStructure\\
LVr &\sqsubseteq BrainStructure\\
CNl &\sqsubseteq BrainStructure\\
CNr &\sqsubseteq BrainStructure\\
PeripheralRegion &\sqsubseteq \exists isPartOf.Brain\\
SubCorticalRegion &\sqsubseteq \exists isPartOf.Brain\\
SmallDeformingTumor &\sqsubseteq BrainTumor \sqcap \exists hasEnhancement.NonEnhanced\\
PeripheralSmallDeformingTumor &\sqsubseteq SmallDeformingTumor \sqcap\\
&\quad \exists isPartOf.PeripheralRegion \sqcap\\
&\quad \exists farFrom.(LVl \sqcup LVr)\\
SubCorticalSmallDeformingTumor &\sqsubseteq SmallDeformingTumor \sqcap\\
&\quad \exists isPartOf.SubCorticalRegion \sqcap\\
&\quad \exists rightOf.CNr\}
\end{aligned}
$$

$$
\begin{aligned}
ABox = \{a &: BrainTumor\\
b &: NonEnhanced\\
\langle a, b\rangle &: hasEnhancement\\
c &: CNr\\
d &: CNl\\
k &: LVr\\
m &: LVl\\
p &: PeripheralRegion\\
s &: SubCorticalRegion\}
\end{aligned}
$$

Figure 6 shows a slice of the global segmentation of the observed image. The instances in the ABox such as a, c, d, k, m, p and s are associated with subsets of the concrete domain. These regions are presented in Figures 7 and 8.

We aim at extracting a high level description of the tumor in the terminological language. As illustrated in Section 2, the most specific concept of $a^{\mathcal{I}}$ is formalized as $BrainTumor \sqcap \exists hasEnhancement.NonEnhanced$. The semantic tableau method is then applied for generating consistent hypotheses. A possible hypotheses set is: *SmallDeformingTumor*, *PeripheralSmallDeformingTumor*, *Sub-*

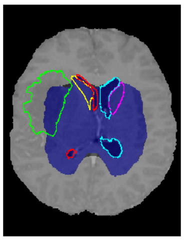
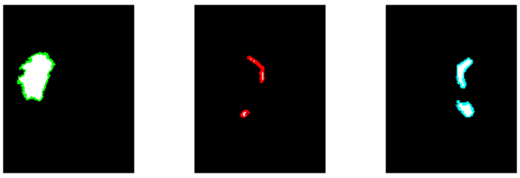


Figure 6: A segmentation of the observed MRI volume. The color shapes represent different structures segmented in the MRI volume. The blue region (respectively the gray region) represents the subcortical region (respectively the peripheral region) in the brain.



(a) Segmented brain tumor. (b) Right lateral ventricle. (c) Left lateral ventricle.

Figure 7: Segmented brain tumor and lateral ventricles.

CorticalSmallDeformingTumor. Obviously, *PeripheralSmallDeformingTumor*, *SubCorticalSmallDeformingTumor* are two more specific hypotheses compared with *SmallDeformingTumor*. In addition, the spatial restriction criterion is considered to find the *best* explanation. The two hypotheses are represented in CNF:

$$
\begin{aligned}
PeripheralSmallDeformingTumor \sqcap & SmallDeformingTumor \sqcap \\
& \exists isPartOf.PeripheralRegion \sqcap \\
& (\exists farFrom.LVl \sqcup \\
& \exists farFrom.LVr) \\
SubCorticalSmallDeformingTumor \sqcap & SmallDeformingTumor \sqcap \\
& \exists isPartOf.SubCorticalRegion \sqcap \\
& \exists rightOf.CNr
\end{aligned}
$$

For example, the hypothesis *PeripheralSmallDeformingTumor* consists of three spatial descriptions: $\exists isPartOf.PeripheralRegion$, $\exists farFrom.LVl$, $\exists farFrom.LVr$. Therefore, we can estimate the satisfaction degree of inclusion between fuzzy representations of $BrainTumor$ and $PeripheralRegion$ as well as the distance between fuzzy representations of $BrainTumor$ and LVl, LVr. Here, we take the volume based method to estimate the satisfaction degree of the inclusion, and we measure the distance using the average Euclidean distance. The overall aggregation satisfaction degree of *PeripheralSmallDeformingTumor* is $\min(e^a_p(isPartOf)$, $\max(e^a_k(farFrom)$, $e^a_m(farFrom))) = \min(0.89, \max(0.13, 0.34)) = 0.34$. Similarly, the satisfaction degree of *SubCorticalSmallDeformingTumor* is $\min(e^a_s(isPartOf)$, $e^a_c(rightOf))$

(a) Right caudate nucleus. (b) Left caudate nucleus.

(c) Subcortical region (containing the gray nuclei). (d) Peripheral region (containing the cortex).

Figure 8: Caudate nuclei, subcortical region and peripheral region.

$= \min(0.81, 0.11) = 0.11$. As a result, the *PeripheralSmallDeformingTumor* is considered as the *best* explanation.

6 Conclusion

We have exploited abductive reasoning based on spatial concrete domains for image interpretation. The proposed framework involves Description Logics for knowledge representation and a semantic tableau method for generating consistent hypotheses for abduction. The *best* explanation is measured based on a spatial restriction criterion in the image domain. The quantitative estimation is computed based on fuzzy representations of spatial relations in concrete domains (image domain) for qualitative spatial relations defined in the knowledge base. This contribution illustrates a concrete criterion in decision making for image interpretation to tackle the semantic gap.

At this stage, the semantic tableau method produces a large number of hypotheses. However, most of them are irrelevant or unsatisfiable. In addition, the concrete domains are only used in the explanation selection. In order to reduce the size of the hypotheses set and improve the efficiency of the inference process, an iterative method will be considered in the future to integrate the selection process using concrete domains in an iterative way. Instead of adding all internalized concepts to the tableau, only relevant axioms are added to corresponding branches that cause a closure. This action can avoid generating unsatisfiable hypotheses. Since the observation is a conjunction of the concepts, the partial hypotheses in each branch will be ordered according to the minimality criterion based on concrete domains. The

selection process for the "best" explanation will be directly embedded into the tableau construction process.
Fuzzy logic is also a useful ingredient in knowledge representation dealing with imprecision and vague information. Another strategy to integrate fuzzy set theory into knowledge representation is to add fuzzy values to terminological and assertional knowledge at the logical level.

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