A New Local Measure of Disagreement between Belief Functions – Application to Localization

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Abstract. In the theory of belief functions, the disagreement between sources is often measured in terms of conflict or dissimilarity. These measures are global to the sources, and provide few information about the origin of the disagreement. We propose in this paper a "finer" measure based on the decomposition of the global measure of conflict (or distance). It allows focusing the measure on some hypotheses of interest (namely the ones likely to be chosen after fusion). We apply the proposed so called "local" measures of conflict and distance to the choice of sources for vehicle localization. We show that considering sources agreement/disagreement outperforms blind fusion.

1 Introduction

Multi-sensor systems are used in many applications such as classification, image processing, change detection, object trajectory localization. Usually the information provided by each sensor is prone to imperfections, such as imprecision and uncertainty, and fusion procedures aim at making better decisions by combining multi-sensor information. Belief Functions (BF) are suitable for modeling imprecision and uncertainty, and handle belief on the power set of the frame of discernment (set of hypotheses). A disagreement between sources makes the system unstable and can impact the decision. Many techniques have been developed to measure the disagreement between sources. A review can be found in [4] or [5]. One method consists in observing the so-called "Demspter's conflict" [10] resulting from the conjunctive combination of the basic belief functions. However, the non-idempotence

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of the usual conjunctive rule can create a non-zero conflict for the combination of two equal belief functions. Other measures are based on distances between mass functions. The distances derived from L1 or L2 norms measure the inter-sources disagreement taking into account all elements of the space of discernment.

In this paper, we aim at exploiting the conflict or distance to provide a diagnosis of the system status. For this we need a more precise measurement than the "Dempster's conflict" or global dissimilarity between sources. Thus we propose a new measure which is related to the different elements of the discernment space, that we call "local" measure. After recalling some notations and basic elements on mass function decompositions and distance measures in Section 2, the proposed measure is introduced and analyzed in Section 3. It is then illustrated on a vehicle localization problem described in Section 4. Results are provided in Section 5.

2 Background

In the following, we denote by Ω the frame of discernment, by 2^{Ω} the power set of Ω, and by m_j a Basic Belief Assignment (BBA) on 2^{Ω} associated with a source S_j. Plausibility and communality are denoted by Pls and q, respectively. Smets proposed a canonical decomposition of every non-dogmatic BBA, as a unique conjunctive combination of simple support functions (SSF) [9]:

$$m_j = \bigcirc\!\!\!\!\cap_{A\subset\Omega} A^{w_j(A)}, \tag{1}$$

where $A^{w_j(A)}$ is a SSF, i.e. a function with only two focal elements A and Ω, such that $A^{w_j(A)}(\Omega) = w_j(A)$, $A^{w_j(A)}(A) = 1 - w_j(A)$, and $A^{w_j(A)}(B) = 0, \forall B \in 2^{\Omega} \setminus \{A, \Omega\}$. If $w_j(A) \leq 1$ then $A^{w_j(A)}$ is a BBA, and if $w_j(A) \leq 1, \forall A \subset \Omega$, m_j in Eq. 1 is a separable BBA (SBBA). The weight $w_j \in \mathbb{R}^+$ is expressed from the commonalities as follows:

$$\forall A \subset \Omega,\ w_j(A) = \prod_{B\supseteq A} q_j(B)^{(-1)^{|B|-|A|+1}}. \tag{2}$$

For the conjunctive combination of N BBAs, two main rules are generally considered depending on whether the sources are "cognitively independent", and can be expressed using the canonical decomposition: Smets' combination [9] (sometimes simply called conjunctive rule because of its authority): $m_{\bigcirc\!\!\!\!\cap} = \bigcirc\!\!\!\!\cap_{A\subset\Omega} A^{\prod_{j=1}^{N} w_j(A)}$, and the cautious rule [2]: $m_{\bigcirc\!\!\!\!\wedge} = \bigcirc\!\!\!\!\cap_{A\subset\Omega} A^{\bigwedge_{j=1}^{N} w_j(A)}$, where $\bigwedge$ denotes the minimum operator.

The dissimilarity between BBAs is often used for computing their disagreement. It is generally estimated from a conflict or distance measure (the reader can refer to [5] or [4] for an overview). These measures involve all elements of 2^{Ω}.

In this study, we focus on the conflict (as a diagnostic tool of the system) between different sources. Besides, in the estimation of a disagreement (conflict), to avoid the bias due to the individual source auto-conflict, we consider sources with null auto-conflict, namely modelled using consonant BBAs, as proposed in [6].

3 Proposed Local Measures of Sources Disagreement

3.1 Local Conflict

We note $\Upsilon = \{\{A_i\}_j, A_i \subseteq \Omega,\ j \in \{1,...,N\}\}$ the multi-set containing the elements of the canonical decomposition of the BBAs to be combined, where $\{A_i\}_j$ is the set of elements of the canonical decomposition of m_j for which $w_j(A_i) \neq 1$. From Eq. 1, the mass of the empty set resulting from the combination of N SBBAs defined on Ω (with $|\Omega| > 2$) is:

$$m_{\textcircled{\cap}}(\emptyset) = 1 + \sum_{B \subseteq \Omega, B \neq \emptyset} (-1)^{|B|} \prod_{k=1}^{|\Upsilon|} \sum_{B \subseteq A} w_k(A). \tag{3}$$

In the following, we focus on the case of two consonant BBAs. If Υ is not a consonant set, then conflict appears. Now for two BBAs the conflict can be brought by different hypotheses. We propose to analyze the origin of the conflict by decomposing it on pairs of elements. For this we consider the canonical decomposition of $m_{1 \textcircled{\cap} 2}$ and we analyze the conflict between the pairs of elements (singletons or compound hypotheses) of this decomposition.

We introduce the following function $f_\emptyset$ on $2^\Omega \times 2^\Omega$ for conflict decomposition:

$$\forall (A,B)\ /\ A \cap B \neq \emptyset\ \ ,\ \ f_\emptyset(A,B) = 0, \tag{4}$$

$$\forall (A,B)\ /\ A \cap B = \emptyset\ \ , \tag{5}$$

$$\begin{aligned} f_\emptyset(A,B) &= \frac{1}{2} \sum_{g=1}^{|\Upsilon|} \sum_{l=1}^{|\Upsilon|} \mu_g(A) \times \mu_l(B) \times \sum_{\substack{\{X_1,...,X_{|\Upsilon|-2}\} \\ \in \{\Upsilon_{\supseteq A} \cup \Upsilon_{\supseteq B}\}}} \prod_{\substack{k\,=\,1 \\ k\,\neq\,\{g,l\}}}^{|\Upsilon|} \mu_k(X_k), \\ &= \frac{1}{2} \sum_{g=1}^{|\Upsilon|} \sum_{l=1}^{|\Upsilon|} \mu_g(A) \times \mu_l(B) \times \prod_{\left\{\substack{k=1, \\ k \neq \{g,l\} \\ X_k \in \{\Upsilon_{\subset A} \cup \Upsilon_{\subset B}\}}\right.}^{|\Upsilon|} \mu_k(\Omega), \end{aligned} \tag{6}$$

where $\Upsilon_{\supseteq A}$ is the set of elements of Υ including A: $\Upsilon_{\supseteq A} = \{X \in \Upsilon\ /\ A \subseteq X\}$ and $\Upsilon_{\subset A}$ is the set of elements being strictly included in A: $\Upsilon_{\subset A} = \{X \in \Upsilon / X \subset A\}$. $\mu_j(A) = A^{w_j(A)}$ if A is an element of the decomposition of m_j and $\mu_j(A)$ is the vacuous BBA (such that $m(\Omega) = 1$) if A is not an element of the decomposition of m_j.

For each element of 2^Ω we define the conflict brought by this element as:

$$\forall A_i \in 2^\Omega,\ Mes_\emptyset(A_i) = \sum_{A_j \in 2^\Omega} f_\emptyset(A_i, A_j). \tag{7}$$

The mass on the empty set (Eq. 3), is thus:

$$m_{\cap}(\emptyset) = \sum_{A \in 2^{\Omega}} Mes_{\emptyset}(A). \tag{8}$$

Example: Let m_1 and m_2 be two consonant SBBAs defined on $\Omega = \{a,b,c\}$ (see the table below). Here $\Upsilon = \{\{a\},\{b\},\{a\cup c\},\{b\cup c\}\}$. After conjunction, $\forall A_i \in \Upsilon$, $A_i^{\prod_{j=1}^{2} w_j(A_i)}$ is a SSF.

	$\{a\}$	$\{b\}$	$\{a\cup b\}$	$\{c\}$	$\{a\cup c\}$	$\{b\cup c\}$	$\{\Omega\}$	$\{\emptyset\}$
m_1	0.3	0	0	0	0.6	0	0.1	0
m_2	0	0.3	0	0	0	0.6	0.1	0
$m_{1\cap 2}$	0.03	0.03	0	0.36	0.06	0.06	0.01	0.45
w_1	0.7	1	1	1	0.1429	1	1	1
w_2	1	0.7	1	1	1	0.1429	1	1
μ_1	0.3	0	0	0	0	0	0.7	0
μ_2	0	0.3	0	0	0	0	0.7	0
μ_3	0	0	0	0	0.8571	0	0.1429	0
μ_4	0	0	0	0	0	0.8571	0.1429	0

$\forall A_i \neq \Omega, \mu_i(A_i) = 1 - \prod_{j=1}^{2} w_j(A_i)$ and $\mu_i(\Omega) = \prod_{j=1}^{2} w_j(A_i)$. From Eq. 6, the decomposition of $m_{1\cap 2}(\emptyset)$ can be written as:

Decomposition of $m_{1\cap 2}(\emptyset)$	Pairs of conflicting hypotheses
$\mu_1(\{a\}) \times \mu_2(\{b\})$	$(\{a\},\{b\})$
$\mu_1(\{a\}) \times \mu_2(\{\Omega\}) \times \mu_4(\{b\cup c\})$	$(\{a\},\{b,c\})$
$\mu_1(\{\Omega\}) \times \mu_1(\{b\}) \times \mu_3(\{a\cup c\})$	$(\{b\},\{a,c\})$

The result of the conflict decomposition is:

	$\{a\}$	$\{b\}$	$\{a\cup b\}$	$\{c\}$	$\{a\cup c\}$	$\{b\cup c\}$	$\{\Omega\}$	$\{\emptyset\}$
$Mes_{\emptyset}$	0.27	0.27	0	0	0.18	0.18	0	0

For this example, we note that the conflict is mainly due to the couple of hypotheses $\{a\}$ and $\{b\}$.

3.2 Local Pseudo-distance

In Section 3.1, we introduced the notion of "local " conflict induced by a hypothesis. In a similar way, we introduce a local pseudo-distance:

$$Dist_{Pl_{1,2}}(A,B) = \frac{1}{2} \mid (Pl_1(A) - Pl_2(A)) + (Pl_2(B) - Pl_1(B)) \mid, \tag{9}$$

where Pl_j, is the plausibility function associated with $m_j, j = \{1,2\}$, and A and B denote two elements of 2^{Ω}. This defines a pseudo-metric: it is non-negative and symmetrical by construction, $\forall A \in 2^{\Omega}, Dist_{Pl_{1,2}}(A,A) = 0$ and satisfies the triangular inequality: $\forall (A,B,C) \in (2^{\Omega})^3$, $Dist_{Pl_{1,2}}(A,C) + Dist_{Pl_{1,2}}(C,B) \geq Dist_{Pl_{1,2}}(A,B)$.

Note that the detection of a partial conflict between BBAs and the detection of a high distance have very different interpretations. In the first case, we aim at selecting the hypotheses mainly inducing conflict in order to specify the conflict (origin, type of conflict, etc.). In the second case, we aim at restricting the measure of distance to a sub-part of 2^{Ω} (pairs of elements) because our interest focuses on some hypotheses (typically those that can be selected when making the decision).

4 Application to the Localization Problem

4.1 Localization Problem

In this section, we apply the previously presented measures to the problem of vehicle localization using different sources j, here algorithms providing localization estimates from vehicle sensors (odometers, camera). Odometers provide the distance travelled by each wheel independently. Using the wheel parameters (radius, length of the rear axle, tick number) and assuming a rigid structure of the vehicle, we can compute its displacement (longitudinal and rotational components). From the camera data, features (interest points i.e. SURF, SIFT points, etc.) are tracked in several images, both to infer the scene structure (3D) and the camera movement [1]. In our experiments, the longitudinal and rotational components of displacement are estimated using three different algorithms. The first one (S_1) exploits only odometer data. The second one (S_2), FastSLAM algorithm [7], exploits both odometer and camera. Finally, the third algorithm (S_3), exploits only images. The estimates from these three algorithms are more or less precise depending on the physical world and the movement of the vehicle. A wheel sliding may induce an error in the estimates of the algorithms using odometer data; an homogeneous environment or a mismatch between features may induce an error for the algorithms using camera data.

4.2 Fusion Model

At each instant the movement is described by a couple $(\delta_s, \delta_\Theta)$ (longitudinal and rotational components), whose values are bounded by the motor vehicle features. Each hypothesis of Ω represents a pair of values $(\delta_s, \delta_\Theta)$. We denote the measurement provided by a given source at instant t by $\overrightarrow{\delta(t)} = (\delta_s(t), \delta_\theta(t))^t$, and the measurement associated to a hypothesis H by $\overrightarrow{\delta_H} = (\delta_s(H), \delta_\theta(H))^t$. The considered measure between $\delta_s(H)$ and $\delta_\theta(H)$ is the Mahalanobis distance $d^2(\overrightarrow{\delta_t}, \overrightarrow{\delta_H}) = \begin{pmatrix} \delta_s[H] - \delta_s(t) \\ \delta_\theta[H] - \delta_\theta(t) \end{pmatrix}^T \Sigma^{-1} \begin{pmatrix} \delta_s[H] - \delta_s(t) \\ \delta_\theta[H] - \delta_\theta(t) \end{pmatrix}$, where Σ is the covariance matrix.

We assume longitudinal and rotational components of the movement are decorrelated, and thus Σ is diagonal. We also assume that the more the acceleration is important, the less accurate are the movement estimations by the considered algorithms, and thus the higher are the Σ terms: in our model Σ depends on the movement estimate itself. The ellipsoid centered at $\overrightarrow{\delta_t}$ models the movement of the vehicle. The probability of a hypothesis H, $H \in \Omega$, is calculated conditionally to $\overrightarrow{\delta_t}$:

$$P(H \mid \overrightarrow{\delta_t}) = \frac{1}{2\pi \times |\Sigma|^{\frac{1}{2}}} \exp\{\frac{-d^2(\overrightarrow{\delta_t}, \overrightarrow{\delta_H})}{2}\} \tag{10}$$

The higher the distance between hypothesis H and source estimate at t, the lower the probability of H. The mass allocation proposed by Dubois [3] builds a consonant BBA (the less committed BBA having given a pignistic probability) centred on the hypothesis maximizing Eq. 10. For consonant BBAs, the number of focal elements is $|\Omega|$, and the auto-conflict [6] is null.

As second main hypothesis about the data model, we assume the sampling of data ($30Hz$) is high relatively to the acceleration so that $(\delta_s(t), \delta_\theta(t))$ varies slowly versus time. This so called "regularity assumption" allows us to consider $(\delta_s(t-1), \delta_\theta(t-1))$ as sources for the estimation of the vehicle movement at t, even if less reliable than measurements at t. We will see in the next section how such $(t-1)$ sources are used in the data fusion process.

Finally, for combination, recall that S_1 and S_2, which both use odometer data, are not independent, and that S_2 and S_3, which both use camera data, are also not independent. Independence between sources can only be assumed for S_1 and S_3. In this study, our aim is to show the interest of the conflict measurement, and sources are combined at the same time. Therefore we consider that the sources are at least partially correlated and we use the cautious combination proposed by Denoeux [2].

4.3 Exploitation of Conflict

As said in Section 4.2, the precision of sources is time varying (e.g. mainly depends on the acceleration), and so is its reliability. In this work, we estimate dynamically the reliability of the sources to improve the fusion robustness. The estimation of conflict (Eq. 7) is "local" to the candidate to be chosen by the fusion. If this latter is conflictual, we try to remove the "unreliable" sources. Using three sources, we could have chosen a majority criterion to decide the reliable sources. However, sources being partially correlated, we prefer to base the detection of reliable sources on "regularity assumption", based on the local distance (Eq. 9), between successive instant measurements. It allows us to focus on the information concerning the hypotheses of interest (the ones selected by the sources).

Precisely, if we denote by $H_1, H_2, H_3, H_{\textcircled{\wedge}}$ the singleton elements maximizing the plausibility function of respectively $m_1, m_2, m_3, m_{\textcircled{\wedge}}$, where m_1, m_2, m_3 are the consonant BBAs associated with sources S_1, S_2, S_3 described in Section 4.2 and $m_{\textcircled{\wedge}}$ is the BBA after combination of m_1, m_2, m_3 by the cautious rule, the exploitation of conflict is composed of three steps:

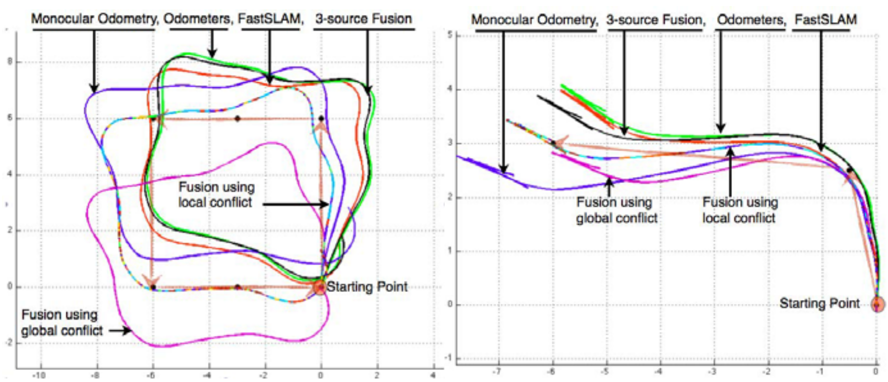
1. Compute the level of conflict introduced by the singleton element chosen by the decision step: $Mes(H_{\circledwedge}) = \sum_{B \subseteq \Omega, H_{\circledwedge} \subset B} Mes_{\emptyset}(B)$.
2. If $Mes(H_{\circledwedge}) > T_M$, then search the sources which do not respect the assumption of regularity: $(Dist_{Pl_n}(H_n(t), H_n(t-1)) > T_D$, with

$$Dist_{Pl_n}(H_n(t), H_n(t-1)) = \frac{1}{2} \mid (Pl_{n_t}(H_n(t)) - Pl_{n_{t-1}}(H_n(t))) + (Pl_{n_{t-1}}(H_n(t-1)) - Pl_{n_t}(H_n(t-1))) \mid ,$$

   where $n = \{1,2,3\}$ is the source index, t and $t-1$ two successive times, and Pl_{n_t} is the plausibility of source n at time t. The threshold values T_M and T_D have been fixed experimentally to 0.1 and 0.5.
3. Combine the sources which have been found as reliable using the two conditions.

5 Results and Conclusion

In this section we present the results obtained in the case of two various trajectories. The first one includes a strong acceleration at the beginning of the trajectory, inducing a sliding of the wheels. During the second trajectory, there is an acceleration at a turn. Figure 1 presents a 2D top view of the 3D physical world. On both trajectories, we remark a wrong odometer estimation either at the beginning, or at the turn, due to the sliding of the wheels. The monocular vision algorithm shows



(a) Trajectory one (b) Trajectory two

Fig. 1 Two different trajectories. On each we can observe respectively in red, green and blue the integration of the movement estimation by odometer data (S_1), FASTSLAM (S_2) and visual odometry (S_3) algorithms. The trajectory in black represents the integration of movement estimated by the fusion of sources S_1, S_2 and finally the multi-color and purple trajectories correspond to the integration of the movement estimation exploiting the local conflict and the global conflict (process derived from [8]), respectively.

also some limitations due to some imprecision in the camera mode (parameters) and some matching errors in the presence of a white wall. These causes of errors also occur for the FASTSLAM algorithm that uses both kinds of data.

We observe that the conflict as defined in Section 4.3 allows us to estimate a movement close to the ground truth even in extreme cases. We also observe that it outperforms the result of the three source fusion not considering their reliability.

In conclusion, this paper introduces a "local" measure to compute the disagreement between sources. Theoretical and experimental examples show that a global measure like "Demspter's conflict" or dissimilarity do not always allow a fine analysis of source reliability and origin of conflict, while the proposed local measure does. Further analysis of the properties of the local measures of conflict, potential extension to non-consonant BBA, more experiments on localization and other applications are planned for our future work.

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